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ROCKING RESPONSE AND STABILITY ANALYSIS OF AN ARRAY OF FREE-STANDING COLUMNS CAPPED WITH A FREE-STANDING RIGID BEAM

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Abstract. *This paper investigates the planar rocking response of an array of free-standing columns capped with a freely supported rigid beam in an effort to explain the appreciable seismic stability of ancient free-standing columns which support heavy epistyles together with the even heavier frieze atop. Following a variational formulation the paper concludes to the remarkable result that the dynamic rocking response of an array of free-standing columns capped with a rigid beam is identical to the rocking response of a single free-standing column with the same slenderness; yet with larger size—that is a more stable configuration. Most importantly, the study shows that the heavier the freely supported cap-beam is (epistyles with frieze atop), the more stable is the rocking frame regardless the rise of the center of gravity of the cap-beam; concluding that top-heavy rocking frames are more stable than when they are top-light. This "counter intuitive" finding renders rocking isolation a most attractive alternative for the seismic protection of bridges with tall piers; while its potential implementation shall remove several of the concerns associated with the seismic connections of prefabricated bridges.*

1 INTRODUCTION

Under base shaking slender objects and tall rigid structures may enter into rocking motion that occasionally results in overturning. Early studies on the seismic response of a slender rigid block were presented by Milne [1]; however, it was Housner [2] who uncovered a size-frequency scale effect which explained why: (a) the larger of two geometrically similar blocks can survive the excitation that will topple the smaller block; and (b) out of two same acceleration amplitude pulses, the one with the longer duration is more capable to induce overturning. Following Housner's seminal paper a number of studies have been presented to address the complex dynamics of one of the simplest man-made structures—the free standing rigid column.

Yim et al. [3] conducted numerical studies by adopting a probabilistic approach, Aslam et al.[4] confirmed with experimental studies that the rocking response of rigid blocks is sensitive to system parameters; while Psycharis and Jennings [5] examined the uplift of rigid bodies supported on viscoelastic foundation. Subsequent studies by Spanos and Koh [6] investigated the rocking response due to harmonic steady-state loading and identified "safe" and "unsafe" regions together with the fundamental and suharmonic modes of the system. Their study was extended by Hogan [7],[8] who further elucidated the mathematical structure of the problem by introducing the concepts of orbital stability and Poincare sections. The transient rocking response of free-standing rigid blocks was examined in depth by Zhang and Makris [9] who showed that there exist two modes of overturning: (a) by exhibiting one or more impacts; and (b) without exhibiting any impact. The existence of the second mode of overturning results in a safe region that is located on the acceleration-frequency plane above the minimum overturning acceleration spectrum. The fundamental differences between the response of a rocking rigid column (inverted pendulum) and the response of the linear elastic oscillator (regular pendulum) led to the development of the rocking spectrum (Makris and Konstantinidis [10]). More recent studies pertinent to the rocking response of rigid columns have focused on more practical issues such as representation of the impact (Prieto et al. [11]), the effect of the flexibility-yielding of the supporting base (Apostolou et al. [12], Palmeri and Makris [13]) or the effect of seismic isolation (Vassiliou and Makris [14]).

In this paper we investigate the planar rocking response of an array of free-standing columns capped with a freely supported rigid beam as shown schematically in Figure 1. Herein we use the term "rocking frame" for the one degree of freedom structure shown in Figure 1. Sliding does not occur either at the pivot points at the base or at the pivot points at the cap-beam. Our interest to this problem was partly motivated from the need to explain the remarkable seismic stability of ancient free-standing columns which support heavy free standing epistyles together with the even heavier frieze atop. As an example, Figure 2 shows the entrance view of the late archaic temple of Aphaia in the island of Aegina nearby Athens, Greece. Dates ranging from 510BC to 470BC have been proposed for this temple. All but three of the 32 outer columns of the temple are monolithic and they have been supporting for 2.5 millennia the front and back epistyles together with the heavy frieze (triglyph and metope).

The understanding of the rocking response and stability of the configuration shown in Figure 1 is also pertinent to the growing precast bridge construction technology where bridge piers supporting heavy decks are allowed to rock atop their foundation in order to achieve re-centering of the bridge bent after a seismic event.

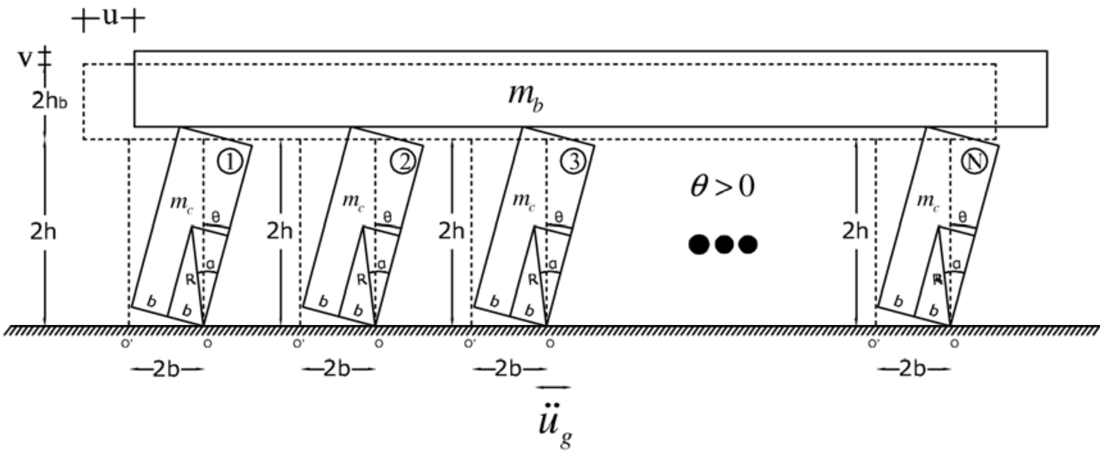

Figure 1: Rocking array of free-standing columns capped with a freely supported rigid beam.

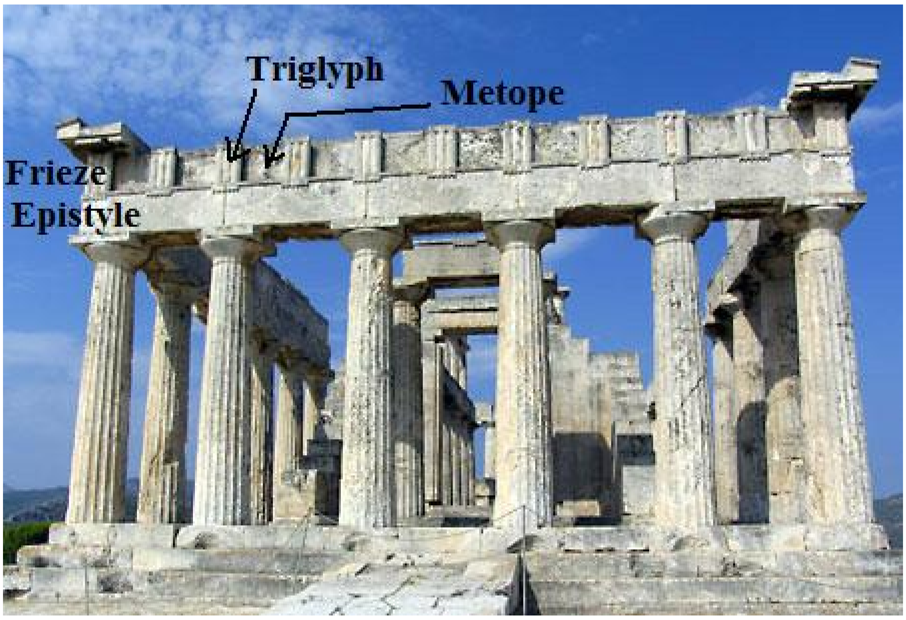

Figure 2: View of the Temple of Aphaia, in Aegina, Greece. Its monolithic, free-standing columns support massive epistyles and the frieze atop and the entire rocking frame remains standing for more than 2500 years in a region with high seismicity.

2 GENERAL SPECIFICATIONS

With reference to Figure 3 and assuming that the coefficient of friction is large enough so that there is no sliding, the equation of motion of a free standing block with size $R = \sqrt{h^2 + b^2}$ and slenderness α=atan(b/h) subjected to a horizontal ground acceleration, $\ddot{u}_g(t)$, when rocking around O and O' respectively is (Yim et al. [3], Hogan [7], Makris and Roussos [15] , Zhang and Makris [9] among others)

$$I_o\ddot{\theta}(t) + mgR\sin[-\alpha - \theta(t)] = -m\ddot{u}_g(t)R\cos[-\alpha - \theta(t)], \quad \theta(t) < 0 \tag{1}$$

$$I_o\ddot{\theta}(t) + mgR\sin[\alpha - \theta(t)] = -m\ddot{u}_g(t)R\cos[\alpha - \theta(t)], \quad \theta(t) > 0. \tag{2}$$

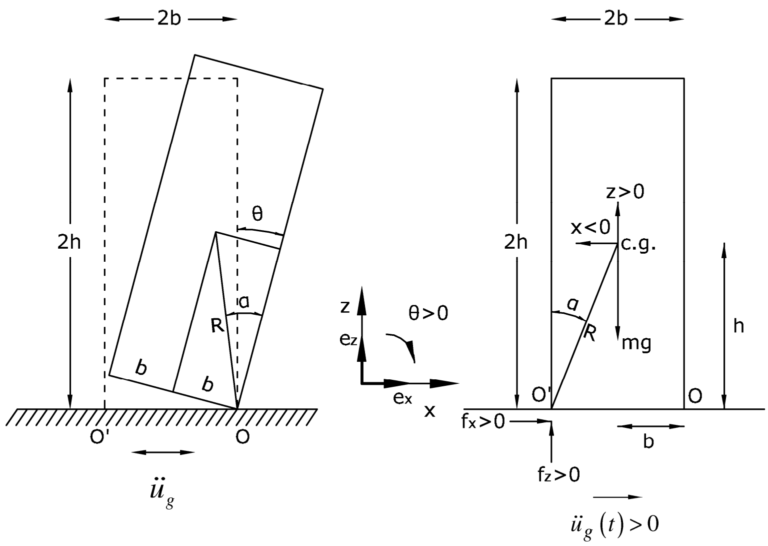

Figure 3: Left: Geometric characteristics of the model considered. Right :Free-body diagram of a free-standing block at the instant that it enters rocking motion.

In order for rocking motion to be initiated, $\ddot{u}_g(t) > g\tan\alpha$ at some time of its history. For rectangular blocks, $I_o = (4/3)mR^2$; and the above equations can be expressed in the compact form

$$\ddot{\theta}(t) = -p^2\left\{\sin\left[\alpha\,\mathrm{sgn}\left(\theta(t)\right)-\theta(t)\right]+\frac{\ddot{u}_g}{g}\cos\left[\alpha\,\mathrm{sgn}\left(\theta(t)\right)-\theta(t)\right]\right\}. \tag{3}$$

The oscillation frequency of a rigid block under free vibration is not constant, because it strongly depends on the vibration amplitude (Housner [2]). Nevertheless, the quantity $p = \sqrt{\frac{3g}{4R}}$ is a measure of the dynamic characteristics of the block. For the 7.5m×1.8m free-standing column of the Temple of Appolo in Corinth, *p =1.4 rad/s*, and for a household brick, *p ≈ 8 rad/s*.

Figure 4 shows the moment-rotation relationship during the rocking motion of a free-standing block. The system has infinite stiffness until the magnitude of the applied moment reaches the value $mgR\sin\alpha$, and once the block is rocking, its restoring force decreases monotonically, reaching zero when $\theta = \alpha$. This negative stiffness, which is inherent in rocking systems is most attractive in earthquake engineering given that such systems do not resonate.
During the oscillatory rocking motion of a free-standing rigid column, the moment-rotation curve follows the curve shown in Figure 4 without enclosing any area. Energy is lost only during impact, when the angle of rotation reverses. When the angle of rotation reverses, it is assumed that the rotation continues smoothly from points *O* to *O'* and that the impact force is concentrated at the new pivot point, *O'*. With this idealization, the impact force applies no moment around *O'*, hence the angular momentum around *O'* is conserved. Conservation of angular momentum about point *O'* just before the impact and right after the impact gives

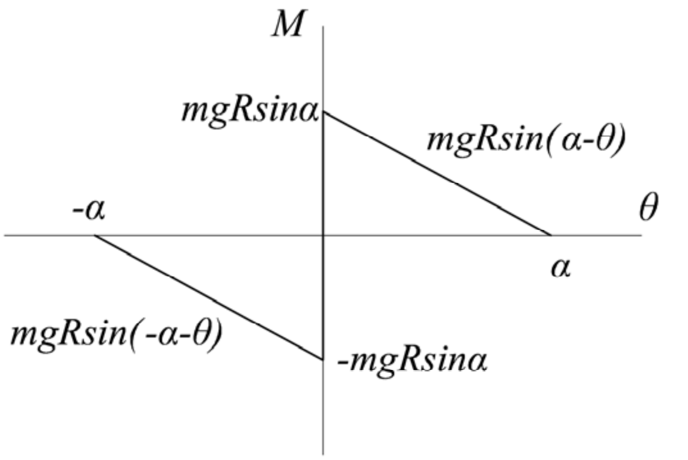

Figure 4: Moment rotation diagram of a rocking object

$$I_o\dot{\theta}_1 - m\dot{\theta}_1 2bR\sin(\alpha) = I_o\dot{\theta}_2 \tag{4}$$

where $\dot{\theta}_1$ = angular velocity just prior to the impact; and $\dot{\theta}_2$ = angular velocity right after the impact. The ratio of kinetic energy after and before the impact is

$$r = \frac{\dot{\theta}_2^2}{\dot{\theta}_1^2} \tag{5}$$

which means that the angular velocity after the impact is only $\sqrt{r}$ times the velocity before the impact. Substitution of (4) into (5) gives

$$r = [1 - \frac{3}{2}\sin^2\alpha]^2 \tag{6}$$

The value of the coefficient of restitution given by (6) is the maximum value of r under which a free-standing rigid block with slenderness α will undergo rocking motion. Consequently, in order to observe rocking motion, the impact has to be inelastic. The less slender a block (larger α), the more plastic is the impact, and for the value of $\alpha = \sin^{-1}\sqrt{2/3} = 54.73^o$, the impact is perfectly plastic. During the rocking motion of slender blocks, if additional energy is lost due to the inelastic behavior at the instant of impact, the value of the true coefficient of restitution r will be less than the one computed from equation (6).

3 EQUATION OF MOTION OF THE ROCKING FRAME

The free standing rocking frame shown in Figure 1 is a single-degree-of-freedom structure with size $R = \sqrt{h^2 + b^2}$ and slenderness $\alpha = atan(b/h)$. The only other parameter that influences the dynamics of the rocking frame is the ratio of the mass of the cap-beam, m_b, to the mass of all the N rocking columns, m_c, $\gamma = \frac{m_b}{Nm_c}$. For the temple of Apollo in Corinth where the frieze is missing, γ is as low as 0.3; whereas in prefabricated bridges $\gamma > 4$. As in the case of the single rocking column, the coefficient of friction is large enough so that sliding does not occur at the pivot point at the base and at the cap-beam. Accordingly, the horizontal translation displacement $u(t)$ and the vertical lift $v(t)$ of the cap-beam are functions of the single degree of freedom $\theta(t)$. For a positive horizontal ground acceleration (the ground is accelerating to the right), the rocking frame will initially rock to the left (*θ(t)*<0). Assuming that the rocking frame will not topple, it will recenter, impacts will happen at the pivot points (at the base and at the cap-beam) and subsequently it will rock to the right (*θ(t)*>0). During rocking the dependant variables *u(t)*, *v(t)* and their time derivatives are given for *θ(t)*<0 and *θ(t)*>0 by the following expressions.

$$u = \mp 2R\left(\sin\alpha - \sin\left(\alpha \pm \theta\right)\right) \tag{7}$$

$$\dot{u} = 2R\cos\left(\alpha \pm \theta\right)\dot{\theta} \tag{8}$$

$$\ddot{u} = 2R\left(\mp\sin\left(\alpha \pm \theta\right)\left(\dot{\theta}\right)^2 + \cos\left(\alpha \pm \theta\right)\ddot{\theta}\right) \tag{9}$$

and

$$v = 2R\left(\cos\left(\alpha \pm \theta\right) - \cos\alpha\right) \tag{10}$$

$$\dot{v} = \mp 2R\sin\left(\alpha \pm \theta\right)\dot{\theta} \tag{11}$$

$$\ddot{v} = -2R\left(\cos\left(\alpha \pm \theta\right)\left(\dot{\theta}\right)^2 + \sin\left(\alpha \pm \theta\right)\ddot{\theta}\right) \tag{12}$$

In the equations above, whenever there is a double sign (say $\pm$) the top sign is for *θ(t)*<0 and the bottom sign is for *θ(t)*>0.

During rocking motion Langrange's equation must be satisfied,

$$\frac{d}{dt}\left(\frac{dT}{d\dot{\theta}}\right) - \frac{dT}{d\theta} = Q\,. \tag{13}$$

In equation (13), *T* is the kinetic energy of the system and Q is the generalized force acting on the system

$$Q = -\frac{dW}{d\theta} \tag{14}$$

in which *W* is the work done by the external forces acting on the rocking frame during an admissible rotation *δθ*. During this admissible rotation *δθ,* the variation of work is

$$\delta W = \frac{dW}{d\theta}\delta\theta \tag{15}$$

In either case were *θ(t)*<0 or *θ(t)*>0 the kinetic energy of the system is

$$T = N\frac{1}{2}I_o\left(\dot{\theta}\right)^2 + \frac{1}{2}m_b\left(\left(\dot{u}\right)^2 + \left(\dot{v}\right)^2\right) \tag{16}$$

Using equations (8) and (11), equation (16) reduces to

$$T = \left(\frac{N}{2}I_o + 2m_b R^2\right)\left(\dot{\theta}\right)^2 \tag{17}$$

Our analysis proceeds by first investigating the rocking motion of a free-standing frame subjected to a horizontal ground acceleration $\ddot{u}_g\left(t\right)$ when *θ(t)<0*. During this segment of the motion the variation of the work, *W*, is

$$\delta W = \left(m_b + \frac{N}{2} m_c \right)\left(\ddot{u}_g \delta u + g \delta v \right) \tag{18}$$

or

$$\delta W = \left(m_b + \frac{N}{2} m_c \right)\left(\ddot{u}_g \frac{du}{d\theta} + g \frac{dv}{d\theta} \right)\delta\theta \tag{19}$$

The combination of equations (15) and (19) gives

$$\frac{dW}{d\theta} = \left(m_b + \frac{N}{2} m_c \right)\left(\ddot{u}_g \frac{du}{d\theta} + g \frac{dv}{d\theta} \right) \tag{20}$$

which simplifies to

$$\frac{dW}{d\theta} = 2R\left(m_b + \frac{N}{2} m_c \right)\left(\ddot{u}_g \cos(\alpha+\theta) - g\sin(\alpha+\theta) \right), \tag{21}$$

after using the expression given by (7) and (10).

The substitution of equations (17) and (21) into Lagrange's equation given by (13) results to the equation of motion of the rocking frame for *θ(t)*<0.

$$\left(\frac{\frac{I_o}{2m_c R} + 2\gamma R}{\left(\gamma + \frac{1}{2}\right) g} \right)\ddot{\theta} = \sin(a+\theta) - \frac{\ddot{u}_g}{g}\cos(\alpha+\theta), \tag{22}$$

where $\gamma = \frac{m_b}{N m_c}$ is the ratio of the mass of the cap-beam (epistyle), m_b, to the mass of all the N columns= Nm_c.

For the case where the rotation is positive *θ(t)*>0 the variation of the work is

$$\delta W = \left(m_b + \frac{N}{m_c} m_c \right)\left(\ddot{u}_g \delta u + g \delta v \right) \tag{23}$$

and equation (14) takes the form

$$\frac{dW}{d\theta} = 2R\left(m_b + \frac{N}{2} m_c \right)\left(\ddot{u}_g \cos(\alpha+\theta) + g\sin(\alpha+\theta) \right). \tag{24}$$

The substitution of equation (17) and (24) into Lagrange's equation given by (13) offers the equation of motion of the rocking frame for *θ(t)*>0.

$$\left(\frac{\frac{I_o}{2m_c R} + 2\gamma R}{\left(\gamma + \frac{1}{2}\right) g} \right)\ddot{\theta} = -\sin(a-\theta) - \frac{\ddot{u}_g}{g}\cos(\alpha-\theta). \tag{25}$$

For rectangular columns $I_o=(4/3)mR^2$; and equations (22) and (25) can be expressed in a single compact form

$$\ddot{\theta} = -\frac{1+2\gamma}{1+3\gamma} p^2 \left(\sin\left[a\,\mathrm{sgn}\left(\theta(t)\right) - \theta(t) \right] + \frac{\ddot{u}_g(t)}{g} \cos\left[a\,\mathrm{sgn}\left(\theta(t)\right) - \theta(t) \right] \right) \tag{26}$$

Equation (26) which describes the planar motion of the free-standing rocking frame is precisely the same as equation (3) which describes the planar rocking motion of a single free-standing rigid column with the same slenderness α, except that in the rocking frame the term p^2 is multiplied with the factor $\frac{1+2\gamma}{1+3\gamma}$. Accordingly, the frequency parameter of the rocking frame, $\hat{p}$, is

$$\hat{p} = \sqrt{\frac{1+2\gamma}{1+3\gamma}}\, p \tag{27}$$

where $p = \sqrt{\frac{3g}{4R}}$ is the frequency parameter of the rocking column and $\gamma = \frac{m_b}{Nm_c}$ is the mass of the cap-beam to the mass of all N columns.

For a light cap-beam ($\gamma = \frac{m_b}{Nm_c} \rightarrow 0$), the multiplication factor $\frac{1+2\gamma}{1+3\gamma} \rightarrow 1$ and the array of free standing columns coupled with a light epistyle exhibit precisely the dynamic rocking response of the solitary free-standing column. On the other hand, as the mass of the epistyle increases

$$\lim_{\gamma \to \infty} \frac{1+2\gamma}{1+3\gamma} = \frac{2}{3}. \tag{28}$$

Accordingly, the dynamic behavior of a rocking frame with a very heavy cap-beam supported on columns with slenderness α and frequency parameter, $p = \sqrt{\frac{3g}{4R}}$, is identical to the dynamic rocking response of a single rigid column with slenderness α and frequency parameter $\hat{p} = \sqrt{\frac{2}{3}}p$ – that is a smaller frequency parameter; therefore a larger, more stable column.

This remarkable result offered by equation (26) – that the heavier the cap-beam is, the more stable is the free standing rocking frame despite the rise of the center of gravity of the cap-beam – has been also confirmed by obtaining equation (26) for a pair of columns with the algebraically-intense direct formulation after deriving the equations of motion of the two-column frame through dynamic equilibrium.

According to equation (26) the rocking response and stability analysis of the free-standing rocking frame with columns having slenderness, α, and size R, is described by all the past published work on the rocking response of the free-standing single block ([2], [3], [4], [6], [9], [10], [14] among others), where the block has the same slenderness, α, and a larger size $\hat{R}$ given by

$$\hat{R} = \frac{1+3\gamma}{1+2\gamma} R = \left(1 + \frac{\gamma}{1+2\gamma}\right) R \tag{29}$$

Figure 5 plots the value of $\hat{R}$ as a function of the mass ratio $\gamma = \frac{m_b}{Nm_c}$. When replacing the rocking frame with the larger-size, equal slenderness solitary column, the maximum coefficient of restitution is given by equation (43)

4 MINIMUM ACCELERATION NEEDED TO INITIATE UPLIFT OF A ROCKING FRAME

With reference to Figure 1 during an admissible rotation $\delta\theta$, the application of the principle of virtual work gives:

$$m_b \ddot{u}_g \delta u + Nm_c \ddot{u}_g R(\cos\alpha)\delta\theta = m_b g \delta v + Nm_c g R(\sin\alpha)\delta\theta \tag{30}$$

where

$$\delta u = \frac{du}{d\theta}\delta\theta \quad \text{and} \quad \delta v = \frac{dv}{d\theta}\delta\theta \tag{31}$$

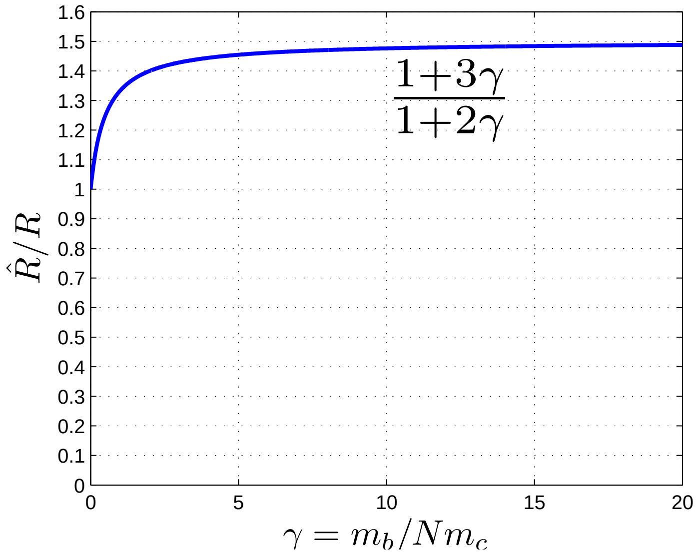

Figure 5: Values of the semidiagonal $\hat{R}$ of a free-standing rigid column with slenderness α that has identical dynamic properties and response as the free-standing rocking frame with N columns having slenderness α, semi-diagonal R, and mass m_c, supporting a cap-beam with mass m_b.

Without loss of generality we assume that the rocking frame undergoes positive rotations ($\theta(t)>0$); and according to equations (7) and (10) with their bottom signs

$$\delta u = 2R\cos(\alpha - \theta)\delta\theta \tag{32}$$

$$\delta v = 2R\sin(\alpha - \theta)\delta\theta \tag{33}$$

Substitution of equations (32) and (33) into equation (30) and after cancelling the admissible rotation $\delta\theta$ one obtains

$$m_b \ddot{u}_g 2\cos(\alpha - \theta) + Nm_c \ddot{u}_g \cos\alpha = m_b g 2\sin(\alpha - \theta) + Nm_c g \sin\alpha \tag{34}$$

At the initiation of uplift, θ=0, and equation (34) simplifies to

$$\left(2m_b + Nm_c\right)\ddot{u}_g^{up}\cos\alpha = \left(2m_b + Nm_c\right)g\sin\alpha \tag{35}$$

which shows that the minimum acceleration needed to initiate uplift of a rocking frame is

$$\ddot{u}_g^{up} = g\tan\alpha\,. \tag{36}$$

According to equation (36) the minimum uplift acceleration of the rocking frame depends solely on the slenderness of its columns and is entirely independent of the mass of the cap-beam (epistyles and frieze atop). This result was expected from the previous analysis on the rocking motion of the free-standing frame (see equation 26) which showed that its dynamic rocking response is identical to the rocking response of a single column which has the same slenderness, α, as the columns of the rocking frame, but larger size, $\hat{R} = \frac{1+3\gamma}{1+2\gamma}R$ (see Figure 5).

5 MAXIMUM COEFFICIENT OF RESTITUTION

The maximum coefficient of restitution of the rocking frame during the impact which happens when the rotation $\theta(t)$ alternates sign is calculated by applying the angular momentum-impulse theorem on one column of the frame before and after the impact. As in the case of the solitary rocking column, the angular momentum of one column of the rocking frame with respect to the imminent pivot point O' (see Figure 6) is

$$H_1 = \left(I_o - 2m_c bR\sin\alpha\right)\dot{\theta}_1\,, \tag{37}$$

where $\dot{\theta}_1$ is the angular velocity of the rocking column just before the impact. Upon impact, the angular momentum of the column with respect to the new pivot point O' is

$$H_2 = I_o\dot{\theta}_2 \tag{38}$$

where $\dot{\theta}_2$ is the angular velocity of the rocking column immediately after the impact.

The main difference between the conservation of angular momentum before and after the impact of the free-standing rocking frame with a freely supported cap-beam and the rocking of a free-standing solitary column, is that, upon impact happens, additional forces are acting on the columns of the rocking frame which were absent in the solitary column. These forces appear when the reactions of the cap-beam (epistyle) shift from point P' to point P as the pivot points at the base shift from point O to O' (see Figure 6)

Given that the cap-beam is rigid and that during the rocking motion of the frame, the motion of the cap-beam is only a translation (no rotations), it is assumed that the impact forces at all columns are equal. Accordingly, the change of the linear momentum of the cap-beam in the horizontal and vertical direction before and after the impact is

$$N\int_{\substack{duration\\ of\ impact}} F_{xI}\,dt = 2m_b R\cos\alpha\left(\dot{\theta}_1 - \dot{\theta}_2\right) \tag{39}$$

and

$$N \int_{\substack{duration\\ of\ impact}} F_{zI} dt = 2 m_b R \sin\alpha \left(\dot{\theta}_1 + \dot{\theta}_2\right) \tag{40}$$

Application of the angular momentum – impulse theorem before and after the impact gives

$$H_1 - 2b \int_{\substack{duration\\ of\ impact}} F_{zI} dt + 2h \int_{\substack{duration\\ of\ impact}} F_{xI} dt = H_2 \tag{41}$$

After substituting equation (37)-(40) into (41) and using that $I_o=4/3m_cR^2$one obtains

$$\left(\frac{4}{3}m_c - 2m_c\sin^2\alpha - \frac{4}{N}m_b\sin^2\alpha + \frac{4}{N}m_b\cos^2\alpha\right)\dot{\theta}_1 = \left(\frac{4}{N}m_b + \frac{4}{3}m_c\right)\dot{\theta}_2 . \tag{42}$$

Further simplification of equation (42) gives that the ratio of kinetic energy of the rocking frame after and before the impact is

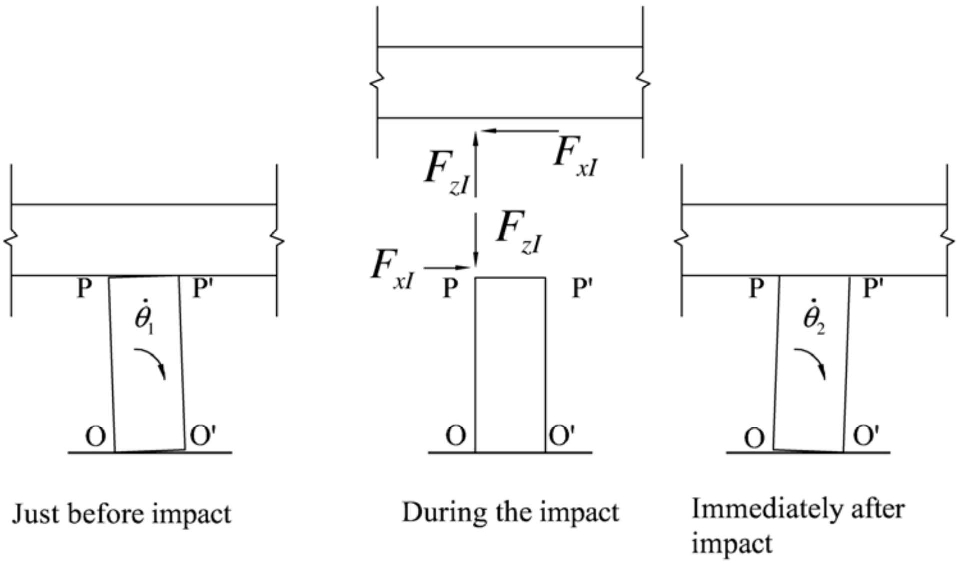

Figure 6: Configuration of the rocking column of the free standing frame just before and immediately after the impact together with the impact forces which develop at point P as the pivoting transfers to point O'.

$$r = \left(\frac{\dot{\theta}_2}{\dot{\theta}_1}\right)^2 = \left(\frac{1 - \frac{3}{2}\sin^2\alpha + 3\gamma\cos 2a}{1+3\gamma}\right)^2 . \tag{43}$$

Equation (5.6) indicates that the angular velocity of the rocking frame after the impact is only $\sqrt{r}$ times the velocity before the impact. Figure 7 plots the value of the minimum coefficient of restitution $\sqrt{r} = \frac{1 - \frac{3}{2}\sin^2\alpha + 3\gamma\cos 2a}{1+3\gamma}$ as a function of the slenderness α for different values of the mass ratio $\gamma = \frac{m_b}{Nm_c}$.

Figure 7 indicates that the maximum coefficient of restitution $\sqrt{r}$ of the rocking frame is always smaller than the maximum coefficient of restitution of the solitary column $=1-\frac{3}{2}\sin^2\alpha$, indicating that when a free-standing frame engages into rocking motion, it dissipates more energy than the equal slenderness equivalent solitary free-standing column with size $\hat{R}$ due to the aaditional impacts that happen between the columns and the cap-beam (epistyles and frieze)

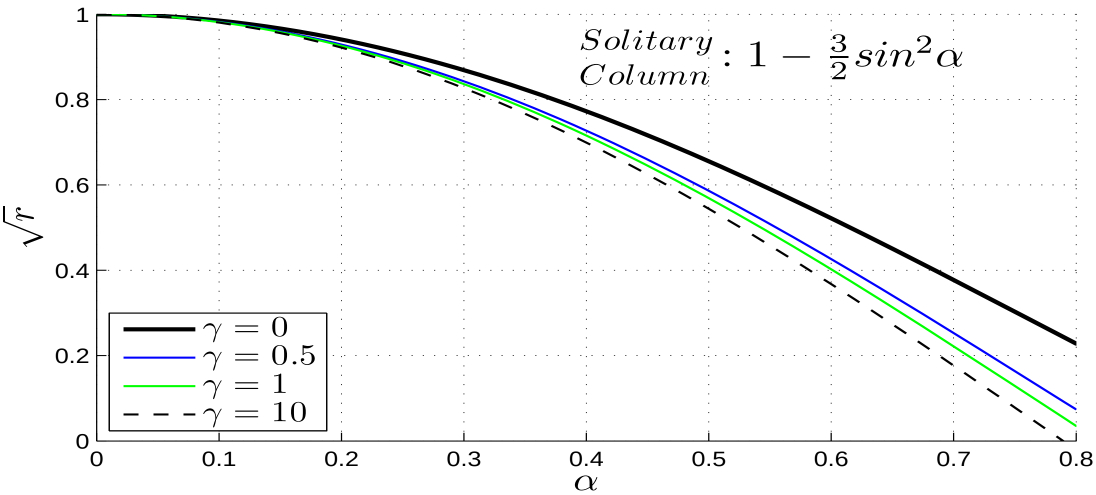

Figure 7: Values of the maximum coefficient of restitution as a function of the slenderness, α of the columns of the rocking frame for different values of the mass ratio $\gamma = \frac{m_b}{Nm_c}$.

6 OVERTURNING SPECTRA – SELF SIMILAR RESPONSE

The relative simple form yet destructive potential of near source ground motions has motivated the development of various closed form expressions which approximate their dominant kinematic characteristics. The early work of Veletsos et al. [16] was followed by the papers of Hall et al. [17], Makris [18], Makris and Chang [19], Alavi and Krawinkler [20] and more recently by the papers of Mavroeidis and Papageorgiou [21] and Vassiliou and Makris [22]. Physically realizable pulses can adequately describe the impulsive character of near-fault ground motions both qualitatively and quantitatively. The minimum number of parameters of the mathematical pulse is two, which are the acceleration amplitude, a_p and the duration T_p. The more sophisticated model of Mavroeidis and Papageorgiou [21] involves 4 parameters which are the pulse period, the pulse amplitude, the pulse phase and the number of half cycles. Recently, Vassiliou and Makris [22] used the Mavroeidis and Papageorgiou model [21] in association with wavelet analysis to develop a mathematically formal and objective procedure to extract the time scale and length scale of strong ground motions.

The current established methodologies for estimating the pulse characteristics of a wide class of records are of unique value, since the product, $a_pT_p^2=L_p$ is a characteristic length scale of the ground excitation and is a measure of the persistence of the most energetic pulse to generate inelastic deformation (Makris and Black [23],[24]). It is emphasized that the persistence of the pulse, $a_pT_p^2=L_p$, is a different characteristic than the strength of the pulse which is measured with the peak pulse acceleration, a_p. The reader may recall that among two pulses with different acceleration amplitude (say $a_{p1}>a_{p2}$) and different pulse durations (say $T_{p1}<T_{p2}$)

the inelastic deformation does not scale with the peak pulse acceleration (most intense pulse) but with the strongest length scale (larger $a_pT_p^2$ = most persistent pulse), (Makris and Black [23][24], Karavassilis et al. [25])

The heavy dark line in Figure 8 (Top) which approximates the long-period acceleration pulse of the NS component of the 1992 Erzincan, Turkey, record is a scaled expression of the second derivative of the Gaussian distribution, $e^{-t^2/2}$, known in the seismological literature as the symmetric Ricker wavelet (Ricker [26][27])

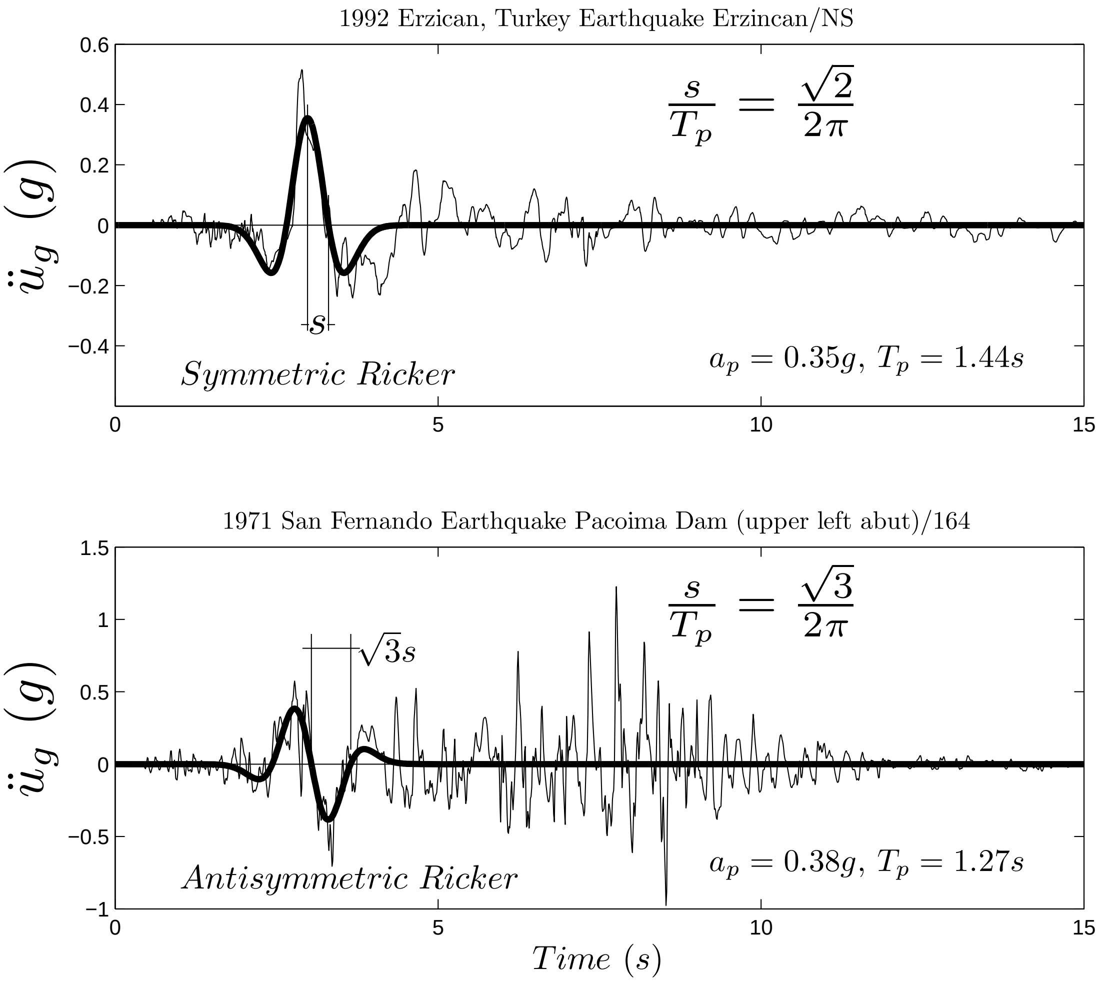

Figure 8: Top: North-South components of the acceleration time history recorded during the 1992 Erzican, Turkey earthquake together with a symmetric Ricker wavelet . Bottom: Fault-normal component of the acceleration time history recorded during the 1971 San Fernando earthquake, together with an antisymmetric Ricker wavelet.

$$\psi(t) = a_p(1 - \frac{2\pi^2 t^2}{T_p^2})e^{-\frac{1}{2}\frac{2\pi^2 t^2}{T_p^2}} \tag{44}$$

The value of $T_p = \frac{2\pi}{\omega_p}$, is the period that maximizes the Fourrier spectrum of the symmetric Ricker wavelet. Similarly, the heavy dark line in Figure 7 (Bottom) which approximates the long-period acceleration pulse of the Pacoima Dam motion recorded during the February 9, 1971 San Fernando, California earthquake is a scaled expression of the third derivative of the Gaussian distribution $e^{-\frac{t^2}{2}}$.

$$\psi(t) = \frac{a_p}{\beta}\left(\frac{4\pi^2 t^2}{3T_p^2} - 3\right) t e^{-\frac{1}{2}\frac{4\pi^2 t^2}{3T_p^2}} \tag{45}$$

in which β is a factor equal to 1.3801 that enforces the above function to have a maximum equal to a_p.

The choice of the specific functional expression to approximate the main pulse of pulse–type ground motions has limited significance in this work. What is important to recognize is that several strong ground motions contain a distinguishable acceleration pulse which is responsible for most of the inelastic deformation of structures (Hall et al. [17], Makris and Chang [19], Alavi and Krawinkler [20], Makris and Psychogios [28], Karavassilis et al. [23] among others). A mathematically rigorous and easily reproducible methodology based on wavelet analysis to construct the best matching wavelet has been recently proposed by Vassiliou and Makris [22].

Consider the free-standing rocking frame shown in Figure 1 that is subjected to an acceleration pulse (like those shown in Figure 8) with acceleration amplitude a_p and pulse duration, $T_p = \frac{2\pi}{\omega_p}$. From equation (26) it results that the response of a free-standing rocking frame subjected to an acceleration pulse is a function of six variables

$$\theta(t) = f\left(p, \alpha, \gamma, g, a_p, \omega_p\right) \tag{46}$$

The seven variables appearing in equation (46) involve only two reference dimensions; that of length [L] and time [T]. According to Buckingham's Π-Theorem, the number of dimensionless products with which the problem can be completely described is equal to [number of variables = 7] – [number of reference dimensions = 2]=5. Herein we select as repeating variables the characteristics of the pulse-excitation, a_p and ω_p and the five independent Π-products are: $\Pi_\theta=\theta$, $\Pi_\omega=\omega_p/p$, $\Pi_\alpha=\tan\alpha$, $\Pi_\gamma=\gamma$ and $\Pi_g=a_p/g$. With these five dimensionless Π-products, equation (46) reduces to

$$\theta(t) = \varphi\left(\frac{\omega_p}{p}, \tan\alpha, \gamma, \frac{a_p}{g}\right) \tag{47}$$

The rocking response of the free-standing frame shown in Figure 1 when subjected to a horizontal base acceleration history $\ddot{u}_g(t)$ is computed by solving equation (26) in association with the minimum energy loss expression given by equation (43) which takes place at every impact.

Figure 9 shows the minimum overturning acceleration spectra of a free-standing rocking frame when subjected to a symmetric Ricker pulse (left) and an antisymmetric Ricker pulse (right) for different values of the mass ratio $\gamma = \frac{m_b}{Nm_c}$. The top plots are for values of the column slenderness α=10^o and the bottom plots are for α=14^o.

In constructing Figure 9, the frequency parameter p is the frequency parameter of the columns of the frame (not $\hat{p}$) and the enhanced stability of the rocking frame due to (a) the corresponding larger size, $\hat{p}=\sqrt{\dfrac{1+2\gamma}{1+3\gamma}}p$, and (b) the reduced coefficient of restitution (see equation (43)) is given by the curves for each given value of γ.

Figure 9 indicates that up to values of ω_p/p=4 the additional stability of the rocking frame versus the stability of the equal slenderness solitary column is marginal.

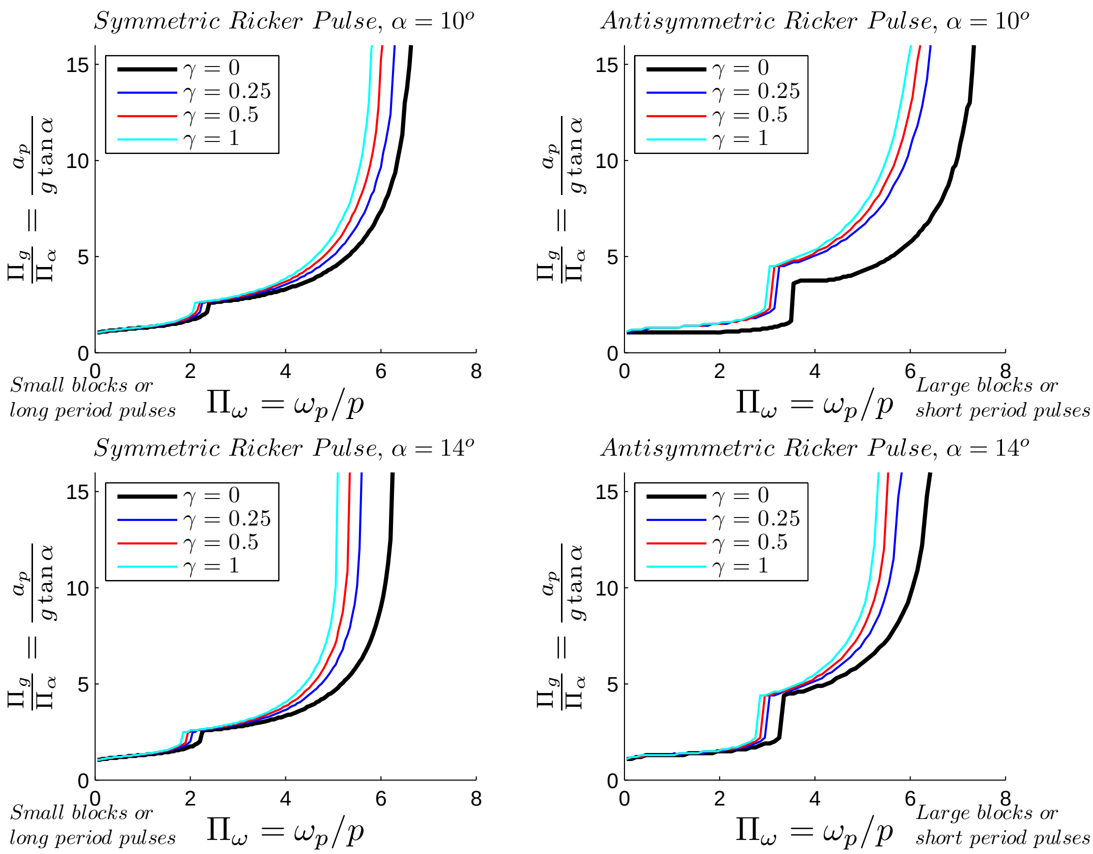

Figure 9: Minimum overturning acceleration spectra of the free-standing rocking frame shown in Figure 1 when subjected to a symmetric Ricker pulse (left) and an antisymmetric Ricker pulse (right) for different values of the mass ratio $\gamma=\dfrac{m_b}{Nm_c}$. Top: α=10°, Bottom α=14°. The values of the coefficient of restitution are give by equation (43).

For values of ω_p/p>4 (larger columns or shorter period pulses) the minimum acceleration overturning spectra of the rocking frame are higher than the corresponding spectrum of the solitary rocking column showing the enhanced seismic stability of the top-heavy rocking frame. This enhanced seismic stability is indifferent to the height of the center of gravity of the cap-beam .

7 SEISMIC STABILITY OF ANCIENT COLUMNS SUPPORTING EPISTYLES AND THE FRIEZE ATOP.

In ancient Greek temples the epistyles are positioned from the vertical axis of one column to the vertical axis of the neighboring column; therefore, the joint of the epistyles are along the vertical axis of the column (see Figure 2). With this configuration during lateral loading of the peristyle of the temple, each epistyle in addition to the horizontal translation, u, shown in

Figure 1 it will also experience a small rotation. Nevertheless, the tendency of the epistyle to rotate is partially prevented from the presence of the neighboring high-profile epistyle and the heavy stone of the frieze atop which goes over the joint of the epistyles. According to this construction pattern with very tight joints the ancient builders constructed a nearly continuous and massive structure atop the columns which according to this study enhanced appreciably the seismic-rocking stability of the peristyle of the temples.

Two of the strongest ground motions recorded in Greece are the 1973 Lefkada record and the 1995 Aigion record (Vassiliou and Makris [22]). Both records exhibit distinguishable acceleration pulses with durations T_p≈0.6s. We concentrate on the Temple of Apollo in Corinth where its 7.5m×1.8m monolithic columns remain standing since 540BC in an area with high seismicity. The dimensions of its columns yield a frequency parameter $p = \sqrt{\frac{3g}{4R}} = 1.4 rad/s$ and a slenderness $\alpha = \tan^{-1}(b/h)$=13.5°. By taking the pulse duration T_p=0.6s of the nearby Aigion record, the dimensionless term Π_ω assumes the value $\frac{\omega_p}{p} = \frac{2\pi}{pT_p} = 7.5$. For such large value of $\omega_p/p \approx 7.5$ the bottom plots of Figure 9 give for the solitary free-standing column (γ=0 line) an overturning ground acceleration a_p>15$g$tanα=15g×0.24=3.6g – which is an unrealistically high acceleration. Consider now the extreme situation for Greece, where the predominant pulse of the ground shaking exhibits a period T_p=0.9s. A pulse period T_p=0.9s may be a rare event for the fault size and earthquake magnitude that prevail in Greece; nevertheless, it helps one understanding the appreciable seismic stability of rocking structures.

With T_p=0.9s and p=1.4rad/s, ω_p/p=5 and according to the bottom plots of Figure 9 which are for slenderness α=14°, the minimum overturning acceleration of a rocking frame with γ=0.25 exceeds the value of $a_p \approx 5g \times 0.24 = 1.2g$. This analysis shows that the free-standing peristyles of ancient temples can survive acceleration pulses as long as 0.9s and as intense as 1.2g. While this is a physically realizable pulse (Loh et al. [26]), it is an unlikely strong shaking for the seismicity of Greece that apparently never happened over the 2500 years of the lifespan of the temple shown in Figure 2.

8 ROCKING ISOLATION OF BRIDGES – PROOF OF CONCEPT

The concept of allowing the piers of bridges to rock in not new. For instance, the beneficial effects that derive from uplifting and rocking have been implemented since the early 1970s in the South Rangitikei bridge in New Zealand (Beck and Skinner [30]). During the last decade, the benefits /challenges associated with the rocking of bridge piers have been receiving increasing attention partly because of growing interest in the prefabricated bridge technology (Wacker et al. [31], Pang et al. [32], Cohagen et al. [33] and references reported therein) and partly because of the need for the bridge structure to recenter after a strong seismic event (Sakai et al. [34], Cheng [35] among others).

In the prefabricated bridge technology the bridge piers and the deck are not free standing. The structural system is essentially a hybrid system (see Wacker et al. [31], Cheng [35]) where the bridge pier is connected to its foundation and at the deck with a post-tensioned tendon that passes through the axis of the column together with longitudinal mild steel reinforcement running near the circumference of the column. During earthquake loading the majority of deformation is concentrated at the pier-foundation and pier-cap-beam interfaces and the overall deformation pattern of the post-tensioned pier-cap-beam system resembles the deformation pattern of the free-standing rocking frame that is under investigation in this study.

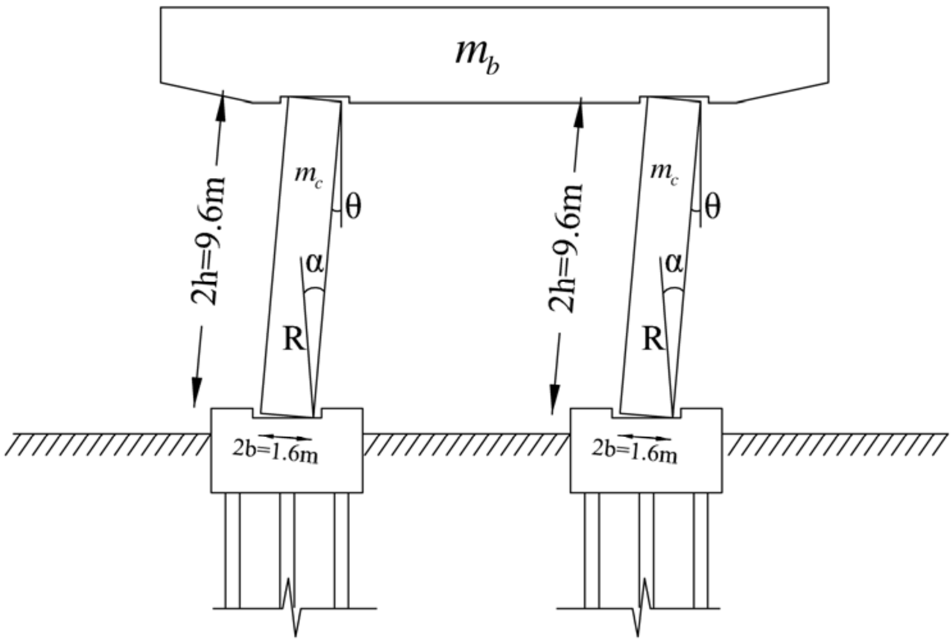

Figure 10: Free Standing rocking bridge bent. Potential sliding during impact is prevented with the recess shown. No vertical post-tensioning, no continuation of the longitudinal reinforcement of the columns through the rocking interfaces at the pile-caps and the cap-beam.

Nevertheless, the post-tensioning tendons and the mild-steel longitudinal reinforcement that extends into the foundation and the cap-beam contributes appreciably to the lateral moment capacity of the system and in most prefabricated bridge applications, the moment-rotation curve of the hybrid systems follows a positive slope.

Within the context of a proof-of-concept, in this study we present the planar rocking response of a free-standing two-column bridge bent where its moment rotation curve follows a negative slope given that the frame is entirely free to rock (see Figure 4). Figure 10 shows schematically the free-standing two-column bridge bent of interest in its deformed configuration. Sliding at the pivot point during impact is prevented with a recess at the pile-cap an the cap-beam as shown in Figure 10. In this numerical application the cylindrical piers of the free-standing bridge bent are 9.6m tall with a diameter $d=2b$=1.6m. These are typical dimensions of bridge piers for highway overpasses and other bridges in Europe and USA. Taller bridge piers will result to even more stable configurations. With $2h$=9.6m and $2b$=1.6m, the slenderness of the bridge pier is $\tan\alpha = \frac{1}{6} = 0.166$ and its frequency parameter p=1.23.

Depending on the length of the adjacent spans and the per-length weight of the deck, the mass ratio $\gamma = \frac{m_b}{2m_c}$ assumes values from 4 and above (γ>4). The larger the value of γ (heavier deck), the more stable is the free standing rocking frame (see Figure 9). The seismic response analysis of the rocking frame has been studied until this section by using as ground excitation acceleration pulses described either by symmetric or the antisymmetric Ricker wavelets. The acceleration amplitude, a_p, and the duration, T_p, of any distinct acceleration pulse allow the use of the dimensional analysis presented in this work and the derivation of the associated Π-products which improve the understanding of the physics that govern the problem together with the organization and presentation of the response. Nevertheless, in order to stress the main finding of this study – that top-heavy free-standing rocking frames enjoy ample seismic stability – we examine the planar seismic response of the free standing two column bridge bent shown in Figure 10 when subjected to six strong-motion historic records listed in Table 1. The values of the acceleration amplitude, a_p, and pulse period, T_p, shown in last two columns of Table 1 have been determined with the extended wavelet transform (Vassiliou and Makris [22]).

Figure 11 plots the time histories of the normalized rotation, θ/α, together with the vertical uplift, $v(t)$, and the horizontal drift, $u(t)$, of the free standing rocking bridge bent shown in Figure 10 with $\gamma = \frac{m_b}{2m_c} = 4$. Note that for all six strong ground motions selected in this analysis the frame rotation, θ, is less than 1/3 of the slenderness, α, of the columns ($\theta/\alpha<0.33$); therefore the free-standing rocking frame exhibits ample seismic stability.

The peak horizontal displacement $u_{\max}$ ranges from 20cm to 50 cm; while the vertical uplift is as high as 5cm. The evaluation of these response quantities shall be conducted in association with the equivalent response quantities from vertically post-tensioned hybrid frames (Wacker et al. [31], Pang et al. [32], Cheng [35]) and seismically isolated decks (Constantinou et al. [36], Makris and Zhang [37], Buckle et al. [38] among others) after considering the effects of the end-conditions of the deck at the abutments. This comparison/evaluation is the subject of an ongoing study which also examines other practical issues such as the effect of the crushing of the pivoting points of the columns (Roh and Reinhorn [39], [40]) and the accommodation of the deck uplift at the end-abutments.

The main conclusion of this study is that heavy decks freely supported on free-standing piers exhibit ample seismic stability and that the heavier is the deck (even if the center of gravity rises) the more stable is the rocking frame. This conclusion may eventually lead to the implementation of the free-standing rocking frame – a structural configuration where all the issues associated with seismic connections such as buckling and fracture of the longitudinal reinforcing bars or spalling of the concrete cover [31-35] are removed as they are not an issue in the ancient temple shown in Figures 2.

Earthquake	Record	Magnitude (Mw)	Epicentral Distance (km)	PGA (g)	PGV (m/s)	a_p (g)	T_p (s)
1966 Parkfield	CO2/065	6.1	0.1	0.48	0.75	0.41	0.6
1971 San Fernando	Pacoima Dam/164	6.6	11.9	1.23	1.13	0.38	1.27
1986 San Salvador	Geotech Investig. Center	5.4	4.3	0.48	0.48	0.34	0.8
1992 Erzican,	Erzincan/EW	6.9	13	0.50	0.64	0.34	0.9
1994 Northridge	Jensen Filter Plant/022	6.7	6.2	0.57	0.76	0.39	0.5
1995Kobe	Takarazuka/000	6.9	1.2	0.69	0.69	0.50	1.1

Table 1: Earthquake records used for the seismic response analysis of the free-standing rocking bridge bent.

9 CONCLUSIONS

This paper investigated the planar rocking response of an array of free-standing columns capped with a freely supported rigid beam. Following a variational formulation, the paper concludes to the remarkable result that the dynamic rocking response of an array of free

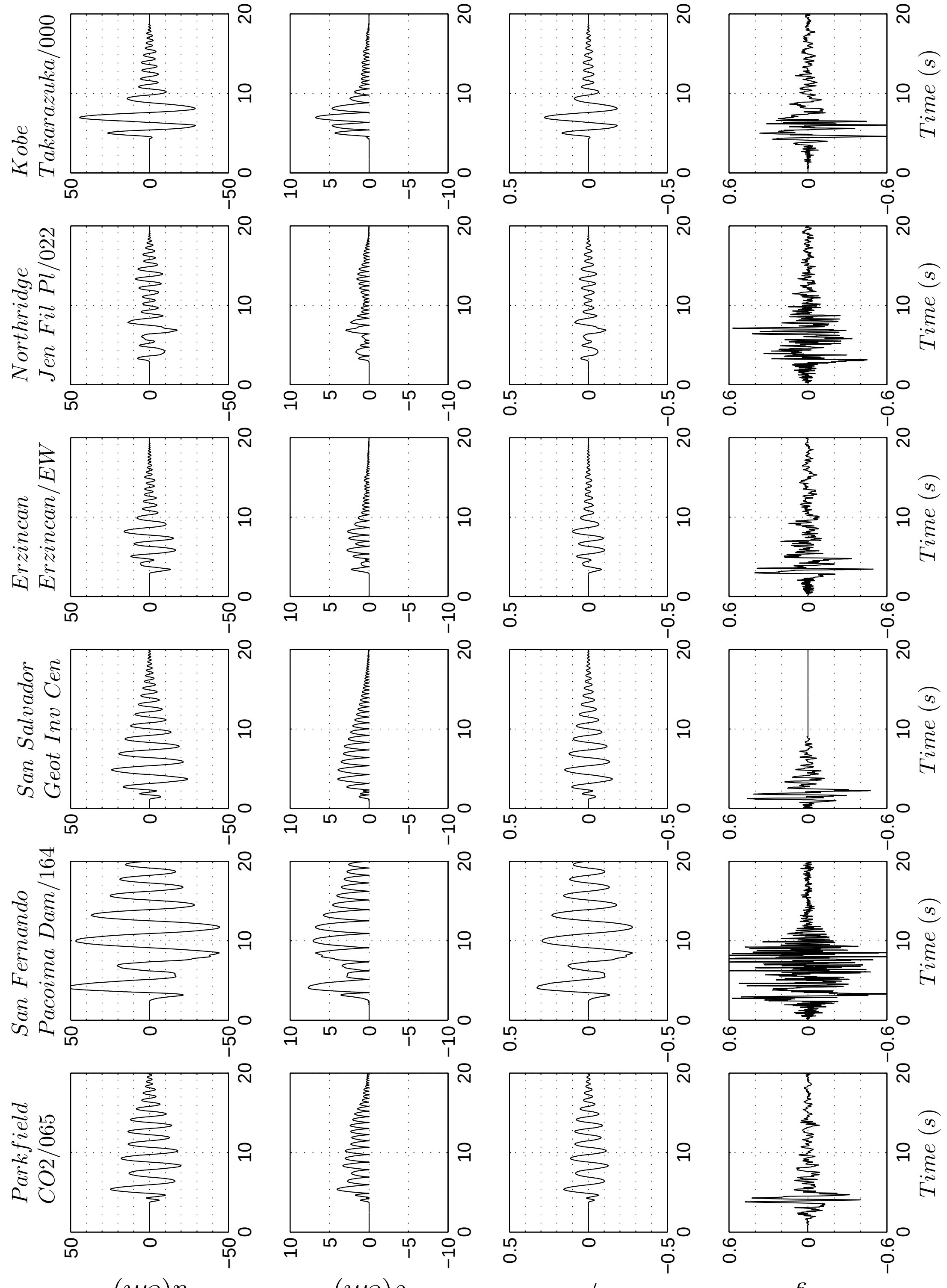

Figure 11: Rotation, vertical and horizontal displacement histories of the free standing rocking frame shown in Figure 10 (p=1.23, tanα=1/6, γ=4) when subjected to the recorded ground motions listed in Table 1.

standing columns capped with a rigid beam is identical to the rocking response of a single free standing column with the same slenderness as the slenderness of the columns of the rocking frame; yet with larger size and more energy loss during impacts. A larger size rocking column corresponds to a more stable configuration; therefore, the presence of the freely supported cap-beam renders the rocking frame more stable despite the rise of the center of gravity.

Most importantly, the study shows that the heavier the freely supported cap-beam is, the more stable is the rocking frame concluding that top-heavy rocking frames are more stable than when they are top-light. The stability of the rocking frame is independent to the number of columns and depends only on the ratio of the weight that is transferred to the column to the weight of the column together with the size and the slenderness of the columns.

The acceleration needed to create uplift of the rocking frame is independent to the mass and the height of the center of gravity of the cap-beam and depends only on the slenderness, α, of the columns ($u_g^{up} = g\tan\alpha$).

The findings above render rocking isolation a most attractive alternative for the seismic protection of bridges given that the heavier is the deck the more stable is the rocking bridge. The future implementation of a truly rocking frame where there is neither post-tensioning nor continuation of the longitudinal reinforcement through the rocking interfaces shall remove several of the concerns associated with the seismic connections of prefabricated bridges such as buckling and fracture of the longitudinal reinforcing bars or spalling of the concrete cover.

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