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UNCERTAINTIES IN MULTISTAGE ROTORS MODELED BY THE POLYNOMIAL CHAOS EXPANSION

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Abstract. *In this work the effects of uncertainties in turbomachinery rotors is assessed using the Polynomial Chaos Expansion. The multistage cyclic symmetry approach is used to reduce the size of the problem and the uncertainties are modeled as variations in the material properties of the blades. A numerical example consisting in an assembly of two bladed disc is presented. Results obtained by polynomial chaos are validated by comparisons with Monte-Carlo simulations. As a consequence of the included uncertainties changes in nature of modes are observed.*

1 INTRODUCTION

The assessment of variability induced by uncertainties is now recognized as an important part in the design process of any mechanical system. The methodology proposed in this work allows to include uncertainties in the analysis of turbomachinery rotors using the Polynomial Chaos Expansion (PCE) [1]. The rotors considered are composed of several stages of bladed discs for which uncertainties may arise from in-use wear of the blades, temperature changes or manufacturing tolerances. As a first approach, only uncertainties that are the consequence of uniform in-use wear of the set of blades will be included.

Recent designs in turbomachinery tend to have more flexible inter-stage rims and to be more lightly damped, resulting in configurations where modes may not be confined to only one stage. This dynamic coupling between stages is discussed, for example, in [2]. To capture all possible multistage dynamic couplings the computationally costly analysis of the whole structure becomes mandatory. A recently introduced multistage cyclic symmetry approach [3, 4] can be used for reducing the cost of modeling rotors composed of several stages even when the stages have different numbers of sectors. This approach takes advantage of the inherent cyclic symmetry of each stage and uses a specific assumption that results in decoupled subproblems for each spatial Fourier harmonic.

The considered uncertainties are modeled as global variation of the material properties of the set of blades. The multistage cyclic symmetry approach can then be used to reduced the size of the problem because the underlying assumption of identical sectors is respected. In bladed disc systems the modes shapes are considered to be important design parameters and in this work their variability is assessed by solving the stochastic modal analysis using the PCE, as in [5]. The positiveness of the random matrices involved is assured by using gamma-distributed random variables which imply the use of Laguerre's polynomials as basis for the polynomial chaos.

2 MODELING THE DYNAMICS OF MULTISTAGE ROTORS

The rotors considered are composed of several stages of bladed discs. If considered separately, each stage has the property of cyclic symmetry. This allows to carry out the analysis of a stage by modeling only one reference sector. Multistage rotors as a whole are not cyclic symmetric because the number of blades in each stage is generally different.

The equation of motion of the multistage structure at rest is

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F} \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrix respectively, $\mathbf{x}$ is the displacement vector and $\mathbf{F}$ the vector of external forces. All quantities are expressed in physical coordinates.

Using a recently developed modeling technique called multistage cyclic symmetry [3, 4], the analysis of the whole structure can be carried out by modeling one reference sector of each stage. First, for each stage the displacement in the circumferential direction is decomposed using a discrete Fourier transform as in classical cyclic symmetry. This yields uncoupled subproblems for each Fourier order which are defined on the reference sector. Second, an assumption of compatible Fourier orders of different stages is used to couple this subproblems into fundamental multistage subproblems

$$\tilde{\mathbf{M}}_f\ddot{\tilde{\mathbf{x}}}_f + \tilde{\mathbf{C}}_f\dot{\tilde{\mathbf{x}}}_f + \tilde{\mathbf{K}}_f\tilde{\mathbf{x}}_f = \tilde{\mathbf{F}}_f \tag{2}$$

where f is associated to the fundamental multistage harmonic and the symbol $\tilde{\cdot}$ indicates quantities expressed in cyclic coordinates.

The total number of fundamental multistage subproblems to be solved is given by the stage with the smallest number of sectors. Also, the Fourier orders that are assumed compatible are defined using a rule that is based on the aliasing of the Fourier transform of this stage. Accordingly each fundamental multistage subproblem may group one or more Fourier orders of a particular stage. Displacements $\tilde{\mathbf{x}}_f$ (or other quantities) in cyclic coordinates are easily transformed into physical coordinates of the whole structure.

The normalized modal analysis of the undamped system for the fundamental multistage Fourier order f is governed by

$$\tilde{\mathbf{K}}_f \boldsymbol{\phi}_l = \omega_l^2 \tilde{\mathbf{M}}_f \boldsymbol{\phi}_l \quad \text{with} \quad \boldsymbol{\phi}_l^T \tilde{\mathbf{M}}_f \boldsymbol{\phi}_l = 1 \tag{3}$$

where ω_l and $\boldsymbol{\phi}_l$ are the lth natural frequency and mode shape in cyclic coordinates respectively.

3 POLYNOMIAL CHAOS EXPANSION (PCE) REPRESENTATIONS

For simplicity uncertainties are introduced by considering a random Young's modulus $\hat{E}(\tau)$. In order for the Young's modulus $\hat{E}(\tau)$ to remain positive it is modeled as

$$\hat{E}(\tau) = \xi(\tau) \langle \hat{E}(\tau) \rangle \tag{4}$$

where $\xi(\tau)$ is a positive random variable that follows a gamma distribution of shape parameter $\alpha > -1$. Argument τ and the symbol $\hat{\bullet}$ are used to indicate the random character of the variables. The variation coefficient of the random Young's modulus δ_E^2, which is assumed to be known, determines the shape parameter of the distribution as $\alpha = -1 + 1/\delta_E^2$. Random stiffness matrix $\hat{\tilde{\mathbf{K}}}_f(\tau)$ is written using the $\hat{E}(\tau)$ as

$$\hat{\tilde{\mathbf{K}}}_f(\tau) = \xi(\tau) \tilde{\mathbf{K}}_f(\langle \hat{E}(\tau) \rangle) \tag{5}$$

3.1 Modal analysis

The random modal analysis is obtained by using random stiffness matrix $\hat{\tilde{\mathbf{K}}}_f$

$$\hat{\tilde{\mathbf{K}}}_f \hat{\boldsymbol{\phi}}_l = \hat{\omega}_l^2 \tilde{\mathbf{M}}_f \hat{\boldsymbol{\phi}}_l \quad \text{with} \quad \hat{\boldsymbol{\phi}}_l^T \tilde{\mathbf{M}}_f \hat{\boldsymbol{\phi}}_l = 1 \tag{6}$$

with $\hat{\omega}_l$ and $\hat{\boldsymbol{\phi}}_l$ being the lth random natural frequency and mode shape. Quantities $\hat{\omega}_l$ and $\hat{\boldsymbol{\phi}}_l$ are expressed in the Polynomial Chaos representation truncated to $p+1$ terms as

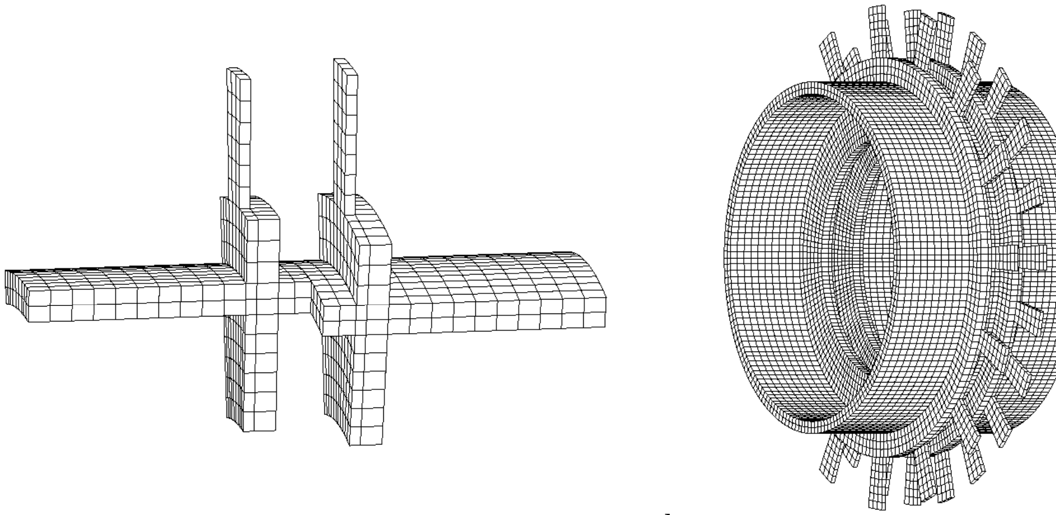
$$\hat{\omega}_l = \sum_{i=0}^{p} \psi_i w_l^{(i)}, \quad \hat{\boldsymbol{\phi}}_l = \sum_{i=0}^{p} \psi_i \boldsymbol{\phi}_l^{(i)} \tag{7}$$

where $\psi_i(\xi)$ are the Laguerre's polynomial which are functions of the random variable and form an orthogonal basis with respect to the gamma distribution [6].

Solution of the random modal problem consists in determining coefficients

$$\left\{ w_l^{(0)} \quad , \quad \ldots \quad , \quad w_l^{(p)} \quad , \quad \boldsymbol{\phi}_l^{(0)} \quad , \quad \ldots \quad , \boldsymbol{\phi}_l^{(p)} \right\} \tag{8}$$

which are deterministic quantities that completely describe the stochastic behavior of $\hat{\omega}_l$ and $\hat{\boldsymbol{\phi}}_l$.



a b

Figure 1: Finite element discretization. a.- one reference sector per stage, b.- complete model.

The solution method consists in substituting the expressions of $\hat{\hat{\mathbf{K}}}_f$ and $\hat{\omega}_l$ and $\hat{\boldsymbol{\phi}}_l$ into Eq. 6 and projecting into the basis of Laguerre's polynomials ψ_m for $m = 0, \ldots, p$. This results in a system of $(p+1)(n+1)$ nonlinear equations, n being the size of $\boldsymbol{\phi}_l$, that is solved by an iterative method such as Newton-Raphson.

In order to study the changes in the modes induced by the uncertainties, the random mode shapes $\hat{\boldsymbol{\phi}}_l$ are decomposed a posteriori onto the deterministic modal basis [5] as

$$\hat{\boldsymbol{\phi}}_l = \sum_{i=1}^{n} \hat{e}_i^l \boldsymbol{\phi}_i \tag{9}$$

where $\hat{e}_i^l$ are random coefficients. The statistics of coefficients $\hat{e}_i^l$ allow to quantify the contributions of the random mode shapes to the deterministic mode shapes.

4 NUMERICAL CASE STUDY

A rotor composed of two bladed discs having 15 and 23 blades is considered. Using the multistage cyclic symmetry assumption only one sector per stage is modeled. Figure 1 shows the finite element discretization of one reference sector per stage and the complete model. The main physical characteristics are, for the discs: thickness 15 mm, inner/outer diameter 150/250 mm; and for the shaft (drum): inner/outer diameter 200/215 mm, total length 220 mm. The length of the blades is 66 mm in stage 1 and 60 mm in stage 2. The material properties used are: density 7820 kg/m^3 and Young's modulus 210 GPa.

The deterministic natural frequencies of the multistage system are plotted as a function of the fundamental Fourier order in Figure 2. The lines drawn help distinguish between families of modes predominantly having blade dominated motion or disc dominated motion. Families of blade dominated modes appear as nearly horizontal lines while the slanted lines correspond to disc dominated modes. Modes corresponding to global deformations of the shaft are not assimilated to any of the families. The stochastic modal analysis for the six modes pointed out in Figure 2, which correspond to Fourier order 3, is shown next.

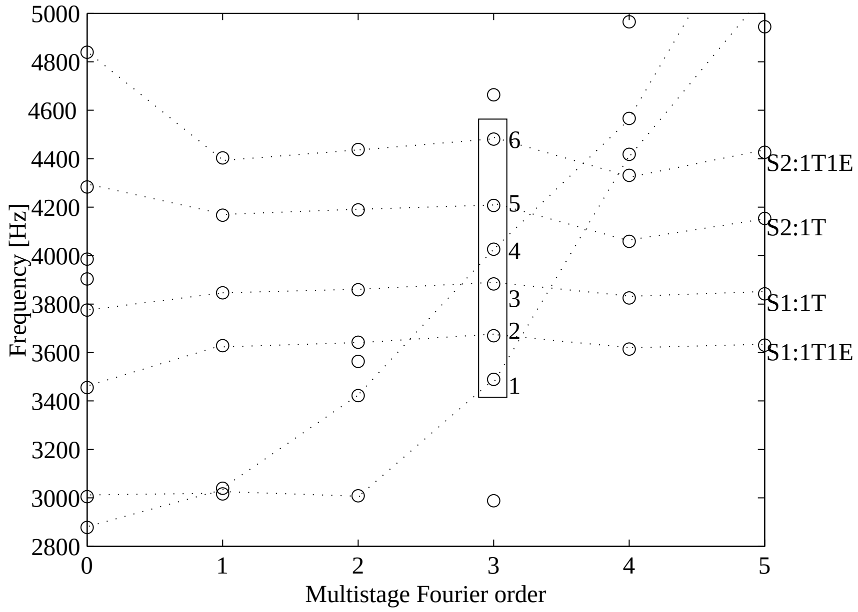


Figure 2: Natural frequencies vs multistage Fourier orders. Families of blade deformations are: S2:1T1E stage 2, torsion–edgewise bending; S2:1T stage 2, 1st blade torsion; S1:1T stage 1, 1st blade torsion; S1:1T1E stage 1, torsion–edgewise bending.

Table 1: Statistical moments of natural frequencies calculated using PCE and Monte-Carlo simulations

	PCE			Monte-Carlo		
Mode	$\langle\hat{\omega}\rangle$ [Hz]	σ_ω [Hz]	δ_ω [%]	$\langle\hat{\omega}\rangle$ [Hz]	σ_ω [Hz]	δ_ω [%]
1	3488.36	5.66	0.16	3488.36	5.63	0.16
2	3669.02	33.61	0.92	3668.99	33.41	0.91
3	3882.75	40.91	1.05	3882.72	40.67	1.05
4	4025.99	4.14	0.10	4025.99	4.10	0.10
5	4206.87	43.26	1.03	4206.83	43.01	1.02
6	4480.33	44.77	1.00	4480.30	44.51	0.99

4.1 Random frequencies and mode shapes

Only uncertainties arising from uniform in-use wear of the set of blades are included. These are modeled by considering a random Young's modulus for the set of blades with a variation coefficient of $\delta_E = 2.5\%$. Hence, only the portion of the stiffness matrix that corresponds to the blades is considered as random.

The Polynomial Chaos expansion of second order ($p = 2$) is used and the obtained results are compared to Monte-Carlo simulations carried out by solving the modal problem for 5000 independent samples of the random variable. Excellent accuracy is found for frequency mean and standard deviation as shown in Table 1. The standard deviations for the modes 1 and 4 are small because they correspond to modes with predominantly disc deformations, however light coupling in frequency between disc and blade dominated modes is evidenced by their non-zero values.

Probability density functions of the frequencies can be obtained using Eq. 7 to calculate the frequencies for a large number of independent samples of the random variable by inexpensive evaluations of polynomials.

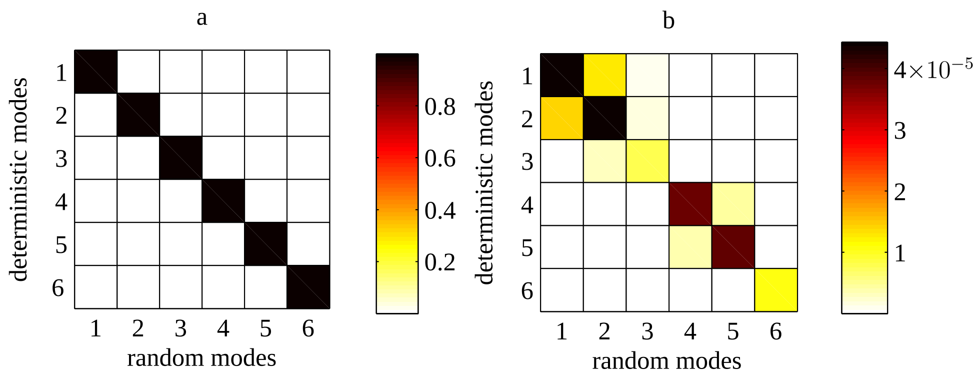


Figure 3: Contributions of the random mode shapes to the deterministic mode shapes. a.- mean $\langle \hat{e}_i^l \rangle$, b.- variance×mean $\sigma^2(\hat{e}_i^l) \times \langle \hat{e}_i^l \rangle$.

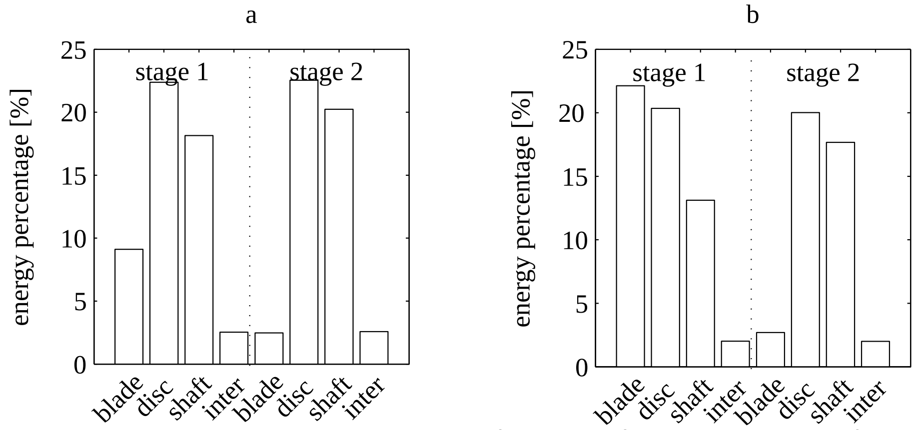


Figure 4: Energy distribution across the components. a.- deformation of mean mode 1, b.- deformation of mode 1 for stochastic realization $\hat{E} = 0.925 \times \langle \hat{E} \rangle$.

The contributions of the random mode shapes to the deterministic mode shapes are illustrated in Figure 3 using the statistics of coefficient $\hat{e}_i^l$ from Eq. 9. The two quantities represented are the mean $\langle \hat{e}_i^l \rangle$ and the variance×mean $\sigma^2(\hat{e}_i^l) \times \langle \hat{e}_i^l \rangle$. Figure 3(a) shows that the mean random modes are very similar to the deterministic modes while Figure 3(b) suggests that changes in the modes induced by the uncertainties are possible. For example, it appears from Figure 3(b) that stochastic mode 1 may have contributions from deterministic modes 1 and 2. Since the later correspond to deformations of the discs and the blades respectively, it is expected that stochastic mode 1 be a combination of both. To further illustrate this, a particular representation for mode 1 of the mean and one stochastic realization is shown in Figure 4. The representation consists on the energy distribution across different components, the components being the blades, disc, shaft and inter-stage region for either stage 1 or 2. The energy distribution for the mean mode 1 suggests a disc dominated mode, however the energy in the blades of stage 1 is significantly increased for the stochastic realization. Furthermore, this change in the nature of the mode is associated to only a very small variation in frequency.

5 CONCLUSIONS

A methodology for solving the stochastic modal analysis using the Polynomial Chaos expansion is briefly presented and illustrated using a numerical example. The size of the underlying problem is reduced using the multistage cyclic symmetry approach and the PCE is applied to the quantities associated to each multistage Fourier order. The uncertainties introduced are modeled using positive gamma-distributed random variables. Frequency mean and standard deviation results obtained by PCE are compared to Monte-Carlo simulations results showing excellent correspondence. Finally, the analysis shows how small uncertainties in the material properties of the blades may induce significant changes in the nature of the modes which might not be visible if looking at variations in frequency only.

REFERENCES

[1] R Ghanem and P Spanos. *Stochastic finite elements: a spectral approach*. Courier Dover Publications, 2003.

[2] R. Bladh, M. P. Castanier, and C. Pierre. Effects of multistage coupling and disk flexibility on mistuned bladed disk dynamics. *Journal of Engineering for Gas Turbines and Power*, 125(1):121–130, 2003.

[3] D Laxalde, J-P Lombard, and F Thouverez. Dynamics of multistage bladed disks systems. *Journal of Engineering for Gas Turbines and Power*, 129(4):1058–1064, 2007.

[4] D. Laxalde and C. Pierre. Modelling and Analysis of Multi-Stage Systems of Mistuned Bladed Disks. *Computers & Structures*, 89(3-4):316–324, 2010.

[5] R. Ghanem and D. Ghosh. Efficient characterization of the random eigenvalue problem in a polynomial chaos decomposition. *International Journal for Numerical Methods in Engineering*, 72(4):486–504, 2007.

[6] C. Soize and R. Ghanem. Physical systems with random uncertainties: Chaos representations with arbitrary probability measure. *SIAM Journal on Scientific Computing*, 26(2):395–410, 2004.