APPLICABILITY OF TOTAL STRESS SEISMIC GROUND RESPONSE ANALYSIS UNDER LARGE EARTHQUAKES

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Abstract. Equivalent linear and truly nonlinear analyses, and total stress and effective stress analyses are compared at 268 sites under 11 earthquake motions in order to grasp general feature of the applicability of the equivalent linear method and total stress analysis. SHAKE and YUSAYUSA are used for equivalent linear and truly nonlinear methods, respectively. SHAKE shows larger peak acceleration and smaller peak displacement than YUSAYUSA (total stress analysis), but other indices such as instrumental seismic intensity, peak velocity, and spectral intensity are nearly same to each other. Therefore, it is applicable when truly nonlinear analysis is applicable. All earthquake motion indices are overestimated by the total stress analysis at the liquefiable sites. Therefore, it is recommended to use effective stress analysis at these sites. However, as indices are overestimated, use of the total stress can be said to result in conservative design. It is also noted that displacement or shear strain is underestimated in the total stress analysis.
1 INTRODUCTION

Static analysis based on the seismic coefficient has been made long time. Recently, however, seismic response analysis becomes a powerful tool for the design of structures. There are several reasons that the seismic response analysis becomes important. One of the big reasons is that the input earthquake motion becomes large, under which design based on the static analysis frequently results in too conservative. In order to make the design based on the seismic response analysis, seismic response analysis of ground also becomes important as a tool to obtain the input earthquake motions to the structure.

Input earthquake motion was increased significantly in Japan, especially after the 1995 Kobe earthquake and more after the 2007 Niigata-chuetsu-oki earthquake. Magnitude of the design earthquake motion, for example, exceeds 1 G in the design of nuclear power plants. Design input earthquake motions with maximum acceleration more than 0.5 m/s² is not an extraordinary case, but frequently appears in the practical design of civil and building structures. In these situations, maximum shear strain frequently reaches a few percent or more.

Various methods and computer programs have been proposed for the seismic response analysis of ground.

One of the frequently used computer programs is SHAKE [2], which employs an equivalent linear technique. The name SHAKE is now used as if it is a common noun, and it is also used as common noun in this paper. Unlike the name "equivalent", however, it is an approximate method. Therefore, it is believed that it cannot be applied at large strains. For example, Ishihara [3] suggests constitutive models and methods of the seismic response analysis related to different shear strain ranges and soil properties as shown in Figure 1. If maximum shear strain yields several percent, therefore, it is recommended to use the truly nonlinear method because the equivalent linear method is not applicable. However, engineers have a tendency to use SHAKE partly because of its easy handling compared with the truly nonlinear method, and partly because of the huge accumulation of experience to use the equivalent linear analysis.

Next problem is soil liquefaction. Considering that the definition of the soil liquefaction by means of shear strain is usually 5 % double amplitude axial strain under the tri-axial test condition, which corresponds to 3.75 % shear strain, effective stress analysis considering the excess porewater pressure generation is necessary if maximum strain comes several percent. However, use of the effective stress analysis is more difficult than that of total stress analysis. Therefore, total stress analysis is frequently carried out even when shear strain becomes very large.

There were not a few reports that compares various analytical method, and some reported that, for example, SHAKE is applicable, some does not. There seem at least two reasons to make this confliction. The one is that comparison is made on very few number of case studies, usually one or two, in which case obtained conclusion can be accidental except that the mechanism is clearly explained such

<table>
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<tr>
<th>Shear strain</th>
<th>10^-6</th>
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<th>10^-4</th>
<th>10^-3</th>
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<td>Effect of load-repetition</td>
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Model | Linear elastic model | Visco-elastic model | Load history tracing type model

Method of response analysis | Linear method | Equivalent linear method | Step-by-step integration method

Figure 1 Change in soil properties with shear strain and corresponding modelling principle and method of response analysis [1]
as ref. [4] which shows two shortages of SHAKE and solution against them. The second reason is that there is no common method to evaluate accuracy of the result of the seismic response analysis.

Looking at past researches to show applicability of the seismic response analysis, one of the indices such as agreement of the maximum acceleration, similarity of waveforms, etc. has been used. Now it is well known that maximum acceleration is not a good ground motion index for long period structures. Detailed investigation is difficult only from the similarity of the waveform. Applicability must be evaluated by using objective and effective ground motion indices.

In this paper, accuracy of the total stress analysis by equivalent linear and truly nonlinear methods is evaluated by comparing the effective stress analysis for more than 250 sites and under various types of input earthquake motions.

2 ANALYZED SITES AND MODELING OF MECHANICAL PROPERTIES

Figure 2 shows soil profiles and SPT-N value of the ground used in this paper; in total, 268 sites are used. These sites were the sites used to evaluate applicability of the conventional method to evaluate onset of identification during the 1995 Kobe earthquake [5]. Many sites in Japan are included here.

Soils are classified into 6 categories, i.e., sand, silt, clay, gravel, humus, and others. Here, others include concrete and cobble, etc. In the modeling in this paper, humus soil are treated same as clay, and others are neglected by replacing them by the soil below it. The SPT-N values are averaged in the same soil layer. If there is no N value data in a layer, it is set same with the one below the layer. These modeling may not be good if actual behavior of the ground is investigated, but are not because the purpose of this paper is to investigate different ground model.

Unit weight of each layer was included in the original data. The shear wave velocity $V_s$ is evaluated based on the specifications for highway bridges [6], i.e.,

\[
V_s = 100N^{1/3} \quad \text{(clayey soil)}
\]

\[
V_s = 80N^{1/3} \quad \text{(sandy soil, other soils)}
\]

Shear strength of the clayey soil is evaluated as an average value suggested by JGS [7]

\[
\tau_f = 19N \quad \text{(kPa)}
\]

On the other hand, the internal friction angle $\phi$ is evaluated based on Hatanaka and Uchida [8] as

\[
\phi = 20 + 20N_i \quad \text{(degree)}
\]

where $N_i$ is corrected $N$ value considering the effective over burden stress, and is evaluated as [6]

\[
N_i = \frac{170N}{\sigma_i' + 100} \quad \text{($\sigma_i'$ in kPa)}
\]

The hyperbolic model

\[
\tau = \frac{G_0\gamma}{1 + \tau_f\gamma/G_0}
\]
is used for shear stress $\tau$ - shear strain $\gamma$ relationships, and the Masing's rule is used to obtain damping characteristics, where $G_0$ is an elastic shear modulus.

Liquefaction strength of the sandy soils is evaluated based on the design specification of road bridges [6]. Shear stress ratio $R_L$ is calculates as

$$R_L = \begin{cases} 
0.0882\sqrt{N_a/1.7} & (N_a < 14) \\
0.0882\sqrt{N_a/1.7} + 1.6 \times 10^{-6}(N_a - 14)^{4.5} & (N_a \geq 14)
\end{cases}$$

(6)

where $N_a$ is corrected SPT-N value considering fines contents $F_c$ as

$$N_a = C_1N_i + C_2$$

(7)

where
\[
C_1 = \begin{cases} 
1 & (0 \% \leq F_c < 10 \%) \\
(F_c + 40) / 50 & (10 \% \leq F_c < 60 \%) \\
F_c / 20 - 1 & (60 \% \leq F_c \leq 100 \%)
\end{cases}, \quad C_2 = \begin{cases} 
0 & (0 \% \leq F_c < 10 \%) \\
(F_c - 10) / 18 & (10 \% \leq F_c \leq 100 \%)
\end{cases}
\] 
(8)

and \( N_1 \) is corrected SPT-N value considering the effective overburden stress obtained from Eq. (4).

This shear stress ratio (liquefaction strength) corresponds to 20 cycles of loading. Shear stress ratio at other cycles of loading is evaluated based on Seed et al. [9].

3 EARTHQUAKE MOTIONS

Input earthquake motion must have various and typical natures in order to obtain the general feature of the response. Eleven recorded earthquake motions which have different characteristics are employed so as to satisfy this requirement. They are as follows.

1) Inland earthquake
   - Port Island GL-33 m, NS, 1995 Kobe eq.
   - JMA Kobe, NS, 1995 Kobe eq.
   - Hakuta, NS, 1999 Tottoriken-seibu eq.
   - JMA Kawaguchi, EW, 2004 Niigataken-chuetsu eq.
   - TEPCO Kashiwa-Kariwa Service hole SG1, NS, 2007 Niigataken-chuetsu-oki eq.
   - Ichinoseki-nishi, EW, 2008 Iwate-Miyagi-nairiku eq.

2) Ocean trench earthquake
   - Kaihoku bridge EW, 1978 Miyagiken-oki eq.
   - Akita port, 1983 Nihonkai-chubu eq.
   - Taiki, EW, 2003 Tokachi-oki eq.

These waves are applied as the earthquake motion of the outcropped base. Layers with \( V_s = 300 \) m/s is added at the bottom of the ground in Figure 2. The shear wave velocity of the base layer is confirmed not to affect the behavior in the subsoil [10].

Waveforms of the input motion are shown in Figure 3. Generally speaking, duration of the earthquake is shorter in the inland earthquakes, but intensity is larger. Figure 4 shows response spectra of the input motions under 5% damping.

4 METHOD OF ANALYSIS AND ITS CHARACTERISTICS

Two different total stress analyses, equivalent linear method and truly nonlinear method, and one effective stress analysis are used. A computer program DYNEQ [11] is used for the equivalent linear analysis. DYNEQ is an open source program that has various functions based on complex moduli method, among which technique same with SHAKE is used in the calculation, and is referred as SHAKE. A computer program YUSAYUSA-2 [12], which is also an open source program, is used for the truly nonlinear and effective stress analyses. It will be referred as YUSAYUSA in the followings.

These two programs are examined under the same condition, i.e., the same ground model and the same stress-strain relationships. The only difference is that velocity proportional damping is considered in YUSAYUSA for the stability of numerical integration. Since the Rayleigh damping is employed, viscous coefficient matric \([c]\) is calculated from the mass matrix \([m]\) and stiffness matrix \([k]\) as
\[ [c] = \alpha[m] + \beta[k] \]  \hspace{1cm} (9)

where \( \alpha \) and \( \beta \) are constants and are set 0.090 and 0.00091, respectively, so that average

Figure 3  Input earthquake motions

(a) Inland earthquakes

(b) Ocean trench earthquakes

Figure 4  Response spectra of input motion
The damping ratio between 0.5 Hz and 6 Hz becomes 1.43% ($Q=35$).

Method of each analysis is briefly explained in the followings.

### 4.1 SHAKE

SHAKE [2] is based on the multiple reflection theory; equation of motion is solved in the frequency domain. Equivalent linear method is used in order to consider nonlinear behavior, in which stiffness and damping ratio at the effective stress $\gamma_{\text{eff}}$ are used in whole analysis. The effective stress is evaluated from the maximum strain $\gamma_{\text{max}}$ as

$$\gamma_{\text{eff}} = \alpha \gamma_{\text{max}}$$

where $\alpha$ is an adjusting parameter and 0.65 is usually used.

Two shortages are known in SHAKE [4] in addition to the fact that it cannot express the accurate stress-strain behavior because it is produced by the complex moduli.

The first shortage of SHAKE is overestimation of the maximum acceleration. The mechanism is shown in Figure 5. Since stress-strain relationships is a linear line passing the point $A$ ($\gamma_{\text{eff}}, \tau = G\gamma_{\text{eff}}$), maximum stress at $\gamma = \gamma_{\text{max}}$ ($\tau_1$ at point $C$) is larger than the actual stress ($\tau_2$ at point $B$), which overestimation of the maximum shear stress result in overestimation of the maximum acceleration (See section 6.2 on the mechanism).

![Figure 5](image1)

**Figure 5** Mechanism of overestimation of maximum shear stress [4]

The second shortage is typically seen in Figure 6. Amplification is significantly underestimated at high frequency. Maximum acceleration is underestimated by this mechanism.

These two factors work in the opposite way. The first mechanism overestimates maximum acceleration and the second mechanism underestimates maximum acceleration. Degree of effects depends on the degree of nonlinearity. This may be one of the reasons why evaluations of SHAKE scatter.

### 4.2 YUSAYUSA

YUSAYUSA [12] is a truly nonlinear seismic response analysis program. It uses hyperbolic model, Eq. (5), for backbone curve and the Masing's rule is used to develop hysteresis curve. Both total and effective stress analysis can be made. Excess porewater pressure generation is evaluated in the effective stress analysis by specifying the stress paths in the shear stress-effective overburden stress plane as shown in Figure 7, where $B_p$ and $B_u$ are parameters to control excess porewater pressure generation. They are evaluated so that $R_{20}$ and $R_5$ coincide with the liquefaction strength defined in chapter 2.
5 EARTHQUAKE MOTION INDEX

Many earthquake motion indices have been proposed, which indicates difficulty to express
the intensity of the ground motion in one index. In this paper, several frequently used indices
are used.

1) Peak acceleration

Peak acceleration is the most frequently used index because it directly related to inertia
force of the structure. However, it is sometimes too sensitive.

2) Peak velocity

Applicability of the peak acceleration as an earthquake motion index becomes less for
longer period structure. Instead of it, peak velocity becomes a good index. Here, this velocity
is absolute velocity, but not relative velocity from the base although relative velocity is fre-
quently used as outputs because it is directly obtained by solving the equation of motion.

3) Peak displacement

Displacement is a very important index for the design of the underground structures. Com-
parison of the relative displacements with recorded one is very difficult because vertical array
records are necessary and because it depends on the method of integration as will be seen in in
Figure 12. However, it can be done between analyses.

4) Instrumental seismic intensity

Seismic intensity has been used to express the degree of damage or ground shaking. Be-
cause of its nature, it takes time to evaluate the seismic intensity. After the 1995 Kobe earth-
quake, Japan Meteorological Agency made an seismic intensity which can be calculated from
the acceleration time history and which is consistent with traditional JMA seismic intensity
[13]. Evaluation of the seismic intensity can be done very quickly. Instrumental seismic inten-
sity $I_{JMA}$ is evaluated as

$$I_{JMA} = 2 \log a_0 + 0.94$$  \hspace{1cm} (11)
where \( a_0 \) is the acceleration in cm/s\(^2\); sum of the duration where acceleration exceeds \( a_0 \) is just 0.3 s in the filtered acceleration-time history. It is defined by using three directional components, but only one-directional component is used in this paper.

5) Spectral Intensity

Spectral intensity \( SI \) is first proposed by Housner [14] by integrating the response spectrum from 0.1 s to 2.5 s. It is modified to divide 2.4 s in order to make the dimension of the seismic intensity to be same with velocity. Then it is calculated as

\[
SI = \frac{1}{2.4} \int_{0.1}^{2.5} S_{v,0.2} dt
\]

where \( S_{v,0.2} \) is a velocity response spectrum under 20 \% damping. Equation (12) indicates that \( SI \) is an average of the velocity spectrum between 0.1 and 2.5 s. Pseudo velocity spectrum is sometimes used instead of velocity spectrum, but velocity spectrum is used in this paper.

6 COMPARISON AND DISCUSSION

6.1 Equivalent linear vs. truly nonlinear

Result under 11 earthquake motions are shown in one figure in the followings, in which result of each earthquake motion is distinguished by the symbols shown in Figure 8.

Figure 9 compares \( PGA \)s by all analyses, where subscript \( EQ \) indicates equivalent linear method (SHAKE) and \( NL \) indicates truly nonlinear method (YUSAYUSA). Two lines are drawn in the figure whose gradients are 1:1 and 1.5:1. \( PGA \) by SHAKE is always larger than that by YUSAYUSA except only several cases. \( PGA \) is overestimated more than 1.5 times in SHAKE in many cases which agrees with the discussion in Ref. [4] and Figure 5.

Figure 10 shows ratio of \( PGA \) in each soil profile. The same symbols seem to lie nearly horizontally although \( PGA \) scatters widely as seen in Figure 9.

Peak ground velocity, \( PGV \), is compared in Figure 11. Both \( PGV \)'s are nearly the same although that by the equivalent linear method seems to be a little larger than that by the nonlinear analysis. There are, however, two exceptions. \( PGV \) by nonlinear method shows much larger value than that of equivalent linear method under Kawaguchi (\( \triangle \)) and Kaihoku (\( \blacktriangle \)). This is not caused from the difference of the analytical methods but other reason.

Input earthquake motions are integrated to obtain absolute velocity by Fourier transform in SHAKE and Newmark's \( \beta \) method (\( \beta=0.25 \)) in YUSAYUSA, respectively. Zero-th order or constant term is neglected in SHAKE, but is also integrated in YUSAYUSA. Figure 12 compares maximum velocity integrated by the Fourier transform and by the Newmark's \( \beta \) method. Almost all earthquake motion shows the same maximum velocity in two methods, but maximum velocity by the Newmark's \( \beta \) method is much larger than that by Fourier transform for Kaihoku and JMA Kawaguchi, which is the reason why \( PGV \)'s by YUSAYUSA are much larger than those by SHAKE in Figure 11.

Considering these it can be concluded that \( PGV \)'s are nearly the same for both SHAKE and YUSAYUSA although those by SHAKE is a little larger than those by YUSAYUSA.

Figure 13 compares displacements at the ground surface relative to base, \( PGD \). \( PGD \) by SHAKE is in general smaller than that by YUSAYUSA. This behavior can be explained by the mechanism in Figure 5, too. Since stiffness is estimated large in SHAKE, resulting stress becomes small. It is also seen that degree of difference depends on the earthquakes as the same symbol shows similar tendency. Both analyses, for example, show similar PDG under Kaihoku. PDGs under Hachinohe also seem similar although \( PGD \) by SHAKE becomes
smaller when PDG becomes larger than 0.1 m. In addition, agreement is better under the ocean type earthquake than under the inland earthquake, except under Taiki.

Figures 14 and 15 compare JMA seismic intensity $I_{JMA}$ and spectral intensity $S_I$, respectively. Same with $PGV$, both indices are nearly the same although those by SHAKE are a little larger than that by YUSAYUSA. It is also pointed out that data under the same earthquake
motion show similar characteristics. Figure 16 shows comparison of the maximum stresses by both methods. Here, the largest strain in each ground is used as the maximum strain. Therefore, it may not be the same layer. If the mechanism shown in Figure 5 works, the shear strain by SHAKE is smaller than that by YUSAYUSA. Validity of this mechanism was confirmed through Figure 5 and Figure 13 in general. However, there are many data that strains by SHAKE are larger than those by YUSAYUSA. It may indicate that vibration modes are different under each analysis.

![Figure 13 Comparison of PGD](image)

![Figure 14 Comparison of JMA seismic intensity](image)

![Figure 15 Comparison of SI value](image)

![Figure 16 Comparison of maximum strains](image)

### 6.2 Total stress vs. effective stress

Effective stress analysis and total stress truly nonlinear analysis are compared here. Both calculations are made by YUSAYUSA; the only difference is consideration of excess porewater pressure generation or not. The subscript \( EF \) and \( NL \) indicate effective stress and total stress analyses, respectively.

The peak ground acceleration \( PGA \) is compared in Figure 17. Many data lie on the line \( PGA_{EF} = PGA_{NL} \). They are the case where there is no liquefiable layer, in which case results becomes identical. Beside these data, \( PGA \) by effective stress analysis is smaller than that by total stress analysis. Sometimes it reaches 1/3 or less, which can be seen in Figure 19.
The peak ground velocity \( PGV \) is compared in Figure 18. Again magnitude by the effective stress analysis is much smaller than that by the total stress analysis. This is quite different feature compared with the comparison between equivalent linear and truly nonlinear analyses in Figure 11, in which \( PGVs \) are similar.

![Figure 17 Comparison of PGA](image1)

![Figure 18 Comparison of PGV](image2)

![Figure 19 Acceleration ratios (Effective stress/Total stress)](image3)

Instrumental seismic intensity \( I_{JMA} \) is compared in Figure 20 and \( SI \) is compared in Figure 21. Similar with \( PGA \) and \( PGV \), those by the effective stress analysis are frequently much smaller than those by the total stress analysis. These observations can be explained by considering the upper bound ground motion caused by the nonlinear behavior [15].

Left side of Figure 22 shows equilibrium of a infinitesimally small element, from which equation of motion or equation of wave motion can be derived. On the other hand, right side of Figure 22 shows equilibrium of a soil column above the weak layer. If shear stress of the weak layer reaches shear strength, average acceleration above this layer reaches upper bound. This mechanism can explain the reason why \( PGA \) by the effective stress analysis is smaller than that by the total stress analysis.

There are also upper bounds on \( I_{JMA} \) and \( SI \) values, but the mechanism is somewhat different from the upper bound of \( PGA \). \( I_{JMA} \) uses frequency between 0.5 to 8 Hz and \( SI \) uses period between 0.1 to 2.5 s. Period of the wave become longer as nonlinear behavior becomes signif-
icant, and if period becomes longer than the period that is used in these indices, earthquake motion does not contribute these indices. Indices by the effective stress analysis are sometimes much smaller than that by the total stress analysis, which indicates that nonlinear behavior is significant when liquefaction occurs.

Peak ground displacements are compared in Figure 23 and maximum shear strains are compared in Figure 24. Those by the effective stress analysis are sometimes much larger than those by the total stress analysis, which also suggests significant nonlinear behavior.
7 CONCLUDING REMARKS

Applicability of the equivalent linear analysis (SHAKE) and total stress analysis under large earthquakes are examined by using more than 250 sites and 11 earthquakes. Since typical grounds and earthquake motions are included, the conclusions obtained here shows general feature of the analysis. The following conclusions are obtained.

1) $PGA$ is overestimated by the equivalent linear method. Sometimes, it is more than 1.5 times larger than that by the truly nonlinear method.
2) $PGD$ is underestimated by the equivalent linear method. It is frequently less than a half compared with that by the truly nonlinear method.
3) Other indices such as $PGV$, seismic intensity, and spectral intensity are nearly the same between the equivalent linear method and truly nonlinear method although evaluation by the equivalent linear method is a little larger than that by the truly nonlinear method.
4) All earthquake motion indices become significantly small when soil liquefaction occurs, except PGD and the maximum shear strain which becomes large.

Considering these, it is recommended to use effective stress analysis at the liquefiable site, but total stress analysis may be applicable because it results in larger earthquake motion to the structures. However, as predominant period of the wave becomes long, it may affect the structure with long natural period. It is also noted that shear strain and relative displacement is underestimated in the total stress analysis, which affects the design of underground structures such as pipe and pile lines.

Equivalent linear method is applicable in many fields if liquefaction does not occur because it shows a little conservative result, except the case that maximum displacement and maximum acceleration are interested at.

REFERENCES


