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BASIC DISPLACEMENT FUNCTIONS IN DYNAMIC ANALYSIS OF AN ARCH DAM AS A CURVED BEAM RESTING ON A CONTINUES ELASTIC FOUNDATION

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Abstract. *In recent years several researches have been done on different ways of analyzing and designing arch dams but most of them were involved with cumbersome calculations and heavy loads of computations. In this paper a novel approach for dynamic analysis of arch dams is presented. The most commonly accepted method for analyzing arch dams assumes that the horizontal water load is divided between arches and cantilevers so that the arch and cantilever deflections are equal at conjugate points in all parts of structure. In this the arch dam is modeled as non-prismatic curved beam resting on continues elastic foundation. Based on structural and mechanical principals, a flexibility based method is used to evaluate exact structural matrices and by introducing the concept of basic displacement functions (BDFs), it is shown that dynamic shape functions are derived in terms of BDFs. The flexibility basis ensures the true satisfaction of equilibrium equations at any interior point of the curved element. Dynamic stiffness matrix is evaluated by solving the governing equation of motion. Differential Transform method, a powerful numerical tool in solving of ordinary differential equations, is used for this purpose. The method is capable of modeling any curved element whose cross-sectional area and moment of inertia vary along beam with any two arbitrary functions and any type of cross-section with just few numbers of elements so that it can be used in most of engineering applications concerning non-prismatic curved beams and arch dams in particular. In order to verify the competency of the method, a numerical example are presented and the results and convergence of them are compared with other methods in the literature.*

1 INTRODUCTION

An arch dam is a curved dam which carries a major part of its water load horizontally to the abutments by the arch action, the part so carried being primarily dependent on the amount of curvature. The most commonly accepted method of analyzing arch dams assumes that the horizontal water load is divided between arches and cantilevers so that the arch and cantilever deflections are equal at conjugate points in all parts of the structure. In this study this theory is used and the arches are modeled as a non-homogeneous elastic foundation which prevents mentioned cantilevers from free displacements so the structure is modeled as a Non-prismatic curved beam resting on a non-homogeneous continuous elastic foundation. Moreover, in this study a new formulation for non-prismatic curved beams is derived which is based on flexibility method and respectively a new series of dynamic shape functions are obtained which also are based on applied load's frequency and finally new elements for analyzing curved beams are generated. Correspondingly, the stiffness and consistent mass matrices have been derived. The results show the efficiency and accuracy of the presented formulation in comparison with the methods in the technical literature. By using the method for modeling and analyzing of arch dams with simply employing just one element, same results are almost achieved comparing to using numerous normal elements. Flexibility basis ensures the true satisfaction of equilibrium equations at any interior point of the element in analyzing these types of structures. The accuracy and competency of this method is proven through a numerical example.

2 SHAPE FUNCTIONS FOR STATIC ANALYSIS

In this paper a novel method for analyzing and modeling of arch dams (Such as single curvature arch dams, double curvature arch dams or multiple arch dams) is proposed which is based on flexibility concept. Assume that a non-prismatic curved beam has two fix ends and subjected to arbitrary distributed load. This curved beam can be modeled with two cantilever beams which one is subjected to the arbitrary distributed load and the second beam is subjected to reactions (Figure 1 and 2).

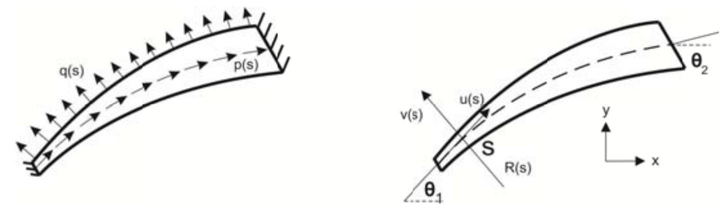

Figure 1: Non-Prismatic Curved Beam

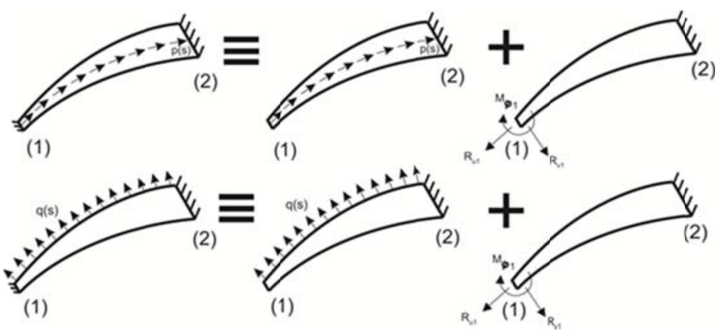

Figure 2: Non-Prismatic Curved Beam Modeling

With Regard to Figure 1, support reactions are obtained as equation (1).

$$\{F_a\}=\begin{Bmatrix} N_1 \\ V_1 \\ M_1 \\ N_2 \\ V_2 \\ M_2 \end{Bmatrix}=\begin{Bmatrix} K_{11} & 0 \\ 0 & K_{22} \end{Bmatrix}=\int p(s)\begin{Bmatrix} b_{u1}(s) \\ b_{v1}(s) \\ b_{\varphi 1}(s) \\ b_{u2}(s) \\ b_{v2}(s) \\ b_{\varphi 2}(s) \end{Bmatrix}ds \quad \{F_b\}=\begin{Bmatrix} N_1 \\ V_1 \\ M_1 \\ N_2 \\ V_2 \\ M_2 \end{Bmatrix}=\begin{Bmatrix} K_{11} & 0 \\ 0 & K_{22} \end{Bmatrix}=\int q(s)\begin{Bmatrix} c_{u1}(s) \\ c_{v1}(s) \\ c_{\varphi 1}(s) \\ c_{u2}(s) \\ c_{v2}(s) \\ c_{\varphi 2}(s) \end{Bmatrix}ds \quad (1)$$

Where $\{b\}$ and $\{c\}$ are the matrixes of axial and vertical deformations due to subjected unit load and also $\{K\}$ is the nodal stiffness matrix. The definitions of $\{b\}$ and $\{c\}$ are shown in Figure 3. In order to obtain them, these structures have to be analyzed by applying unit load method.

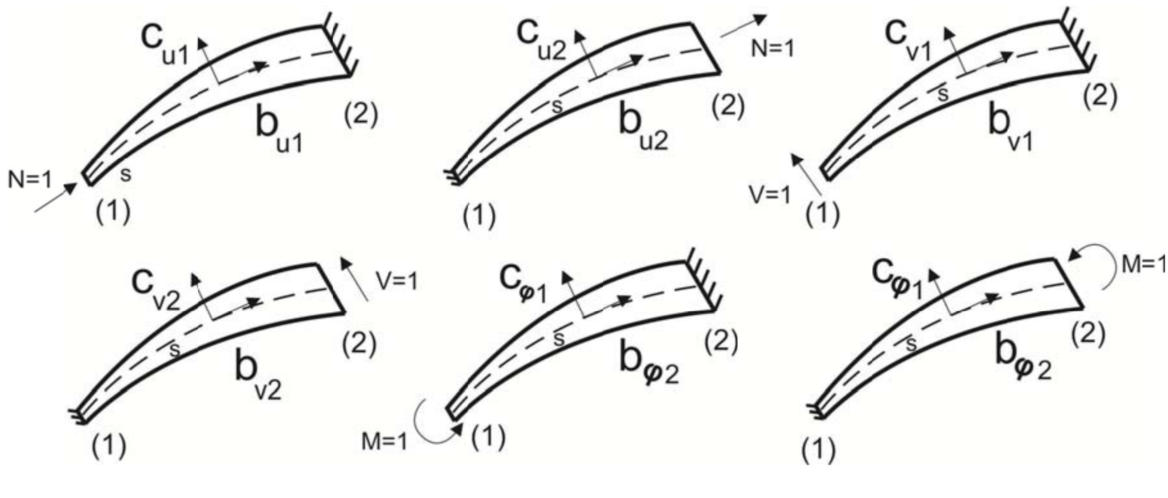

Figure 3: Definitions of $\{b\}$ and $\{c\}$

In accord with Figure 1 and 3, the necessary equations for obtaining $\{b\}$ and $\{c\}$ are presented in Table 1. Moreover, nodal stiffness matrixes are presented in Table 3.

$b(s)$	$c(s)$
$b_{u1}(s)=\int_s^1 \frac{m_1(t)M_{b1}(s,t)}{EI(t)}dt+\int_s^1 \frac{n_1(t)N_{b1}(s,t)}{EA(t)}dt$ $b_{v1}(s)=\int_s^1 \frac{m_2(t)M_{b1}(s,t)}{EI(t)}dt+\int_s^1 \frac{n_2(t)N_{b1}(s,t)}{EA(t)}dt$ $b_{\varphi1}(s)=\int_s^1 \frac{m_3(t)M_{b1}(s,t)}{EI(t)}dt+\int_s^1 \frac{n_3(t)N_{b1}(s,t)}{EA(t)}dt$ $b_{u2}(s)=\int_0^s \frac{m_4(t)M_{b2}(s,t)}{EI(t)}dt+\int_s^1 \frac{n_4(t)N_{b2}(s,t)}{EA(t)}dt$ $b_{v2}(s)=\int_0^s \frac{m_5(t)M_{b2}(s,t)}{EI(t)}dt+\int_s^1 \frac{n_5(t)N_{b2}(s,t)}{EA(t)}dt$ $b_{\varphi2}(s)=\int_0^s \frac{m_6(t)M_{b2}(s,t)}{EI(t)}dt+\int_s^1 \frac{n_{66}(t)N_{b2}(s,t)}{EA(t)}dt$	$C_{u1}(s)=\int_s^1 \frac{m_1(t)M_{c1}(s,t)}{EI(t)}dt+\int_s^1 \frac{n_1(t)N_{c1}(s,t)}{EA(t)}dt$ $C_{v1}(s)=\int_s^1 \frac{m_2(t)M_{c1}(s,t)}{EI(t)}dt+\int_s^1 \frac{n_2(t)N_{c1}(s,t)}{EA(t)}dt$ $C_{\varphi1}(s)=\int_s^1 \frac{m_3(t)M_{c1}(s,t)}{EI(t)}dt+\int_s^1 \frac{n_3(t)N_{c1}(s,t)}{EA(t)}dt$ $C_{u2}(s)=\int_s^1 \frac{m_4(t)M_{c2}(s,t)}{EI(t)}dt+\int_s^1 \frac{n_4(t)N_{c2}(s,t)}{EA(t)}dt$ $C_{v2}(s)=\int_s^1 \frac{m_5(t)M_{c2}(s,t)}{EI(t)}dt+\int_s^1 \frac{n_2(t)N_{c2}(s,t)}{EA(t)}dt$ $C_{\varphi2}(s)=\int_s^1 \frac{m_6(t)M_{c2}(s,t)}{EI(t)}dt+\int_s^1 \frac{n_6(t)N_{c2}(s,t)}{EA(t)}dt$
$M_{b1}(s,t)=sin\theta\,(s)\big(x(t)-x(s)\big)-cos\theta(s)(y(t)-y(s))$ $N_{b1}(s,t)=-cos\theta\,(s)cos\theta(t)-sin\theta(s)\sin\theta(t)$	$M_{b2}(s,t)=sin\theta\,(s)\big(x(s)-x(t)\big)-cos\theta(s)(y(s)-y(t))$ $N_{b2}(s,t)=cos\theta\,(s)cos\theta(t)+sin\theta(s)\sin\theta(t)$
$M_{c1}(s,t)=cos\theta\,(s)\big(x(t)-x(s)\big)+sin\theta(s)(y(t)-y(s))$ $N_{c1}(s,t)=sin\theta\,(s)cos\theta(t)-cos\theta(s)\sin\theta(t)$	$M_{c2}(s,t)=cos\theta\,(s)\big(x(s)-x(t)\big)+sin\theta(s)(y(s)-y(t))$ $N_{c2}(s,t)=-sin\theta\,(s)cos\theta(t)+cos\theta(s)\sin\theta(t)$
$m_1(t)=sin\theta_1\,(x(t)-x_1)-cos\theta_1(y(t)-y_1)$ $n_1(t)=-cos\theta_1\,(s)cos\theta(t)-sin\theta_1(s)\sin\theta(t)$	$m_2(t)=cos\theta_1\,(x(t)-x_1)+sin\theta_1(y(t)-y_1)$ $n_2(t)=sin\theta_1\,cos\theta(t)-cos\theta_1\sin\theta(t)$
$m_3(t)=-1$ $n_3(t)=0$	$m_4(t)=sin\theta_2\,\big(x_2-x(t)\big)-cos\theta_2(y_2-y(t))$ $n_4(t)=cos\theta_2\,cos\theta(t)+sin\theta_2\sin\theta(t)$
$m_5(t)=cos\theta_2\,\big(x_2-x(t)\big)-sin\theta_2(y_2-y(t))$ $n_5(t)=-sin\theta_2\,cos\theta(t)+cos\theta_2\sin\theta(t)$	$m_6(t)=1$ $n_6(t)=0$

Table 1: Equations regarding $\{b\}$ and $\{c\}$

Nodal Stiffness Matrix in Section 1	Nodal Stiffness Matrix in Section 2
$K_{11}=F_{11}^{-1}=\begin{bmatrix} f_{11}^{(1)} & f_{12}^{(1)} & f_{13}^{(1)} \\ f_{21}^{(1)} & f_{22}^{(1)} & f_{23}^{(1)} \\ f_{31}^{(1)} & f_{32}^{(1)} & f_{33}^{(1)} \end{bmatrix}^{-1}$	$K_{22}=F_{22}^{-1}=\begin{bmatrix} f_{11}^{(2)} & f_{12}^{(2)} & f_{13}^{(2)} \\ f_{21}^{(2)} & f_{22}^{(2)} & f_{23}^{(2)} \\ f_{31}^{(2)} & f_{32}^{(2)} & f_{33}^{(2)} \end{bmatrix}^{-1}$
$f_{ij}^{(1)}=\int_0^1 \frac{m_i(t)m_j(t)}{EI(t)}dt+\int_0^1 \frac{n_i(t)n_j(t)}{EA(t)}dt$	$f_{ij}^{(2)}=\int_0^1 \frac{m_{i+3}(t)m_{j+3}(t)}{EI(t)}dt+\int_0^1 \frac{n_{i+3}(t)n_{j+3}(t)}{EA(t)}dt$
m and n are defined in Table 1	

Table 2: Nodal Stiffness Matrixes

With respect to definition of external loads vector in finite element method, the new shape functions in non-prismatic curved beams are defined as Equation (2).

$$\{N_u\}=\begin{Bmatrix} N_{u1}^u \\ N_{v1}^u \\ N_{\varphi1}^u \\ N_{u2}^u \\ N_{v2}^u \\ N_{\varphi2}^u \end{Bmatrix}=\begin{Bmatrix} K_{11} & 0 \\ 0 & K_{22} \end{Bmatrix}=\begin{Bmatrix} b_{u1}(s) \\ b_{v1}(s) \\ b_{\varphi1}(s) \\ b_{u2}(s) \\ b_{v2}(s) \\ b_{\varphi2}(s) \end{Bmatrix} \qquad \{N^v\}=\begin{Bmatrix} N_{u1}^v \\ N_{v1}^v \\ N_{\varphi1}^v \\ N_{u2}^v \\ N_{v2}^v \\ N_{\varphi2}^v \end{Bmatrix}=\begin{Bmatrix} K_{11} & 0 \\ 0 & K_{22} \end{Bmatrix}=\begin{Bmatrix} c_{u1}(s) \\ c_{v1}(s) \\ c_{\varphi1}(s) \\ c_{u2}(s) \\ c_{v2}(s) \\ c_{\varphi2}(s) \end{Bmatrix} \tag{2}$$

Where u and v are displacement vectors.

3 SHAPE FUNCTIONS FOR DYNAMIC ANALYSIS

In a non-prismatic curved beam which is subjected to dynamic loads the equation of motion is assumed to be governed by

$$\frac{1}{R}\frac{\partial}{\partial s}\left(EI\left(\frac{\partial^2 v}{\partial s^2}-\frac{\partial\left(\frac{u}{R}\right)}{\partial s}\right)\right)-\frac{\partial}{\partial s}\left(EA\left(\frac{\partial u}{\partial s}+\frac{v}{R}\right)\right)+\rho A\frac{\partial^2 u}{\partial t^2}+ke_x u=p(x,t) \tag{3}$$

$$\frac{\partial^2}{\partial s^2}\left(EI\left(\frac{\partial^2 v}{\partial s^2}-\frac{\partial\left(\frac{u}{R}\right)}{\partial s}\right)\right)+\frac{1}{R}EA\left(\frac{\partial u}{\partial s}+\frac{v}{R}\right)+\rho A\frac{\partial^2 v}{\partial t^2}+ke_y v=q(x,t) \tag{4}$$

Where *s*= longitude of curvilinear, *u*= tangential displacement, *v*= vertical displacement, *EI* = flexural stiffness, *EA* = axial stiffness, *ke* = stiffness of elastic foundation, *ρA* = mass per unit length and *R*= radius of curvature. If the subjected dynamic load defines as a harmonic load, the deflection of structure can also be assumed as harmonic.

$$\frac{1}{R}\frac{d}{ds}\left(EI\left(\frac{d^2 v}{ds^2}-\frac{d\left(\frac{u}{R}\right)}{ds}\right)\right)-\frac{d}{ds}\left(EA\left(\frac{du}{ds}+\frac{v}{R}\right)\right)+\left(ke_x-\omega^2\rho A\right)u=p(s) \tag{5}$$

$$\frac{d^2}{ds^2}\left(EI\left(\frac{d^2 v}{ds^2}-\frac{d\left(\frac{u}{R}\right)}{\partial s}\right)\right)+\frac{1}{R}EA\left(\frac{du}{ds}+\frac{v}{R}\right)+\left(ke_y-\omega^2\rho A\right)v=q(s) \tag{6}$$

Where ω is the angular frequency of harmonic load.

Definitions of dynamic shape functions are similar to definition of static shape function as presented in the last section; the only difference is that in order to obtain the dynamic shape functions the equations of motion have to be solved. For this purpose in this study a numerical method called "Differential Transform Method" is employed. Differential transform of a function and differential inverse transform are as follow

$$\bar{Y}(k)=\frac{1}{k!}\left[\frac{d^k y(\xi)}{d\xi^k}\right]_{\xi=0} \quad \text{And} \quad y(\xi)=\sum_{k=0}^{N}\bar{Y}(k)\xi^k \tag{7}$$

Where $y(\xi)$ the original is function and $\bar{Y}(k)$ is the transformed function. In accord with Equation (7), it is readily proven that transformed functions complied with following basic mathematic operations

$$y(\xi)=u(\xi)\pm v(\xi) \quad \Rightarrow \quad \bar{Y}(k)=\bar{U}(k)\pm\bar{V}(k) \tag{8}$$

$$y(\xi)=C.u(\xi) \qquad \Rightarrow \qquad \overline{Y}(k)=C.\overline{U}(k) \tag{9}$$

$$y(\xi)=u(\xi).v(\xi) \qquad \Rightarrow \qquad \overline{Y}(k)=\sum_{r=0}^{k}\overline{U}(r)\overline{V}(k-r) \tag{10}$$

$$y(\xi)=\frac{d^{j}u}{d\xi^{j}} \qquad \Rightarrow \qquad \overline{Y}(k)=(k+1)(k+2)...(k+j)\overline{U}(k+j) \tag{11}$$

By applying the variable change $\varepsilon=\frac{x}{l}$, Equation (5) and (6) are rewritten as follow

$$\frac{1}{R}\frac{1}{L}\frac{d}{d\xi}\left(EI\left(\frac{1}{L^2}\frac{d^2v}{d\xi^2}-\frac{1}{L}\frac{d\left(\frac{u}{R}\right)}{d\xi}\right)\right)-\frac{1}{L}\frac{d}{d\xi}\left(EA\left(\frac{1}{L}\frac{du}{d\xi}+\frac{v}{R}\right)\right)+\left(ke_{\xi}-\omega^2\rho A\right)u=0 \tag{12}$$

$$\frac{1}{L^2}\frac{d^2}{d\xi^2}\left(EI\left(\frac{1}{L^2}\frac{d^2v}{d\xi^2}-\frac{1}{L}\frac{d\left(\frac{u}{R}\right)}{d\xi}\right)\right)+\frac{1}{R}EA\left(\frac{1}{L}\frac{du}{d\xi}+\frac{v}{R}\right)+\left(ke_{\xi}-\omega^2\rho A\right)v=0 \tag{13}$$

Applying differential transform to Equations (12) and (13) besides the theorems presented at Equations (8) to (11), the following recursive expressions are obtained

$$\sum_{r=0}^{k}\overline{\rho}(k-r)\overline{v}(r)-\frac{1}{L}(k+1)\overline{n}(k+1)+\sum_{r=0}^{k}k\overline{e}(k-r)\overline{U}(r)=\omega^2\sum_{r=0}^{k}\rho\overline{A}(k-r)\overline{U}(r) \tag{14}$$

$$\frac{1}{L}(k+1)(k+2)\overline{m}(k+2)+\sum_{r=0}^{k}\overline{\rho}(k-r)\overline{n}(r)+\sum_{r=0}^{k}k\overline{e}(k-r)\overline{V}(r)=\omega^2\sum_{r=0}^{k}\rho\overline{A}(k-r)\overline{V}(r) \tag{15}$$

Where$\overline{EI}$, $\overline{EA}$, $\overline{\rho A}$, $\overline{U}$ and $\overline{V}$ are the transformed functions of EI , EA , ρA , U and V and ρ is the reverse of radius of curvature (R). $\overline{m}$, $\overline{n}$ and $\overline{v}$ are the transformed functions of bending moment, axial force and shear force which are defined as follow

$$\overline{n}(k)=\sum_{r=0}^{k}E\overline{A}(k-r)\overline{\varepsilon}(r),\overline{\varepsilon}(r)=\frac{1}{L}(k+1)\overline{U}(k+1)+r=\sum_{r=0}^{k}\overline{\rho}(k-r)\overline{V}(r) \tag{16}$$

$$\overline{m}(k)=\sum_{r=0}^{k}E\overline{I}(k-r)\overline{\psi}(r),\overline{\psi}(r)=\frac{1}{L}(k+1)\varphi(k+1) \tag{17}$$

$$\varphi(k)=\frac{1}{L}(K+1)\overline{V}(K+1)-\frac{1}{L}\sum_{r=0}^{k}\overline{\rho}(k-r)\overline{U}(r) \tag{18}$$

$$\overline{v}(k)=\frac{1}{L}(K+1)\overline{m}(K+1) \tag{19}$$

Regarding the definitions of basic displacement functions and Figure 3, all boundary conditions are tabulated in Table 3.

c_{u1} و b_{u1}	c_{u2} و b_{u2}	c_{v1} و b_{v1}
$\sum_{k=0}^{N}\bar{U}(k)=0\quad \bar{n}(0)=-1$ $\sum_{k=0}^{N}\bar{V}(k)=0\quad \bar{v}(0)=0$ $\sum_{k=0}^{N}\bar{\varphi}(k)=0\quad \bar{m}(0)=0$	$\sum_{k=0}^{N}\bar{n}(k)=1\quad \bar{U}(0)=0$ $\sum_{k=0}^{N}\bar{v}(k)=0\quad \bar{V}(0)=0$ $\sum_{k=0}^{N}\bar{m}(k)=1\quad \bar{\varphi}(0)=0$	$\sum_{k=0}^{N}\bar{U}(k)=0\quad \bar{n}(0)=-1$ $\sum_{k=0}^{N}\bar{V}(k)=0\quad \bar{v}(0)=1$ $\sum_{k=0}^{N}\bar{\varphi}(k)=0\quad \bar{m}(0)=0$
c_{v2} و b_{v2}	$c_{\varphi1}$ و $b_{\varphi1}$	$c_{\varphi2}$ و $b_{\varphi2}$
$\sum_{k=0}^{N}\bar{n}(k)=0\quad \bar{U}(0)=0$ $\sum_{k=0}^{N}\bar{v}(k)=-1\quad \bar{V}(0)=0$ $\sum_{k=0}^{N}\bar{m}(k)=1\quad \bar{\varphi}(0)=0$	$\sum_{k=0}^{N}\bar{U}(k)=0\quad \bar{n}(0)=0$ $\sum_{k=0}^{N}\bar{V}(k)=0\quad \bar{v}(0)=0$ $\sum_{k=0}^{N}\bar{\varphi}(k)=0\quad \bar{m}(0)=-1$	$\sum_{k=0}^{N}\bar{n}(k)=0\quad \bar{U}(0)=0$ $\sum_{k=0}^{N}\bar{v}(k)=0\quad \bar{V}(0)=0$ $\sum_{k=0}^{N}\bar{m}(k)=1\quad \bar{\varphi}(0)=0$

Table 3: Boundary conditions in order to obtain $\{b\}$ and $\{c\}$

Once the first two terms of series $\bar{U}$ and first four terms of $\bar{V}$ derived, concerning mentioned boundary conditions, all terms of transformed functions of $\bar{U}$ and $\bar{V}$ are obtained by using the recursive Equations (14) and (15). Having $\{b\}$ and $\{c\}$ matrixes, nodal stiffness matrixes are calculated. With respect to section 2 of this paper, dynamic shape functions and stiffness matrixes are similarly obtained.

$$K_{11}=\begin{bmatrix} b_{u1}(0) & c_{u1}(0) & \frac{1}{L}\frac{dc_{u1}}{d\xi}\Big|_{\xi=0}-\frac{b_{u1}(0)}{R(0)} \\ b_{v1}(0) & c_{v1}(0) & \frac{1}{L}\frac{dc_{v1}}{d\xi}\Big|_{\xi=0}-\frac{b_{v1}(0)}{R(0)} \\ b_{\varphi1}(0) & c_{\varphi1}(0) & \frac{1}{L}\frac{dc_{\varphi1}}{d\xi}\Big|_{\xi=0}-\frac{b_{\varphi1}(0)}{R(0)} \end{bmatrix}^{-1},K_{22}=\begin{bmatrix} b_{u2}(L) & c_{u2}(L) & \frac{1}{L}\frac{dc_{u2}}{d\xi}\Big|_{\xi=L}-\frac{b_{u1}(L)}{R(L)} \\ b_{v2}(L) & c_{v2}(L) & \frac{1}{L}\frac{dc_{v2}}{d\xi}\Big|_{\xi=L}-\frac{b_{v2}(L)}{R(L)} \\ b_{\varphi2}(L) & c_{\varphi2}(L) & \frac{1}{L}\frac{dc_{\varphi2}}{d\xi}\Big|_{\xi=L}-\frac{b_{\varphi2}(L)}{R(L)} \end{bmatrix}^{-1} \tag{20}$$

4 CANTILEVER AND ARCH ACTION

The most commonly accepted method of analyzing arch dams assumes the horizontal water load is divided between the arches and the cantilevers so that the calculated cantilever and arch deflections are equal at all conjugate points in all parts of structure. The cantilever elements are assumed to be fixed at foundation and the arch elements fixed at abutments. In this paper we used this theory and presented a new method for modeling the arch dams with dam-reservoir interaction. We model the arches as a non-homogeneous elastic foundation that prevents arch dam's cantilevers from free displacements. So we model the structure as a Non-prismatic cantilever curved beam resting on a continuous elastic foundation (Figure 4).

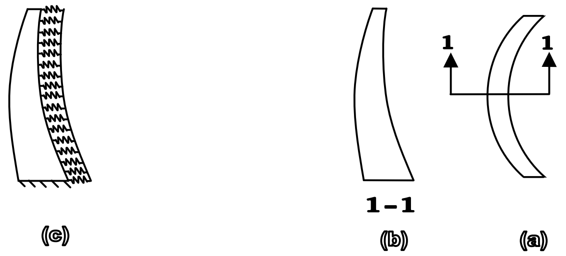

Figure 4: (a) Arch Element (b) Cantilever Element (c) Cantilever Beam Resting On an Elastic Foundation

5 NUMERICAL EXAMPLE

In order to verify the accuracy and competency of the presented method the double arch dam “Komarnica” in Yugoslavia with 195 m of height is chosen as a numerical example. Some of geometric properties of this dam are tabultaed in Table 4.

Radius (m)	Central Angle (degree)	Cord (m)	Height (m)
121	130	220	0
105	126	188	19.5
82	118	141	58.5
69	110	112	97.5
50	102	77	136.5
38	94	56	175.5
32	90	45	195

Table 4: Geometric Properties of Komarnica Dam

Considering the central cantilever of the structure, the thickness of this dam is assumed to be governed by

$$t(\xi) = -2.4\xi^2 - 17.7\xi + 29.3 \tag{21}$$

With regard the geometry of arches, a continuous elastic foundation is defined as

$$ke(\xi) = -4.71e10\xi^3 + 9.45e10\xi^2 - 6.07e10\xi + 1.26e10 \tag{22}$$

Now, with respect to section 4, the central arch of this structure can be modeled as non-prismatic cantilever curved beam which rests on a continuous elastic foundation (Figure 5).

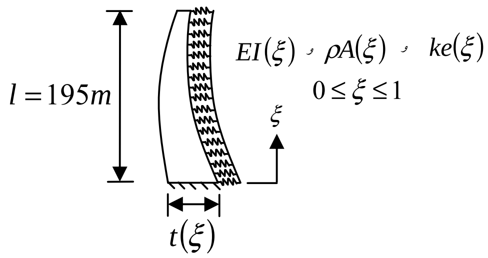

Figure 5: Modeling The Central Cantilever Of Komarnica Dam As a Curved Cantilever Resting On An Continuous Elastic Foundation

If the harmonic load $q(x,t)$ and material properties define as below, by obtaining $EI(\xi)$, $\rho A(\xi)$ and $ke(\xi)$ functions, the dynamic shape functions are obtained.

$$q(x,t) = q_0 \sin(\omega t)$$

$$p(x,t) = 0$$

$$\omega = 50 \qquad q_0 = 10^6 kg$$

$$\rho = 2400 \frac{kg}{m^3}$$

$$E_{static} = 2e9 \frac{kg}{m^2}$$

$$E_{dynamic} = 3.5e9 \frac{kg}{m^2} \tag{23}$$

By using mentioned dynamic shape functions the mass matrix, the stiffness matrix and the load vector are obtained. Finally, we analyzed the cantilever curved beam by using the New-mark method with coefficients $\beta = 0.25$ and $\gamma = 0.5$ and the time step $\Delta t = 0.01s$ with zero initial conditions. Maximum displacement of top of the structure wich calculated by applying this method and the comparison of it with normal beam elements are presented in Table 5 and Figure 6.

Case Number	Number of Ordinary Elements	Maximum Displacement of Top of the Structure (m)
1	5	0.0026952
2	10	0.0014896
3	20	0.0012011
4	50	0.0010523
5	100	0.0009597
6	Present Method (One Element)	0.0008982

Table 5: Maximum Displacement of Top of the Structure

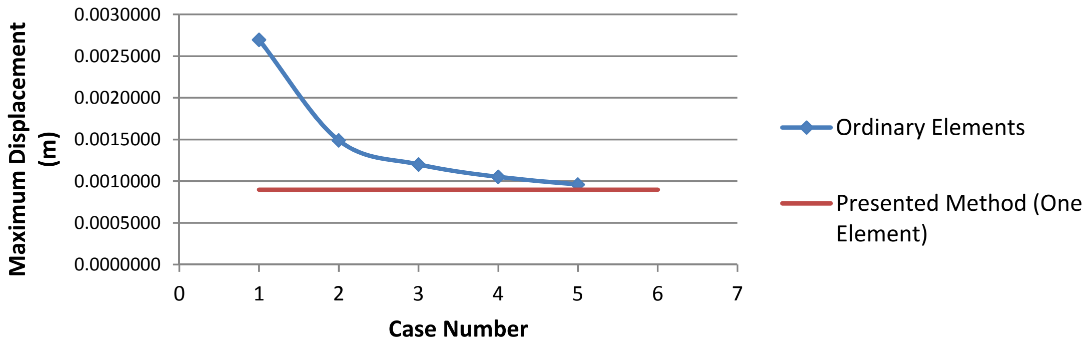

Figure 6: Convergence of the Maximum Displacement of Top of the Structure

6 CONCLUSIONS

- According to the numerical example the presented method has high accuracy and high performance for analyzing non-prismatic beams.
- The form of the defined static shape functions and the method that used for obtaining dynamic shape functions for non-prismatic curved beams, make this method capable of modeling any continues variation in geometric properties of the dams.
- Because the functions of thickness of the dam and stiffness of the elastic foundation are continues, this method is very practical for analyzing of arch dams.
- Concerning to the results of the numerical example highly acceptable results can be achieved by employing just one element.

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