COMPDYN 2013
4th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, V. Papadopoulos, V. Plevris (eds.)
Kos Island, Greece, 12–14 June 2013

# EFFICIENT DYNAMIC INTERACTION ANALYSIS OF HIGH-SPEED TRAIN AND RAILWAY STRUCTURE DURING AN EARTHQUAKE

**M. Tanabe[1], M. Sogabe[2-1], H. Wakui[2-2], N. Matsumoto[2-3] and Y. Tanabe[3]**

[1]Kanagawa Institute of Technology
Atsugi, Kanagawa, 243-0292, Japan
e-mail: tanabe@sd.kanagawa-it.ac.jp

[2] Railway Technical Research Institute
Kokubunji, Tokyo, 185-8540, Japan
e-mail: [2-1]sogabe@rtri.or.jp, [2-2]wakui@rtri.or.jp, [2-3]nobel@rtri.or.jp

[3] Laboratory for Computational Mechanics Inc.
3-14-18 Kurihara-chuo, Zama, 252-0014, Japan

**Keywords:** Dynamic interaction, Train, Railway structure, Derailment, Contact-impact, Earthquake.

**Abstract.** *An efficient computational method to solve the dynamic interaction between a high-speed train and the railway structure including derailment during an earthquake is given. The motion of the train is expressed in multibody dynamics. Efficient mechanical models to express contact-impact behaviors between wheel and the track structure including derailment during an earthquake are given. Rail and track elements with multibody dynamics and FEM combined are described. The motion of a railway structure is modeled with various finite elements and rail and track elements. A modal reduction is applied to solve the problem effectively. An exact time integration scheme has been developed that is free from the round-off error for very small time increments needed to solve the interaction between wheel and railway structure including derailment during an earthquake. Numerical examples are demonstrated.*

# 1 INTRODUCTION

There is a very sophisicated interaction between a high-speed train and railway structure anticipated during an earthquake. The impact force between the wheel and rail may leads to a radical dynamic phenomenon such as lifting of wheel, derailing, touching down on the track surface and the impact on surface creating a high-frequency response higher than several hundred Hz mixed with the foundamental low-frequency response of railway structure and train. This is so called a multiscale phenomenon of the frequency and is hard to get the numerical solution as the time increment needed to solve the nonlinear problem becomes very small for the convergency during each time increment that causes a round-off error in the numerical integration. To avoid the round-off error for the very small time increment in the numerical integration, the scaling techniquie has been developed [1]. It is very important to develop an efficient method to solve the dynamic interaction between the train and railway structure including derailment and post-derailment behaviors during an earthquake to build an earthquake-safe railway system.

Computational methods to solve the dynamic interaction of a high-speed train and railway structure have been developed by using mutibody dynamics together with finite element method, and various mechanical models for the car, the railway structure and the interaction between wheel and rail have been developed based on the purposes and applications to design railways [2-5]. However, very little work related to derailment and post-dearilment behaviors of the train on the railway structure during an earthquake has been reported so far [6-7].

In this paper, a simple and efficient computational method to solve the dynamic interaction of a Shinkansen train (high-speed train in Japan) and railway structure including derailment behavior during an earthquake is given.

The motion of the train is modeled in multibody dynamics with nonlinear springs and dampers to connect all components. Efficient mechanical models to express contact-impact behaviors between wheel and rail before derailment and also between wheel and the track structure after derailment are given to solve the interaction during an earthquake effectively. Rail and track elements have been developed using multibody dynamics and finite element method combined to solve the interaction between wheel and long railway components such as rail and track effectively.

The motion of railway structure is modeled with various finite elements such as truss, beam, shell, solid, and nonlinear spring and damper elements, and also with rail and track elements. The nonlinear dynamic response during an earthquake is obtained by solving equations of motions of the train and railway structure subjected to interactions between wheel and the track structure including derailment and post-derailment behaviors. A modal reduction to equations of motions is made to solve the large-scale nonlinear problems effectively. The response calculation for the nonlinear equations of the train and railway structure during an earthquake requires very small time increments due to the high-frequency impact behavior between wheel and railway structure. An exact time integration scheme has been developed that is free from the round-off error for very small time increments needed to solve the radical interaction between wheel and railway structure in derailment and post-deralment during an earthquake.

Based on the present method a computer program, DIASTARS, has been developed for the simulation of a Shinkansen train running at high speed on the railway structure including derailment during an earthquake. Numerical examples are demonstrated.

# 2 MECHANICAL MODEL OF A SHINKANSEN TRAIN

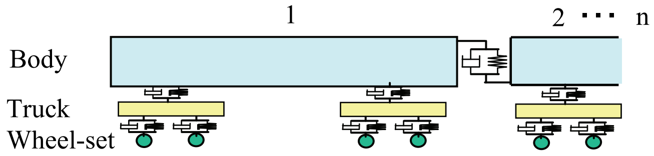



Fig. 1 Mechanical model of a Shinkansen train

A Shinkansen train is modeled with rigid components of car body, truck and wheel-set connected by nonlinear springs and dampers as shown in Fig. 1. Assuming that the train runs at a constant speed, the equation of 3D motion of the train with n cars connected is derived with 31n degrees of freedom and written in a familiar matrix form as [8]

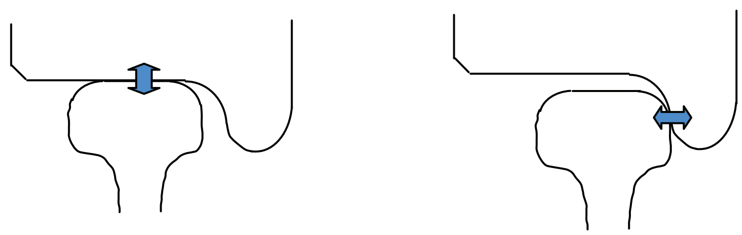
$$M^V \ddot{X}^V + D^V \dot{X}^V + K^V X^V = F^V \tag{1}$$

where $X^V$ and $F^V$ are displacement and load vectors of the train, and $M^V$ , $D^V$ and $K^V$ are the mass, damping and stiffness matrices, respectively.

# 3 INTERACTION BETWEEN WHEEL AND TRACK

## 3.1 Contact between wheel and rail before derailment

Assuming that the yawing and rolling of wheel-set are relatively small for the contact behavior between wheel and rail considered here, two dimensional geometries of the cross sections of wheel and rail are considered, and the contact-impact behavior in the normal direction on the contact surface between wheel and rail is modeled simply in two modes of the contact in the vertical and transverse directions as shown in Fig. 2.



(a) Vertical direction (b) Transverse direction

Fig. 2 Contact modes between wheel and rail

Assuming that wheel and rail are stiff enough, regarding the vertical mode of the contact the contact displacement between wheel and rail in the vertical direction, $\delta_Z$ , is expressed as a function of displacements of the rail $z_R$ and the wheel $z_w$ in the vertical direction and also the relative displacement on the contact surface between wheel and rail in the transverse direction $d_y$ depending on the geometry of the rail and wheel as follows

$$\delta_Z = \delta_Z(z_R, z_W, d_y)\ . \tag{2}$$

However, assuming the rolling and yawing of wheel set is relatively small, geometries of rail and wheel are defined by the cross-section, where the curve to define the cross-section is expressed by arcs and straight lines connected so that the slope of the curve is continuous at any places.

The contact displacement $\delta$ in the normal direction on the contact surface between wheel and rail is obtained from the contact angle at the contact position. When wheel contacts on rail, there is a contact force created on the contact surface. The contact force on the contact surface between wheel and rail in the normal direction, H, is expressed as a function of $\delta$ and $d_y$ as follows

$$H = H(\delta, d_y) \ . \tag{3}$$

Regarding the transverse mode of the contact between wheel and rail, the contact displacement in the transverse direction $\delta_y$ is also expressed as a function of $d_y$ and $\delta_Z$ depending on the geometry of the cross sections of wheel and rail as

$$\delta_y = \delta_y(d_y, \delta_Z) \ . \tag{4}$$

When a wheel contacts on the rail in the transverse direction, the contact force is obtained in the same manner as the contact mode in the vertical direction described above.

Regarding the tangential and longitudinal directions on the contact surface between wheel and rail, constitutive equations to describe the relationship between creep forces and slipping rates of wheel are given [9]. The creep force in the tangential direction $Q_c$ and yaw moment $T_c$ due to the creep force in the longitudinal direction on the contact surface between wheel and rail are described mathematically here as functions of slipping rates of wheel in the longitudinal and tangential directions, $S_x$ and $S_t$, and also of the spin rate around the normal vector on the contact surface, $S_n$, as [10]

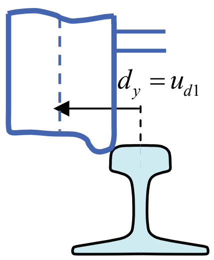
$$Q_c = Q_c(S_x, S_t, S_n) \tag{5}$$

$$T_c = T_c(S_x) \ . \tag{6}$$

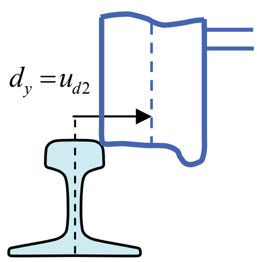
When a wheel lifts on the rail, there is no impact and creep forces created between the wheel and the rail.

### 3.2 Derailment Criterion

When the relative displacement between wheel and rail in the transverse direction, $d_y$, exceeds derailment criterions $u_{d1}$ or $u_{d2}$, it is detected that the derailment in the field side or gauge side is initiated, respectively as shown in Fig. 3 that leads to post-derailment behaviors of the wheel on the track structure. Once derailment of a wheel occurs, it is assumed here that




(a) Field-side derailment




(b) Gauge-side derailment

Fig. 3 Derailment criterion of left wheel

the wheel never returns to the normal running on the rail. After derailment of wheel from rail during an earthquake the wheel touches down on the track structure and there is the interaction between wheel and the track structure.

### 3.3 Contact between wheel and guard after derailment

Guards are attached on the track structure to prevent wheel deviating from the track after derailment during an earthquake to build an earthquake-safe railway system as shown in Fig. 4. After the derailment of a wheel, it contacts on the guard of the track structure in the transverse direction. The contact force $Q_{Gy}$ is expressed here as a function of the embedded area $A_{Gy}$ between the wheel and the guard as

$$Q_{Gy} = Q_{Gy}\,(A_{Gy}\,). \tag{7}$$

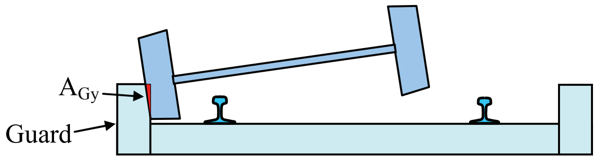


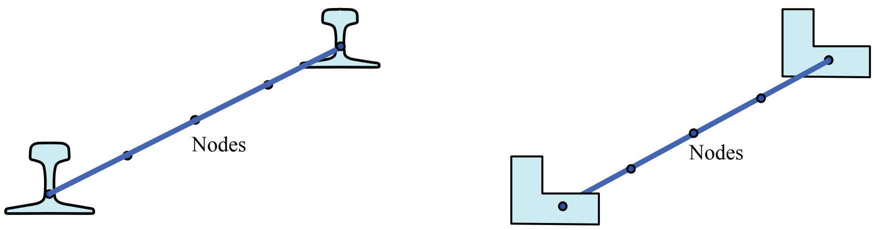

Fig. 4 Contact between wheel and guard on the track structure in the transverse direction

If the guard has an enough height and strength for the impact force between wheel and guard, the wheel is guided well between left and right guards in the rail direction after the derailment preventing wheel deviating from the track during an earthquake.

## 4. MECHANICAL MODEL OF RAILWAY STRUCTURE

### 4.1 Rail and track elements

Long railway components in the rail direction such as rail and track are considered to move as rigid bodies of the motion in plane of the cross-section. Rail and track elements have been developed to solve contact-impact behaviors between wheel and rail in the pre-derailment and between wheel and the track structure in the post-derailment effectively for the actual railway structure. Fig. 5 shows rail and track elements where motions in plane of cross-sections are expressed by multibody dynamics (MD) and motions in out-of-plane are expressed by beam elements given along the rail. The elements mixed with MD and FEM are very effective to solve contact-impact problems between wheel and a long track structure in the rail direction including derailment with a small numbers of DOFs.




(a) Rail element (b) Track element

Fig. 5 Rail and track elements

## 5 NUMERICAL METHOD

A railway structure is modeled with various finite elements such as truss, beam, shell, solid, mass, and nonlinear spring and damper elements, and also with rail and track elements for long railway components such as rail and track. Assembling all elements in the model, the equation of motion of a railway structure is obtained in a familiar matrix form as

$$M^b \ddot{X}^b + D^b \dot{X}^b + K^b X^b = F^b \tag{8}$$

where $X^b$ and $F^b$ are the displacement and load vectors of the railway structure, and $M^b$, $D^b$ and $K^b$ are the mass, damping and stiffness matrices, respectively. Note that $F^b$ includes nonlinear forces, and is a nonlinear function of $X^b$, $\dot{X}^b$, $X^v$ and $\dot{X}^v$.

### 5.1 Modal reduction

A modal reduction is applied to displacement vectors of the train and the railway structure to solve the practical problem with a long railway structure to allow the train to run at high speed during an earthquake effectively as

$$X^v = \Phi^v Z^v \tag{9}$$
$$X^b = \Phi^b Z^b \tag{10}$$

where $\Phi^v$ and $\Phi^b$ are rectangular matrices made of the mode vectors of the train and railway structure, and $Z^v$ and $Z^b$ are the modal coordinates, respectively. Moving the nonlinear terms in eqs. (1) and (8) to the right sides of equations, and operating the modal transformation by $\Phi^v$ and $\Phi^b$ respectively, equations of motions of the train and the railway structure are derived in the modal coordinates as

$$\ddot{Z}^V + \widetilde{C}^V \dot{Z}^V + [(\omega_i^V)^2] Z^V = \widetilde{F}^V \tag{11}$$
$$\ddot{Z}^B + \widetilde{C}^B \dot{Z}^B + [(\omega_i^B)^2] Z^B = \widetilde{F}^B \tag{12}$$

where $\omega_i^V$ and $\omega_i^B$ are angular frequencies of i-th mode in the train and railway structure, respectively, and $[(\omega_i^V)^2]$ denotes a diagonal matrix with i-th diagonal element of $(\omega_i^V)^2$.
However

$$\widetilde{C}^V = (\Phi^V)^T C^V \Phi^V \tag{13}$$
$$\widetilde{C}^B = (\Phi^B)^T C^B \Phi^B \tag{14}$$
$$\widetilde{F}^V = (\Phi^V)^T F^V \tag{15}$$
$$\widetilde{F}^B = (\Phi^B)^T F^B \tag{16}$$

where the superscript T denotes the transpose of the matrix. Note that the nonlinear terms for the train and railway structure are included in $\widetilde{F}^V$ in eq. (11) and $\widetilde{F}^B$ in eq.(12), respectively.

### 5.2 Exact time integration

Equations of motions of the train and the structure are solved in the modal coordinates for each time increment by the exact time integration scheme by approximating the right side term with m-th degree of polynomial and applying the exact time integration for each small time increment as the numerical time integration may cause the round-off error for very small time increments needed to solve the radical dynamic interaction in the derailment during an earthquake. However, since the equations are strongly nonlinear, iterative calculations are needed during each time increment until the unbalanced force between the train and railway structure becomes small enough within a tolerance specified.

Based on the present method, a computer program DIASTARS has been developed for the simulation of a Shinkansen train at high speed on the railway structure including derailment during an earthquake.

## 6 NUMERICAL EXAMPLES

### 6.1 One dimensional nonlinear dynamic problem

Fig. 6 shows a simple one dimensional nonlinear dynamic problem with the natural frequency of $\omega$, damping constant of h, the gravity force of g due to the unit mass where a contact element with the gap G and the spring constant K is attached.

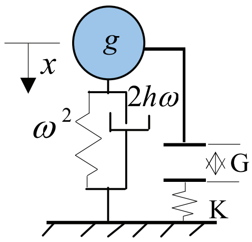



Fig. 6 One dimensional nonlinear dynamic problem

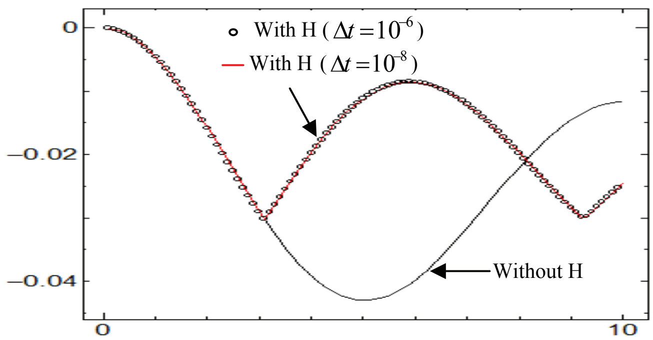



Fig. 7 Displacement response by the exact time integration

For the problem of $\omega=2\pi/10$ rad/sec, h=0.1, $K = 5\times10^3$ N/m, G=0.03m and g=9.8N, the response was obtained by the exact time integration scheme mentioned in the section 5.2 where the linear approximation to the right side terms of equations is made. Fig. 7 shows the displacement response for very small time increments of $\Delta t = 10^{-6}$ and $10^{-8}$ for the case with the contact force H and also for the case without the contact force. For very small time increments, the exact time integration scheme gives the exact solution. On the other hand, the numerical time integration by Newmark method failed due to the round-off error for both cases with H and without H for very small time increments of both $\Delta t = 10^{-6}$ and $10^{-8}$.

### 6.2 Simulation of a Shinkansen car running on the viaduct with guards attached during an earthquake

The simulation of a Shinkansen car running at a speed of 200km/h on the ladder track with guards attached on the five spanned viaduct with the height of 10m, the span-length of 8m and the width of 11.6m during an earthquake as shown in Fig. 8 has been conducted. The ladder track is made of ladder-shaped composite concrete beams to support rails tied with steel pipes where guards are attached to prevent wheel deviating from the track even after derailment during a strong earthquake [11-12].

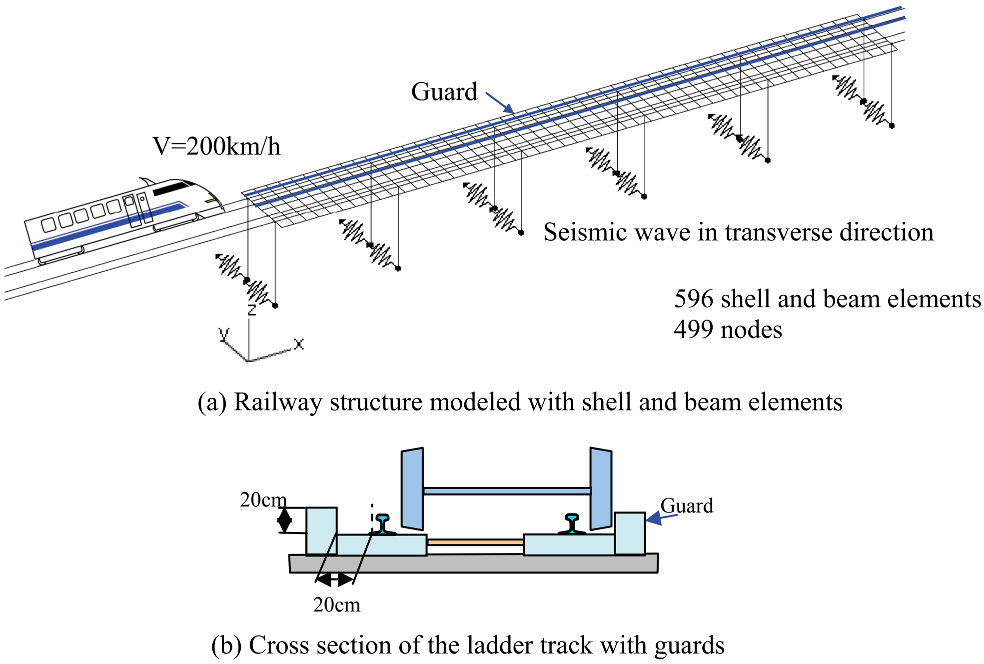



Fig. 8 Simulation of a Shinkansen car on the five spanned viaduct during an earthquake

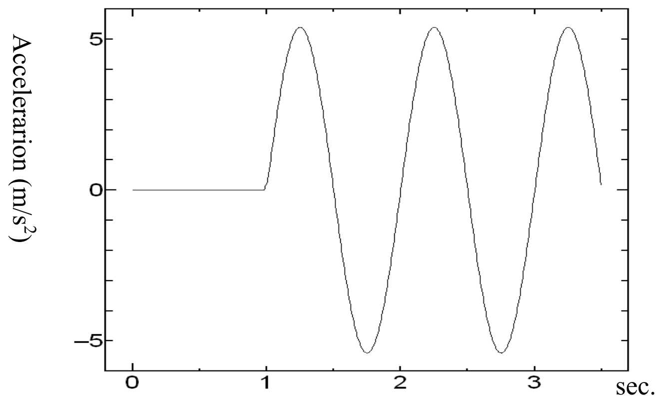



Fig. 9 Transverse acceleration at the base of the 3rd pier

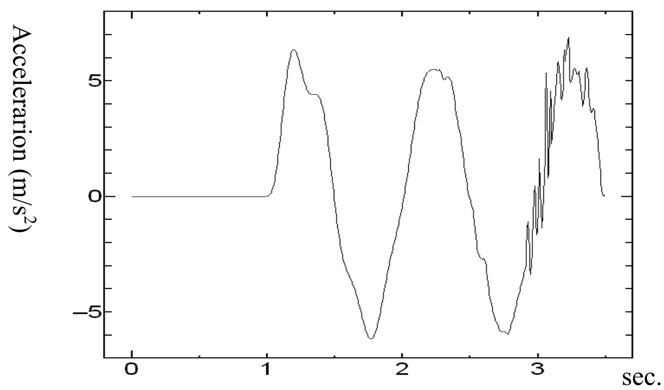



Fig.10 Transverse acceleration at the top of the 3rd pier

A sinusoidal seismic wave with the maximum acceleration of 5.4 m/sec$^2$, the wave number of five, and the frequency of 1Hz was given from the basis of all piers in the transverse direction. The frequency of 1Hz is close to the resonance frequency of rolling motion of the car body, truck and wheelset leading to derailment of wheel from the rail. The frame structure of the viaduct is modeled with beam elements and the concrete slab with shell elements. The rail and the ladder track with guards attached were modeled with rail and track elements mentioned earlier, respectively.

Fig. 9 and 10 show transverse acceleration at the base and top of the 3rd pier, respectively. The maximum acceleration at the top of the pier is amplified about 30% to the maximum acceleration of 5.4 m/s$^2$ given from the base. Fig. 11 shows the vertical displacement response of the right wheel at the 1st wheel-set. It is shown that the wheel runs onto the rail about 7 cm in height due to the impact between wheel and rail in the vertical direction, derails, touches down on the track, and lifts on the track surface during the earthquake. Fig. 12 shows the relative displacement response between right wheel and rail in the transverse direction, $\delta_y$ , at the 1st wheel-set. When $\delta_y$ exceeds $u_{d1}$ (7cm), the derailment to the field side is initiated, the wheel touches down on the surface of the ladder track in the vertical direction, impact on the guard of the track in the transverse direction, rebounds and gets back to the rail without deviating from the track. The track with the guards attached was shown to be effective to prevent the wheel deviating from the track even after derailment during the earthquake.

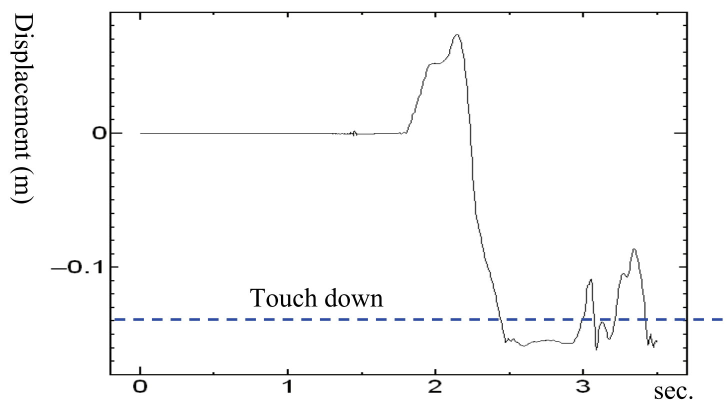



Fig. 11 Vertical displacement of the right wheel of the 1st wheel-set

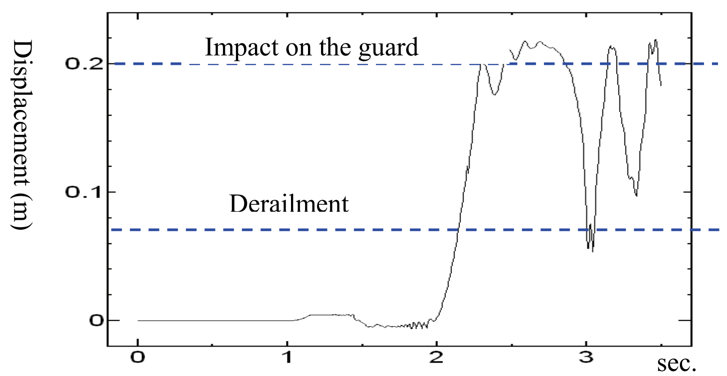



Fig. 12 Relative displacement between the right wheel and rail in the transverse direction

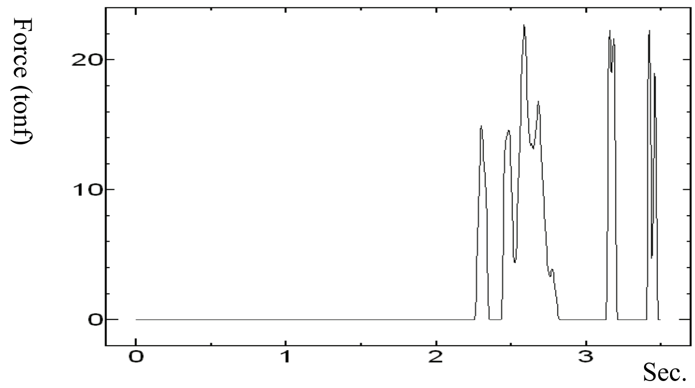



Fig. 13 Impact force of the right wheel on the guard in the transverse direction

Fig.13 shows the impact force of the right wheel on the guard. The maximum impact force of the wheel on the guard is shown to be about 22.5 tonf.

## 7 CONCLUSIONS

A simple and efficient computational method to solve the combined dynamic response of a high-speed train and railway structure including derailment during an earthquake was given. Efficient mechanical models to solve contact-impact behaviors between wheel and the track structure including derailment were described. Modal reduction was applied to equations of motions of train and railway structures to solve practical problems effectively. The exact time integration scheme has been developed to avoid the numerical error for very small time increments needed to solve the radical dynamic interaction between wheel and track structure during an earthquake. Simulation of a Shinkansen car on the five spanned viaduct during an earthquake was demonstrated. The computational method developed here would be effective to design an earthquake-safe railway system.

REFERENCES

[1] O. A. Bauchau, A. Epple and C. L. Bottasso, Who's afraid of high index DAES? Scaled and augmented Lagrangian formulations in multibody dynamics, *8th World Congress on Computational Mechanics*, Venice, Italy, 2008.
[2] M. Tanabe, H. Wakui and N. Matsumoto, The finite element analysis of dynamic interactions of high-speed Shinkansen, rail, and bridge, *Computers in Engineering,* Book No. G0813A, 17-22, ASME, 1993.
[3] A. Jaschinski, G. Schupp and H. Netter, Demonstration of simulation potentials in railway vehicle system dynamics through selected examples, *Proceedings of World Congress on Railway Research*, vol. D, 15-23, 1997.
[4] S. Bruni, A. Collina, R. Corradi and G. Diana, Numerical simulation of train-track-structure interaction for high speed railway systems, *Proceedings of Structures for High-Speed Railway Transportation*, *IABSE Symposium*, Antwerp, Belgium, 2003.
[5] R. Simoes, R. Calcada and R. Delgado, Track-bridge interaction in railway lines: Numerical modelling and application, *Track-Bridge Interaction on High-Speed Railways*, 205-216, FEUP Porto Portugal, 2007.

[6] M. Tanabe, N. Matsumoto, H. Wakui, M. Sogabe, H. Okuda and Y. Tanabe, A simple and efficient numerical method for dynamic interaction analysis of a high-speed train and railway structure during an earthquake, J. Computational and Nonlinear Dynamics, Vo.3/ 041002, ASME, 2008.
[7] M. Tanabe, N. Matsumoto, H. Wakui, M. Sogabe, Simulation of a Shinkansen train on the railway structure during an earthquake, *Japan J. Indust. Appl. Math.,* 28, 223-236, 2011.
[8] M. Tanabe, S. Komiya, H. Wakui, N. Matsumoto and M. Sogabe, Simulation and visualization of a high-speed Shinkansen train on the railway structure, *Japan J. Indust. Appl. Math.* vol.17, 309-320, 2000.
[9] J. J. Kalker, *Three-dimensional elastic bodies in rolling contact,* Kluwer, 1990.
[10] *Dynamics of railway vehicle,* JSME, 1994.
[11] H. Wakui, Technological innovation in railway structure system with ladder sleeper, *Concrete Journal,* Vol.36, No.5, 8-16, 1998.
[12] K. Asanuma, M. Sogabe, T. Watanabe, J. Okayama and H. Wakui, Development of a Ballasted ladder track equipped with a vehicle guide device, *RTRI Report* Vol.23, No.2, 27-32, 2009.