

THREE-DIMENSIONAL FINITE ELEMENT MODELLING OF DYNAMIC PILE–SOIL–PILE INTERACTION IN TIME DOMAIN

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Abstract. *Dynamic analysis of embedded foundations like pile groups have received considerable attention in the past. Most existing approaches are established in the frequency domain, not allowing the simulation of material and geometric nonlinearities. In this paper, transient analyses are conducted to calculate the impedance of piles and pile groups, using a full three-dimensional numerical model for both the soil and the structure. Based on a combined viscous boundary and infinite elements, defining an efficient non-reflecting boundary, a finite element model is developed. Modelling guidelines, in terms of domain and element size, are presented and appear to be less severe in time domain than in frequency analysis. The proposed three-dimensional model can be processed with no excessive computational resources. Besides boundary efficiency, wave propagation effects are studied through dispersion diagrams in the case of homogeneous and layered soil. In particular, dissipative conditions are described, with the emphasis on the proper representation of wave propagation. These analyses thoroughly validate the proposed modelling approach for transient excitations. Viscoelastic model of soils are therefore developed including piles group considered as concrete material. A decay function is chosen for the excitation defined in the time domain, allowing a large covering of the input force spectral content. The implementation and validation of the numerical model for predicting the dynamic stiffness and damping of pile groups are presented. Vertical and lateral pile and pile group impedance examples are given and compared to a published analytical model. The proposed analysis methodology provides a convincing example of the capability of the finite element method to reproduce foundation response to dynamic loading.*

1 INTRODUCTION

With the growing development of society, the construction of buildings is increasingly affected by the soil constitution and becomes problematic in some circumstances (soft soil, lack of space, large design loads, ...). Significant research has been reported on the static and dynamic analysis of deep foundations, and especially the pile groups, in order to master the soil–structure interaction and to propose pile designs adapted to the soil configuration. Foundations relying on driven piles often have groups of piles connected by a pile cap (a large concrete block into which the heads of the piles are embedded) to distribute loads which are larger than one pile can bear.

Dynamic analysis of piles and pile groups interaction with soils has received considerable attention in the past. The main difficulty is to represent a large structure with finite dimensions coupled to a medium considered as infinite. The challenge is not only to define the accurate interaction properties in the pile–soil contact surface but also the transmission of static and dynamic loads from the concrete structures and the ground wave propagation generated by their kinematics. The dynamic pile–soil interaction can be typically associated with a problem of ground wave propagation. Several procedures to model soils used in the various soil dynamics fields (traffic ground vibration, blasting, earthquake studies) can be applied to the pile–soil–pile systems. Most of the existing approaches contribute to the understanding of ground vibration behaviour and provide interesting findings. Nevertheless, dynamic soil–structure interaction is difficult to take into account for complex structure geometries, advanced mechanical behaviour models and realistic soil configurations. The objective is to define an approach which represents the structure in a rigorous manner, while still using a reasonable number of degrees of freedom.

Soil dynamics analysis is often defined as a problem where the region of interest is small compared to the surrounding medium, which is considered as unbounded. The first modelling approaches were based on Winkler's formulation largely used in many geotechnical applications due to its implementation simplicity. Gazetas [1] provided a detailed description of these models for soil–foundation interaction problems. The main difficulty is obviously the identification of model parameters, often obtained by fitting results from numerical or experimental studies. In the case of pile–soil–pile configurations, this formulation is not able to purely represent the associated interaction but it is possible to make a reasonable assessment of the dynamic stiffness, as these proposed by Dai and Roesset [2] with approximate expressions for horizontal loading in pile groups or by Gazetas et al. [3] for their beam–on–dynamic–Winkler–foundation model.

Because of the rapid progress of computers, the numerical models abound in literature today and are considered as one of the most attractive tools for the soil–structure interaction problems. The finite element method (FEM) is efficient in dealing with complex geometries and numerous heterogeneities but it requires special local boundaries in order to mimic the geometrical radiation to infinity. Numerical models of soils have received particular attention in recent times with important contributions to the modelling of the radiation conditions. Lou et al. [4] mentioned in their review the many possible ways to define efficient radiation conditions for application in FEM. Boundary element methods (BEMs) have proved their efficiency to solve the problem with infinite dimensions but they are limited to weak heterogeneities and simple constitutive laws, and require extra mathematical developments. For example, Batista de Paiva and Tronci [5] used a BEM formulation for solving capped and uncapped pile groups, representing each pile by a polynomial function. The combination of the two methods, well known as FEM/BEM approach [6–8], allows an accurate description of the near field (FEM model) and

a reliable estimation of the far-field (BEM model). Researchers have also proposed the use of finite difference method (FDM) [9], the infinitesimal finite element cell method (CIFECM) [10] or the finite layer theory [11].

The dynamic behaviour of the piles strongly depends on the characteristics of the pile itself but also on the soil characteristics (rigidity, half-space, layered medium, material damping, ...). The use of the numerical soil modelling initially developed by Kouroussis et al. [12, 13] for railway-induced ground vibrations could be an interesting example of the capacity of the FEM to reproduce the response of foundations to dynamic loads. This paper presents the implementation and the validation of a similar model for predicting the dynamic stiffness and the damping of piles groups. The purpose of this study is to compare the numerical results with those obtained by a published simple analytical method. After a brief review of this method, an overview of the existing boundaries conditions associated to the FEM is presented, also mentioning the associated mesh rules in frequency and in time domain. The dispersion properties are checked for typical homogeneous and layered soil examples. Numerical impedances are afterwards introduced, with the corresponding findings.

2 REVIEW OF THE SIMPLE ANALYTICAL MODEL

In 1988, Dobry and Gazetas [14] presented an original simple method for predicting the dynamic impedance of pile groups. This method requires as input the related dynamic impedance of a single pile (which can be easily found in the literature) and three soil parameters: the shear waves velocity c_s , the Poisson's ratio ν and the hysteretic damping η . The geometrical dimensions (spacing S , length L , and radius r_0) are shown in Figure 1.

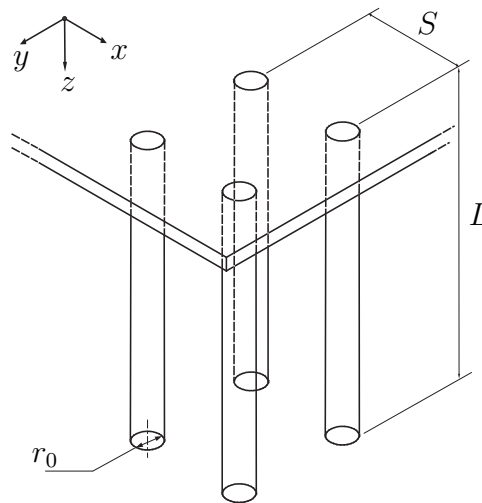


Figure 1: Typical dimensions of a pile group

The studied problem has as assumption that the single pile impedance is known, by any of several available numerical methods or from experimental tests. The impedance of the pile group is defined as the ratio between force F^G and displacement w^G at the head of the pile group (the cap is assumed to be massless and rigid):

$$K_a^G = F^G/w^G = k_a^G + ja_0c_a^G, \quad (1)$$

the terms k_a^G and c_a^G being interpreted as equivalent spring and dashpot coefficients at the head of the pile group ($a = v$ or h for vertical and horizontal motions respectively). The proposed

method defines interaction factors

$$\alpha_a(\omega) = \frac{w_{qp}}{w_{qq}} = \frac{\text{additional displacement of pile } q \text{ caused by pile } p}{\text{additional displacement of pile } q \text{ under own dynamic load}} \quad (2)$$

and expresses for each motion type as a function of the circular frequency ω [14]:

$$\alpha_v(\omega, S) = \sqrt{\frac{r_0}{S}} e^{-\eta\omega S/c_S} e^{-j\omega S/c_S} \quad (3)$$

$$\alpha_h(\theta, \omega, S) = \sqrt{\frac{r_0}{S}} e^{-\eta\omega S/c_{La}} e^{-j\omega S/c_{La}} \cos^2 \theta + \alpha_v(\omega, S) \sin^2 \theta. \quad (4)$$

As seen, the horizontal motion depends on the angle θ between the line of the two piles and the direction of the applied force. The velocity c_{La} is the so-called Lysmer analogue velocity, equal to $3.4c_S/(\pi(1-\nu))$. The motion of each group is calculated by accounting for all possible interactions between piles. Table 1 summarizes the various applications developed by the authors, assuming the knowledge of the single pile impedance K_a^S .

| Pile group | axial impedance K_v^G | horizontal impedance K_h^G |
|-------------------------|---|--|
| 1×2 piles | $\frac{2K_v^S}{1+\alpha_v(S)}$ | |
| 2×2 pile group | $\frac{4K_v^S}{1+2\alpha_v(S)+\alpha_v(\sqrt{2}S)}$ | $\frac{4K_h^S}{1+\alpha_v(S)+0.5\alpha_v(\sqrt{2}S)+\alpha_h(0^\circ, S)+0.5\alpha_h(0^\circ, \sqrt{2}S)}$ |

Table 1: Examples of dynamic stiffness functions of various pile groups

3 FINITE ELEMENT MODELLING

3.1 Modelling seismic wave propagation

Soil dynamics analysis is often defined as a problem where the region of interest is small compared to the surrounding medium, which is considered unbounded. Compared to the BEM, the FEM presents an interesting alternative only if absorbing boundary conditions to avoid spurious reflections are well defined. These conditions are known as transmitting boundaries, which are sub-categorised by Wang et al. [15] as elementary boundaries (Figures 2(a) and 2(b)), local boundaries (Figure 2(c)) or consistent boundaries (typically the infinite elements). The goal is to obtain an accurate description of the near field, including eventual complex geometries or nonlinear behaviours, and an accurate radiation condition estimating the far field.

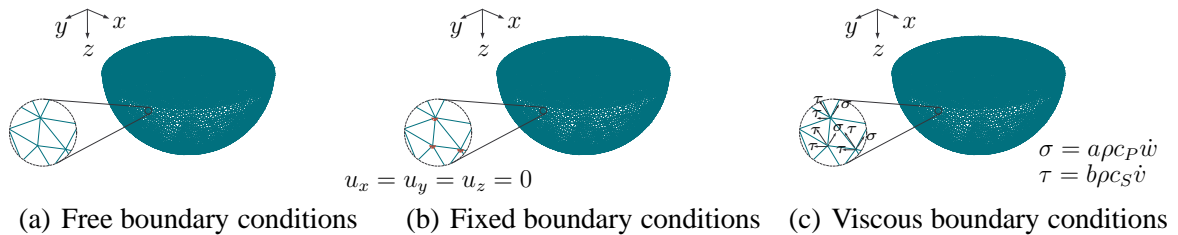


Figure 2: Classical boundary conditions

3.2 On the use of combined viscous boundary and infinite elements

Many local boundary formulations exist but the viscous boundary proposed by Lysmer and Kuhlemeyer [16, 17] is the most commonly used in this case, consisting of dashpots attached to the boundaries. The authors have investigated different possibilities for expressing an efficient boundary condition analytically and have found that the most promising way is to express it by the conditions

$$\sigma = a\rho c_P \dot{w} \quad (5)$$

$$\tau = b\rho c_S \dot{v} \quad (6)$$

where σ and τ are respectively normal and shear stresses, depending on the normal and tangent velocities \dot{w} and \dot{v} of the boundary (Figure 3). Parameters a and b are dimensionless and can both vary from 0 (defining in this way a free boundary) to ∞ (rigid boundary). In accordance with [16, 17], a and b are chosen equal to 1; these values give maximum absorption, in the case of body wave reflections. Physically, Eq. (5) and (6) define a border supported on infinitesimal dashpots with normal and tangential orientations with respect to the boundary. The study of Rayleigh waves is more complex. Lysmer and Kuhlemeyer showed that, in the case of steady-state problems, the damping, constant for body waves, depends on the depth of the soil for Rayleigh waves.

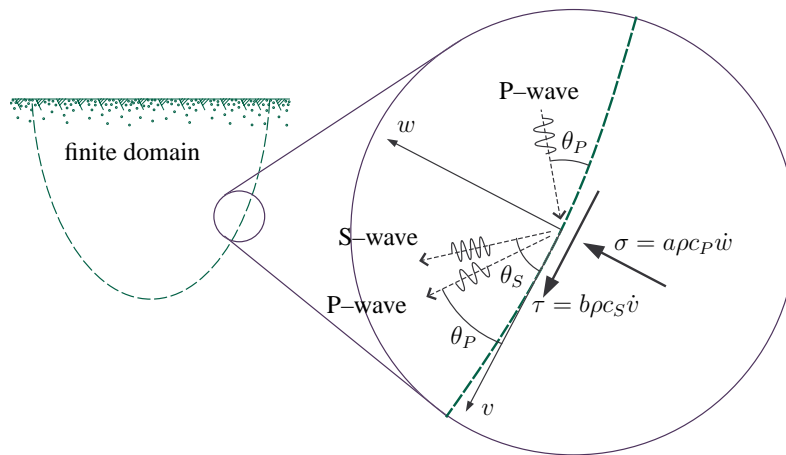


Figure 3: Reflection on the viscous boundary

In this study, an infinite/finite element model is used under the **ABAQUS** software, with a mix of local and consistent boundaries. Since the software does not present a procedure to implement this boundary condition through its mesh library, an individual **MatLab** script has been developed to edit the **ABAQUS** input file. A spherical geometry form is chosen allowing a convex shape at the border, for easy meshing and to guarantee the condition of non-crossing infinite elements (in order to avoid non-unique mappings). Figure 4 shows a typical model based on the aforementioned conditions.

3.3 Meshing rules

Classical rules exist [18], concerning the element size T_e and the domain dimension T_d : a minimum of 10 elements per Rayleigh wavelength λ_R and a domain size T_d greater than at least $3\lambda_R$. The number N_e of elements depends therefore on these two parameters. In soil dynamic

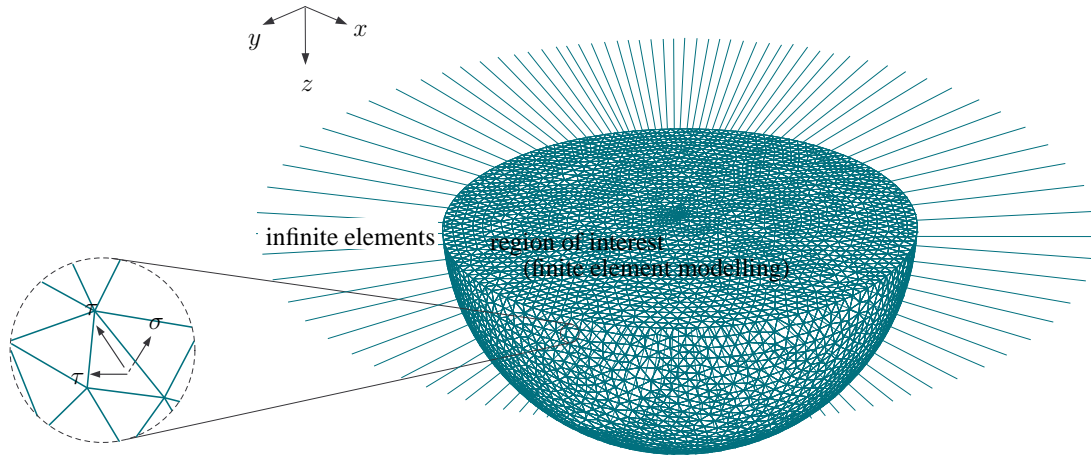


Figure 4: Combined viscous boundary conditions / infinite elements

analysis, the frequency range reaches up to 100 Hz, implying an excessive number of elements (more than 10^9 for a three-dimensional analysis). The well-known condition about element size is critical and must be kept. To make the FEM model practicable on usual computers, it is necessary to reduce the domain dimension. Recently, Kouroussis et al. [12] have shown that the time domain simulation gives the possibility to work with a reduced domain dimension. The influence of the domain dimension was investigated, considering a mesh adapted to the problem yet without loss of accuracy nor excessive number of elements.

4 ANALYSIS OF DISPERSION CHARACTERISTICS

In [13], a detailed validation of a finite element model working in the time domain was performed through the physical modelling of the generation and propagation of ground vibration. The obtained results were compared to elementary boundary conditions for various domain sizes. It turned out that the residual reflected wave energy is less than 5% in all domain dimension cases. Additional results are presented in this section and, complementing this previous study, additional results from checking the wave types are presented in this section.

The dispersion diagram is a useful tool to analyse the propagating modes of the ground as a function of frequency. If the wavenumber k for each mode is plotted as a function of frequency, the dispersion curve for the wave type is generated. Such a curve is generally identified by analytical methods [19] but an alternative method is to calculate the Fourier transformed displacements generated by a load for different frequencies and wavenumbers. With a transient force adjusted so as to realistically represent an impulse load, the ground surface is excited by a uniform magnitude in a large frequency range. Figure 5 presents dispersion diagrams for four examples of soil configuration where the shear velocity c_s and the viscous damping parameter β vary. The response at each frequency is normalized to the maximum value at that frequency, as proposed by Triepaischajonsak et al. [20], allowing an easy identification of the associated wavenumbers. The maximum indicates the modal wave numbers that are vertically excited and follows straight lines (from the origin to the point on its dispersion curve at that frequency) corresponding to the inverse of wave speed. For each case, expected results are found in the frequency range [0 – 100 Hz]. Figures 5(a) to 5(c) show the waves modes for homogeneous soil configuration. Only a single propagating mode exists with a wave speed close to that of the Rayleigh waves. The effect of soil rigidity and damping is also depicted. Figure 5(d) presents the results for the layered soil configuration. It can be seen that the high frequency

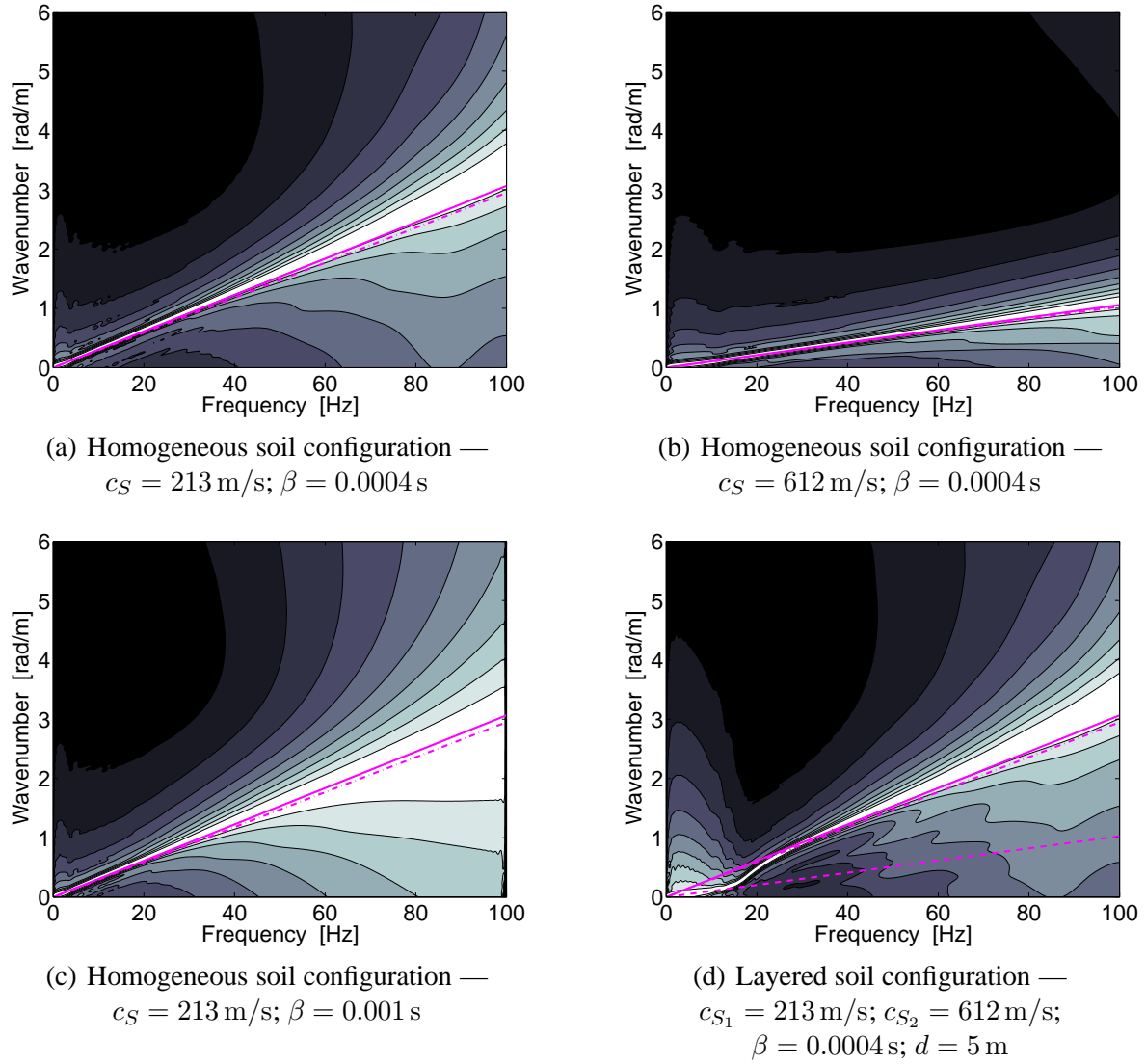


Figure 5: Dispersion diagrams (light=high amplitude; solid line: Rayleigh wave speed of upper layer, dash-dot line: shear wave speed of upper layer, dashed line: shear wave speed of the substratum)

content follows the characteristics of the upper layer while the low frequencies are associated to the substratum. The passage between the two reference lines corresponds to the oscillation frequency of the soil surface, characterized by [21]

$$f_{\text{layer}} = \frac{c_P}{4d} \quad (7)$$

where c_P is the compression wave velocity of the first layer of depth d . This frequency is equal to 20 Hz for the example proposed in Figure 5(d).

These results confirm the potentiality of the FEM model in time domain to capture the modal phenomena of soils. It can be applied to the characterisation of the pile–soil dynamic impedance.

5 APPLICATION TO THE SOIL–PILE INTERACTION

The numerical determination of dynamic impedance has rarely been performed with a 3D FEM model, due to the difficulty of modelling simultaneously the infinite conditions of soil

and the three-dimensional nature of the problem. With the developed approach (time domain simulation), the numerical model can be easily built.

We consider the following dimensions for the pile :

$$2r_0 = 1 \text{ m} ; \quad L = 15 \text{ m}$$

and the spacing is $S = 10 \text{ m}$ for a 2×2 group. The single pile is also modelled separately, to obtain its dynamic impedance (note that the results of Dobry and Gazetas are in dimensionless form, where the group impedance is normalized by the sum of the static stiffnesses of the individual single piles).

Table 2 gives the dynamic parameters of the soil and piles (considered as concrete material). The ratio E_s/E_p is equal to 1000. Subscripts p and s denote the pile and the soil, respectively.

| | Soil (subscript s) | Pile (subscript p) — concrete material |
|-------------------------|------------------------|---|
| Young's modulus E | 30 MN/m | 30 GN/m |
| Poisson's ratio ν | 0.30 | 0.20 |
| Density ρ | 2000 kg/m ³ | 2500 kg/m ³ |
| Viscous damping β | 0.0004 s | <i>no damping</i> |

Table 2: Soil and pile dynamic parameters

Figure 6 displays the model established for the simulation, considering a single pile and a 2×2 pile group configuration. A half-sphere containing the piles is used to model the soil. Tie constraints are imposed at the surface contact between soil and pile parts to assemble them. In this way, the FEM model of the soil subsystem consists of 200,000 and 550,000 finite/infinite elements, for the single and 2×2 pile configurations, respectively (that is 215,000 *dof* for a single pile problem and 410,000 *dof* for a 2×2 pile group, a pile needing around 15,000 elements). The area around the piles, where the forces are applied, is finely described. The analysis is performed in the time domain, considering the following decay function for the excitation:

$$f_{\text{excitation}}(t) = \begin{cases} 0 & \text{if } t < t_0 \\ A e^{[-(t-t_0)/t_d]} & \text{if } t \geq t_0 \end{cases} \quad (8)$$

with $A = 1 \text{ N}$, $t_0 = 0.05 \text{ s}$ and $t_d = 0.001 \text{ s}$.

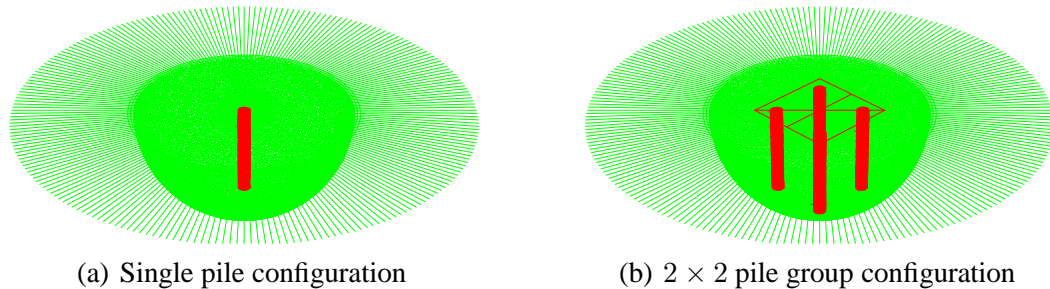


Figure 6: Modelling of piles under ABAQUS

5.1 Vertical response

The first analysed results are related to the vibrations of the pile–soil system at various instants after the load described by Eq. (8) is applied vertically on the pile cap. With our approach, the dimension of the soil (i.e. the radius of the half-sphere) is reduced to 20 m.

Figure 7 gives the time history of the vertical displacement of the cap, for the two studied cases. It clearly and naturally appears that the maximum value is divided by nearly four in the 2×2 pile group, compared with the single pile group maximum. The curve shape is moreover different, with additional oscillations after the first maximum. They are stemming from the pile–to–pile interaction induced by low–frequency waves, which are enable by the large inter–pile spacing of $S/r_0 = 20$. For this spacing, the pile group cannot be considered as an isolated embedded foundation.

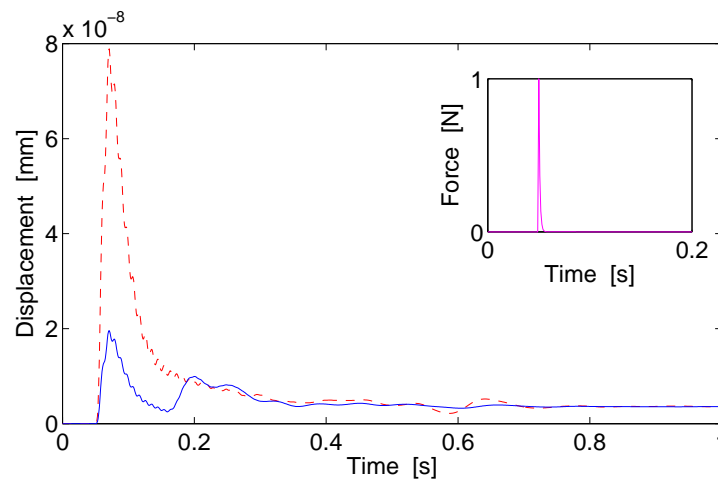


Figure 7: Time history of the vertical displacements obtained with ABAQUS (dash line: single pile; solid line: 2×2 pile group)

5.2 Horizontal response

In the same way, we analyse the horizontal vibrations. Specific boundary conditions are imposed to the cap to allow only the horizontal motion along the x –direction. It is worth mentioning that the horizontal and rocking degrees of freedom of a pile are coupled. For the free field, we impose only the horizontal motion (rocking motion is locked).

Figure 8 displays the time history of the horizontal displacement of the cap, for the two studied cases. As previously, the same observation can be made with the decrease of the maximum amplitude: the maximum value of the single pile group is divided by approximately four in the 2×2 pile group. Some comments can be made regarding the shape of the curve. Although in the preceding results (for the vertical motion), the two curves reach up to the same value after $t = 0.2$ s, the curves have more deviations from each other. The next section presents the results in the frequency domain for a deeper understanding of the problem.

5.3 Comparison with a simple analytical model

The analytical method proposed by Dobry and Gazetas needs damping in the form of the hysteretic parameter η . Unfortunately, the time domain simulation performed for our numerical model limits the intrinsic damping to the Rayleigh type, usually defined as a combination of the

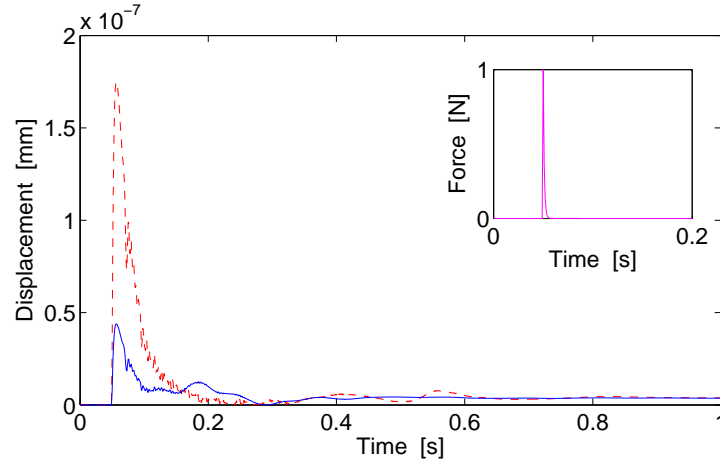


Figure 8: Time history of the horizontal displacements obtained with ABAQUS (dash line: single pile; solid line: 2×2 pile group)

mass \mathbf{M} and stiffness \mathbf{K} matrices of the system

$$[\mathbf{C}] = \alpha [\mathbf{M}] + \beta [\mathbf{K}] . \quad (9)$$

In this formulation, coefficients α and β are usually chosen so as to match the damping ratio at two specific frequencies. Viscous Kelvin–Voigt damping β can be used in the two methods since $\eta = \beta\omega$ in the frequency domain (α is therefore equal to 0). To compare the results in the frequency domain, a discrete Fourier transform is applied on the time history of pile responses.

The dynamic stiffnesses are decomposed in the standard manner as described in Eq. (1). Dynamic stiffness and dashpot group factors are defined as the ratio of the spring k_a and damping coefficient c_a to the sum of the static stiffnesses K_{a0}^S of the individual single piles. These curve are plotted in function of the frequency parameter

$$a_0 = \frac{\omega L}{c_S} \quad (10)$$

for the vertical and horizontal motions, in Figures 9 and 10. Single pile and 2×2 pile group are analysed. To make the deviation more visible, the curves are replotted enlarged in the low frequencies (up to $a_0 = 1$). The solutions of Dobry and Gazetas, denoted analytical solutions, are also on the same figures, considering viscous damping. Good agreement is obtained between simple analytical model results and the numerical values for the 2×2 pile group. Several comments can be highlighted from the results of these figures:

- The sum of the static stiffnesses of the individual piles is larger than the static stiffness of the pile group. The magnitude of the dynamic stiffness of the pile group for a specific frequency can be smaller or larger than that of the sum of the contributions of the individual piles. For dynamic loads, pile–soil–pile interaction is caused by the waves emitted from each pile and propagating to the neighbouring piles with resulting refractions and reflections. The pile group dynamics is strongly frequency dependent.
- The comparison with analytical results is not straightforward, because the viscous damping β intervenes only in the imaginary part although the whole dynamic stiffness K_a is multiplied by $1 + 2j\eta$ in the case of hysteretic damping.

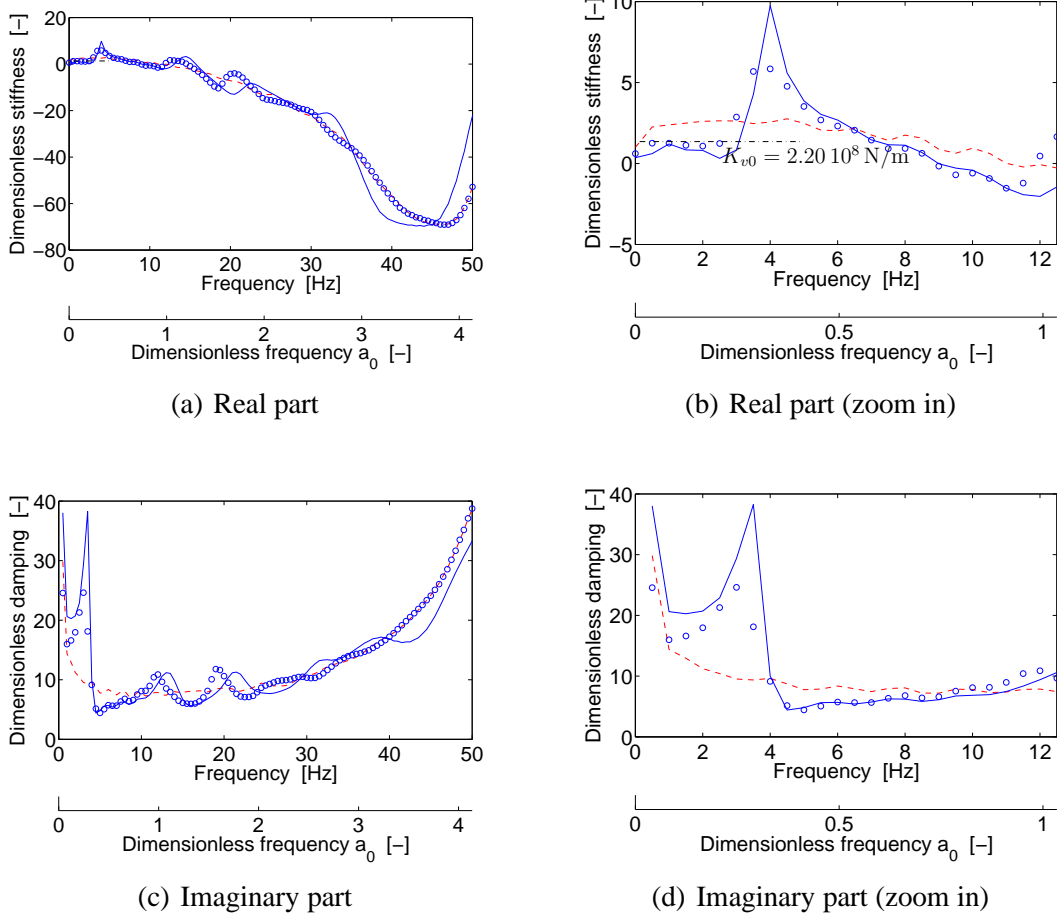


Figure 9: Vertical dynamic stiffness and damping group factors as a function of frequency (○: analytical solution; dash line: single pile; solid line: 2×2 pile group)

- For the 2×2 pile group, the analytical solution uses the numerical one pile results. The agreement with this approximative solution is good. To make the deviations more visible, real and imaginary parts are plotted in the frequency range $[0 - 12 \text{ Hz}]$, which represents the realistic range of the pile study.

Despite the fact that a reference result for the single pile motion is not available, it is possible to make a reasonable assessment of the static value. The static stiffnesses of a pile in a homogeneous halfspace is given by [22]:

$$K_{v0} = 3.8r_0E_s \left(\frac{L}{4r_0} \right)^{0.67} \quad (11)$$

$$K_{h0} = 2r_0E_s \left(\frac{E_p}{E_s} \right)^{0.21} \quad (12)$$

for the vertical and horizontal motion, respectively. The cone model allows obtaining results for the vertical motion in $17.5E_sr_0$ and for the horizontal motion in $7.4E_sr_0$. Figures 9(b) and 10(b) display these static values, showing a good correspondence with the numerical results, both for the vertical value and the horizontal one.

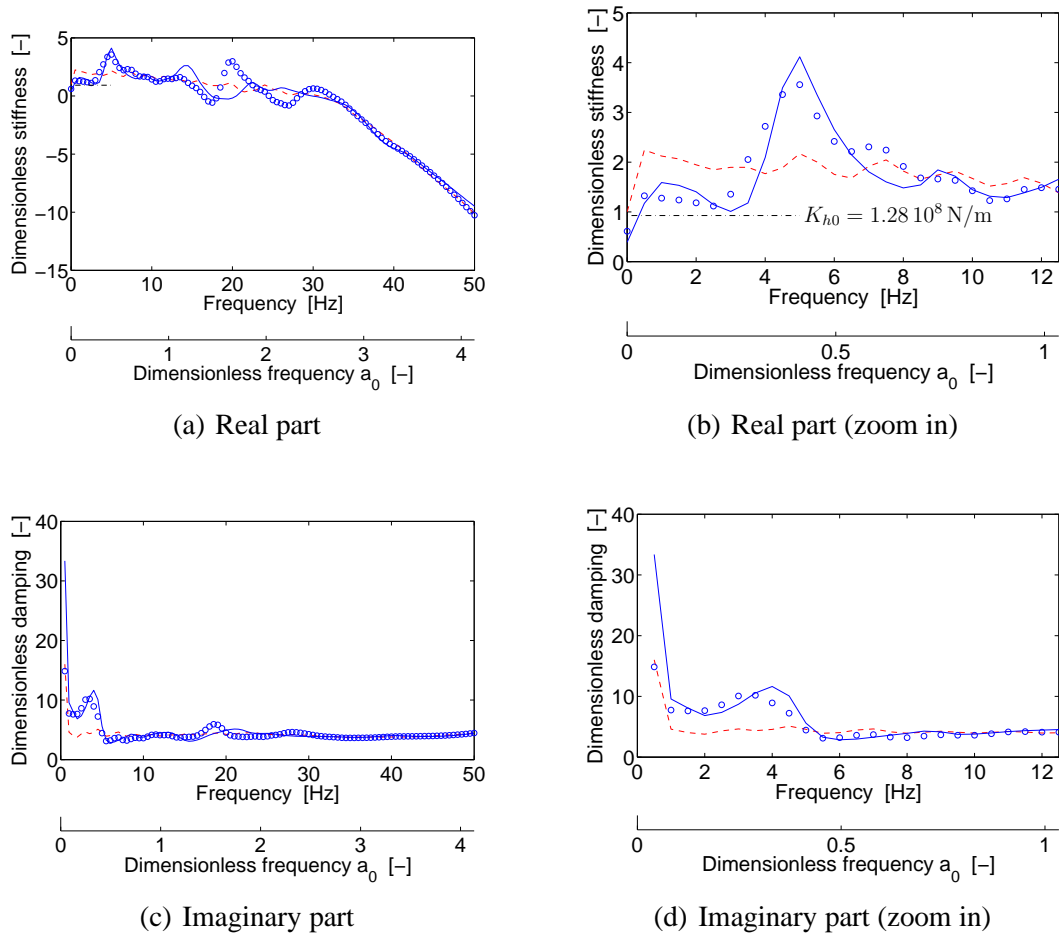


Figure 10: Horizontal dynamic stiffness and damping group factors as a function of frequency (\circ : analytical solution; dash line: single pile; solid line: 2×2 pile group)

6 CONCLUSIONS

In the past, to analyse the dynamic behaviour of a group of vertical piles with their heads connected by a rigid cap, the interaction of the individual piles via the soil was taken into consideration with the help of analytical solutions. BEM has also been used but to a limited extent due to the complex geometry. With the developed FEM in time domain simulation, a new kind of solution can be easily computed, as proposed in this paper. The wave propagation is underlined in the results, showing the performance of the non-reflecting boundary (viscous boundary and infinite elements) attenuating the reflected waves to the maximum.

The limitation imposed by the time domain is not restrictive since the obtained results from a Fourier transform of an “impulse” response are very satisfactory and encouraging, through the validation of dispersive characteristics. Interesting results are obtained and it all leads to believe that advanced analyses could be performed with the proposed FEM approach, as well in pile dynamics as well as in other applications in geotechnics.

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