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ECCENTRIC STEEL BUILDINGS DESIGNED FOR UNIFORM DUCTILITY DEMANDS UNDER EARTHQUAKE ACTIONS

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Abstract. *Research in the past few years has indicated that code designed eccentric buildings exhibit ductility demands under strong earthquake motions that are unevenly distributed throughout the structure. More specifically, it is found that elements at the so called "flexible" sides of the buildings exhibit substantially higher ductility demands than elements at the so called "stiff" sides. Such an uneven distribution indicates suboptimal material use and a potential for premature failure of certain members. In the present paper this problem is demonstrated for two L-shaped five story buildings, a torsionally stiff and a torsionally flexible. Subsequently a simple procedure is used to modify the original design that exhibits a substantially improved behavior in terms of ductility demand distribution. The results are quite promising and could be the basis for improving current codes.*

1 INTRODUCTION

One of the open research areas in earthquake resistant design of buildings is associated with torsion caused by strong earthquake excitations when the building is irregular or simply non-symmetric. Torsional motion during earthquakes can be caused due to a number of factors, some of which can be accounted for in design, while some others may be unknown at the time the structure is designed or may be difficult to quantify and be accounted for properly. Examples of such factors are accidental eccentricities due to mass and/or stiffness uncertainties and non coherent ground motion at the supports. Given also that buildings are designed to respond in the inelastic range under strong earthquakes, stiffness changes due to non symmetric yielding that current design and analysis methods cannot reliably predict, induce additional eccentricities and hence extra torsional motion.

Code provisions for designing eccentric buildings have been based mostly on elastic analyses of idealized multistory buildings or on inelastic analyses of highly simplified, one-story, inelastic models of the shear beam type with 3 degrees of freedom [4-10]. In the past decade, however, research on earthquake induced torsion started using more sophisticated, multi story, multi-degree of freedom inelastic models of the plastic hinge type [e.g. 11-18]. This research revealed that the widely used simplified, one story, 3 degree of freedom models used in most of the past studies, could lead to erroneous conclusions, unless model properties were very carefully selected to closely match key properties of the multistory building [18]. Hence code provisions based, in part at least, on such results might be questionable. Moreover, a number of controversies had been generated from such studies and a few publications were devoted to them [1-3,18].

The same studies of torsion with the detailed plastic hinge model also showed that the ductility demand differences between the two edges, “flexible” and “stiff”, were often very large, with the demands at the “flexible” edge being always substantially greater than the demands at the “stiff” edge of the building. This was initially found for concrete buildings, where both rotational ductility factors and damage indices were used as measures of inelastic deformations [13]. Subsequently the same behavior was confirmed for multi-story, eccentric, braced frame, steel buildings with rectangular layouts and a design modification was proposed to alleviate this problem [14-17].

In the present paper the aforementioned problem is demonstrated for two L-shaped, 5-story eccentric buildings, one planned and designed as torsionally stiff and the other as torsionally flexible. The results indicate the same problem for ductility demand distribution, as observed in the buildings with the rectangular layouts, and further that the same modification proposed before works well also for these two buildings.

2 BUILDINGS AND MOTIONS USED

The present investigation was carried out using two 5-story, steel, braced frame buildings, the first torsionally stiff and the second torsionally flexible. Both buildings are L shaped, and their typical layouts as well as typical frame elevations can be seen in Figures 1, 2 and 3. Note that each building is formed by 5 frames along the x axis, (FR-X01 to FR-X05) and 6 frames along the y axis (FR-Y01 to FR-Y06). In order to create a torsionally stiff and a torsionally flexible building just for the purpose of our work, braces were used to stiffen specific bays as shown in Figures 1 and 2, respectively. The stiffened bays in the second building (Fig. 2) were selected just to create a torsionally flexible variant of the first building. Both buildings have a typical story height of 3.00m and ground story height 4.00m.

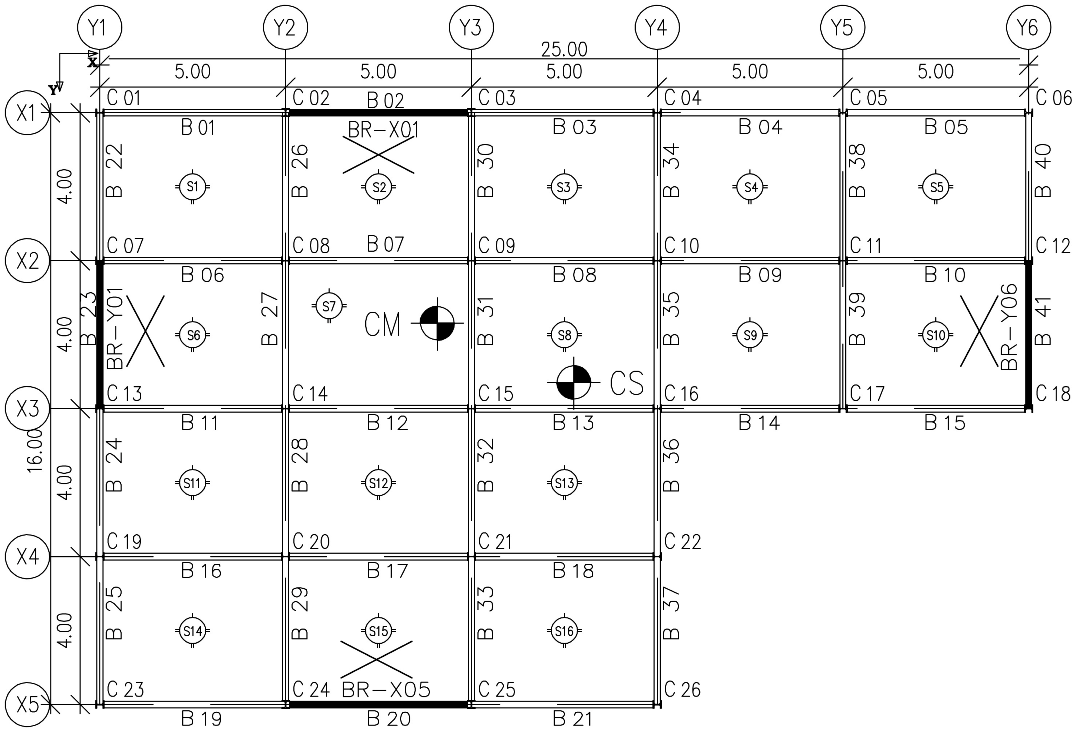


Figure 1: Typical layout of the 5-story torsionally stiff steel building.

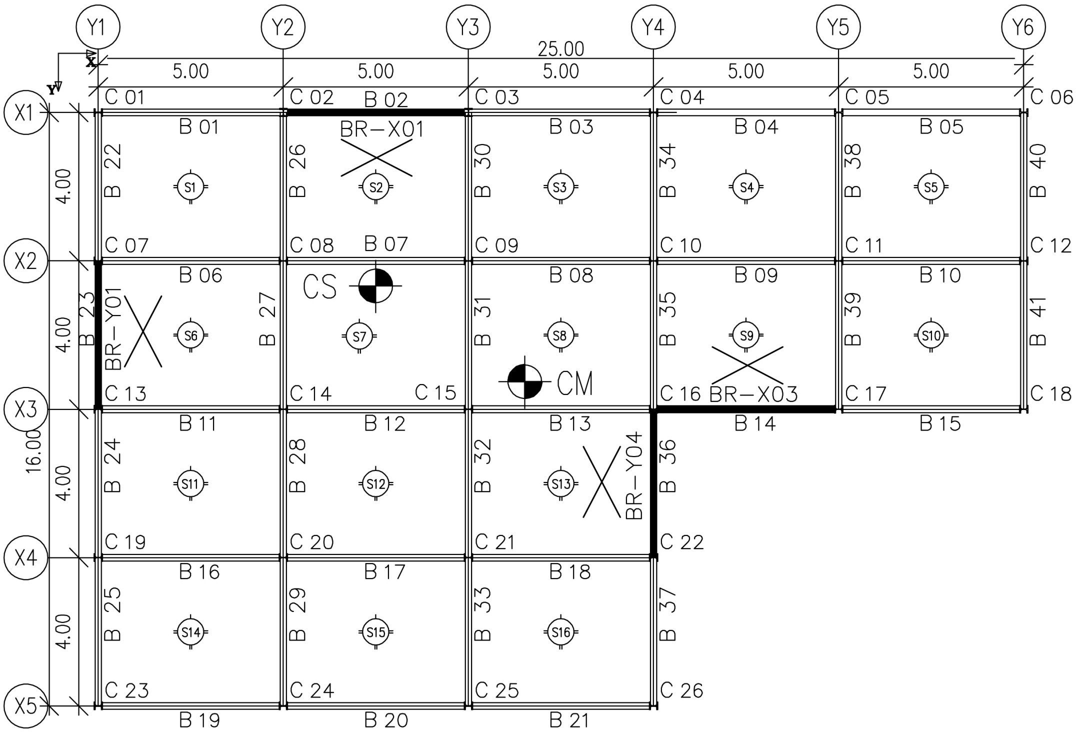


Figure 2: Typical layout of the 5-story torsionally flexible steel building.

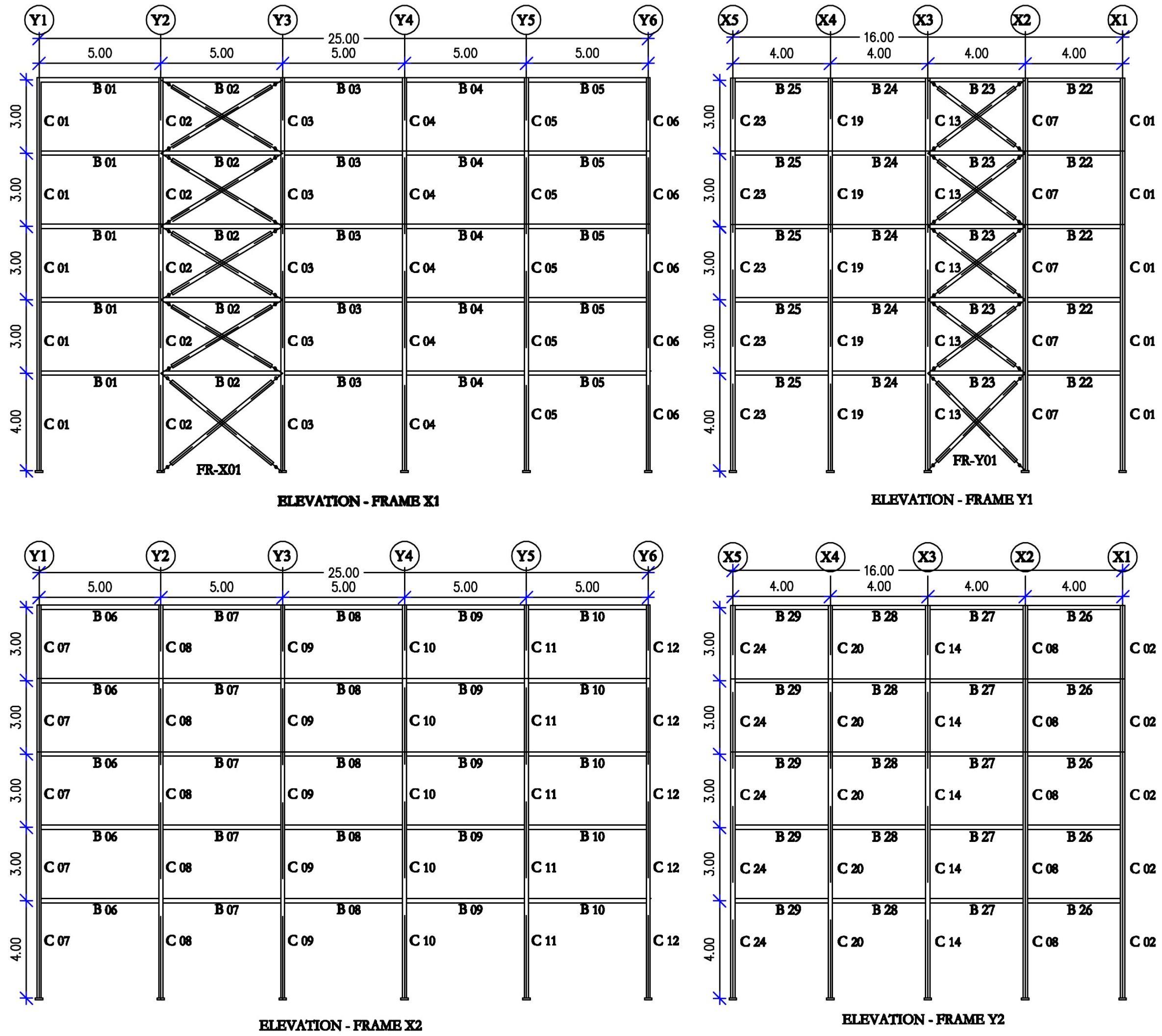


Figure 3: Elevations in X and Y directions of the 5-story torsionally stiff and flexible steel buildings.

Using appropriate distributions of the floor loads, e.g. through non-symmetric live load distribution, non-symmetric balconies (common causes of mass eccentricity in typical buildings, not shown in the given layout), non-symmetric joint masses were assigned at each floor, and mass eccentric floor plans, with $e_m \sim 0.15L$ in all five stories were generated.

The models used for both design and analyses are 3-D models with masses lumped at the joints and the floors acting as diaphragms. All buildings were designed as spatial frames for gravity and earthquake loads using the dynamic, response spectrum method, according to Eurocodes EC3 -steel structures- and EC8 -earthquake resistant design. The uneven mass distribution led to member forces and corresponding sections which created stiffness eccentricities and thus our buildings are both mass and stiffness eccentric. Earthquake actions were described by the design spectrum specified by the Greek Code for ground acceleration PGA=0.24g and soil category II.

As input for the nonlinear dynamic analyses, ten sets of two component semi-artificial motion pairs were used. They were generated from a group of five, two-component, real earthquake records, to closely match the code design spectrum (with a descending branch $1/T^{2/3}$), using a method based on trial and error and Fourier transform techniques [19]. Results were excellent, as may be seen in Figure 4 where the mean response spectrum of the ten semi-artificial motions is compared with the target, code design spectrum. Each synthetic motion pair, derived from the two horizontal components of each historical record, was applied twice by mutually changing the components along the x and y system axes. Thus, each design case was analyzed for ten sets of 2-component motions and mean values of peak response indices were computed. In this manner, the effects of individual motions are smoothed and the conclusions become less dependent on specific motion characteristics.

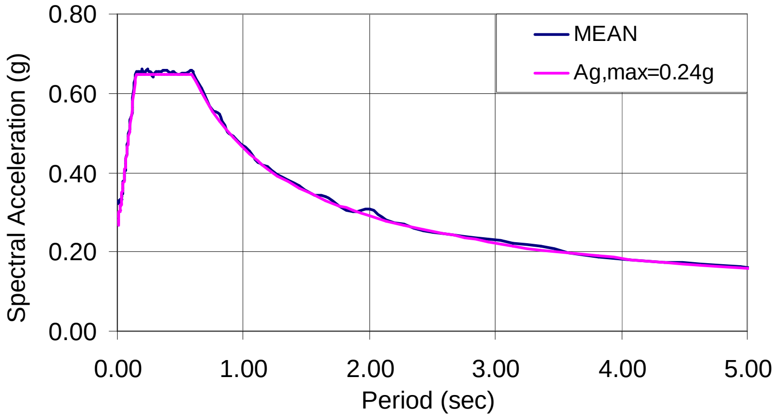


Figure 4: Design spectrum and mean spectrum of the ten semi-artificial motions.

The three lowest periods of the 5-story torsionally stiff building are T_x =0.82 sec, T_y =0.80 sec and T_θ=0.45 sec, while the mean natural eccentricities in each horizontal direction are ε_x=0.10 and ε_y=0.10. Notice that the first torsional period is lower than the two translational periods, as it is expected for a torsionally stiff building. The torsionally flexible building has a fundamental torsional mode with period T_θ=0.90 and two translational modes with periods T_x= T_y=0.82 sec, while its average natural eccentricities are ε_x=0.07 and ε_y=0.10. It is noted that in multistory buildings, the CR cannot be really defined, except under very restrictive conditions. Thus, an approximate CR was computed herein for reference purposes, on a floor by floor basis as follows :

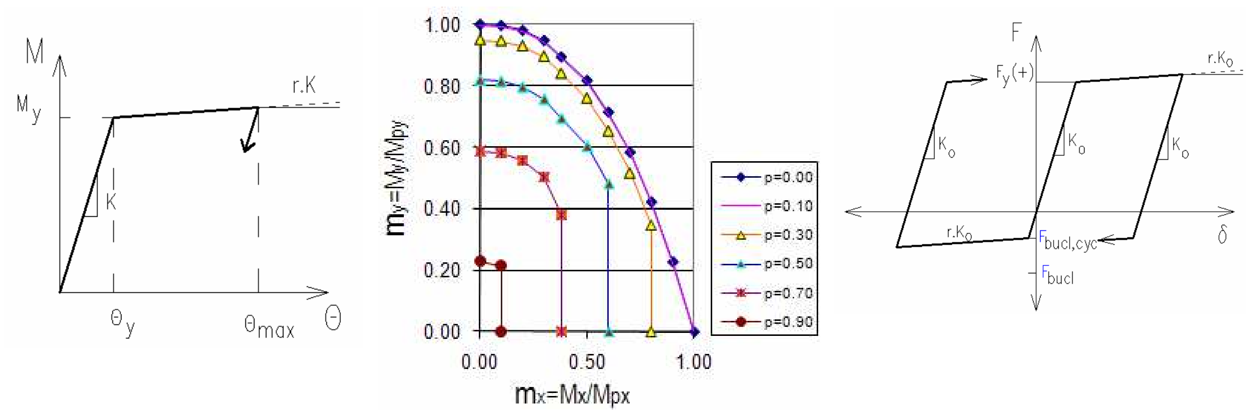
$$e_{sx} = \frac{\sum_{i=1}^{m} K_{f-iy}\, x_i}{\sum_{1}^{m} K_{f-iy}} \qquad e_{sy} = \frac{\sum_{i=1}^{n} K_{f-ix}\, y_i}{\sum_{1}^{n} K_{f-ix}} \tag{1}$$

$$K_{f-i} = \frac{24E}{h^2}\left[\frac{2}{\sum K_c} + \frac{1}{\sum K_{ba}} + \frac{1}{\sum K_{bb}}\right]^{-1} + \sum \frac{AE}{L}\cos^2\varphi \tag{2}$$

where: e_{sx}, e_{sy} are the x and y coordinates of the approximate stiffness center CR, $K_{f\text{-}i}$ designates the approximate story stiffness of frame i, x and y the directions of the frame axis, m and n the number of frames along the y and x axes, respectively, E = modulus of elasticity, $K_c = I_c / h$, $K_b = I_b / \ell$, I_c, I_b = section moment of inertia of columns and beams, respectively, h = story height, ℓ = beam length, A= area of brace section, L=brace length and φ = angle of brace member and the horizontal plane. The second indices, a and b, in K_{ba} and K_{bb} designate the upper (above) and lower (below) floor beams of the frame in the considered story.

3 NON-LINEAR DYNAMIC ANALYSES

The non-linear analyses were carried out using the program RUAUMOKO [20]. Frame beams and columns were modelled with the well-known plastic hinge model, in which yielding at member ends is idealized with plastic hinges of finite length having bilinear moment-curvature relationship and strain hardening ratio equal to 0.05. A moment-axial force interaction diagram was also employed for columns, giving the yield moment as a function of the applicable axial force on the column section. Bracing members, yielding in tension and buckling in compression, were modelled with a non-symmetric bilinear force-axial deformation relationship (Figure 5).



(a) (b) (c)

Figure 5: (a) Nonlinear moment-rotation relations for beam-columns, (b) Column M-N interaction diagram and (c) nonlinear force deformation diagram for braces.

The basic measure used to assess the severity of inelastic response is the ductility factor of the various members. For bracing members the ductility factor is defined as:

$$\mu_u = 1 + \left(\frac{u_p}{u_y} \right) \tag{3}$$

where u_p is the maximum plastic member elongation and u_y the elongation at first yield.
For beams and beam-columns the rotational ductility factor has traditionally been defined as:

$$\mu_\theta = 1 + \left(\frac{\theta_p}{\theta_y} \right) \tag{4}$$

where θ_p is the maximum plastic hinge rotation at either end of a member (beam or column) and θ_y is a normalizing "yield" rotation, typically set equal to $\theta_y = M_y \ell / 6EI$. For columns, the yield moment M_y is usually taken to correspond to the yield moment under the action of gravity loads. In the present study, an alternative definition of the rotational ductility factor, based on the post yield plastic moment, has been used [21]

$$\mu = 1 + \left(\frac{\Delta M}{p \cdot M_y} \right) \tag{5}$$

where: $\Delta M = M_{max} - M_y$, M_y = yield bending moment and $p=0.05$, the strain hardening ratio.

In addition to the above measures, peak floor displacements and interstory drifts are used to assess the inelastic behavior of the buildings.

4 RESULTS FROM NON-LINEAR ANALYSES OF "AS DESIGNED" BUILDINGS.

Results from time history analyses of the two buildings are presented in terms of mean values of the peak response parameters over the ten pairs of applied motions. In the case of the beam ductility factors, the response parameter averaged over the ten pairs of motion is the maximum rotational ductility demand in any of the beams in the considered frame and floor. Following standard terminology based on static application of the lateral load in eccentric buildings, the edge where the displacement from rotation is added to the pure floor translation is called "flexible" edge, while the opposite edge, where the displacement due to rotation is subtracted from the translation is called "stiff" edge. Since the examined buildings have biaxial eccentricity, the edge distinction just mentioned applies to both the X and Y horizontal directions of the buildings. Thus, results are presented for each edge frame and each direction. In torsionally flexible buildings, however, it is not necessarily the "flexible" edge that experiences the largest translation but it could well be the "stiff" edge, depending on the relative values of the torsional and translational periods and on the input characteristics.

4.1. Five-story torsionally stiff building

The "flexible" and "stiff" edges of the five-story torsionally stiff buildings are presented in Figure 6. Displacement results and ductility demands for braces and beams of the "flexible" and "stiff" edges can be in Figures 7 and 8, respectively. Ductility demands are presented only for beams and brace members because the columns remained essentially elastic. Looking into Figure 7, we can see that displacements in the Y direction at the "flexible" edges of the torsionally stiff eccentric building are substantially greater than those at the "stiff" edges due to the induced earthquake rotations. The same is true for ductility demands of the braces and the beams (Figure 8).

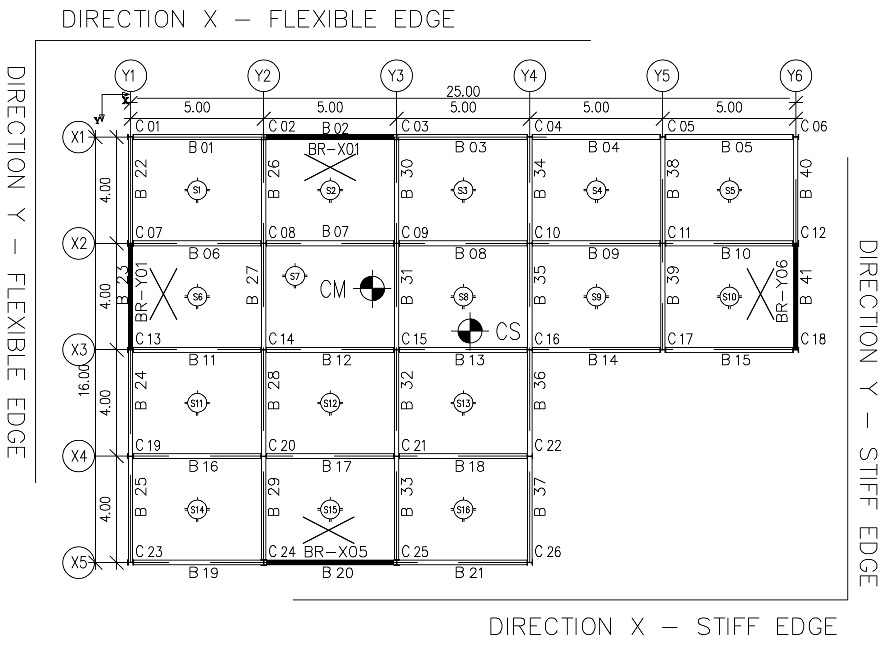


Figure 6: "Flexible" and "stiff" edges in torsionally stiff building.

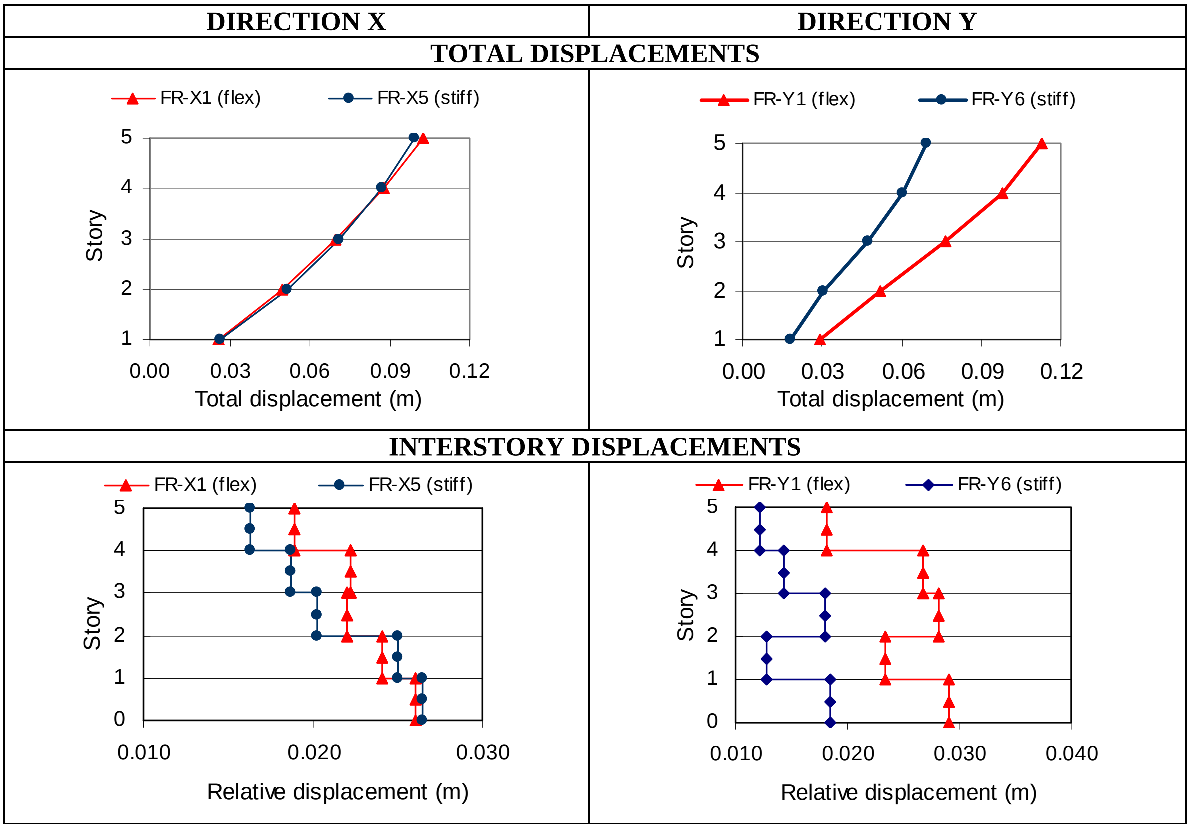


Figure 7: Total displacements and interstory drifts of 5-story torsionally stiff building (FR-X1 & FR-Y1: “flexible” edges, FR-X5 & FR-Y6: “stiff” edges).

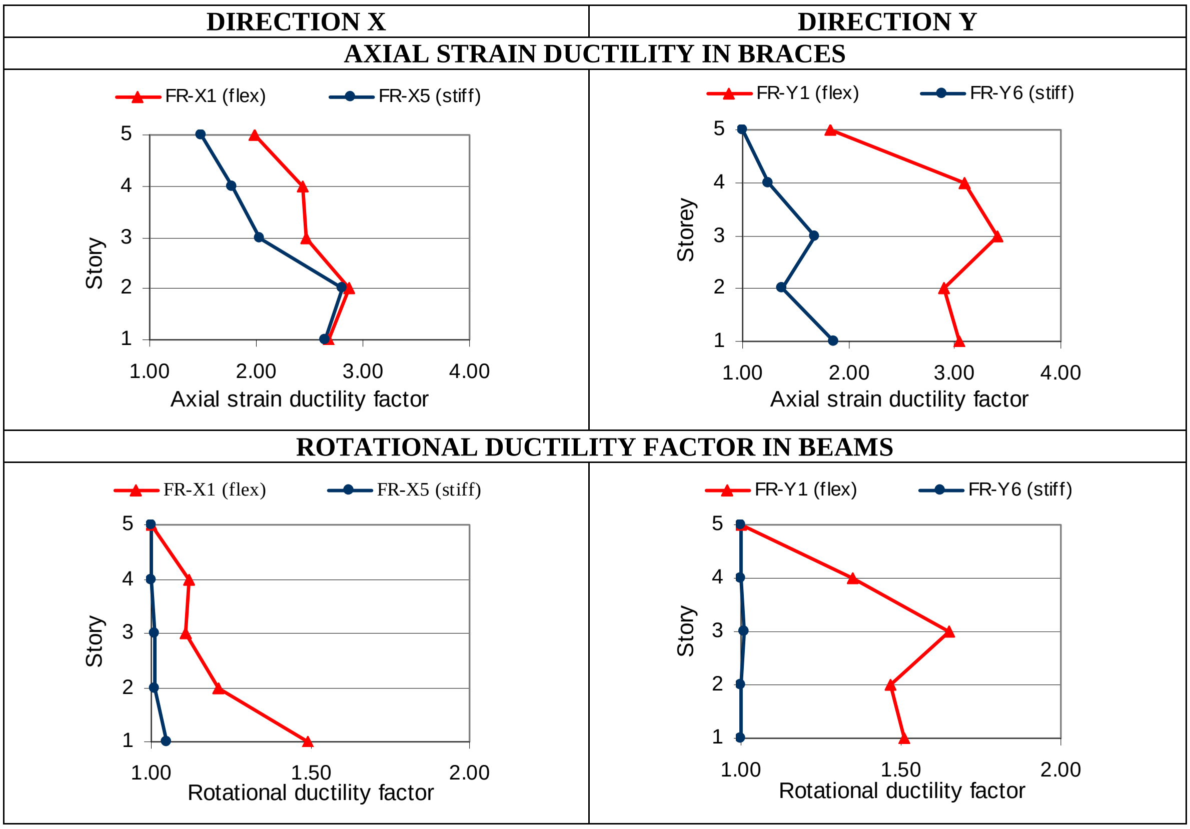


Figure 8: Member ductility demands of 5-story torsionally stiff building (FR-X1 & FR-Y1: “flexible” edges, FR-X5 & FR-Y6: “stiff” edges).

4.2. Five-story torsionally flexible building

The "flexible" and "stiff" edges of the five-story torsionally flexible buildings are presented in Figure 9. Displacement results for the torsionally flexible building are shown in Figure 10 and ductility demands in Figure 11. Compared to the results of the torsionally stiff building, quantitative differences aside, the behavior pattern is similar, with displacement and ductility demands being larger in the frames at the "flexible" edges or sides. It is further noticed that the differences in ductility demands in the braces here are much smaller than in the torsionally stiff building, since the braces in this case are placed near the core of the building.

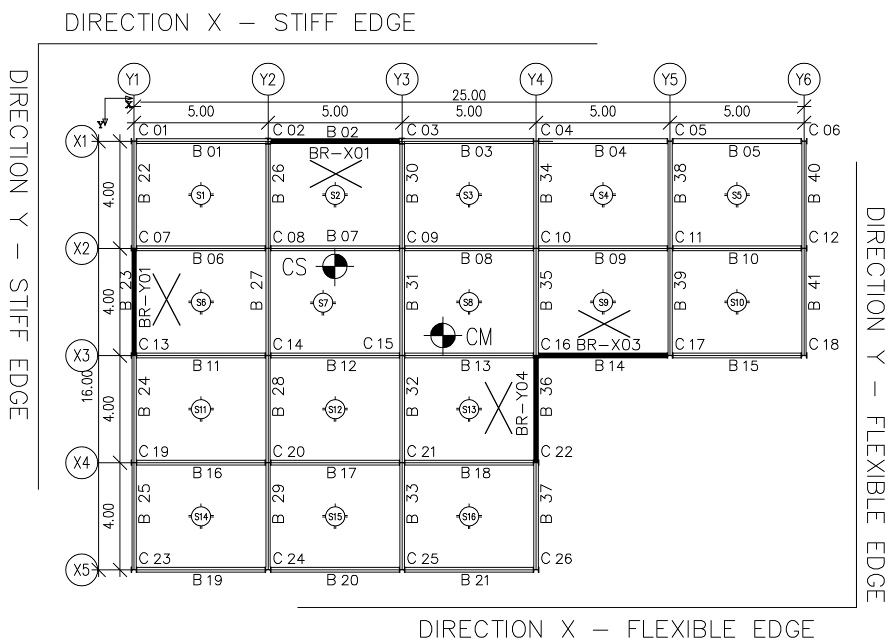


Figure 9: "Flexible" and "stiff" edges in torsionally flexible building.

5 MODIFICATION PROCEDURE

A structural design can be characterized as satisfactory when the limiting values of the controlling response parameters do not have wide variations within the groups of structural members to which they apply. In the opposite case, suboptimal use of material may be present as well as a potentially higher risk of failure in cases of unexpected overloads. Thus the observed substantial differences in ductility demands between the opposite edges of the examined buildings, point to the need for a design modification that would eliminate or reduce these differences. The modifications for the torsionally stiff building are the same as in Refs. [14,16]. A modified version of this modification will be applied to the torsionally flexible building [17].

5.1. Modification procedure and results for the torsionally stiff building.

The modification procedure for the torsionally stiff building aims at increasing the stiffness and strength of the bracing members at the "flexible" edges and reducing the same at the "stiff" edges without affecting the strength of the other structural elements (columns, beams). The first step for application of this modification is to obtain the top story displacements at the "flexible" and "stiff" edges of the building in both horizontal directions due to the earthquake loading considered and then compute the following factors in each horizontal direction:

$$f_{i,flex} = 2 \cdot \frac{u_{i,flex}}{(u_{i,flex} + u_{i,stiff})}$$
$$f_{i,stiff} = 2 \cdot \frac{u_{i,stiff}}{(u_{i,flex} + u_{i,stiff})} \tag{6}$$

where $u_{i,flex}$ is the top story displacement of the “flexible” edge in the i - direction and $u_{i,stiff}$ the top story displacement of the “stiff” edge also in the i - direction. These displacements are obtained by the dynamic response spectrum method for the seismic combinations considered. The factors are ratios of the top story displacements at the “flexible” and “stiff” edges in a given direction (x or y), to their mean values. The design modification that was subsequently applied was to multiply the axial areas of the bracing members in both the “stiff” and “flexible” edges by the corresponding factors in each direction. The values of these factors, for modifying frames in direction x, are 0.91 for the “stiff” edge and 1.09 for the “flexible” edge. Similarly the values for modifying frames in direction y are 0.80 for the “stiff” edge and 1.20 for the “flexible” edge. After this modification, each structure was checked again for full compliance with the applicable codes. The new, modified structures were again subjected to the same two component earthquake set and their responses were again computed as before.

Figures 12 and 13 show ductility demands for the torsionally stiff building for the initial and the modified designs. If we compare the results obtained from the modified design with that of the original design, we see a substantial improvement of response in all cases: the overall maximum ductility demand factor in each group is reduced and so are the differences between “flexible” and “stiff” edges, producing more uniform distribution of such demands.

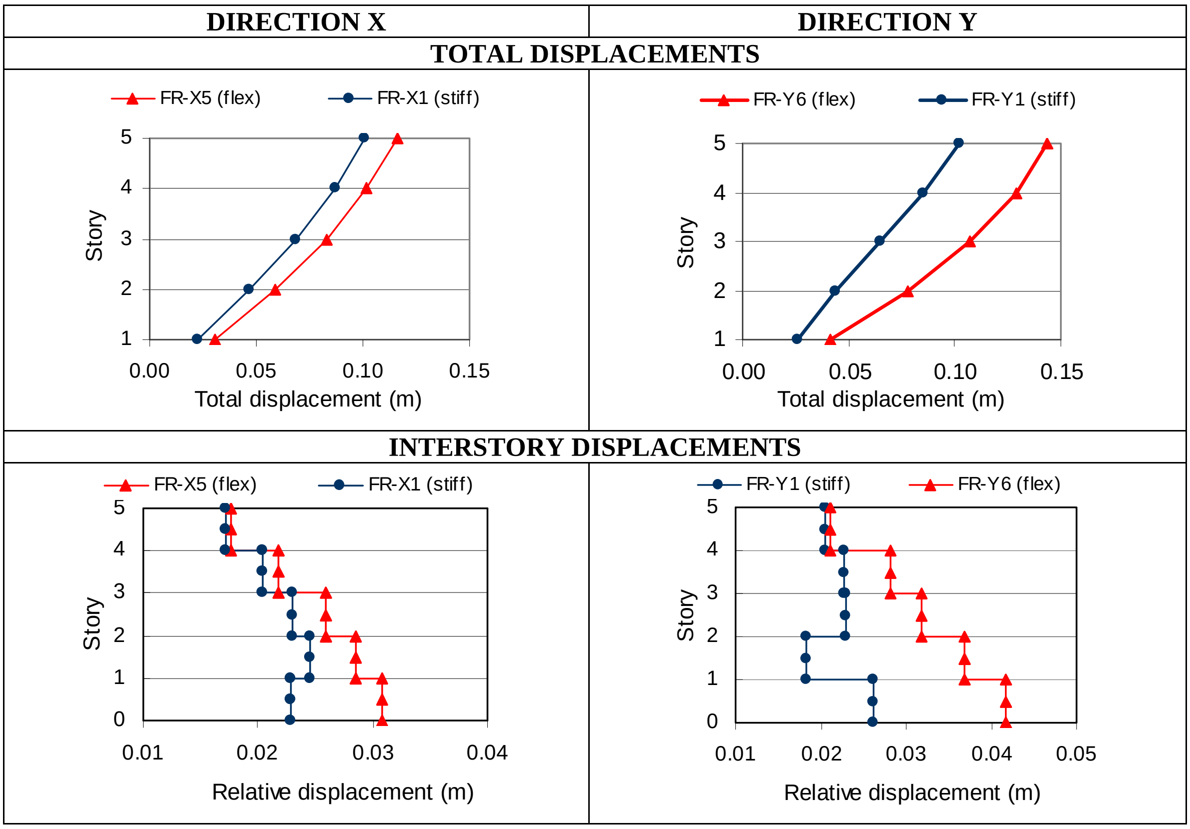


Figure 10: Total displacements and interstory drifts of 5-story torsionally flexible building (FR-X5 & FR-Y6: “flexible” edges, FR-X1& FR-Y1: “stiff” edges).

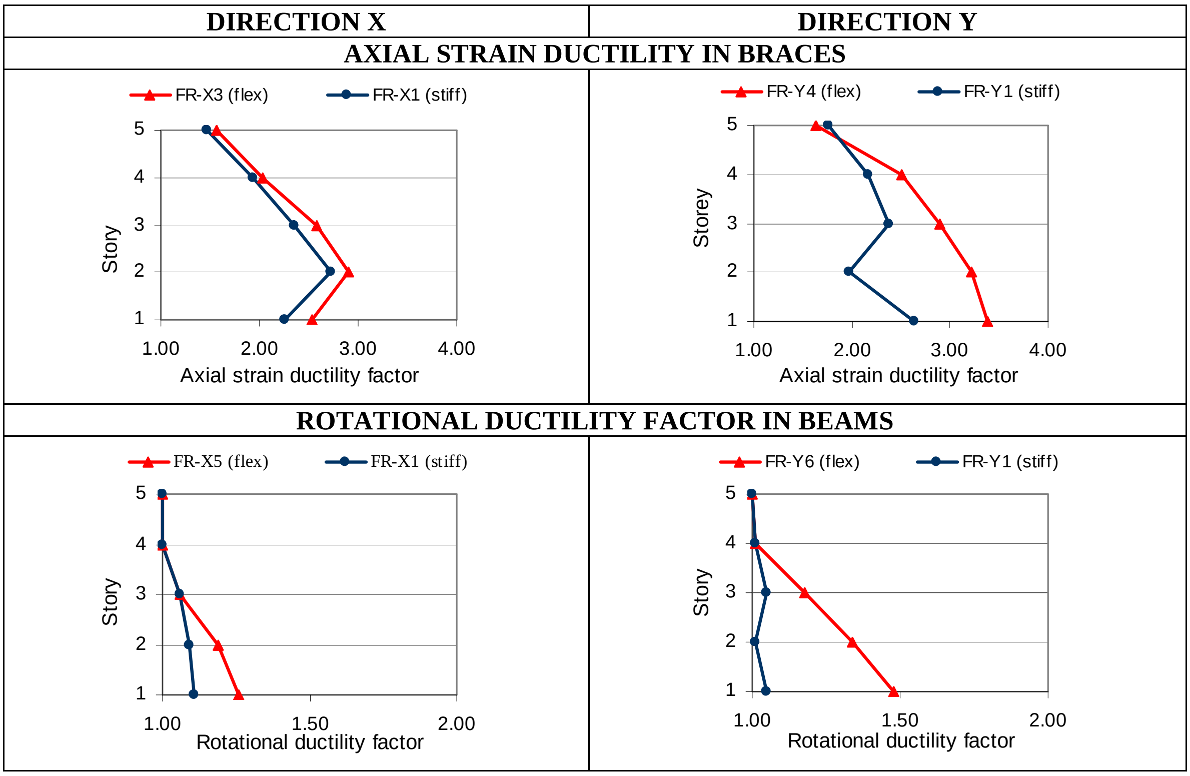


Figure 11: Member ductility demands of 5-story torsionally flexible building (FR-X5,X3 & FR-Y4,Y6: "flexible" sides, FR-X1 & FR-Y1: "stiff" sides).

5.2. Modification procedure and results for the torsionally flexible building

The modification procedure for the torsionally flexible building aim at increasing the stiffness and strength of the structural members (columns, beams and braces) at the "flexible" edges and reducing the stiffness and strength of the braces at the "stiff" edges without affecting the strength of the other structural elements (columns, beams). The ratios of the top story displacements $f_{i,\,flex}$ and $f_{i,stiff}$ (Eq. 6) are also used, but now the displacements are obtained by the equivalent static method for the seismic combinations considered. The design modification that was subsequently applied was to multiply the axial areas of the bracing members at both the "stiff" and "flexible" edges by the corresponding factors in each direction and to do the same for the beam and column sections, but **only** at the "flexible" edges. The cross sections of columns and beams of the "stiff" edges are not reduced, as their strength is controlled mainly by gravity loads. After this modification, each structure was checked again for full compliance with the applicable codes. This procedure gave modification factors of 0.88 and 0.79 for the braces in the stiff x and y sides, respectively and factors 1.12 and 1.21 for beams and columns in the "flexible" edges along the x and y directions, respectively. However, contrary to what happened in the torsionally stiff building, now the reduction in the brace sections at the stiff side proved excessive and these two factors were increased from 0.88 and 0.79 to 0.90 and 0.88, respectively, to satisfy all the code required checks. The new, modified structures were again subjected to the same two component motion earthquake set and their responses were again computed. Figures 14 and 15 show ductility demands for the torsionally flexible building for the initial and the modified designs. If we compare the results obtained from the initial and the modified designs, we see again some noticeable improvement of response: the overall maximum ductility demand factor in each group is reduced and so are the differences between "flexible" and "stiff" edges, producing more uniform distribution of such demands.

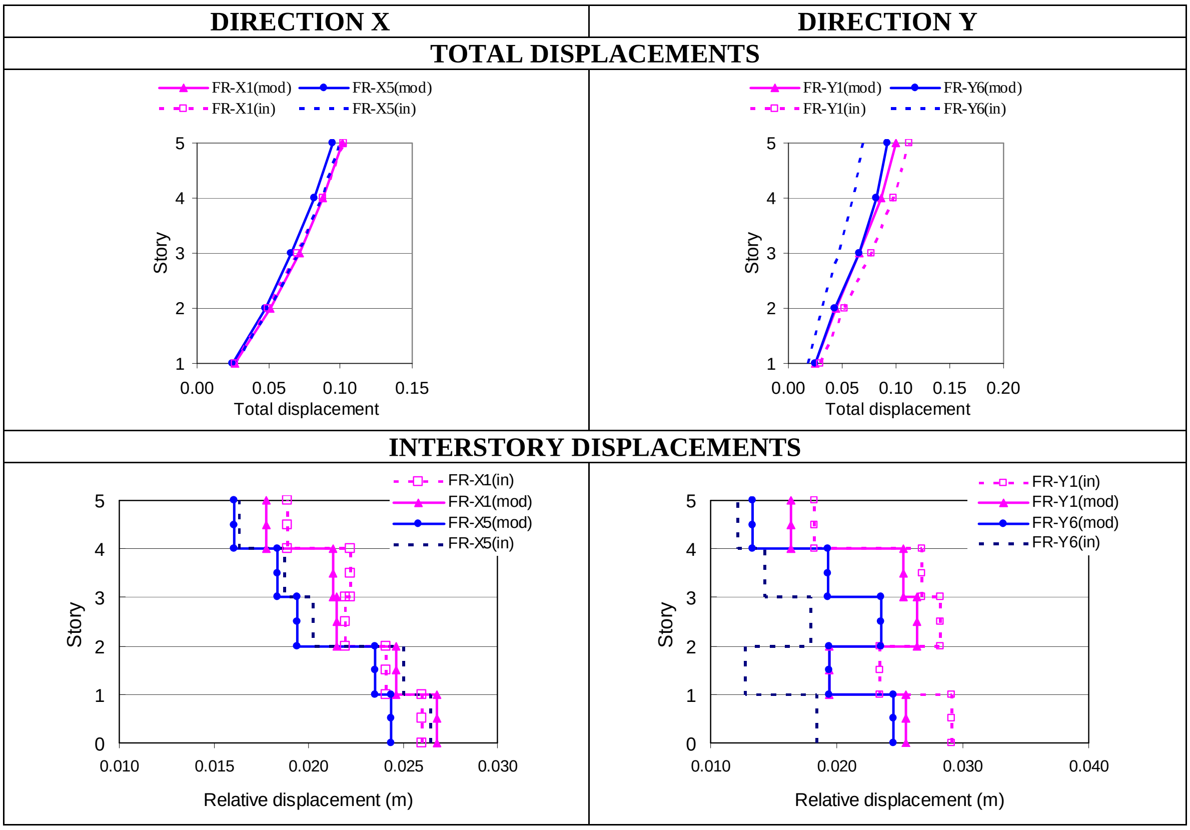


Figure 12: Comparison of total displacements and interstory drifts of 5-story torsionally stiff building, for the initial and modified design (FR-X1 & FR-Y1: "flexible" edges, FR-X5 & FR-Y6: "stiff" edges).

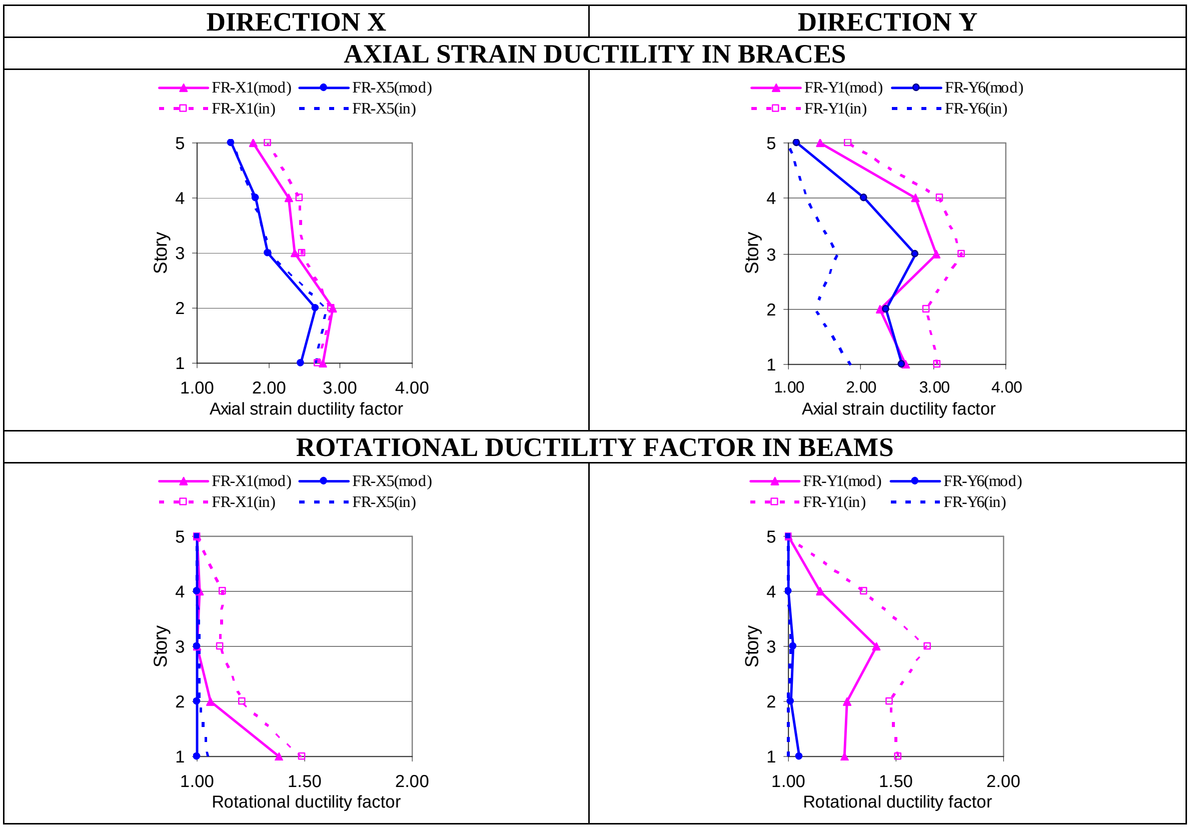


Figure 13: Comparison of ductility demands of 5-story torsionally stiff building, for the initial and modified design (FR-X1 & FR-Y1: "flexible" edges, FR-X5 & FR-Y6: "stiff" edges).

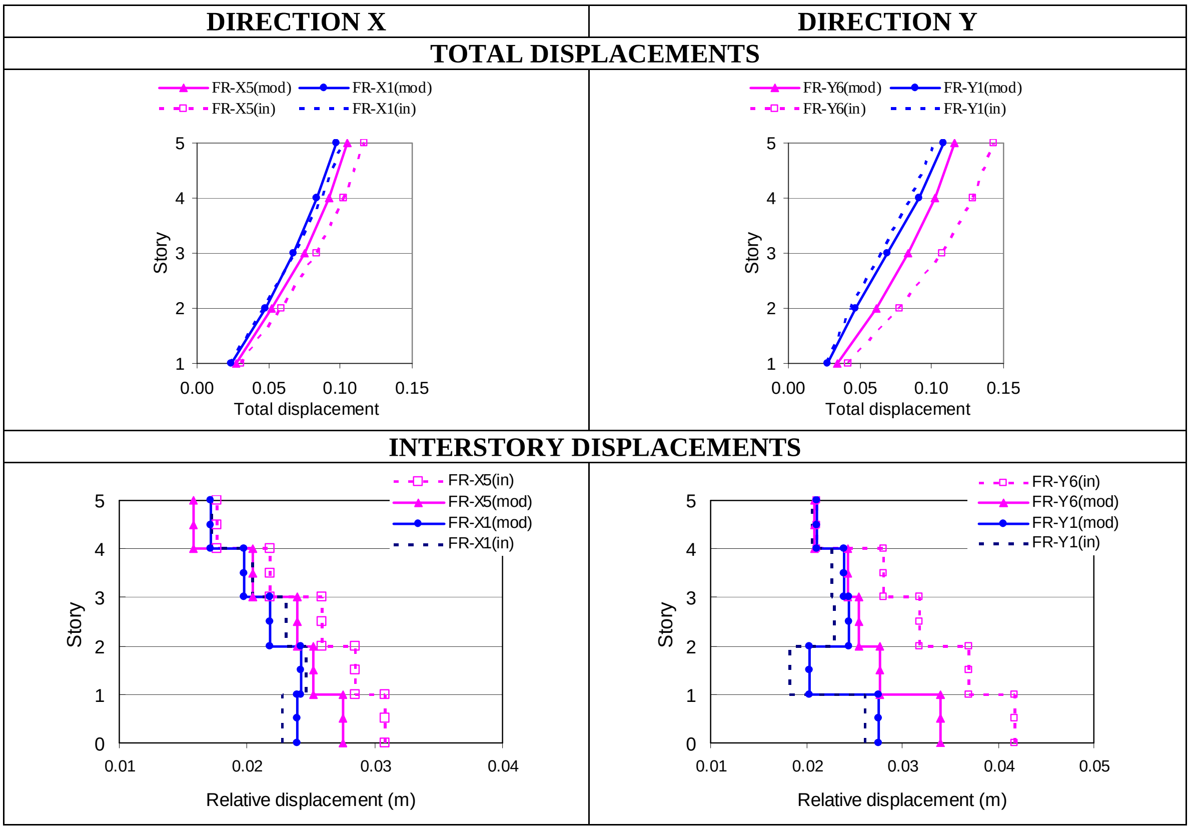


Figure 14: Comparison of total displacements and interstory drifts of 5-story torsionally flexible building, for the initial and modified design (FR-X5 & FR-Y6: “flexible” edges, FR-X1 & FR-Y1: “stiff” edges).

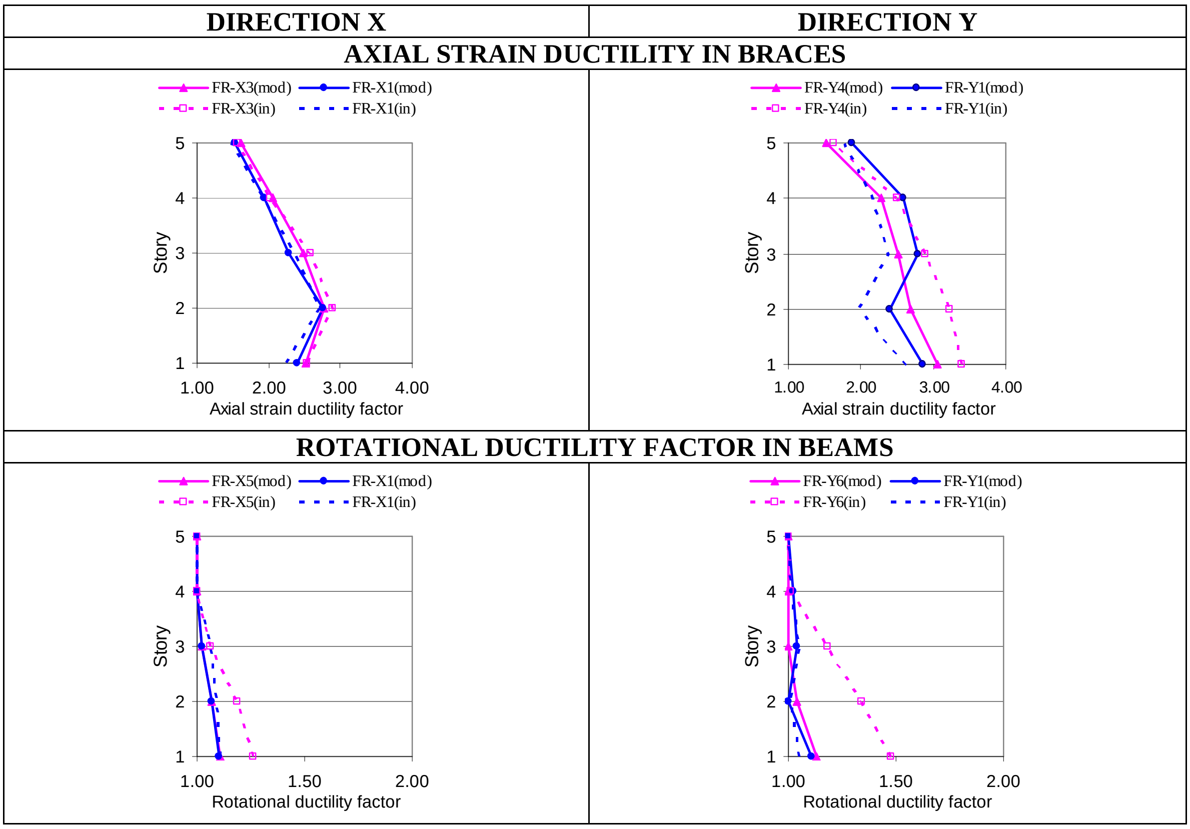


Figure 15: Comparison of ductility demands of 5-story torsionally flexible building, for the initial and modified design (FR-X3,X5 & FR-Y4,Y6: “flexible” sides, FR-X1 & FR-Y1: “stiff” sides).

6 COMMENTS ON THE PROPOSED MODIFICATION

To get more insight about the consequences of the proposed modification, the new stiffness centers of the modified buildings were computed using equations 1 and 2 and compared with those of the original designs. Results are presented in Table 1. We can see that the proposed modification brings the approximate stiffness center of each story closer to the mass center and thus the torsional motions are reduced. This reduction is obviously greater for the "flexible" edge and hence the reduction in the observed differences of ductility demands between "flexible" and "stiff" edges. We must note here that bringing the stiffness center as close as possible to the mass center, in other words trying to minimize the physical eccentricity, is a well known design objective in earthquake engineering as it minimizes torsional motion. The proposed modification is thus a "blind" way of achieving this without significant extra effort.

	MEAN NATURAL ECCENTRICITY			
	INITIAL DESIGN		MODIFIED DESIGN	
	ε_x	ε_y	ε_x	ε_y
5-st torsionally STIFF	0.10	0.10	0.04	0.03
5-st torsionally FLEXIBLE	0.07	0.10	0.03	0.06

Table 1: Mean natural eccentricities for the initial and modified designs of the 5-story buildings.

7 CONCLUSIONS

In the present paper the earthquake response of two irregular L shaped steel, braced frame buildings, one torsionally stiff and the other torsionally flexible, both designed in accordance with Eurocodes EC3 and EC8 was examined and similar, overall, results were obtained compared to earlier findings for eccentric rectangular, steel and reinforced concrete frame buildings. More specifically, it was found that under the action of two horizontal component earthquake loadings, compatible with the design spectra, ductility demands at the "flexible" edges were greater than ductility demands at the "stiff" edges. Subsequently, the original designs were modified using two slightly different procedures for each building and it was found that the response of the new designs was improved: ductility demands at the flexible edges generally decreased and the differences between the two sides diminished, so that a more uniform distribution of ductility demands was achieved.

On the basis of these findings, a code modification may appear desirable. However, additional studies covering other types of irregular buildings and a wider spectrum of parameters will be required, before any firm recommendation is put forward.

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