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ANALYTICAL AND NUMERICAL ESTIMATE OF FOOTBRIDGES' MAXIMUM DYNAMIC RESPONSE TO UNRESTRICTED PEDESTRIAN TRAFFIC

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Abstract. *The assessment of footbridge vibrations due to normal unrestricted pedestrian traffic is a topical problem in the serviceability analysis of these structures. Based on a probabilistic model of the parameters involved, the authors of the present paper have analytically derived an equivalent spectral model for the loading induced by pedestrian groups, modeled as a stationary random process, and simple closed-form expressions for the evaluation of the mean value of the maximum dynamic response. This paper provides a complete assessment of the random walking parameters which mainly influence the serviceability analysis of footbridges, and a comparison between the analytical predictions provided by the simple expressions for the estimate of the maximum dynamic response and the results of Monte Carlo simulations.*

1 INTRODUCTION

Modern footbridges can be very sensitive to walking-induced vibrations because of their increasing slenderness and flexibility. The assessment of footbridge vibrations due to normal unrestricted pedestrian traffic is a topical problem in the serviceability analysis of these structures (see, e.g., [1]). Real pedestrian traffic conditions should be modeled probabilistically, considering several sources of randomness among which pedestrian arrivals, step frequencies and velocities, force amplitudes and pedestrian weights. Based on a probabilistic model of the involved parameters, the authors have derived analytically an equivalent spectral model for the loading induced by pedestrian groups, modeled as a stationary random process under suitable simplifying assumptions ([2], [3]). Furthermore, starting from the assumption that the footbridge dynamic response is a narrow-band random process, simple closed-form expressions for the evaluation of the mean value of the maximum dynamic response have been provided [3]. According to the proposed spectral model, the main force parameter affecting the structural response is the probability distribution of the pedestrian step frequency, in terms of mean value and coefficient of variation: pedestrian arrivals, walking velocity, dynamic load factor and pedestrian weight may be assumed as deterministic and coincident with their mean value. The statistical characterization of these loading parameters is possible through experimental measurements available in the scientific literature, which furnish information concerning the statistical distribution of the step frequency ([1], [4]-[9]).

The present paper has three main objectives: the complete assessment of the random walking parameters which mainly influence the serviceability of footbridges to normal unrestricted pedestrian traffic, a full validation of the simple expressions proposed in [3] for the estimate of the maximum dynamic response, and the analysis of the influence of the variability of walking random parameters on the maximum footbridge acceleration.

Monte Carlo simulations are performed in order to deal with the first two aspects. At first, the variability of all the involved random parameters is analyzed in order to verify the effective role of the parameters that the proposed spectral model assumes as deterministic: the influence of the randomness of walking velocity, dynamic load factor and pedestrian weight on the maximum dynamic response is analyzed. After this first step, extensive analyses are carried out for different values of the relevant statistical parameters, i.e. the non-dimensional mean step frequency and its coefficient of variation: the results of Monte Carlo simulations are compared with the analytical estimate of the mean value of the maximum dynamic response provided by the model proposed by the authors in [3]. The reliability of the formulation adopted for the statistical characterization of the maximum response, based on the narrow band assumption, is assessed through the analysis of the mean maximum dynamic response and its probability density function.

Finally, the role of the main parameters on footbridge serviceability assessment is investigated through extensive parametric analyses. The closed-form expression for the maximum structural response in free pedestrian traffic conditions allows to perform such analysis without the need to carry out burdensome Monte Carlo simulations. Furthermore, the use of the spectral approach allows a synthetic description of the maximum dynamic response as a function of its relevant parameters; its use appears very appropriate for technical purposes.

2 LOAD MODELLING AND MAXIMUM DYNAMIC RESPONSE

2.1 Probabilistic loading model

Dealing with pedestrian traffic on footbridges, a realistic loading scenario is characterized by pedestrians arriving in a random way and able to move undisturbed, each of them with their own characteristics in terms of loading amplitude, frequency, velocity and phase. In such a case, focusing attention only on the first walking harmonic for each pedestrian, the force induced by N_p pedestrians can be expressed as:

$$f\left(x,t\right)=\sum_{i=1}^{N_p}\alpha_i \mathrm{G}_i \sin\left(\Omega_i\left(t-\tau_i\right)+\Psi_i\right)\delta\left[x-c_i\left(t-\tau_i\right)\right]\left[H\left(t-\tau_i\right)-H\left(t-\tau_i-\frac{L}{c_i}\right)\right] \tag{1}$$

where $\delta(\bullet)$ and $H(\bullet)$ are the Dirac function and the Heaviside function, respectively; furthermore, F_i ($=\alpha_i G_i$), Ω_i, Ψ_i, c_i and τ_i are, respectively, the force amplitude, the step circular frequency, the phase-angle, the walking speed and the arrival time of the *i*-th pedestrian, while α_i and G_i are the DLF and the weight of the *i*-th pedestrian. All these quantities have to be probabilistically modeled in order to consider the inter-subject variability in walking forces induced by different pedestrians. In particular, the step circular frequency Ω, the pedestrian weight G, the DLF α and the walking velocity c can be considered as random Gaussian variables. Table 1 reports the results of some experimental tests for the statistical characterization of the step frequency.

References	People tested	Type of event	f_{pm} (Hz)	σ_{fp} (Hz)	V_{fp}
[4]	505	Free field, normal traffic	2	0.173	0.09
[5]	251	Laboratory	1.82	0.12	0.07
[6], [1] Podgorica Bridge	1976	Free field, normal traffic	1.87	0.186	0.1
[1] Reykjavik City Footbridge	38	Free field, normal traffic	1.95	0.15	0.08
[7]	227	Laboratory	1.92		0.07
[8]	400	Free field, normal traffic	1.8	0.11	0.06
[9]	116	Laboratory	1.835	0.172	0.09

Table 1: Statistical characterization of the step frequency.

Starting from the probabilistic characterization of all the parameters involved in Eq. (1), the footbridge dynamic response to unrestricted pedestrian traffic may be numerically determined through Monte Carlo simulations (e.g. [10]).

The authors recently proposed an equivalent spectral model of pedestrian-induced loads, which permits a closed-form estimate of the maximum acceleration due to unrestricted pedestrian traffic [3], without the need to carry out numerical simulations.

Considering a structure with linear behavior and classical damping, the equation of motion of the *j*-th non-dimensional principal coordinate $\tilde{p}_j$ can be expressed in the following non-dimensional form:

$$\ddot{\tilde{p}}_j\left(\tilde{t}\right)+2\xi_j\dot{\tilde{p}}_j\left(\tilde{t}\right)+\tilde{p}_j\left(\tilde{t}\right)=\tilde{F}_j\left(\tilde{t}\right) \tag{2}$$

where ξ_j is the *j*-th modal damping ratio, and the non-dimensional quantities $\tilde{t}$, $\tilde{p}_j$, $\tilde{F}_j$ read [3]:

$$\tilde{t} = \omega_j t, \quad \tilde{p}_j = \frac{p_j}{p_{js}} = \frac{p_j\, M_j \omega_j^2}{\alpha_m G_m}, \quad \tilde{F}_j(\tilde{t}) = \int_0^1 \tilde{f}(\tilde{x},\tilde{t})\, \varphi_j(\tilde{x})\, \mathrm{d}\tilde{x}$$
$$\tilde{f}(\tilde{x},\tilde{t}) = \frac{f(x,t)\, L}{\alpha_m G_m}, \quad \tilde{x} = \frac{x}{L} \tag{3}$$

t being the time, x the abscissa along the structure, L the length of the structure, ω_j and M_j the j-th natural circular frequency and modal mass, α_m the mean dynamic loading factor, G_m the mean pedestrian weight, φ_j the j-th mode shape.

The non-dimensional modal force due to a probabilistically-modeled group of pedestrians is obtained substituting Eq. (1) into Eq. (3), and is given by [2]:

$$\tilde{F}_j(\tilde{t}) = \sum_{i=1}^{N_p} \tilde{\alpha}_i \tilde{G}_i \sin\left(\tilde{\Omega}_i(\tilde{t}-\tilde{\tau}_i) + \Psi_i\right) \varphi_j\left(\tilde{\Omega}_{ci}(\tilde{t}-\tilde{\tau}_i)\right) \left[H(\tilde{t}-\tilde{\tau}_i) - H\left(\tilde{t}-\tilde{\tau}_i - \frac{1}{\tilde{\Omega}_{ci}}\right) \right] \tag{4}$$

where the following non-dimensional parameters are introduced:

$$\tilde{\alpha}_i = \frac{\alpha_i}{\alpha_m}, \quad \tilde{G}_i = \frac{G_i}{G_m}, \quad \tilde{\Omega}_i = \frac{\Omega_i}{\omega_j}, \quad \tilde{\Omega}_{ci} = \frac{c_i}{\omega_j L}, \quad \tilde{\tau}_i = \omega_j \tau_i \tag{5}$$

2.2 Equivalent spectral model

Considering the particular case in which the structural mode shape is sinusoidal, $\varphi_j(\tilde{x}) = \sin(\pi\tilde{x})$, focusing attention only on the randomness of the step frequency and under the assumption that $\tilde{\Omega}_{cm}$ is very small compared with $\tilde{\Omega}_i$, the following expression for the psdf of the non-dimensional modal force is obtained [3]:

$$S_{\tilde{F}_j}(\tilde{\Omega}) \simeq \frac{N_p}{4}\, p_{\tilde{\Omega}}(\tilde{\Omega}) \tag{6}$$

In Eq. (6), $p_{\tilde{\Omega}}(\tilde{\Omega})$ is the probability density function of the non-dimensional circular step frequency $\tilde{\Omega} = \omega/\omega_j$.

The psdf of the j-th non-dimensional principal coordinate is supplied by:

$$S_{\tilde{P}_j}(\tilde{\Omega}) = \left| H_{\tilde{P}_j}(\tilde{\Omega}) \right|^2 S_{\tilde{F}_j}(\tilde{\Omega}) \tag{7}$$

where $H_{\tilde{P}_j}(\tilde{\Omega})$ is the complex frequency response function of the j-th non-dimensional principal coordinate:

$$H_{\tilde{P}_j}(\tilde{\Omega}) = \frac{1}{1 - \tilde{\Omega}^2 + 2\mathrm{i}\xi_j \tilde{\Omega}} \tag{8}$$

i being the imaginary unit.

Finally, the psdf of the acceleration of the j-th non-dimensional principal coordinate $\tilde{p}_j$, $S_{\ddot{\tilde{P}}_j}(\tilde{\Omega})$, is:

$$S_{\ddot{\tilde{P}}_j}(\tilde{\Omega}) = \tilde{\Omega}^4 S_{\tilde{P}_j}(\tilde{\Omega}) \tag{9}$$

2.3 Maximum dynamic response

Starting from the proposed equivalent spectral model of the modal force, the maximum value of the acceleration of the *j*-th non-dimensional principal coordinate $\tilde{p}_j$ can be expressed as follows:

$$\ddot{\tilde{p}}_{j\max} = g_{\ddot{\tilde{p}}_j} \sigma_{\ddot{\tilde{p}}_j} \tag{10}$$

where $\sigma_{\ddot{\tilde{p}}_j}$ and $g_{\ddot{\tilde{p}}_j}$ are, respectively, the standard deviation and the so-called peak factor of $\ddot{\tilde{p}}_j$.

If the non-dimensional mean step frequency is close to a unit value (i.e. the bridge is prone to pedestrian excitation), the dynamic response may be considered as mainly resonant. In such a case, the standard deviation is given by the following simple approximate formula [3]:

$$\sigma^2_{\ddot{\tilde{p}}_j} \cong \frac{\pi}{4\xi_j} S_{\tilde{F}_j}(1) \tag{11}$$

According to Davenport formulation, [13], the peak factor can be expressed as:

$$g_{\ddot{\tilde{p}}_j} = \sqrt{2\ln\left(2\nu_{\ddot{\tilde{p}}_j}\tilde{T}\right)} + \frac{0.5772}{\sqrt{2\ln\left(2\nu_{\ddot{\tilde{p}}_j}\tilde{T}\right)}} \tag{12}$$

where $\tilde{T}$ is the non-dimensional time duration over which the maximum response is estimated, and $\nu^e_{\ddot{\tilde{p}}_j}$ is the non-dimensional expected frequency of $\ddot{\tilde{p}}_j$. They can be given by [13]:

$$\tilde{T} = \omega_j \frac{NL}{c_m} = \frac{N}{\tilde{\Omega}_{cm}} \qquad \nu_{\ddot{\tilde{p}}_j} \simeq \tilde{n}_j = \frac{1}{2\pi} \tag{13}$$

N being the number of consecutive pedestrian groups to be considered in the serviceability analysis in order to satisfy the stationarity hypothesis (a reasonable value seems to be around 10 [3]), $\tilde{\Omega}_{cm}$ being the non-dimensional mean walking velocity, given by Eq. (5) with $c_i = c_m$.

The Davenport expression for the peak factor [11] is based on the assumption that threshold upcrossings are Poisson events. As discussed in [13], a conventional peak factor formula on the basis of the Poisson assumption may still be a reasonable choice because of its conservative nature. This aspect will be analyzed in Section 3.2.

The maximum acceleration of the *j*-th non-dimensional principal coordinate $\tilde{p}_j$ can thus be obtained by considering Eqs. (6), (10) and (11):

$$\ddot{\tilde{p}}_{j\max} = \sqrt{\frac{N_p}{\xi_j}} g_{\ddot{\tilde{p}}_j} \frac{\sqrt{\pi}}{4} \sqrt{p_{\tilde{\Omega}}(1)} \tag{14}$$

Eq. (14) shows that, according to the equivalent spectral model proposed by the authors in [3], the maximum non-dimensional principal coordinate is a function of the non-dimensional mean walking speed $\tilde{\Omega}_{cm}$ through the peak factor; furthermore, it is a function of N_p, ξ_j and of the probability density function of the non-dimensional step circular frequency $p_{\tilde{\Omega}}$, estimated for $\tilde{\Omega}=1$.

The maximum structural acceleration in dimensional form can finally be obtained considering Eqs. (3) and (14) together, and is given by:

$$\ddot{q}_{\max} = \ddot{p}_{j_{\max}} = \frac{\alpha_m G_m}{M_j} \ddot{\tilde{p}}_{j_{\max}} = \frac{\alpha_m G_m}{M_j} \sqrt{\frac{N_p}{\xi_j}} g_{\ddot{\tilde{p}}_j} \frac{\sqrt{\pi}}{4} \sqrt{p_{\tilde{\Omega}}(1)} \tag{15}$$

3 MONTE CARLO SIMULATIONS

Monte Carlo simulations are performed with two main objectives. The first goal is an analysis of the influence of the variability of all the statistical parameters involved on the mean value of the maximum dynamic response in order to verify the effective role of the parameters that the spectral model assumes as deterministic. In particular, walking speed, dynamic load factor and pedestrian weight are analyzed (Section 3.1). The second aim is an assessment of the reliability of the proposed formulation for the statistical characterization of the maximum response. The results of Monte Carlo simulations are compared with the analytical evaluation of the mean value of the maximum dynamic response provided by the spectral model (Section 3.2).

In order to obtain general results without the need to focus on a particular structure, Monte Carlo simulations are performed starting from the non-dimensional expression of the modal force in Eq. (4). The non-dimensional form of the equation of motion of the *j*-th principal coordinate, Eq. (2), is numerically solved and the maximum acceleration of the non-dimensional principal coordinate is extracted from each simulation. For every case analyzed, the mean value of the maximum acceleration is estimated from 10^4 simulations. All simulations are performed assuming N_p=10, ξ_j= 0.005, V_α=0.1.

3.1 Statistical parameters affecting dynamic response

The equivalent spectral model of pedestrian loading, described in Section 2, has been obtained under the assumption that the pedestrian weight, the dynamic loading factor, the walking speed and the pedestrian arrivals could be considered as deterministic and coincident with their mean value: only the randomness of the step frequency has been taken into account. In the following, the influence of the parameters' statistical representation on the maximum structural acceleration is investigated through fully probabilistic simulations. In particular, numerical simulations are performed with the aim of verifying the secondary role of the randomness of the parameters that are assumed as deterministic; moreover, an assessment of the influence of the step frequency statistical distribution on the maximum dynamic response is performed.

The role of the probabilistic distribution of the body weight and of walking speed is analyzed by performing fully probabilistic simulations with different values of the coefficients of the two random variables. Fully-probabilistic Monte Carlo simulations are performed assuming $\tilde{\Omega}_m$=1.

Figure 1 plots the mean value of the maximum acceleration of the principal coordinate for different values of the coefficient of variation of pedestrian weight V_W (Fig. 1a, V_Ω = 0.09, V_c = 0.11) and of the walking speed (Fig. 1b, V_Ω = 0.09, V_W = 0.17). The maximum acceleration of the principal coordinate is almost independent of the variation coefficient of the pedestrian weight and of the walking speed. Thus, the probabilistic distribution of the body weight and of the walking speed practically do not affect the footbridge serviceability: assuming the pedestrian weight and the walking speed as deterministic and coincident with their mean value provides an accurate estimate of the maximum structural acceleration.

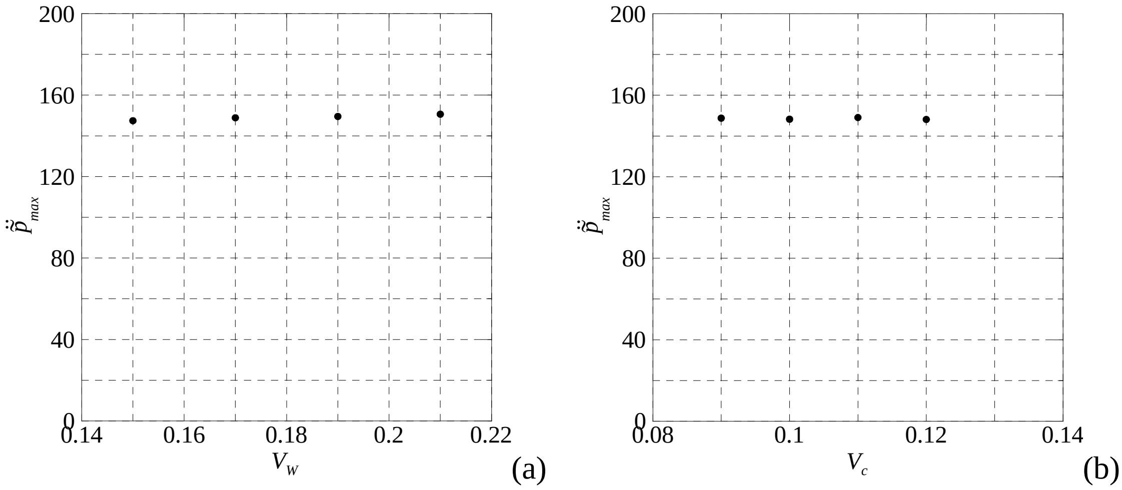

Figure 1: Influence of the probability distribution of the body weight (a) and of the walking speed (b) on the mean value of the maximum acceleration.

Finally, the role of the statistical distribution of the pedestrian step frequency is analyzed. Monte Carlo simulations are performed assuming fixed values for V_c and V_w (V_c = 0.11, V_W=0.17), whereas the coefficient of variation of pedestrian step frequency V_Ω is varied between 0.06 and 0.1, and the non-dimensional mean step frequency $\tilde{\Omega}_m$ between 0.8 and 1.2. Figure 2(a) plots the mean value of the maximum acceleration of the non-dimensional principal coordinate as a function of the variation coefficient of the step frequency V_Ω, for a unitary non-dimensional mean step frequency ($\tilde{\Omega}_m$ =1); Figure 2(b) plots the mean value of the maximum acceleration of the non-dimensional principal coordinate as a function of the mean value of the non-dimensional step frequency $\tilde{\Omega}_m$, for V_Ω=0.09. From Figures 3(a) and 3(b) it is evident that the probabilistic distribution of the step frequency has a strong influence on the maximum dynamic response: both the coefficient of variation and the mean value of the non-dimensional step frequency cause remarkable variations of the maximum dynamic response.

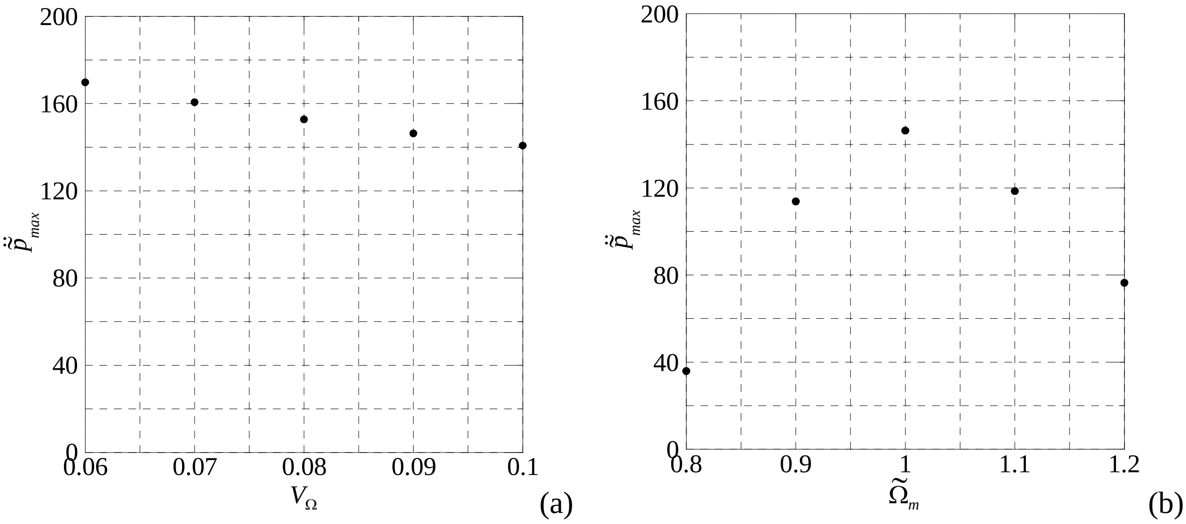

Figure 2: Influence of the probability distribution of the step frequency on the mean value of the maximum acceleration: (a) dependence on V_Ω ($\tilde{\Omega}_m$ =1), (b) dependence on $\tilde{\Omega}_m$ (V_Ω=0.09).

In conclusion, the Monte Carlo simulations presented in this Section have shown that the maximum dynamic response is mainly governed by the probability distribution of the pedestrian step frequency in terms of its mean value and standard deviation. The basic assumption of the spectral model (to adopt the pedestrian weight, the walking speed and the dynamic loading factor as deterministic) appears totally reliable.

3.2 Validation of the proposed expression for the maximum dynamic response

In this Section, Monte Carlo simulations are performed with the aim of validating the proposed closed-form expression in order to estimate the maximum dynamic response, by comparing numerical results with the closed-form expression, Eq. (10).

Monte Carlo simulations are performed assuming fixed values for the variation coefficients of the walking speed, of the dynamic loading factor and of the pedestrian weight (V_c = 0.11, V_α = 0.1, V_W=0.17), whereas the coefficient of variation of pedestrian step frequency V_Ω is varied between 0.06 and 0.1 and the non-dimensional mean step frequency $\tilde{\Omega}_m$ between 0.8 and 1.2.

Figure 3 plots the mean value of the maximum non-dimensional acceleration as a function of the coefficient of variation of the step frequency V_Ω; the different curves correspond to different values of the non-dimensional mean step frequency $\tilde{\Omega}_m$. The results of numerical simulations (symbols) are compared with the closed-form expression (solid and dashed lines). It can be observed that the closed-form expression of the maximum acceleration provides an overestimation of the numerical results when the non-dimensional mean step frequency is approximately one. However, the proposed expession provides a better approximation of the numerical results when the non-dimensional mean step frequency is lower than 0.95 or higher than 1.05: in such cases, the analytical expression slightly overestimates the numerically estimated maximum acceleration.

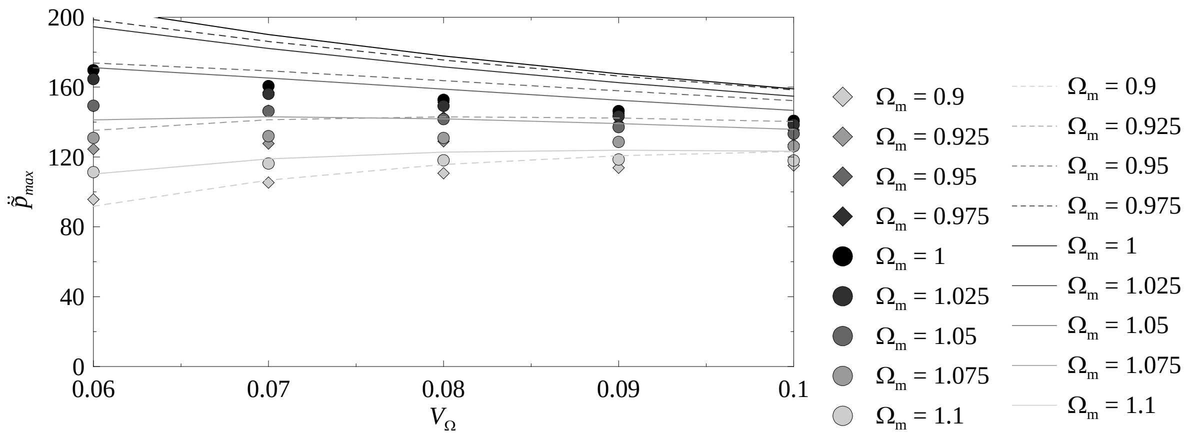

Figure 3: Mean value of the maximum non-dimensional acceleration: comparison between numerical simulations and closed-form expression.

In order to have a better insight into the problem, Figure 4 compares the numerically-estimated probability density function of the maximum non-dimensional acceleration (bar plot) with the one based on Davenport (solid lines) theory, for two different values of the non-dimensional mean step frequency, (a) $\tilde{\Omega}_m$=1 and (b) $\tilde{\Omega}_m$=1.1. Both figures show that the distribution of the maximum obtained from a Monte Carlo simulation (bar plot) is much wider than the one predicted by the Davenport theory, that is very far from the one obtained from

Monte Carlo simulations. Numerical distributions are in fact wide and identifying the maximum response with its mean value may be difficult and, sometimes, unsafe. In any case, due to the width of the numerically-estimated maximum distribution, adopting the Davenport theory may be a good choice for all possible technical values of the non-dimensional mean step frequency: it implies identifying the maximum structural response with a value characterized by a small probability of exceedance.

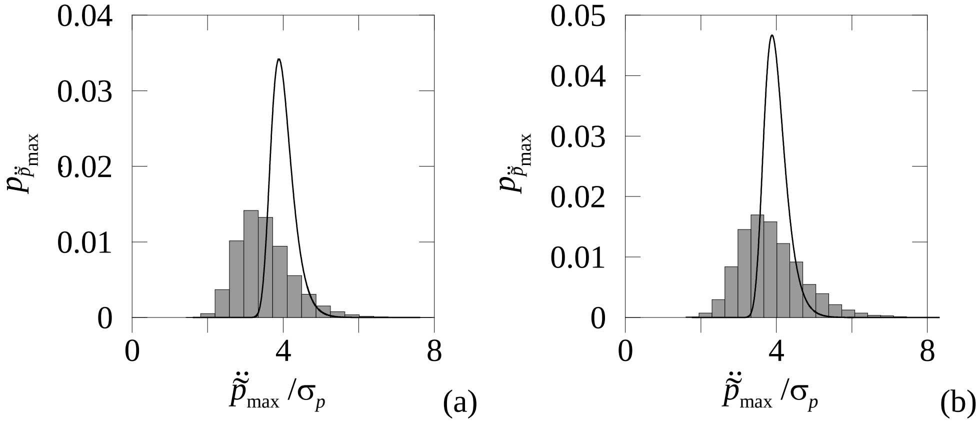

Figure 4: Probability density function of the maximum non-dimensional acceleration: comparison between numerical simulation and analytical model (a) $\tilde{\Omega}_m = 1$, (b) $\tilde{\Omega}_m = 1.1$.

4 PARAMETRIC ANALYSIS

The non-dimensional formulation in Section 2.1 allows a critical analysis of the essential parameters governing footbridge serviceability that have been developed throughout Section 3. Then, the main parameter affecting the dynamic response is the probability density function of the non-dimensional step frequency, which is essentially governed by two parameters: the non-dimensional mean step frequency (i.e. the ratio between the mean step frequency and the structural natural frequency) and the coefficient of variation (i.e. the ratio between the standard deviation of the step frequency and its mean value). In the literature, a variability of such parameters has been observed in real cases (Table 1). In this Section, a parametric analysis is performed starting from the equivalent spectral model of the loading, in order to assess the effective role of the statistical distribution of the step frequency on the maximum footbridge acceleration.

A quantitative estimate of the influence of the stochastic walking parameters on the maximum footbridge acceleration may be obtained by adopting the closed-form solution for the evaluation of the maximum acceleration, Eq. (14). In order to obtain general results without the need to set any value of the structural damping ratio ξ_j and of the pedestrian number N_p, a normalized value $\ddot{\tilde{p}}_{j\mathrm{norm}}$ of the maximum acceleration of the non-dimensional principal coordinate $\ddot{\tilde{p}}_{j\max}$ is defined as follows:

$$\ddot{\tilde{p}}_{j\mathrm{norm}} = \ddot{\tilde{p}}_{j\max}\sqrt{\xi_j / N_p} \tag{16}$$

Substituting Eq. (14) into Eq. (16), $\ddot{\tilde{p}}_{j\mathrm{norm}}$ is given by:

$$\ddot{\tilde{p}}_{j\mathrm{norm}} = g_{\ddot{\tilde{p}}_j}\frac{\sqrt{\pi}}{4}\sqrt{p_{\tilde{\Omega}}\left(1\right)} \tag{17}$$

Eq. (17) shows that the normalized maximum non-dimensional acceleration only depends on the peak factor $g_{\ddot{\tilde{p}}_j}$ and on the probability density function of the non-dimensional step frequency, estimated for $\tilde{\Omega}$=1. The peak factor essentially depends on $\tilde{\Omega}_{cm}$; thus it does not depend on the probabilistic distribution of the step frequency. Furthermore, $\tilde{\Omega}_{cm}$ is certainly a small parameter for real footbridges [3]. For bridges having a span length greater than 30 m, assuming a mean walking speed $c_m = 0.9\tilde{\Omega}_{cm}/(2\pi)$, it is lower than 0.006. The peak factor being a decreasing function of $\tilde{\Omega}_{cm}$, the following evaluations are carried out assuming, on the safe side, $\tilde{\Omega}_{cm}$=0.001.

Figure 5 plots $\ddot{\tilde{p}}_{j\mathrm{norm}}$, estimated from Eq. (17), as a function of the non-dimensional mean step frequency and of the coefficient of variation of the step frequency. The peak factor is set $g_{\ddot{\tilde{p}}_j}$ =4.16, corresponding to $\tilde{\Omega}_{cm}$ =0.001. Fig.5(a) provides a surface plot representation, Fig.5(b) provides the variation of the maximum acceleration of the non-dimensional principal coordinate as a function of the non-dimensional mean step frequency, for different values of the coefficient of variation. Figure 5 clearly shows the combined effect of the two parameters governing the maximum dynamic response. The non-dimensional mean step frequency strongly modifies the maximum value in the plot, but also the coefficient of variation considerably influences the shape of the response curves. Concerning the effect of the mean step frequency, it is evident from Figs. 5(a) and (b) (and also trivial from a theoretical point of view) that the worst situation occurs when the pedestrians' mean step frequency coincides with a natural frequency of the structure (a maximum is always detectable for $\tilde{\Omega}_m$=1). From Fig. 5(b) it can be observed that the influence of the coefficient of variation depends on the value of $\tilde{\Omega}_m$: for $\tilde{\Omega}_m$ close to a unit value, $\tilde{\Omega}_m$ = [0.95 - 1.05], the dynamic response decreases on increasing V_Ω; for $\tilde{\Omega}_m$ far from the unit value ($\tilde{\Omega}_m$< 0.9, $\tilde{\Omega}_m$> 1.1), the dynamic response increases on increasing V_Ω. For intermediate values of $\tilde{\Omega}_m$ ($\tilde{\Omega}_m$~ 0.93, $\tilde{\Omega}_m$~ 1.08), the dynamic response is almost independent of the coefficient of variation.

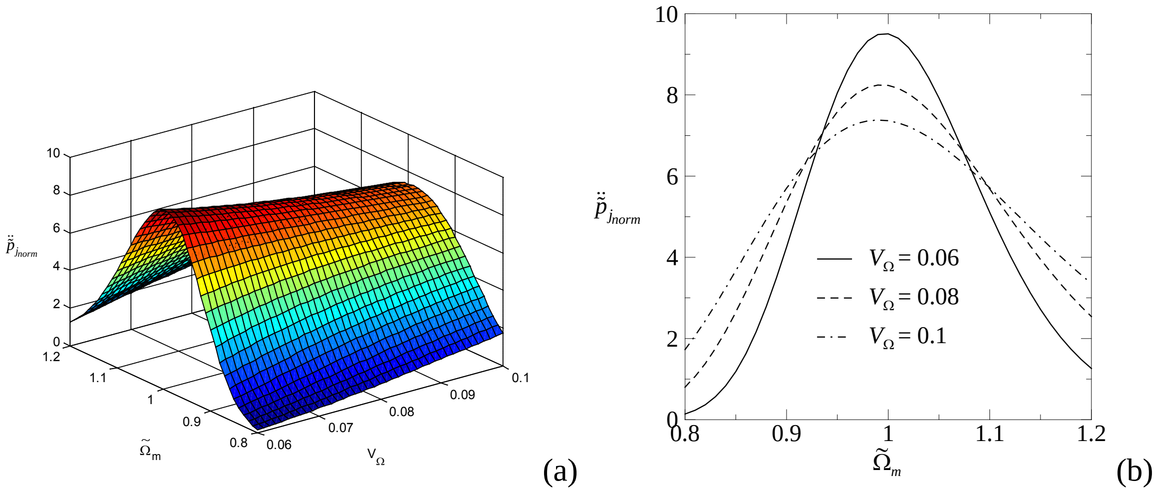

Figure 5: Influence of the of the step frequency distribution on the maximum structural response.

From these observations it can be deduced that, as schematized in Figure 6, if a footbridge has a natural frequency included between 1.8 Hz and 2 Hz (the values observed for the mean step frequency, Table 1), it is recommended to perform the serviceability analysis assuming a mean step frequency coincident with the natural frequency of the footbridge ($\tilde{\Omega}_m = 1$): in such a case, the worst condition is achieved assuming a small value for the coefficient of variation of the pedestrian step frequency (V_Ω=0.06). If the footbridge natural frequency is not in the interval 1.8 Hz - 2 Hz, assuming a mean step frequency coincident with the structural natural frequency could be too conservative. In such a case, it is recommended to assume a mean step frequency which is closer to the footbridge natural frequency, and to assume a low coefficient of variation (e.g. 0.06, Fig. 6) if $\tilde{\Omega}_m$ is close to 1, a large coefficient of variation (e.g., 0.1, Fig. 6) if $\tilde{\Omega}_m$ is far from 1.

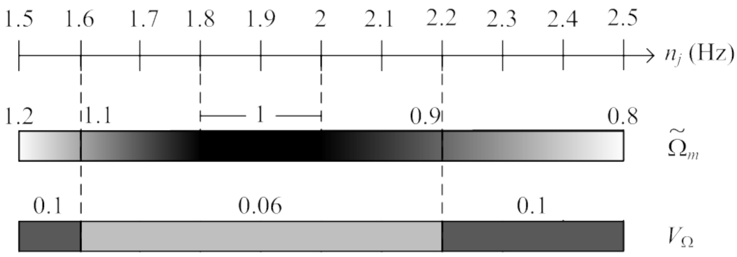

Figure 6: Recommended values of statistical parameters of non-dimensional step frequency.

The clarity and simplicity of the spectral approach allows an approximate, conservative direct estimation of footbridge maximum acceleration by Eq. (15) as a function of the main parameters governing the response. The load effect emerges through the statistical properties of the step frequency, in addition to the number of pedestrians (which is only a magnification factor of the response). The structural parameters (here assumed deterministic) are represented by the modal mass and the damping ratio (in addition to the natural frequency, which influences the assumption of the $\tilde{\Omega}_m$ value). Figure 7 plots the ratio $\ddot{q}_{\max} / \sqrt{N_p}$ as a function of the product $M_j\sqrt{\xi_j}$, setting a classic value for the average weight of pedestrians $\alpha_m G_m$ = 280 N. The different curves correspond to different values of the non-dimensional mean step frequency $\tilde{\Omega}_m$ ($\tilde{\Omega}_m$ is assumed in the range 0.8-1.2); in each case the worst coefficient of variation is selected according to Figure 6 (V_Ω=0.06 for $\tilde{\Omega}_m$ =0.9-1.1, V_Ω=0.1 for $\tilde{\Omega}_m$ outside this interval).

The response curves seem almost linear in a bi-logarithmic scale making the role of the different parameters explicit. Both the non-dimensional mean step frequency $\tilde{\Omega}_m$ and the product $M_j\sqrt{\xi_j}$ strongly affect the maximum structural acceleration, and thus footbridge serviceability. The graph quantifies the well-known beneficial effect on the dynamical behavior due to the increase of the modal mass and of the damping ratio. Moreover, the maximum structural acceleration greatly increases as the non-dimensional mean step frequency $\tilde{\Omega}_m$ moves closer to 1. Bridges characterized by $\tilde{\Omega}_m$ in the neighborhood of unity (0.9-1.1) appear to be very sensitive to vibrations; on the contrary, very small accelerations are expected only for small $\tilde{\Omega}_m$, such as $\tilde{\Omega}_m$ =0.8 (i.e. when the natural frequency of the bridge is much larger than the mean step frequency). Finally, it should be noted that values of 1 m/s^2 on the ordinate of Figure 7 are clearly out-of-comfort as regards any possible criteria (e.g. [12]), since the maximum acceleration caused by N_p pedestrians is obtained by multiplying the ordinate value by $\sqrt{N_p}$ (N_p being in general a number greater than 5). Bridges that exceed this limit are poorly designed from a dynamic point of view.

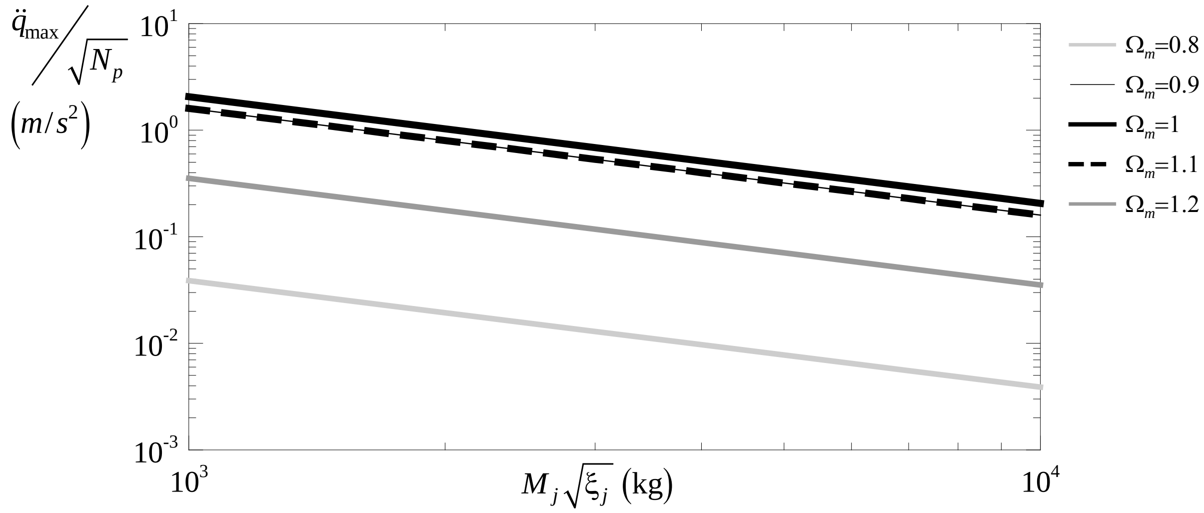

Figure 7: Estimate of the maximum structural acceleration.

5 CONCLUSIONS

In this paper, normal unrestricted pedestrian traffic has been analyzed adopting an equivalent spectral loading model that has been introduced by the authors in [3], starting from a complete probabilistic representation of pedestrians. With the aim of studying the dynamic response of lively footbridges, characterized by a vibration mode with a natural frequency close to the mean step frequency, attention is focused on the effect of the first walking harmonic only; higher walking harmonics are neglected.

Based on the proposed spectral model, the main force parameter affecting the structural response is the statistical characterization of the pedestrian step frequency, in terms of mean value and coefficient of variation. The comparison between the mean value of the maximum structural response estimated from numerical simulations and the analytical estimate has highlighted that the closed-form expression provides a slight overestimate of the numerical maximum. In any case, since the maximum structural response is characterized by a wide probability density function, the closed-form expression provides a safe estimates of the maximum structural response.

Extensive parametric analyses have been carried out in order to investigate the role of the probabilistic distribution of the step frequency on footbridge serviceability assessment. The analysis of the shape of the power spectral density function, structural acceleration and maximum dynamic response points out the two parameters governing the footbridge dynamic sensitivity: the non-dimensional mean step frequency (i.e. the ratio between the mean step frequency and the natural frequency of the structure) and the coefficient of variation of the step frequency (which becomes particularly important when the mean step frequency is not in resonance with a structural mode of vibration). From the analyses performed it can be deduced that, if a footbridge has a natural frequency included between 1.8 Hz and 2 Hz (the values observed for the mean step frequency), it is recommended to check the serviceability assuming a mean step frequency coincident with the natural frequency of the footbridge: in such case, the worst condition is achieved assuming a small value for the coefficient of variation of the pedestrian step frequency. If the footbridge natural frequency is not in the interval 1.8 Hz - 2 Hz, assuming a mean step frequency coincident with the structural natural frequency may even be too conservative. In this latter case, it is recommended to assume a mean step frequency which is closer to the footbridge natural frequency, and to assume a low coefficient of variation if the non-dimensional mean step frequency is close to 1, a high coefficient of variation if the non-dimensional mean step frequency is far from 1.

Finally, it is possible to summarize the results obtained from the spectral approach in a very synthetic way, as a function of mechanical damping and structural modal mass. Using the proposed conservative criterion on the choice of the step frequency variation coefficient, it is thus possible to obtain simple graphs which permit a direct and simple evaluation of footbridge maximum acceleration. These results seem of interest also from a technical point of view, and might apply to codes and guidelines for the evaluation of footbridge serviceability subjected to unrestricted pedestrian traffic.

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