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INVERSE ANALYSIS FOR DETERMINING THE REINFORCED CONCRETE FLEXURAL STIFFNESS OF A STRUCTURE UNDER STRONG COMPRESSION

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Abstract. *In general, the determination of internal forces along the transverse sections of a piece is made assuming the structure is in its undeformed position. This represents a 1st order theory where the deformations undergone by the piece are neglected. However, when the deformations are considered, the internal forces are no longer proportional to the external forces and the problem then requires a 2nd order theory. The problem worsens when the material is non-linear. Concrete reinforcement structures subjected to high normal effort show both, geometric and physical nonlinearities. The stiffness of bars with a geometric effect is resolved analytically and numerically. There is a need to establish therefore the flexural stiffness to be used. In this work, this is obtained by inverse analysis from the frequency of a real structure in conjunction with a numerical model.*

1 INTRODUCTION

In general, the calculation of the internal forces and moments along the transverse sections of a piece is made assuming the structure is in its undeformed position. Thus, there is proportionality between actions and internal efforts induced. This represents a 1^{st} order theory, because the deformation undergone by the piece is implicitly neglected. However, when the deformations are considered, the internal forces and moments are no longer proportional to the external actions and the problem becomes a 2^{nd} order theory.

Strictly speaking, one should always consider the deformed position of a structure to calculate internal forces, thus 2nd order theory represents a higher degree of approximation. However, from a practical standpoint, for many engineering structures a 1st order theory is a satisfactory approximation, while for others, it is a too inexact to determine the internal stresses. For such structures the efforts and displacements induced must be calculated taking into account the deformations undergone by the system.

The problem worsens when the material is non-linear. Non-linearity is an intrinsic property of material and results in non-proportionality between cause and effect (stress and strain), even in 1^{st} order theory. Concrete reinforcement structures subjected to high normal effort show both, geometric and physical nonlinearities. More information about the characteristics of concrete can be found in Fusco [1] and Metha [2].

A satisfactory solution to most engineering problems can arise from two considerations which are easily implemented in analytical formulations and numerical computing: geometric nonlinearity can be solved through the concept of geometric stiffness, and material nonlinearity through flexural stiffness.

The stiffness of bars with geometric effect is resolved analytically and numerically. There is a need to establish therefore the flexural stiffness to be used. In this work, this is obtained by inverse analysis from the frequency of a real structure in conjunction with a numerical model by using the finite element method to adjust the experimental frequency found.

2 THE STRUCTURE INVESTIGATED

Our case involves an elevated reservoir designed to supply drinking water to a housing development. It is 21.654 meters tall and has a circular hollow section former suit with a diameter of 100 cm and 12 cm uniform thickness along the height. The foundation is superficial, a Pad Footings type. The reservoir is made in *fiberglass* shell and has a capacity of 20,000 liters. The concrete section contains 24 bars 12.5 mm diameter steel reinforcement and has a 2.5 cm cover.

Although there is lateral wind action, the structure behaves more akin to a column than to a beam, in view of the predominance of normal force measured by the ratio between the bending moment and the axial force in section further solicited that is 13 cm. Therefore, the normal force is situated at the section central nucleus of inertia.

The modulus of elasticity of *fiberglass* and density are 4.142857 GPa and 1450 kg/m^3, respectively. The structure has an access ladder to the top, which starts 2 m from the floor, inducing an additional distributed mass of 25 kg/m to structure. There are two working platforms, one located at 10 m and another at 15.60 m from the floor, with masses estimated at 350 kg and 250 kg, respectively. There are also feed and discharge PVC pipes of 25 mm diameter with a wall thickness of 1.7 mm and 20 mm in diameter with a wall thickness of 1.2 mm, which run the entire length of the structure, except underground. In order to model numerically better, the set structure/reservoir was divided into foundation, column (hollow section), head of the column (full section), slab reservoir supporting, reservoir and cap. The height between the lower foundation face and the ground surface is 2.15 m.

The characteristics and dimensions of the assembly are shown in Table 1, corresponding to the discretization of the model, in which ϕ_{ext} is the external diameter, diameter of full section or side of square section, as is the case; and t is the thickness of the section if it exists.

Height (m)	ϕ_{ext} (cm)	t (cm)	Height (m)	ϕ_{ext} (cm)	t (cm)	Height (m)	ϕ_{ext} (cm)	t (cm)
21.654	325.00	0.00	13.66	100.00	12.00	6.28	100.00	12.00
21.650	325.00	0.00	13.29	100.00	12.00	5.75	100.00	12.00
19.700	255.00	0.40	12.91	100.00	12.00	5.22	100.00	12.00
17.750	315.00	0.00	12.53	100.00	12.00	4.68	100.00	12.00
17.650	315.00	0.00	12.15	100.00	12.00	4.15	100.00	12.00
17.450	100.00	0.00	11.62	100.00	12.00	3.82	100.00	12.00
17.071	100.00	12.00	11.08	100.00	12.00	3.48	100.00	12.00
16.693	100.00	12.00	10.55	100.00	12.00	3.15	100.00	12.00
16.314	100.00	12.00	10.02	100.00	12.00	2.82	100.00	12.00
15.936	100.00	12.00	9.48	100.00	12.00	2.48	100.00	12.00
15.557	100.00	12.00	8.95	100.00	12.00	2.15	100.00	12.00
15.179	100.00	12.00	8.42	100.00	12.00	1.40	100.00	12.00
14.800	100.00	12.00	7.88	100.00	12.00	0.65	120.00	0.00
14.421	100.00	12.00	7.35	100.00	12.00	0.20	350.00	0.00
14.043	100.00	12.00	6.82	100.00	12.00	0.00	350.00	0.00

Table 1: The characteristics of the structure.

It is interesting to mention that some of the data were provided by the design engineer and executors of the structure, others were collected in the field and some obtained from the manufacturer of the water tank. The slenderness of the structure is 106. Figure 1 shows a photograph of the structure and Fig 2 details the geometry of the structure.

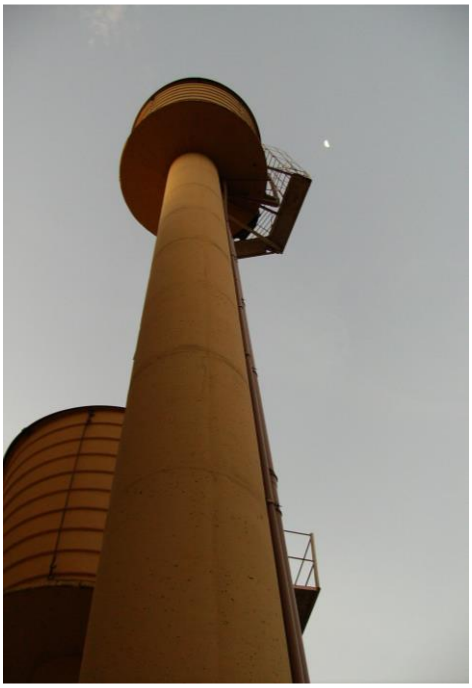


Figure 1: Photograph of the structure.

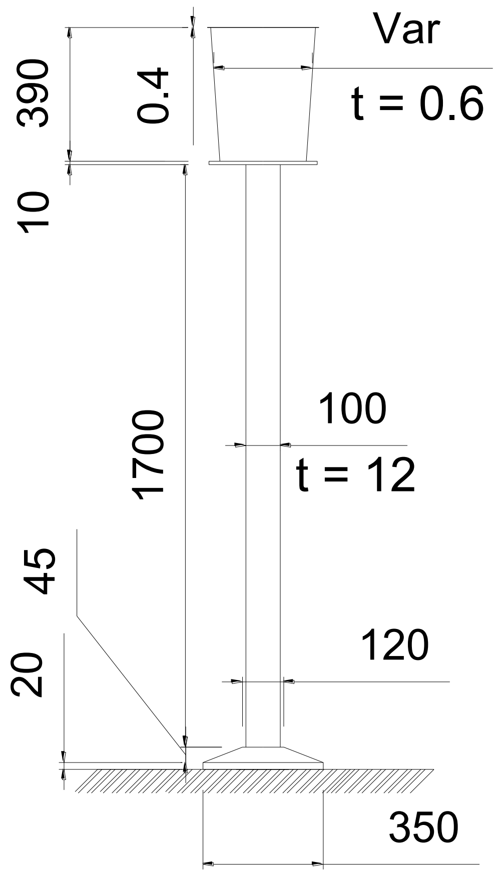


Figure 2: The Geometry of the structure – Measured in centimeters.

The homogenization of the concrete section was necessary due the existence of reinforcement bars in the transverse sections of the column, which was done as follows:

There is a circular ring cross-section with an outside diameter D. A reinforcement bar b_i occupies a position i in the section defined by R_{bi} and θ_i, as shown in Fig.3. R_{bi} determines the center position of each bar in relation to the section center. As all bars have the same radius, for simplicity of notation, $R_{bi} = R_b$, thus:

$$R_b = \frac{D}{2} - \mathrm{cov} - \frac{d_{bi}}{2} \tag{1}$$

where *cov* is the concrete cover of the reinforcement and d_{bi} is the diameter for the i bar.

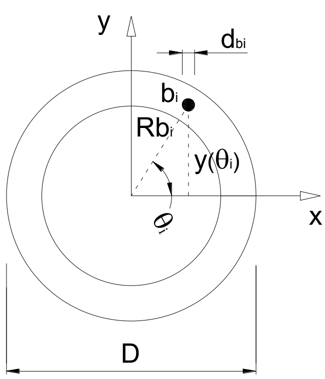


Figure 3: Transversal section homogenizing of the structure analyzed

As θ is the independent variable, the distance between the center of each bar relative to the axis center of inertia of the section is

$$y(\theta_i) = sen(\theta_i)R_b \tag{2}$$

The spacing between the center of each bar section was obtained using $esp = \dfrac{2\pi R_b}{nb}$, where *nb* is the number of longitudinal bars of reinforcement steel, and for angular phase shift between them for $\Delta\theta = \dfrac{esp}{R_b}$. Considering θ varying from *0* to 2π at intervals defined by $\Delta\theta$, the total inertia of the steel bars in relation to the section of the structure could be obtained by the theorem of parallel axes with the expression (3).

$$\mathrm{I_s} = \sum_{\theta}^{2\pi}\left(\frac{\pi d_{bi}{}^4}{64} + y(\theta_i)^2\,\frac{\pi d_{bi}{}^2}{4}\right) \tag{3}$$

The homogenized moment of inertia of the steel bars is therefore:

$$\mathrm{I_{s\,hom}} = \sum_{\theta}^{2\pi} \mathrm{I}(\theta_i)\left(\frac{\mathrm{E_s}}{\mathrm{E_{csec}}} - 1\right) \tag{4}$$

The parcel of concrete inertia is $I_{conc} = I - I_s$, with *I* being the inertia of the circular section. The total homogenized inertia of the section is obtained by $I_{tot} = I_{conc} + I_{s\,hom}$.

Therefore, in order to find a factor *F* which multiplies the nominal inertia of the section in terms of total steel inertia homogenized section we use

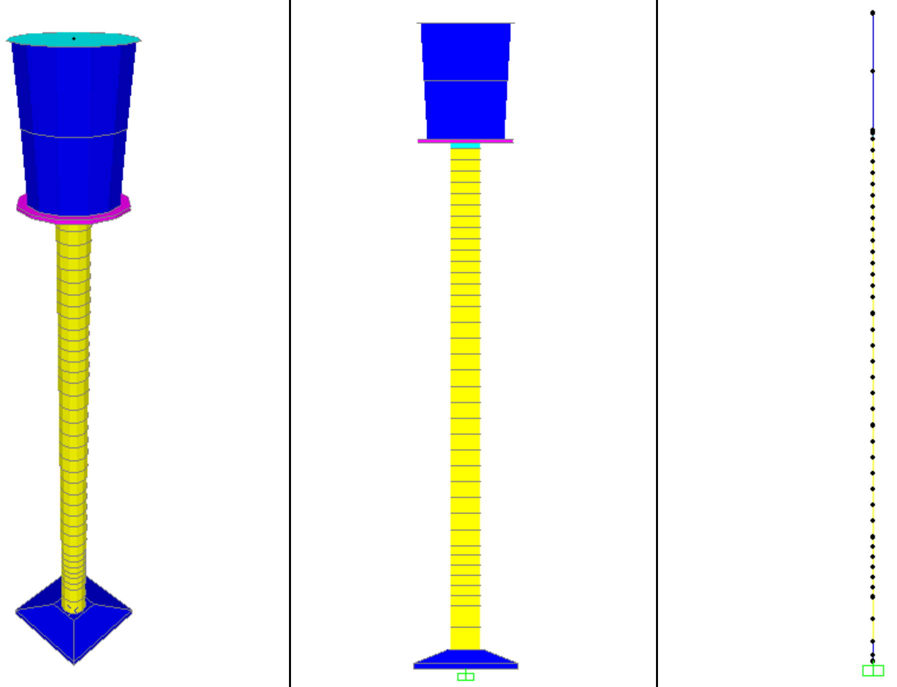
$$F = 1 + \frac{I_{s\,hom}}{I_{tot}} \tag{5}$$

The total homogenized factor of 1.0937 was found to the transversal section of the pillar

3 NUMERICAL MODELLING

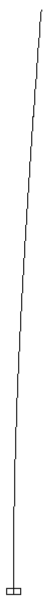
The numerical computational modeling of the structure was performed using the Finite Element Method and followed linear and nonlinear criteria from a geometric and material point of view. For the geometric nonlinear criteria we used the portion of the geometric stiffness matrix processing, which allows us to take into account the effect of normal force on the structural frequency of system. It is useful to recall that the 1st portion of the full stiffness matrix is a function of the flexural stiffness *EI*, parameter manipulated in the mathematical model to obtain the same result experimental. It is disregarded the damping system.

In Figure 4 the Finite Element Model can be seen, it is presented in a three-dimensional and lateral view, and with the discretization of the structure, containing 40 bar elements. The vibration modes and frequencies obtained by Finite Element Method are listed in Figure 5, in which GNL stands for Geometric Non-Linearity and MNL Material Non-Linearity. More details about Finite Element Method are presented by Clough [3], Bucalem [4] and Cook [5].



(a) 3D (b) Lateral View (c) Discretization

Figure 4: Finite Element Model.

Natural modes of vibration	 1^{st} Mode	 2^{nd} Mode	 3^{rd} Mode	 4^{th} Mode	 5^{th} Mode
Linear	0.621513 Hz	4.536457 Hz	9.321836 Hz	9.918722 Hz	11.639488 Hz
GNL	0.606852 Hz	4.511043 Hz	9.298632 Hz	9.918722 Hz	11.610126 Hz
GNL + MNL	0.472836 Hz	3.972319 Hz	8.448274 Hz	9.485529 Hz	10.850828 Hz

GNL is Geometric Non-Linearity and MNL Material Non-Linearity.

Figure 5: Finite Element Processing.

4 EXPERIMENTAL INVESTIGATIONS

The investigation of the natural frequency of the structure under ambient excitement was carried out using a gas-damped DC response accelerometer, manufactured by Bruel & Kjaer, with sensitivity of 1021 mV/g, with an integrated cable, suitable for measuring accelerations between ± 2g (Bruel & Kjaer Product Data, [6]). This device was fixed to the superior extremity of the structure, on the lateral surface of the support slab, as seen in Fig. 6(a). The acquisition of data was carried out by the system ADS2000, AqDados [7], by a Lynx Computer that was connected to a portable computer for the recording of the signals. The equipment was taken to the column top and placed on the superior surface of the support slab, as seen in Fig 6(b). It was registered that the structure was under sufficient excitement of wind, that was able to movement it. Signal acquisition was carried out at the rate of 50 Hz and the elapsed time was 1 h 33 min 22 s.

(a) Accelerometer (b) Data Acquisition System

Figure 6: Signal in frequency domain.

The fundamental frequency of the structure was obtained from the time series of the signals acquisition by Fourier Transform (FFT) in the program AqDAnalysis 7.02 [8]. The obtained result was 0.47 Hz. In Fig. 7 the analysis of the signal in the frequency domain can be seen and the acceleration time series in Fig. 8.

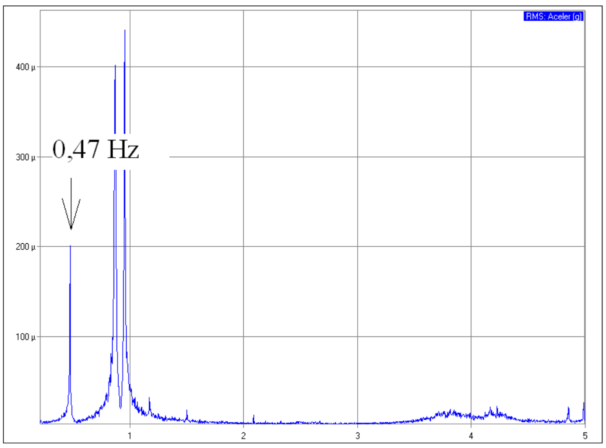


Figure 7: Signal in frequency domain.

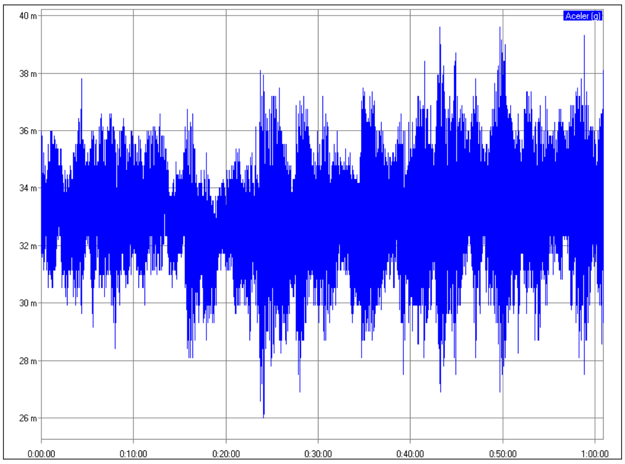


Figure 6: Temporal series.

5 CONCLUSIONS

- The Modulus of Elasticity of concrete was numerically adjusted to obtain the same value as the first vibration frequency mode which obtained experimentally.
- This was possible due to the dependence that the structure stiffness matrix has with the flexural stiffness *EI*, where *E* is the modulus of elasticity of material and *I* is the concrete section inertia homogenized area.
- The mitigation factor of the concrete flexural stiffness used in structures allows us to take into account the non-linear elastic behavior of the material, simplifying structural analysis.
- Similarly, the use of the geometrical stiffness matrix allows us to deal with a geometrically nonlinear problem, linearizing the second-order effects.
- Therefore, the use of the full stiffness matrix structure together with the adjustment factor appropriate for concrete flexural stiffness enables geometric and material nonlinearities solutions, solving the problem without iterative calculations.
- Through inverse analysis, the operator which adjusts the concrete modulus of elasticity, and so to speak the flexural stiffness, in nonlinear analysis, geometric and material, to experimental value, corresponds to 0.62 from linear elastic value, for the specific case studied.
- The first frequency mode of vibration of 0.473 Hz, calculated by the Finite Element Method, with both nonlinear aspects, lies 24% below the purely linear frequency and 2.36% of the frequency with geometric nonlinearity consideration only.
- The structural damping was considered negligible, therefore the adjusted numerical frequency relates to the natural frequency of the system. Uncertainties which may arise from the experimental investigation were discarded as well as the elastic characteristics and soil influence portion situated laterally from structure and above the foundation.

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