

## CONTACT INTERFACE MODELING IN THE DYNAMIC RESPONSE OF RIGID BLOCKS SUBJECTED TO BASE EXCITATION

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**Abstract.** *A proper selection of the percentage of the live load that must be considered as inertia is critical to the seismic analysis and design of certain structures like pile-supported container yards, in which live loads are nearly permanent and exceed the total dead load. This selection is particularly relevant in the case of flexible structures, for which the containers are not likely to experience sufficient accelerations to slide or rock and thus behave as rigid attachments. For this case, there are no clear design guidelines available with regard to the portion of the live load that should be used in dynamic analyses. To shed some light on this problem, a series of tests on a 1:15 scale model subjected to arrays of ground motions is currently underway. Along with this experimental program, numerical simulation is being conducted to develop a simple yet robust finite element (FE) model that can capture not only sliding and rocking, but also inelastic collisions involved in the response of a rigid block to base excitation. The resulting model will later be used to estimate the seismic response of a platform-container system with dynamic characteristics similar to those of the laboratory test model. Thus, a comparison between test and analysis results can be established for FE model calibration purposes and sensitivity studies can ultimately be conducted to assess the effect of live load on the seismic response of pile-supported container yards. This paper mainly deals with one of the most important steps in the development of the FE model: the modeling of the platform-container contact interface. Results from preliminary analyses of rigid blocks subjected to base excitation and exhibiting different response modes (including sliding and rocking) are presented. The results are shown to be in agreement with literature test data.*

## 1 INTRODUCTION

In structures such as the pile-supported container yard schematically shown in Figure 1, live loads are nearly permanent and exceed the total dead load even by a factor of two or more. The contribution of containers to the inertial forces acting on the structure varies depending on how much they slide and rock during a ground motion. This is because the movement of the containers dissipates some of the energy imposed by the ground motion through friction and impact. Therefore, the seismic analysis and design of a pile-supported yard must carefully consider the portion of the live load that should be included as inertia, particularly in the case of flexible structures, for which the containers may not experience sufficient accelerations to slide or rock, and thus behave as rigid attachments. Unfortunately, no clear design guidelines are available with regard to the portion of the live load that should be used in the dynamic analyses of these types of structures [1, 2, 3].

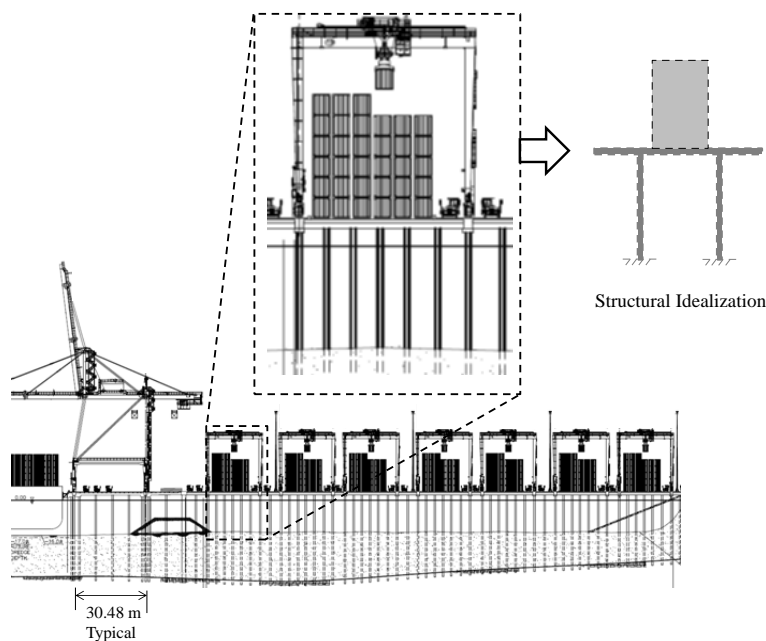


Figure 1: Pile-supported container yard and idealization.

The behavior of rigid bodies- like the containers in a terminal- is a complex problem of dynamics that has been studied extensively for over 50 years. The driving interest has been in the nuclear engineering industry and the need to determine the anchorage mechanisms for containers loaded with sensitive equipment or hazardous materials. The most influential formulation to describe the rocking motion of rigid bodies was introduced by Housner [4], who used basic principles to derive two piecewise equations of motion. Yim et al. [5] developed a numerical procedure to solve the nonlinear equations governing the rocking motion of rigid blocks on a rigid base subjected to horizontal and vertical ground motion. Shenton & Jones [6] proposed a general, two-dimensional formulation for the response of free-standing rigid bodies to base excitation. A series of criteria for initiation of slide, slide-rock and rock body modes from rest conditions for rigid boxes when subjected to horizontal ground pulses was developed by Shenton [7]; his analyses, which assume rigid foundation and Coulomb friction, show a transition of the mode of initiation of the response of the body from at-rest to sliding, rocking, or coupled sliding-rocking depending on the friction coefficient at the interface, the aspect ratio of the block and the magnitude of the horizontal ground acceleration as shown in

Figure 2. More recently, Peña et al. [8] conducted a comprehensive experimental investigation to study the rocking response of single rigid-block structures. They also developed two tools for the numerical simulation of the rocking motion observed in the tests: one analytical and one using the discrete element method (DEM).

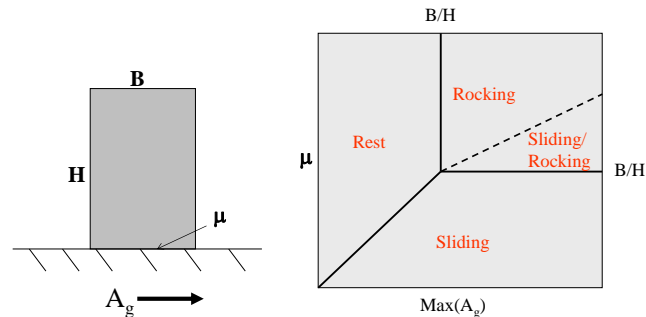


Figure 2: Mode initiation criteria for rigid blocks, after Shenton [7]

## 2 RESEARCH PROGRAM OVERVIEW AND SIGNIFICANCE

In order to shed some light on the determination of the portion of the live load to be considered as inertia in simple structure like the idealization depicted in Figure 1, a series of tests on a 1:15 scale model subjected to an array of ground motions is currently underway. The test structure, designed based on similitude requirements, consists of a platform and four steel columns as a representative portion of a pile-supported terminal. The model will be subjected to base excitation while supporting heavy rigid blocks of different aspect ratios which represent possible container stack configurations.

The experimental program is being supplemented with a simple finite element (FE) model that captures different phenomena involved in the response of a rigid block to base excitation. These phenomena include sliding and rocking of the block, as well as impact with the supporting structure. Other methodologies and formulations to address the different modes of response of rigid blocks to base excitation that are available, although robust, are often not simple for practical use in seismic design. The finite element method (FEM) is advantageous because it can be relatively easy to implement in design processes and also because of the increasing familiarity of structural engineers with the method and commercial FE software.

It is apparent that in order to capture appropriately the different phenomena of interest, the block-foundation contact interface must be carefully represented. This is the most important step in the model development and is the main topic of this paper.

## 3 FINITE ELEMENT (FE) MODEL

In this section, the main features of the FE model, developed using ANSYS release 13.0 (academic version), are described. Although the element types used in the model correspond to this program, the modeling concepts are applicable to other FE software.

### 3.1 Rigid block and foundation

The basic FE model developed consists of a rectangular block resting directly on a rectangular plate that simulates the foundation (Figure 3). If the base acceleration occurs in a direction parallel to one of the horizontal axes of symmetry of the block, and if perfectly symmetric response is assumed, the problem becomes two-dimensional (2-D) and the FE model can be simplified. Thus, both the block (with width  $B$  and height  $H$ ) and the foundation plate (with width  $B_f$  and thickness  $H_f$ ) are modeled using 2-D structural solid ANSYS element

type PLANE182 [9], which can be used either as a plane strain/stress element or an axisymmetric element. This element type is defined by four nodes, each having two translational degrees of freedom (DOF's), and has non-linear geometry capabilities (large strains/deflections). All the DOF's of the foundation plate are restrained to simulate a fixed base.

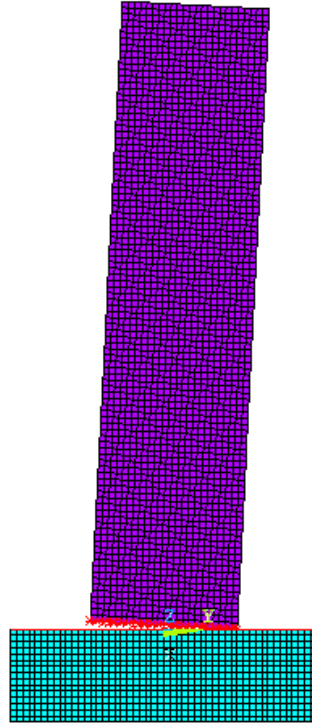


Figure 3: Finite element (FE) model of a rigid block resting on a rigid foundation

### 3.2 Block-foundation contact interface

To capture sliding and rocking of the block under base excitation, as well as impact with the foundation, the contact interface that exists between the block and the foundation plate is modeled via an ANSYS contact pair. This is defined by two surfaces designated as the contact and target surfaces, which can be initially in contact or are expected to make contact within the duration of the motion/response analyzed. Non-linear contact and target elements are used to model the two surfaces in the contact pair. These surfaces share a unique identifier and set of properties that allow ANSYS to detect any potential interpenetration and implement the contact algorithm selected by the user to guarantee geometric and kinematic compatibility.

The target surface is modeled using ANSYS element type TARGE169 [9], which is a 2-D target segment that can be paired with any of the 2-D contact elements available in the program. These target segments are used to overlay the top surfaces of the solid elements in the uppermost layer of the foundation plate mesh, thereby defining the target surface (Figure 4a). Forces and moments, as well as displacements and rotations can be imposed on the target elements.

The contact surface is generated by overlaying the bottom surfaces of the solid elements in the lowermost layer of the block mesh with contact elements (Figure 4b). The selection of this element type depends on the type of structural solid used in the model. There are two element types available in ANSYS that are compatible with PLANE182: CONTA171 and CONTA175 [9]. CONTA171 is a 2-D two-node surface-to-surface contact element used to represent contact and sliding between a 2-D target surface and a deformable surface.

CONTA175 is a 2-D/3-D node-to-surface contact element used to represent contact and sliding between two surfaces, a node and a surface, or a line and a surface.

For both contact elements described above, contact occurs when the element surface penetrates one of the segments (TARGE169) on the specified target surface. Interface friction and delamination can be modeled with both contact elements. Based on element geometry and using guidelines provided in Reference [10], CONTA171 is used in the FE model to analyze sliding-dominated problems, whereas CONTA175 is used to analyze rocking-dominated response. In both cases, a Coulomb friction model is used to capture friction and sliding at the contact interface.

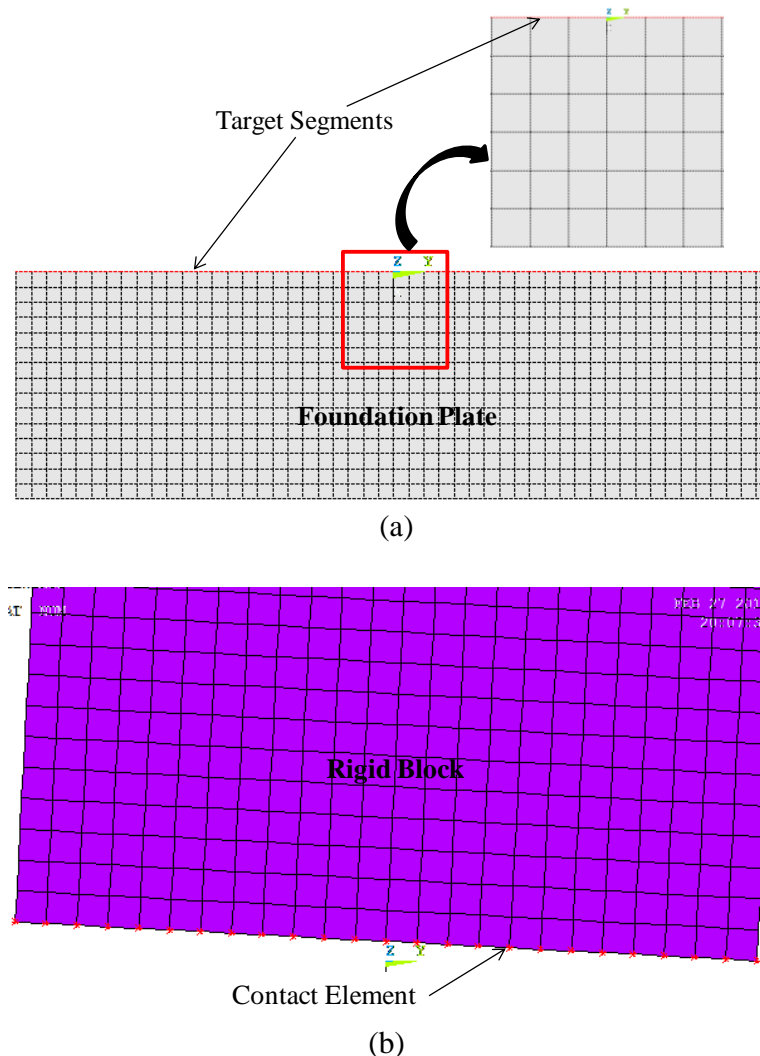


Figure 4: Modeling of block-foundation contact interface: (a) target surface and (b) contact surface

### 3.3 Material properties

Both the block and the foundation plate are modeled using linear-elastic material properties. Thus, only the Young's modulus ( $E$ ), Poisson's ratio ( $\nu$ ), and mass density ( $\rho$ ) need to be input. The contact elements use an isotropic Coulomb friction model [10], which assumes that sliding between two contacting surfaces begins at a threshold shear stress at the interface ( $\tau$ ) given by:

$$\tau = c + \mu p \quad (1)$$

where:

$c$  = Cohesion at the contact interface  
 $\mu$  = Interface friction coefficient  
 $p$  = Contact pressure

For the problem at hand, no cohesion is assumed to exist at the contact interface ( $c = 0$  in Eq. (1)) and, therefore, the frictional shear stress is directly proportional to the contact pressure ( $\tau = \mu p$ ). The friction coefficient  $\mu$  is input as a material property for the contact elements, and it can depend on temperature, time, normal pressure, sliding distance, and sliding relative velocity. Based on the physics of the problem analyzed and for simplicity,  $\mu$  is assumed to depend only on the relative velocity of the two surfaces in contact ( $V_{rel}$ ). ANSYS provides an exponential decay friction model [10] to account for the variation of  $\mu$  with  $V_{rel}$ , as indicated below:

$$\mu = \mu_d \left[ 1 + \left( \frac{\mu_s}{\mu_d} - 1 \right) \cdot e^{-DC \cdot V_{rel}} \right] \quad (2)$$

where:

$\mu_d$  = Dynamic friction coefficient  
 $\mu_s$  = Static friction coefficient  
 $DC$  = Decay coefficient [time/length]

The purpose of Eq. (2) is to provide a smooth transition from a “sticking” condition ( $V_{rel} = 0$  &  $\mu = \mu_s$ ) to a sliding condition ( $V_{rel} \gg 0$  &  $\mu = \mu_d$ ). The decay coefficient  $DC$  is obtained experimentally and, therefore, it is not always available. In this case,  $DC$  may be taken as zero, and Eq. (2) is rewritten to be  $\mu = \mu_s$  for sticking and  $\mu = \mu_d$  for sliding. Since typically  $\mu_s > \mu_d$ , there will be a sudden jump in the value of  $\mu$  when the sliding condition is reached if no value of  $DC$  is specified. This discontinuity may lead to convergence difficulties and should be avoided.

In addition to the energy dissipation mechanism provided by friction, numerical viscous damping is provided to the FE model to account for other sources of energy dissipation such as the multiple impacts of the block with the foundation during rocking response.

### 3.4 Contact algorithm and impact modeling

The basic contact algorithm selected in ANSYS is the augmented Lagrangian method, which consists of an iterative series of Penalty methods [11]. The Penalty method uses normal and tangential springs to relate the two contacting surfaces, with the spring constants called the contact stiffnesses. Contact tractions (pressure and frictional stresses) at the interface are proportional to the contact stiffnesses, and they lead to values of penetration and slip (in the normal and tangential directions, respectively) that are lower than allowable values. This guarantees that the relative motion of the two surfaces and the deformation occurring at the interface are geometrically compatible.

In addition to the displacement constraints imposed by the basic contact algorithm on nodes at the interface, velocity constraints must also be imposed when modeling contact in a transient dynamic analysis. This is to guarantee that nodal velocities and accelerations are consistent with nodal displacements, thereby enforcing conservation of momentum at the contact interface.

### 3.5 Boundary conditions and loading

All the nodes of the foundation plate are restrained against translation. Vertical acceleration is applied to the block to simulate its self-weight, which is the only load that is considered in free rocking motion analysis. Horizontal acceleration is applied to the foundation plate to simulate an excitation at the base of the block resulting from either harmonic motion or an earthquake.

## 4 RELEVANT NUMERICAL SOLUTIONS AND EXPERIMENTAL RESULTS

### 4.1 Numerical solution of SDOF structure supporting a sliding block

Figure 5 shows the structural idealization of a linearly elastic single degree of freedom (SDOF) structure (platform) supporting a rigid block. The SDOF oscillator is characterized by a mass  $m_p$ , damping  $c$ , and a stiffness  $k$ ; whereas the rigid block is characterized by a mass  $m_L$  and a constant static/kinematic friction coefficient  $\mu$  at its contact interface with the structure.

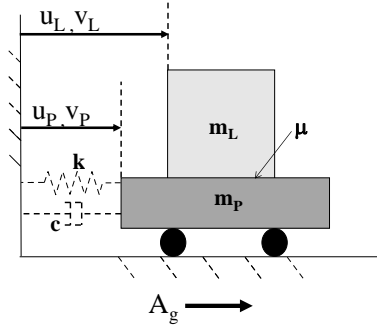


Figure 5: SDOF structure supporting a sliding rigid block

Provided that the rocking mode of the block is precluded, and letting  $u_P$ ,  $u_L$  and  $v_P$ ,  $v_L$  be the displacements and velocities of the SDOF oscillator and the block, respectively, the equations of equilibrium for the system can be written as (time derivatives are indicated with a dot directly above the variable):

$$\begin{aligned} m_L(\dot{v}_L + A_g) + f_x &= 0 \\ m_P(\dot{v}_L + A_g) + c v_P + k u_P - f_x &= 0 \end{aligned} \quad (3)$$

where  $A_g$  is ground acceleration, and  $f_x$  is friction force at the contact interface given by:

$$f_x = \begin{cases} -m_L(\dot{v}_P + A_g), & \dot{v}_P + A_g < \mu g \\ -\mu m_L g \text{SIGN}(v_L - v_P), & \dot{v}_P + A_g > \mu g \end{cases} \quad (4)$$

Equations (3) and (4) were solved using a Runge-Kutta (R-K) numerical algorithm for a SDOF oscillator with the properties listed below:

$$\begin{aligned} \xi &= \frac{c}{2\sqrt{k m_P}} = 0.05 \quad (\text{damping ratio}) \\ \alpha &= \frac{m_L}{m_P} = 2.0 \quad (\text{mass ratio}) \\ T_n &= 2\pi\sqrt{\frac{m_P}{k}} = 0.5s \quad (\text{natural period}) \end{aligned} \quad (5)$$

Excitation at the base consisted of a series of 30 far-fault ground motions corresponding to a seismic scenario with the characteristics indicated in Table 1. The calculated maximum displacements of the SDOF oscillator (platform)  $u_p$  are plotted in Figure 6 for all the ground motions analyzed. These results serve as a benchmark to the FE model in ANSYS for the sliding-only case (Section 5).

Parameter	Far-fault scenario
Magnitude	$6.7 \pm 0.2$
Distance	$25 \pm 5$ km (Joyner-Boore)
NEHRP Soil type	C and D
Highest usable period	4 sec
Number of records	30

Table 1: Selected seismic scenario used to test the sliding mode

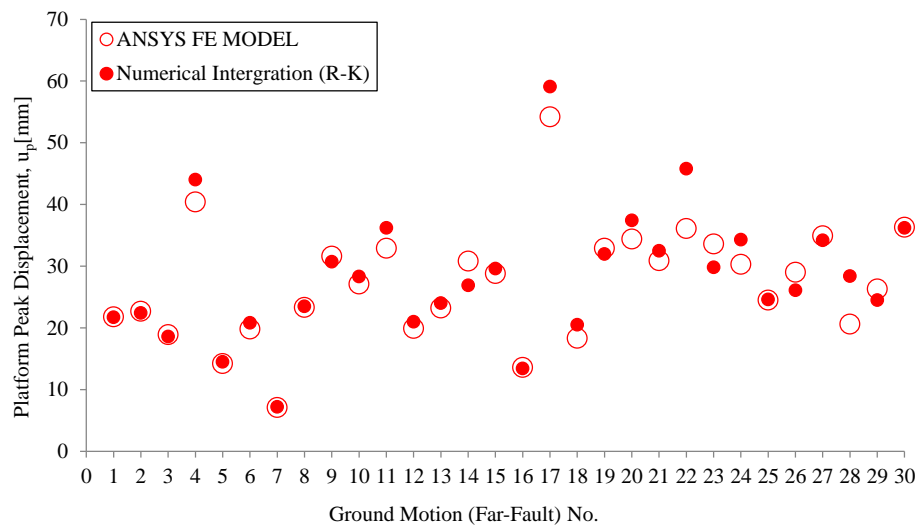


Figure 6: Platform displacements in SDOF structure supporting a sliding box

## 4.2 Experimental and Numerical Results on Rocking Motion of Rigid Blocks

Peña et al. [8] conducted a comprehensive experimental investigation to study the rocking response of single rigid-block structures. The investigation included 275 shaking table tests in total. In these tests, four blue granite stones with different geometric characteristics and supported by a foundation (bolted to the shaking table) made of the same material were subjected to free rocking, and harmonic and random base excitation. The stones had height-width ratios ( $H/B$ ) ranging between 4 and 8, and a constant thickness-width ratio ( $t/B$ ) of 3 to minimize 3-D effects. Light emission diodes (LED's) and high resolution cameras were used to measure displacements (and rotations) of the specimens and the shaking table. Similarly, accelerometers were used to measure corresponding accelerations. The main goal of the experimental program was to address the issues of repeatability of rocking motion response and its stability.

Two analysis tools were developed by Peña et al. [8] for the numerical simulation of the rocking motion observed in their experimental program. One tool is analytical and is referred to as the complex coupled rocking rotations (CCRR) method, while the other is numerical and uses the discrete element method (DEM). A new methodology was also proposed to find the parameters of the DEM model from the classical theory of rigid-block rocking motion



The harmonic/random base motion tests are the main portion of the experimental program mentioned above as they allow studying the dynamics of rigid blocks under rocking motion regime and earthquake excitation. However, the free rocking motion tests are essential for parameter identification and analytical model calibration. This consideration is also applicable to the FE model development described in this paper and, therefore, only free-rocking motion results reported by Peña et al. [8] are included here as an initial benchmark. Measured rocking angle histories for a block with a width of 0.25 m, height of 1.00 m, thickness of 0.75 m, total mass of 503 kg, and initial base rotation of  $3^\circ$  are presented in Figure 7. Comparison of these experimental results with the calibrated numerical and analytical formulation developed by the authors can be found in Peña et al. [8]

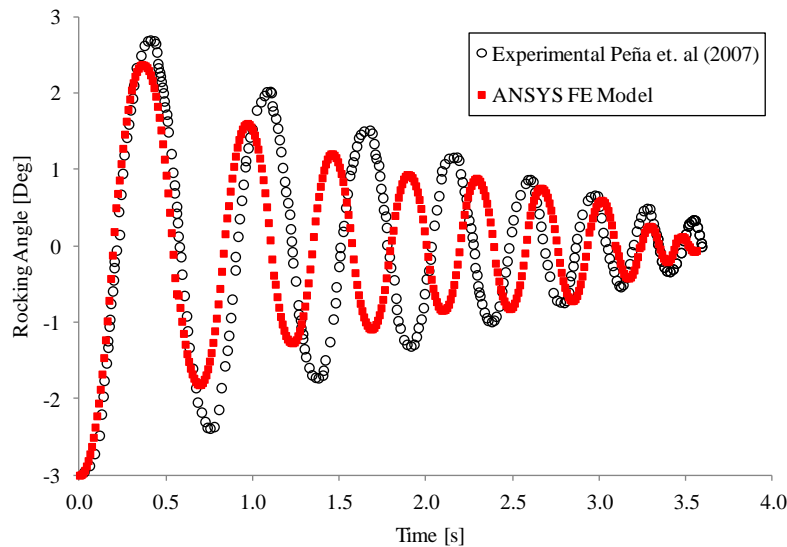


Figure 7: Response history of rigid block under free rocking, after Peña et al. [8]

## 5 RESULTS COMPARISON AND PRELIMINARY CALIBRATION OF THE FE MODEL

The basic FE model described in Section 3 was used to model the response to base excitation of an SDOF oscillator supporting a sliding rigid block as described in Section 4.1. To do so, the horizontal translation nodal restraints applied to the foundation plate (Section 3.5) were removed, and a horizontal spring element with stiffness  $k$  (determined from  $m_P$  and  $T_n$ ) was attached to this plate to simulate the oscillator. The geometry of the rigid block was selected so that the target mass ratio ( $\alpha = 2.0$ ) was matched and the rocking initiation mode of response was precluded in compliance with the criteria presented by Shenton [7]. The maximum displacement  $u_P$  for each ground motion considered was calculated via finite element analysis (FEA) using the ANSYS model developed and it is included in Figure 6. The FEA results are found to be reasonably similar to those obtained with the numerical solution described in Section 4.1. The resulting mean ratio  $u_{P\_FEA}/u_{P\_R-K}$  is 0.98, where  $u_{P\_FEA}$  &  $u_{P\_R-K}$  are the peak displacements of the oscillator calculated via FEA and numerical integration using the Runge-Kutta method, respectively.

The FE model developed was also used to study the free rocking response of a rigid block with a height-to-width ratio ( $H/B$ ) of 4 that was subjected to an initial base rotation of  $3^\circ$  as described in Section 4.2. The rocking angle history of the block was computed using the ANSYS model and it is included in Figure 7 for comparison purposes. The amplitudes of the rocking motion calculated with the model are within 25% of those measured in the tests described in Section 4.2. The period of the calculated response is consistently shorter than the

period measured in the experimental program. The discrepancies observed between the experimental and numerical (FEA) data can be attributed to an inaccurate representation in the FE model of the energy dissipation mechanisms available in the actual specimen/test. To capture these mechanisms more accurately and compute a rocking response closer to the test measurements, the viscous damping provided to the FE model (Section 3.3) through Rayleigh damping [12] must be calibrated. The classical Rayleigh damping model is summarized below:

$$[C] = \lambda[M] + \beta[K] \quad (6)$$

where:

$[C]$  = System damping matrix

$\lambda$  = Mass-proportional Rayleigh damping coefficient

$[M]$  = System mass matrix

$\beta$  = Stiffness-proportional Rayleigh damping coefficient

$[K]$  = System stiffness matrix

Assuming  $\lambda = 0$  for simplicity, the damping provided to the FE model depends on stiffness only. To investigate the effect of damping on the rocking response of the rigid block being analyzed, different values of  $\beta$  were considered. The rocking angle histories of the block computed with the FE model and corresponding to different values of  $\beta$  are shown in Figure 8.

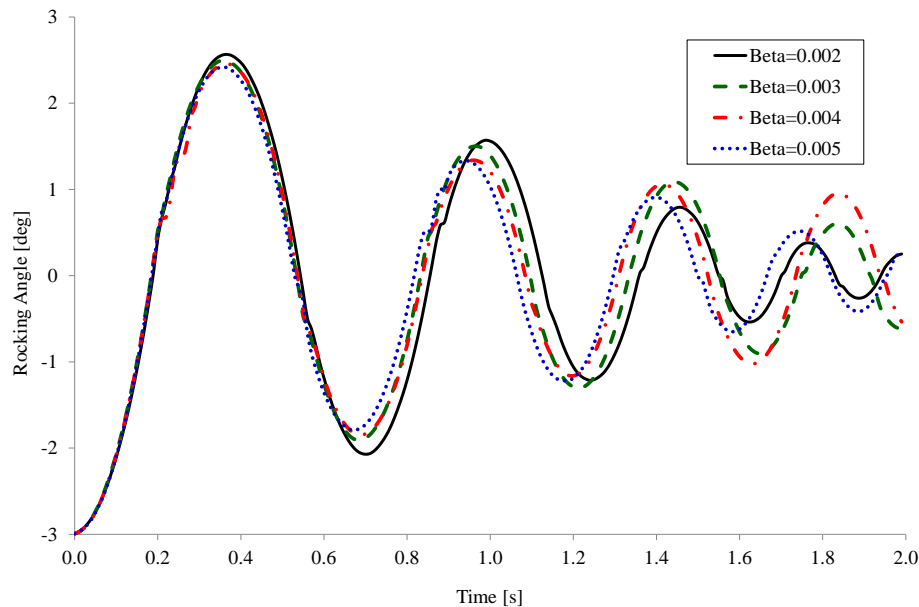


Figure 8: Effect of Rayleigh damping on computed free-rocking response of rigid block

## 6 APPLICABILITY OF THE FE MODEL TO SENSITIVITY STUDIES

Once the numerical model of the dynamic response of a rigid block under base excitation itself is found to be reasonably consistent with the results from experimental programs available in the literature, the FE analyses will proceed with modeling of the platform structure used

in the experimental program of this study. The results obtained in the laboratory will serve to further calibrate the numerical and finite element models and to conduct parametric studies aimed at the development of design recommendations on the portion of the live load that should be included as inertia in platform structures supporting rigid blocks. This will involve an iterative process in which an equivalent linear elastic structure, with the same lateral stiffness but with an attached rigid block of a portion of its actual mass, has an average lateral displacement that is equal to that of the actual structure supporting a sliding/rocking rigid block. It is anticipated that the proportion of the live load to be included in seismic analyses will be a function of dynamic properties of the structure, the aspect ratio of the supported rigid blocks, the maximum acceleration of the ground motion, and the friction coefficient at the interface between the rigid block and the platform. Intuition would suggest that the portion of the live load to be considered as inertia should increase with the fundamental period of the structure and also with the friction coefficient at the rigid block-platform interface.

## 7 CONCLUSIONS

This paper presents some of the preliminary steps taken in the development of a finite element model that can capture the sliding and rocking response of rigid blocks subjected to base excitation. For the case of a single-degree-of-freedom structure supporting sliding blocks, the FE model is found to estimate displacement demands of the platform within 2% on average of the values obtained through the numerical integration of the equation of motion of the system. For the case of free rocking, on the other hand, the FE model estimates the amplitude of the motion within 25% of the values measured in the experimental program conducted by Prieto et al.[8]. Further calibration of the model will be conducted in the near future using the results of an ongoing experimental program with a 1:15 test structure. The final FE model will serve to conduct parametric studies aimed at the development of design recommendations in regards to the portion of the live load that should be considered as inertia in structures supporting nearly permanent and heavy live loads.

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