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PHENOMENOLOGICAL MODELING OF DEFORMATION OF POROUS AND CELLULAR MATERIALS TAKING INTO ACCOUNT THE INCREASE IN STIFFNESS BECAUSE OF THE COLLAPSE OF PORES

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Abstract. *Mathematical model for the description of deformation of a porous material with a random distribution of pores is constructed on the basis of generalized rheological method taking into account the different resistance of a material to tension and compression. Phenomenological parameters of the model are determined by means of the approximate computations for the problem of static loading of a cubic periodicity cell with spherical cavities. Within the framework of this theory the fields of displacements and stresses around expanding cavity of spherical shape in a homogenious space are constructed. It is shown that the porosity does not change at the stage of elastic deformation. When the pressure increases, a zone of plastic compaction is formed in the vicinity of a cavity, and in a part of this zone the collapse of pores takes place. Engineering formulas for calculation of critical pressures of the elastic limit state and the limit state of a medium with open pores, and also the formulas for determining the radiuses of interfaces of zones of plasticity and compaction are obtained. A complex of parallel programs for numerical analysis of the dynamic deformation of porous materials on multiprocessor computer systems is worked out. By means of this complex a series of computations for deformation of the protection elements, made of metal foams, under the action of localized impulsive loads was performed. The random nature of distribution of the pores size is taken into account with the help of computer modeling. Computations allow one to estimate plastic dissipative part of the energy of impact.*

1 INTRODUCTION

A problem of modeling of porous and cellular materials has applications in many technical areas. One of these areas is related to the usage of porous metals (metal foams). Porous metals are new artificial materials, which can be widely used in engineering because of their low density and good damping properties [1]. The ability of porous metals to absorb the energy during plastic deformation opens prospect of their use for the production of car bumpers and the so-called “collapsible zones”. They also can be applied in gearboxes and drives as destructible fuses, which dissipate the energy of dynamic impact, preventing the destruction of mechanical system.

At present the technologies of production of metal foams based on aluminum, copper, nickel, tin, zinc and other metals are well developed. According to the information published in Internet, professor A. Rabiei from the University of North Carolina (USA) in 2010 created a method of producing the most durable foam in the world. High strength of this material is achieved by ensuring that the surface of a thin-walled skeleton in the foam practically has no dislocations, i.e. defects that are initiators of the destruction. Extensive experimental researches of the mechanical properties of such materials are conducted. The diagrams of uniaxial tension and uniaxial compression on the example of an aluminum foam and of a porous copper were obtained in [2, 3]. The problems of durability and cyclic fatigue of porous metals are considered in [4], etc.

The main difficulties in mathematical modeling of the behavior of porous materials are related to the fact that their deformation properties are significantly different in tension and in compression, before and after the collapse of pores. In tension there are the stage of elastic deformation of a porous skeleton and the stage of plastic flow up to the fracture. In compression there are the stages of elastic and plastic deformation of a skeleton until the collapse of pores, and the subsequent stage of elastic or elastic-plastic deformation of a solid material without pores after the collapse. At small sizes of pores the collapse can occur on the elastic stage with the appearance of plasticity only under a sufficiently high level of load.

Theoretical questions of the constructing constitutive equations and of the analysis on this basis the spatial stress–strain state of structural elements of metal foams are poorly understood. At the level of physical and mechanical representations, the deformation of a metal foam is rather complex process. At high porosity the compression leads to elastic-plastic loss of stability of a metal skeleton, at low porosity the stable mechanism of collapse of pores is realized. The collapse is accompanied by a contact interaction of skeleton walls, which is difficult for modeling at the discrete level. Besides, it is necessary to consider the presence of compressed gas in closed pores. It is rather difficult to describe the process of shear when, according to experiments, the volume of a material changes. Even more difficult to construct a universal model of the spatial stress–strain state of a material under complex loading. The performance of adequate computations based on discrete models of a metal foam as a structurally inhomogeneous material is only possible with the use of multiprocessor systems with high productivity and large amount of random-access memory.

In this paper we construct a simple phenomenological model of a porous material, which takes into account the main qualitative and quantitative effects: a significant difference of diagrams of uniaxial deformation before and after the collapse of pores and a significant dissipation of energy at the stage of plastic flow of a material.

2 GOVERNING EQUATIONS

The porosity is defined as the ratio of the pore volume to the volume of a porous material: $\theta_0 = V_0/V$. If ρ denotes the density of a source (solid) metal then, ignoring the weight of the gas in the pores, the density of a porous metal can be calculated as follows: $\rho_0 = \rho\,(V - V_0)/V$. Therefore, $\theta_0 = (\rho - \rho_0)/\rho$. The volumetric strain of highly porous materials, caused by changing the volume of pores, is significantly higher than the volumetric strain of solid skeleton, hence, the pores disappear with the volumetric strain $\theta \approx ((V - V_0) - V))/V = -\theta_0$.

Rheological models of uniaxial stress–strain state of porous materials are constructed in [5]. A simple rheological scheme taking into account the main features of deformation of porous metals is shown in Fig. 1 *a*. The behavior of a material in tension and in compression before the collapse of pores is simulated by an elastic spring with the compliance modulus a, and the increase in stiffness after collapse is simulated by an additional spring with the compliance modulus b. A diagram of uniaxial tension–compression of a porous material is represented in Fig. 1 *b* as a two-segment broken line. Such scheme describes the elastic process that occurs without dissipation of mechanical energy.

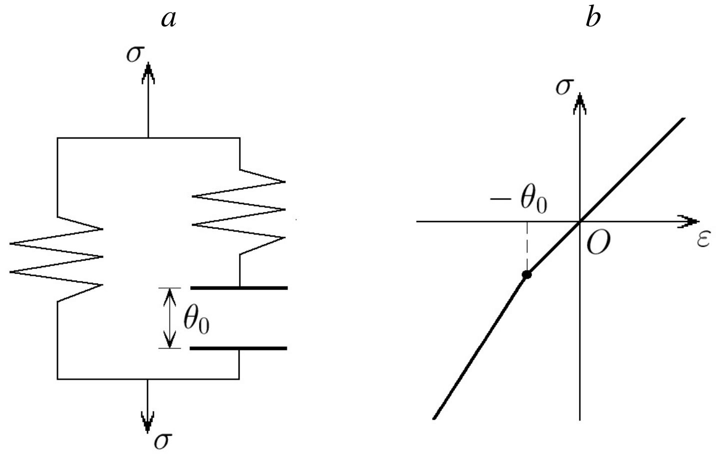

Figure 1: Rheological scheme (*a*) and diagram (*b*) of uniaxial elastic deformation.

More general rheological scheme with a plastic hinge is shown in Fig. 2 *a*. It is assumed that under the tensile stress σ_s^+ a sceleton goes into the state of plastic flow, and under the compressive stress $-\sigma_s^-$ the plastic loss of stability takes place. The stage of elastic-plastic deformation of a solid material after the collapse of pores is described by the rheological scheme of linear hardening. A diagram of uniaxial tension–compression is represented in Fig. 2 *b* as a four-segment broken line. In this model the specific dissipation of energy in the process of collapse can be evaluated by the product $\sigma_s^-\theta_0$.

In the case of spatial stress–strain state in accordance with rheological scheme in Fig. 2 the stress tensor σ is equal to the sum of the tensor σ^p of plastic stresses and the tensor σ^c of additional stresses, acting after the collapse of pores. Elastic compliance of a material is characterized by the fourth-rank tensors a and b, satisfying the usual conditions of symmetry and positive definiteness. Series connection of an elastic spring and a plastic hinge in the scheme corresponds to the Prandtl–Reuss theory of elastic-plastic flow. Constitutive relationships of this theory can be represented in the form of principle of maximum of the energy dissipation rate [5]:

$$(\tilde\sigma - \sigma^p) : (a : \dot\sigma^p - \dot\varepsilon) \ge 0, \qquad \tilde\sigma,\ \sigma^p \in F. \tag{1}$$

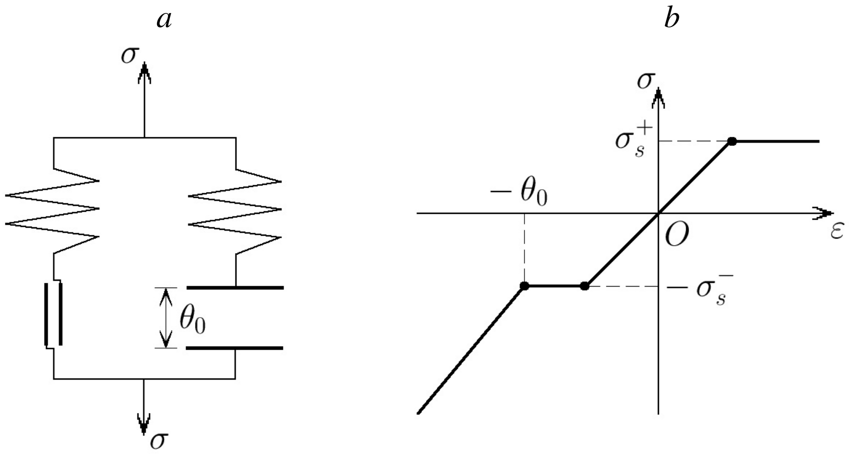

Figure 2: Rheological scheme (*a*) and diagram (*b*) of elastic-plastic deformation.

Here ε is the actual strain tensor, $\tilde{\sigma}$ is an arbitrary admissible variation of the stress, F is the convex set in the stress space, limited by the yield surface of a material. The conventional notations and operations of the tensor analysis are used: a colon denotes double convolution of tensors, a dot over a symbol denotes a derivative with respect to time.

The set F of admissible stresses is defined by means of the Tresca–Saint-Venant or von Mises yield condition. Assuming that the deformation of jumpers of a porous skeleton can be described by the rods model, this set can be approximated as follows:

$$F = \Big\{\sigma \;\Big|\; -\sigma_s^- \le \sigma_k \le \sigma_s^+,\;\; k = 1,2,3\Big\},$$

where σ_k are the principal values of the tensor σ.

Constitutive relationships of a rigid contact are formulated in the form of variational inequality [5]:

$$(\tilde{\sigma} - \sigma^c) : (\varepsilon^c + \varepsilon^0) \le 0, \qquad \tilde{\sigma},\; \sigma^c \in K. \tag{2}$$

Here $\varepsilon^c = \varepsilon - b : \sigma^c$ is the strain tensor of a porous skeleton, $\varepsilon^0 = \theta_0\,\delta/3$ is the spherical tensor of initial porosity (δ is the Kronecker delta), K is the convex cone in the stress space, which serves for modeling the transition from a porous state to a solid state of a material. Further K is the von Mises–Schleicher circular cone:

$$K = \Big\{\sigma \;\Big|\; \tau(\sigma) \le æ\, p(\sigma)\Big\},$$

where æ is the phenomenological parameter of dilatancy, $p(\sigma) = -\sigma : \delta/3$ is the hydrostatic pressure, $\tau(\sigma)$ is the intensity of tangential stresses, calculated via the deviator $\sigma' = \sigma + p(\sigma)\delta$ of stress tensor by the formula $\tau^2(\sigma) = \sigma' : \sigma'/2$.

The variational inequality (2) can be reduced to the next form:

$$(\tilde{\sigma} - \sigma^c) : b : (\sigma^c - s) \ge 0, \qquad \tilde{\sigma},\; \sigma^c \in K. \tag{3}$$

Here s is the conditional stress tensor, which is calculated by the law of linear elasticity taking into account initial strains, namely $b : s = \varepsilon + \varepsilon^0$. If this tensor is admissible, i.e. if $s \in K$, then by (3) the tensor σ^c is equal to s. If s is not admissible ($s \notin K$) and for any $\tilde{\sigma} \in K$ the inequality $\tilde{\sigma} : b : s \le 0$ is valid, which means that the sum $\varepsilon + \varepsilon^0$ of tensors belongs to the cone C of admissible strains, conjugate to K:

$$C = \Big\{\varepsilon \;\Big|\; \tilde{\sigma} : \varepsilon \le 0 \quad \forall\; \tilde{\sigma} \in K\Big\},$$

then $\sigma^c = 0$, as follows from (3). In the general case the variational inequality (3) allows to determine the tensor $\sigma^c = \pi_K(s)$ as the projection of s onto the cone K with respect to the Euclidean norm $|s| = \sqrt{s : b : s}$, and the considered above two variants are particular cases, when the projection coincides with the original tensor and when the projection is the vertex of the cone.

Third variant, when the projection belongs to the conical surface, is realized under the fulfillment of two conditions: $s \notin K$ and $\varepsilon + \varepsilon^0 \notin C$. For an isotropic material the elastic compliance tensor b is characterized by two independent parameters: the bulk modulus k and the shear modulus μ. Formulas for calculating the projection onto the conical surface are as follows [5]:

$$p(\sigma^c) = \frac{\mu\, p(s) + æ\, k\, \tau(s)}{\mu + æ^2 k}, \qquad (\sigma^c)' = æ\, p(s)\, \frac{s'}{\tau(s)}. \tag{4}$$

The cone C, conjugate to the von Mises–Schleicher cone, is defined as

$$C = \Big\{ \varepsilon \;\Big|\; æ\, \gamma(\varepsilon) \le \theta(\varepsilon) \Big\},$$

where $\theta(\varepsilon) = \varepsilon : \delta$ is the volumetric strain, $\gamma(\varepsilon) = \sqrt{2\, \varepsilon' : \varepsilon'}$ is the shear intensity. The condition $\varepsilon + \varepsilon^0 \in C$ means that a rigid contact in rheological scheme is open, i.e. that pores are in the open state. The limit condition of the collapse of pores $æ\, \gamma(\varepsilon) = \theta_0 + \theta(\varepsilon)$ describes the dilatancy of a porous material due to the shear deformation.

3 MATHEMATICAL MODEL

Mathematical model, describing the dynamic deformation of a porous material at small strains and rotations of elements, can be written in the form of the following system:

$$\begin{aligned} &\rho_0\, \dot v = \nabla \cdot \sigma + \rho_0\, f, \\ &(\tilde\sigma - \sigma^p) : (a : \dot\sigma^p - \nabla v) \ge 0, \qquad \tilde\sigma,\ \sigma^p \in F, \\ &b : \dot s = \frac{1}{2}\, (\nabla v + \nabla v^*), \qquad \sigma = \sigma^p + \pi_K(s). \end{aligned} \tag{5}$$

Here v is the velocity vector, f is the vector of body forces, ∇ is the vector of gradient with respect to spatial variables, the asterisk denotes transposition. This system consists of the equation of motion in vector form and the constitutive relationships, which follow from (1) and (2) taking into account the kinematic equation $2\, \dot\varepsilon = \nabla v + \nabla v^*$. The vector v and the tensors σ^p and s are unknown functions in this model.

Assuming $\tilde\sigma = 0$, one can derive from (5) the inequality of internal dissipation of energy:

$$\frac{1}{2}\, \frac{d}{dt} \Big(\rho_0 |v|^2 + \sigma^p : a : \sigma^p + \pi_K(s) : b : \pi_K(s) \Big) \le \nabla \cdot (\sigma \cdot v) + \rho_0\, f \cdot v.$$

The left-hand side of this inequality is the rate of change of kinetic and potential energy, the right-hand side is the power of internal surface forces and active body forces. The difference between right-hand side and left-hand side is equal to the power of plastic dissipation of energy, which can not be negative in accordance with the principles of irreversible thermodynamics.

The initial conditions, describing the natural (unstressed) state of a material, are as follows:

$$v\,\Big|_{t=0} = 0, \qquad \sigma^p\,\Big|_{t=0} = 0, \qquad s\,\Big|_{t=0} = b^{-1} : \varepsilon^0. \tag{6}$$

The boundary conditions can be given in velocities or stresses:

$$v\Big|_{\Gamma} = v^0(x), \qquad \sigma\Big|_{\Gamma}\cdot\nu(x) = q(x),$$

where ν is the normal vector to the boundary, v^0 and q are given functions. Mathematical correctness of the boundary conditions follows from the integral estimates of the difference between two solutions. Let us assume that in some space–time domain two sufficiently smooth solution v, σ^p, s and $\bar{v}$, $\bar{\sigma}^p$, $\bar{s}$ of the system (5) are defined. Substituting $\tilde{\sigma} = \bar{\sigma}^p$ in the variational inequality of (5) and $\tilde{\sigma} = \sigma^p$ in a similar inequality, characterizing the second solution, after summation of the results one can obtain

$$(\bar{\sigma}^p - \sigma^p) : a : (\dot{\bar{\sigma}}^p - \dot{\sigma}^p) \le (\bar{\sigma}^p - \sigma^p) : \nabla(\bar{v} - v).$$

With the help of a standard procedure of scalar multiplication of the equations of motion by the velocity vectors it can be shown that

$$\begin{aligned} \rho_0\,(\bar{v} - v)\cdot(\dot{\bar{v}} - \dot{v}) + (\bar{\sigma}^p - \sigma^p) : a : (\dot{\bar{\sigma}}^p - \dot{\sigma}^p) + \\ + \Big(\pi_K(\bar{s}) - \pi_K(s)\Big) : b : (\dot{\bar{s}} - \dot{s}) \le \nabla\cdot\Big((\bar{\sigma} - \sigma)\cdot(\bar{v} - v)\Big). \end{aligned} \tag{7}$$

As a result of integration of the inequality (7) over the domain of the type of a truncated cone, a priori estimates are obtained, guaranteeing the uniqueness of solution and the continuous dependence on the initial data for the Cauchy problem and for the boundary-value problems with dissipative boundary conditions. For a more general situation such estimates were obtained in the monograph [5]. Dissipativity of boundary conditions in this case means that in each point of the boundary the inequality

$$(\bar{\sigma} - \sigma)\cdot\nu(x)\cdot(\bar{v} - v) \le 0$$

is fulfilled. This inequality holds automatically for the mentioned above main types of boundary conditions.

4 RADIAL EXPANSION OF CAVITIES

The problem of expansion of a cavity in an infinite medium under the action of internal pressure is of great practical importance for applications on different scale levels in aerospace, marine and oil industries, in geomechanics and geodynamics. In classical formulation for isotropic and homogeneous continuum this problem was solved in [6, 7]. The problem for a porous medium has particular importance in the petroleum geophysics. Increasing the pressure in the well leads to the compression and collapse of pores, which impedes the filtration flow near the surface of the well until complete closing of hydrocarbons in a rock mass. In practice, it is important to estimate the size of the blocking zone as a function of pressure for given mechanical parameters of a geomaterial.

Let us describe the expansion of a cavity of the radius r_0 in an infinite porous space under the action of slowly increasing pressure p_0 in the framework of suggested model. In the case of spherical symmetry at elastic stage of the process the following system of equations is valid:

$$\begin{gathered} \frac{d\sigma_r}{dr} + 2\,\frac{\sigma_r - \sigma_\varphi}{r} = 0, \qquad \varepsilon_r = \frac{du_r}{dr}, \qquad \varepsilon_\varphi = \frac{u_r}{r}, \\ \sigma_r + 2\,\sigma_\varphi = 3\,k\,(\varepsilon_r + 2\,\varepsilon_\varphi), \qquad \sigma_\varphi - \sigma_r = 2\,\mu\,(\varepsilon_\varphi - \varepsilon_r). \end{gathered} \tag{8}$$

The solution of (8), satisfying the boundary condition on the surface of a cavity, is given by the formulas:

$$\sigma_r = -2\,\sigma_\varphi = -p_0\Big(\frac{r_0}{r}\Big)^3, \qquad u_r = \frac{p_0\,r_0^3}{4\,\mu\,r^2}. \tag{9}$$

The elastic state is realized up to the limit pressure $p_s = 4\,\tau_s/3$, determined by the Treska–Saint-Venant yield condition (τ_s is the yield point of a porous material under shear):

$$\sigma_\varphi - \sigma_r = 2\,\tau_s. \tag{10}$$

The plastic stage is described by a system which is obtained from (8) by replacing the last equation on the equation (10). From this system one can find

$$\sigma_r = -p_0 + 4\,\tau_s \ln\frac{r}{r_0}, \qquad \sigma_\varphi = 2\,\tau_s + \sigma_r. \tag{11}$$

The displacement in a plastic zone is determined from the equation of elastic change in volume

$$\frac{k}{r^2}\,\frac{d(r^2\,u_r)}{dr} = \frac{\sigma_r + 2\,\sigma_\varphi}{3} = \sigma_r + p_s,$$

the integration of which gives

$$3\,k\,u_r = -p_0\,r + 4\,\tau_s\,r \ln\frac{r}{r_0} + \frac{C_1}{r^2}. \tag{12}$$

At the interface between a zone of plastic deformation and an elastic zone the conditions of continuity of radial stress and radial displacement, and the additional condition of limiting elastic state are fulfilled. The solution in an elastic zone is given by the formulas (9), in which the radius r_0 of a cavity is replaced by the radius r_s of a plastic zone, and the acting pressure p_0 is replaced by the pressure p_s. So, we have

$$r_s = r_0 \exp\frac{p_0 - p_s}{4\,\tau_s}, \qquad C_1 = \Big(\frac{k}{\mu} + \frac{4}{3}\Big)\tau_s\,r_s^3.$$

The expression for the displacement allows to determine the total strain in a plastic zone up to the moment of the pores collapse. Via the stresses (11) by means of Hooke's law one can find the elastic components ε_j^e of strains ($j = r, \varphi$), and then their plastic components $\varepsilon_j^p = \varepsilon_j - \varepsilon_j^e$. The critical pressure $p_c = p_s + k\,\theta_0$ of the collapse of pores on the surface of a cavity is calculated by the equality $\theta = -\theta_0$. This equality describes the collapse in the case of $æ = 0$, when a porous material has not the dilatancy.

Further increase in pressure leads to the formation near the cavity of a zone of compressed, non-porous material ($r < r_c$) within a zone of plastic compaction. According to the constitutive relationships (2), in this zone $\theta(\varepsilon^c) = -\theta_0$ and $\sigma_r^c = \sigma_\varphi^c = -q \le 0$. Thus, the total stresses in a zone of collapse are calculated via the stresses σ_j^p in a plastic element as $\sigma_r = \sigma_r^p - q$, $\sigma_\varphi = \sigma_\varphi^p - q$, where q can be interpreted as the additional pressure caused by the collapse.

Since $\sigma_\varphi - \sigma_r = \sigma_\varphi^p - \sigma_r^p$, then the continuity of radial stress in a plastic zone leads to the continuity of stresses σ_r and σ_φ, hence, the stresses in a whole space are defined by the formulas (11). The difference is only in formulas for strains. According to the rheological scheme, in a zone of collapse $\theta = -\theta_0 - q/k_0$, where k_0 is the bulk modulus of an elastic spring, which is in series connection with a rigid contact. On the other hand,

$$\theta = \frac{\sigma_r^p + 2\,\sigma_\varphi^p}{3\,k} = \frac{\sigma_r + 2\,\sigma_\varphi + 3\,q}{3\,k}.$$

Hence,

$$\Big(\frac{1}{k}+\frac{1}{k_0}\Big)q=-\theta_0-\frac{\sigma_r+2\,\sigma_\varphi}{3\,k}.$$

The volumetric strain satisfies the equation

$$\frac{k+k_0}{r^2}\,\frac{d(r^2\,u_r)}{dr}=-k_0\,\theta_0-p_0+p_s+4\,\tau_s\ln\frac{r}{r_0},$$

the integration of which gives

$$3\,(k+k_0)\,u_r=-(p_0+k_0\,\theta_0)\,r+4\,\tau_s\,r\ln\frac{r}{r_0}+\frac{C_2}{r^2}. \tag{13}$$

The constant C_2 and the radius r_c of a zone of collapse are obtained from the condition of continuity of displacement and from the condition $\theta=-\theta_0$ of collapse on the boundary of a plastic zone. The last condition gives

$$-k\,\theta_0=-p_0+p_s+4\,\tau_s\ln\frac{r_c}{r_0},\qquad r_c=r_0\exp\frac{p_0-p_c}{4\,\tau_s}.$$

The displacement in a plastic zone can be found from (12) after the replacing p_0 by p_c and r_0 by r_c. In a zone of collapse of pores the equation (13) is fulfilled. From the condition of continuity of u_r on $r=r_c$ it follows that

$$k\,C_2=-k_0\,p_s\,r_c^3+(k+k_0)\,C_1.$$

The equation (13) allows to find reversible and irreversible components of strains in this zone.

Typical graphs of distribution of the residual stresses and displacement around a spherical cavity of the radius $r_0=0.01$ m in an infinite porous medium after the increasing pressure to $p_0=0.3$ GPa and the complete unloading, in accordance with the elastic law, are shown in Fig. 3. Phenomenological parameters of a material were taken close to the parameters of a porous (cellular) aluminum [8]: $k=2$, $k_0=70$, $\mu=2.5$, $\tau_s=0.03$ GPa. According to the exact solution, the residual stresses σ_r and σ_φ, which correspond to the curves 1 and 2 in Fig. 3 *a*, do not depend on the initial porosity. Three graphs of the residual displacement in Fig. 3 *b* were obtained for different values of the porosity $\theta_0=0$, $\theta_0=1$ and $\theta_0=10\,\%$ (curves 1, 2 and 3) at equal other parameters.

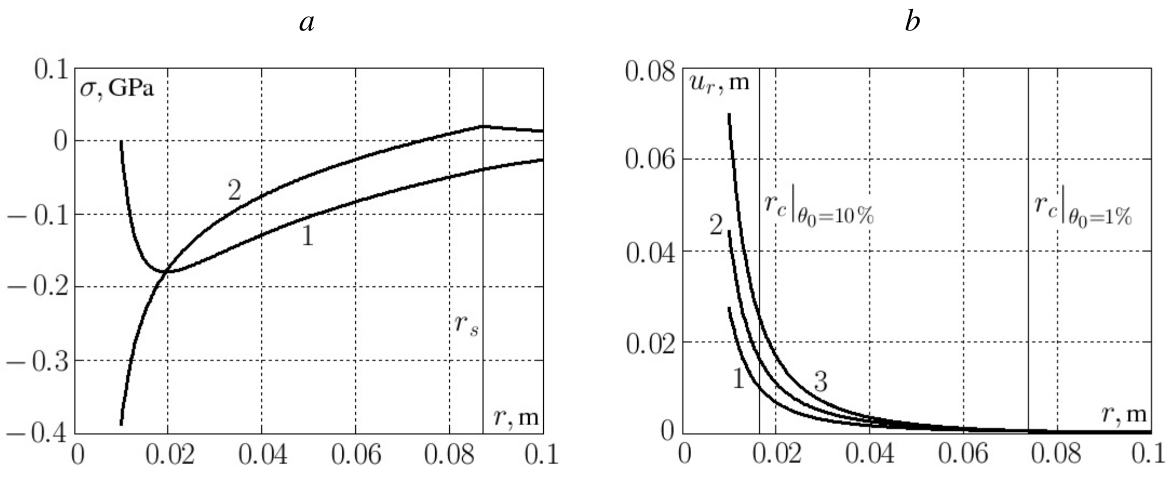

Figure 3: Residual stresses (*a*) and displacement (*b*) around a spherical cavity.

Similarly we obtain the solution of the problem of expansion of a cylindrical cavity in an infinite porous medium. It differs from the solution for a spherical cavity, because in the case of cylindrical symmetry after the appearance of a plastic zone of incomplete plasticity at the neighbourhood of a cavity appears a zone of full plasticity. With increasing pressure a zone of the collapse of pores can occupy some part of a zone of full plasticity or all points of this zone with some part of a zone of incomplete plasticity.

5 COMPUTATIONAL ALGORITHM

Within the framework of proposed mathematical model the parallel computational algorithm was worked out for numerical solution of the problems of dynamic deformation of porous materials. The system (5) is solved by means of the splitting method with respect to spatial variables. Variational inequality is solved by the splitting with respect to physical processes, which leads to a special procedure of solution correction in each node of the computational domain at each time step.

The technology of parallelization of computational algorithm is based on the method of two-cyclic splitting with respect to spatial variables [9]. For the solution of 1D systems of equations an explicit monotone finite-difference ENO–scheme of the "predictor–corrector" type with piecewise-linear distributions of velocities and stresses over meshes, based on the principles of grid-characteristic methods [10], is applied. The parallelizing of computations is carried out using the MPI library, the programming language is Fortran. The data exchange between processors occurs at step "predictor" of the finite-difference scheme by means of the function MPI_Sendrecv. At first each processor exchanges with neighboring processors the boundary values of their data, and then calculates the required quantities in accordance with the explicit finite-difference scheme. Mathematical models are embedded in programs by means of software modules that implement the constitutive relationships, the initial data and boundary conditions of problems. The universality of programs is achieved by a special packing of the variables, used at each node of the cluster, into large one-dimensional arrays. Computational domain is distributed between the cluster nodes by means of 1D, 2D or 3D decomposition so as to load the nodes uniformly and to minimize the number of passing data. Detailed description of the parallel algorithm one can found in [5].

Using this computational algorithm, we worked out the parallel program systems 2Dyn_Granular [11] and 3Dyn_Granular [12] for numerical solution of two-dimensional and three-dimensional dynamic problems, which can be applied particularly for modeling porous materials.

6 CONCLUSIONS

- Mathematical model of a spatial stress–strain state of a porous medium, taking into account the increase in stiffness of a material at the moment of the collapse of pores and the plastic energy dissipation during the deformation of a porous skeleton, was worked out.

- For the problem of expansion of a spherical cavity in an infinite porous medium under the action of internal pressure the exact solution was obtained, which allows to estimate the size of a zone of collapse and can be used for testing and verification of numerical algorithms and computer programs.

- On the basis of this model a software was worked out for multiprocessor computers of the cluster architecture. The developed parallel program system is intended for solving the problems of production of damping elements of constructions, made of porous metals.

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