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REAL-TIME ERROR CORRECTION IN FAST PSEUDODYNAMIC TESTING: A NUMERICAL STUDY

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Abstract. *Undershooting error is a common error in fast pseudodynamic (PSD) tests as a result of inherent actuator lag. This type of systematic error introduces energy into a system, which if not monitored or controlled, can result in test instability and causes the response to grow exponentially. This leads to premature termination of a test and unreliable result. By considering the systematic error as negative damping in a structure, this paper proposes an intuitive error-correction scheme to compensate for the error. The key to this scheme is the introduction of a variable amount of viscous damping at every integration time step during the pseudodynamic test. The amount of time-varying viscous damping is derived by equating the magnitude of the energy error with that dissipated by the introduced damping at every integration time step. The accuracy and simplicity of the proposed error-correction scheme is demonstrated through numerical simulations of a linear-elastic SDOF structure and a two DOF structure subjected to sinusoidal ground motions.*

1 INTRODUCTION

The fast PSD test method is an improvement to the conventional PSD test method as it is able to directly replicate strain-rate dependent material effects and rate-sensitive behaviours [1]. A conventional PSD test numerically models the inertia and damping effects of a structure, allowing otherwise very large dynamic forces on the physical specimen to be applied at a slower rate or even pseudo-statically. A critical prerequisite of a conventional PSD test is that the effect of loading rate on the restoring force of the structure should be secondary. Although this assumption is generally valid for most conventional construction types, this is not the case for many new structural systems. These systems, such as structures with visco-elastic dampers and base-isolated structures, are highly rate dependent [2]. Thus, conventional PSD test is not suitable for simulating earthquake loads on such structures.

In a fast PSD test the specimen is loaded at or near real-time [3]. For example, if an earthquake event is 40 seconds long, then a fast PSD test may take 40 seconds of wall time to perform in real time, or 400 seconds at a 1/10 time scale. The fast PSD test imposes greater challenge in generating error-minimum test results, as every cycle in the test must be completed within a short interval (in the order of milliseconds). Each cycle of a PSD test involves 1) measuring the restoring force, 2) completing the necessary numerical integration, 3) data communication between the computer hosting the calculation and the computer controlling the actuator, and 4) moving the specimen to the target displacement using the actuator. Research shows minimising the errors in PSD tests, even for conventional PSD tests, is essential in ensuring reliable test results [4].

This paper presents a numerical simulation of a PSD test utilising a new intuitive compensation scheme, to minimise the effect of actuator control error in a fast PSD test where inaccurate displacements are applied to the specimen.

2 EXPERIMENTAL ERRORS IN PSD TEST

The PSD test method is a reliable alternative to the shake table test method provided errors are properly mitigated [1]. Experimental errors associated with inaccurate laboratory equipment control or inaccurate measurement should be given special attention as experimental data is intimately used in the step-by-step integration procedure. The effect of any small error in a single time step is compounded at each subsequent test cycle. Furthermore, if the errors are systematic in nature and occur at each time step, it could lead to a significant divergence from the true solution due to error accumulation from the large number of computation steps. Experimental errors can further be classified into random and systematic errors. Systematic errors have a regular pattern of occurrence and they have a greater detrimental effect on the accuracy of the solution and the stability of the solution process when compared to random errors. Random errors are irregular by nature and no specific effects on the solution can be anticipated [4, 5].

2.1 Systematic Displacement Control Error

One common and significant form of systematic errors is displacement control error. This occurs when the displacements applied to a specimen are systematically greater or smaller than the target displacements at the time when force measurements are required by the solution algorithm. Displacement control error commonly occurs to actuators under simple closed loop control and it can be a result of mis-calibration, poor control loop tuning, and truncation error in digital to analogue conversion amongst other causes. Displacement control error leads to one of two conditions, overshooting or undershooting. Overshooting occurs when the incremental displacement applied by the actuator is larger than the incremental displacement target. Conversely, undershooting occurs when the incremental displacement applied by the actuator is less than the incremental displacement target. These conditions affect the energy balance in the system and affect the result of PSD tests [5, 6]

2.2 Effect of actuator delay

Undershooting error is more common in a fast PSD test due to the dynamic interactions between the structure and the test apparatus (servo-controller, servo valve and the hydraulic actuator). Due to these interactions, a time delay exists between when the displacement is commanded and when the specimen actually reaching the target displacement [7].

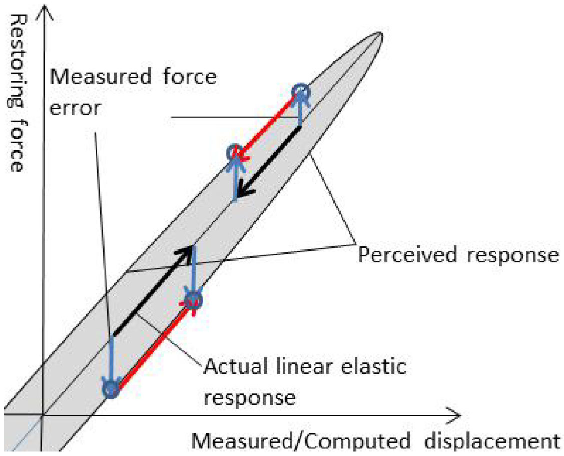


Figure 1: Idealised force-displacement response of systems with undershooting error

In a PSD test, the numerical integration algorithm calculates the time history response using the measured restoring forces from the test specimen and commanded (computed) displacements. This leads to an interesting conundrum as the commanded and measured displacements are not necessarily the same due to the presence of errors. Figure 1 shows a perceived force-deformation relationship of a linear-elastic structure where consistent undershooting errors are present. This results in the measured force lagging behind the command displacement. These errors can be thought as vertical departures of the restoring forces from the straight line. As a result, the system follows a counter-clockwise oval-shaped hysteretic response instead of the true linear response as shown by the bold straight line. This behaviour adds energy into the test specimen, represented by the shaded area of the plot [8]. Research by Horiuchi et al. [9] established that the effect of this response delay is equivalent to introducing negative damping to the system. Tests can become unstable if the negative damping is larger than the inherent structural damping. The paper also presented a method to negate this delay effect by using a simple polynomial extrapolation to predict displacement several time steps ahead to account for actuator delay, using computed displacements at current and several time steps before.

3 INTUITIVE ERROR COMPENSATION UTILIZING VARIABLE VISCOUS DAMPING

It is important that the negative damping from actuator delay is properly compensated to avoid instability and unintentional damage to the structures and the equipment. The proposed compensation scheme aims to dissipate the energy introduced into the structure by the delay error through additional viscous damping proportional to the amount of energy error introduced at every time step. The proposed scheme utilize previous research which has mathematically quantified the amount of energy error [6, 8].

3.1 Review of Hybrid Simulation Error Monitoring (HSEM)

The change in mechanical energy content in a dynamic structural system is the sum of the work done by the inertial, dissipative and elastic restoring components. This balances with the work done by the external force in lieu of any errors. This is expressed mathematically as Equation 1. This assumes the structural system is initially at rest and the system is elastic.

$$\int(\boldsymbol{M}\ddot{u})^T\ du + \int(\boldsymbol{C}\dot{u})^T\ du + \int \boldsymbol{R}^T\, du = \int \boldsymbol{F}^T\ du \qquad (1)$$

where terms have their usual meaning.

The third term in Equation 1 represents the strain energy content. In a PSD test, this quantity depends on the measured restoring force from the test specimen. Any displacement control error will result in erroneous displacement and hence erroneous measured restoring force. This distorts the energy balance in the system described by Equation 1.

Mosqueda et al. [8] proposed an approach to calculate the energy error at each step based on the difference between the actual strain energy stored in the specimen and the strain energy in the specimen as perceived by the numerical integration process. Consider the system at time step *i*, the energy error is mathematically expressed as

$$\boldsymbol{E}_i^{er} = \boldsymbol{E}_i^{E} - \boldsymbol{E}_i^{BE} \qquad (2)$$

where $\boldsymbol{E}_i^E$ is the vector of the change in strain energy across a time step in the deformed specimen as perceived by the numerical integration, while the actual change in strain energy in the specimen is given by $\boldsymbol{E}_i^{BE}$. $\boldsymbol{E}_i^E$ can be approximated using trapezoid rule, as

$$\boldsymbol{E}_i^E = \frac{1}{2}(\boldsymbol{R}_{i-1}^m + \boldsymbol{R}_i^m)^T(u_i^c - u_{i-1}^c) \qquad (3)$$

and $\boldsymbol{E}_i^{BE}$ can be approximated as

$$\boldsymbol{E}_i^{BE} = \frac{1}{2}(\boldsymbol{R}_{i-1}^m + \boldsymbol{R}_i^m)^T(u_i^m - u_{i-1}^m) \qquad (4)$$

In Equation 3 and 4, $\boldsymbol{R}$ and $\boldsymbol{u}$ represents the restoring force and displacements respectively, while the superscripts "*m*" and "*c*" denote "measured" and "computed" quantities respectively. Using the above expressions, Mosqueda et al. [8] proposed a means to monitor in real time, the accumulation of energy error in a PSD test to determine the reliability of the test. Hybrid Simulation Error Monitoring (HSEM) provides a timely warning when the accumulation of energy error in a PSD test exceeds a certain threshold, when the final result of the test would no longer be reliable and the test should be terminated and continued only after corrective measures are taken. Furthermore, this prevents damage to the specimen and/or equipment.

3.2 Calculating the artificial viscous damping

The amount of energy dissipated by viscous damping mechanism is given by the second term on the left-hand side of Equation 1. Using trapezoid rule, this integration can be approximated as

$$\boldsymbol{E}_i^D = \frac{1}{2}(\boldsymbol{C}\dot{u}_{i-1} + \boldsymbol{C}\dot{u}_i)^T(u_i^c - u_{i-1}^c) \qquad (5)$$

where $\boldsymbol{E}^D$ is the energy dissipated by viscous damping mechanism from time step *i* to time step *i+1*. It is assumed that the viscous damping matrix $\boldsymbol{C}$ is constant throughout the simulation. For clarity, this constant viscous damping used here is the initial viscous damping matrix. The variable viscous damping matrix, represented by a new variable $\boldsymbol{C}^{cor}$, compensates for energy error $\boldsymbol{E}^{er}$ from time step *i* to time step *i+1* and can be derived from Equation 5, taking the form,

$$C_i^{cor} = \frac{-2E_i^{er}}{(\dot{u}_{i-1}+\dot{u}_i)^T(u_i^c - u_{i-1}^c)} \quad (6)$$

The energy balance between the structure and the external excitation, incorporating the errors in restoring forces and the new variable damping matrix is thus

$$\int(\boldsymbol{M}\ddot{u})^T\ du + \int(\boldsymbol{C}\dot{u})^T\ du + \int(\boldsymbol{R}^m)^T\ du + \int(\boldsymbol{C}^{cor}\dot{u})^T du = \int \boldsymbol{F}^T\ du \quad (7)$$

The term $\int(\boldsymbol{C}^{cor}\dot{u})^T du$ in Equation 7 compensates the energy error defined by Equation 2, which is inherent in the vector of restoring forces $\boldsymbol{R}^m$ (Equation 1).

It can be seen that similar to the HSEM procedure, the new compensation scheme proposed here only requires data that are readily available at every time step during a fast PSD test. The scheme does not require predicted displacement using polynomial extrapolation such as proposed that by Horiuchi et al. [9], thus eliminating the error associated with the extrapolation procedure.

3.3 Numerical verification

This section presents a series of numerical simulations for validating the proposed error compensation scheme. The simulation considers two systems, a single-degree-of-freedom (SDOF) system and a two DOF system, subjected to sweeping sinusoidal ground acceleration for 40 seconds. The structures emulate two simple shear buildings as shown in Figure 2, and the frequencies of the ground accelerations increases gradually from β = 0.75 to β = 4 in four equal steps. β is the ratio of the excitation frequency to the fundamental natural frequency of the structure. The numerical simulations used a time step size of 0.005 s. Table 1 listed the dynamic properties of both structures. Explicit Newmark integration method is utilized to solve the equation of motion of the structures.

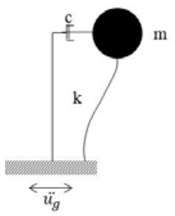


Figure 2a: SDOF structure

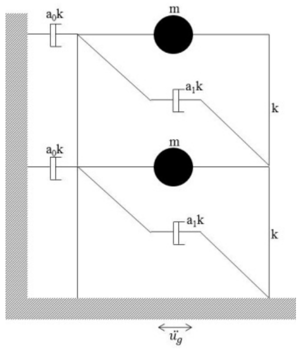


Figure 2b: Two DOF structure

m (kg)	1000	
k (KN/m)	158	
	SDOF	MDOF
Mass	m	$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$
Stiffness	k	$\begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix}$
Damping ratio	0.02	0.02 (all modes)
Natural period (sec)	0.5	$T_1 = 0.81$ $T_2 = 0.31$

Table 1: Properties of structures

In these numerical simulations, the restoring forces of the specimens are evaluated using the simulated stiffness for each specimen as shown in Table 1. Systematic undershooting errors are simulated at each step such that the restoring forces used for subsequent computation are proportionally smaller than the ideal restoring forces according to the size of the displacement increments. Other common sources of errors in a PSD test, such as random noise in force and displacement measurements, or calibration errors in control and measurement devices, are not simulated here.

The results of the simulations where the errors are not compensated are shown in Figure 3 and 4 for the SDOF and the two DOF structures respectively. The figures show the response from the start of the excitation until instability occurs. The results emphasize the importance of compensating actuator delay to avoid such instabilities in real experiments.

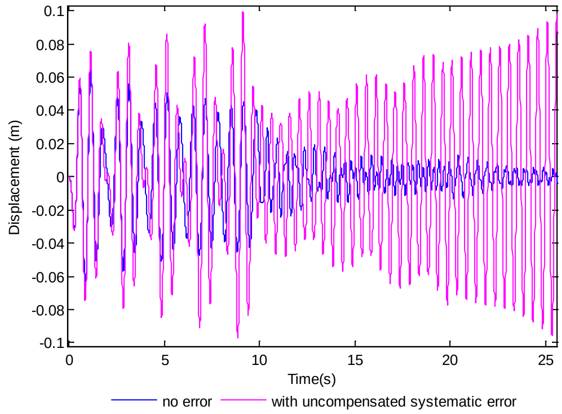


Figure 3: Displacement response of the SDOF structure without displacement error and uncompensated systematic displacement error

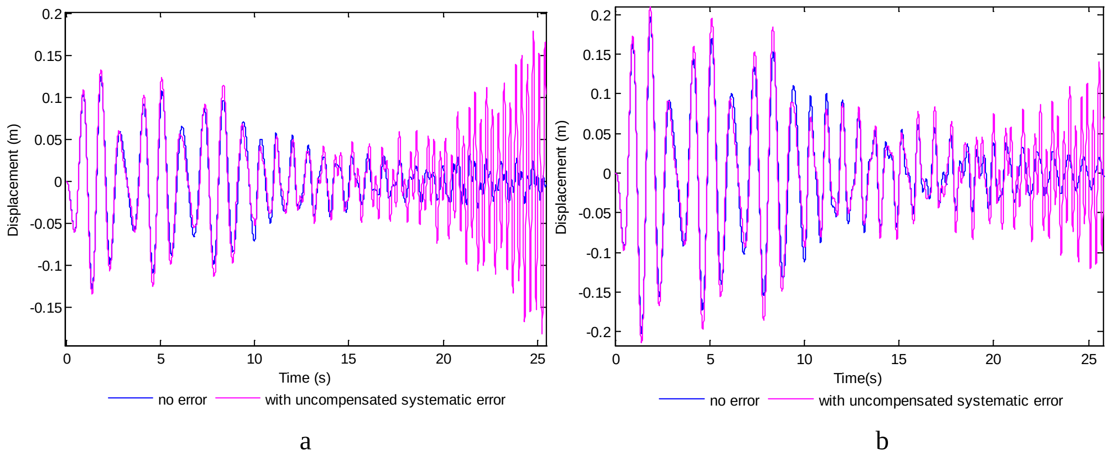


Figure 4: Displacement response of the two DOF structure without displacement error and uncompensated systematic displacement error; a) lower storey response and b) upper story response

Figure 5 and 6 present the displacement response and the simulated force-deformation relationship for each structure, under the same excitation but with the proposed compensation scheme applied. It can be seen that the corrected responses accurately match the ideal response with no error for both structures. In Figure 6a and 6b, small hysteresis can be observed on the corrected force-deformation relationships compared to the linear relationships with no error. These small hysteresis exist as the proposed method does not completely eliminate all energy error at every time step, resulting in small cumulative residual error. The normalized maximum cumulative error, defined as the ratio of the maximum error to the maximum ideal displacement are 0.052 and 0.0445 for the lower and upper storey respectively for the two DOF structure, and 0.0105 for the SDOF structure. In the two DOF structure, the effect of the energy error is aggravated by the presence of a higher second mode. This is because the rate of error propagation is proportional to the magnitude of the natural frequency.

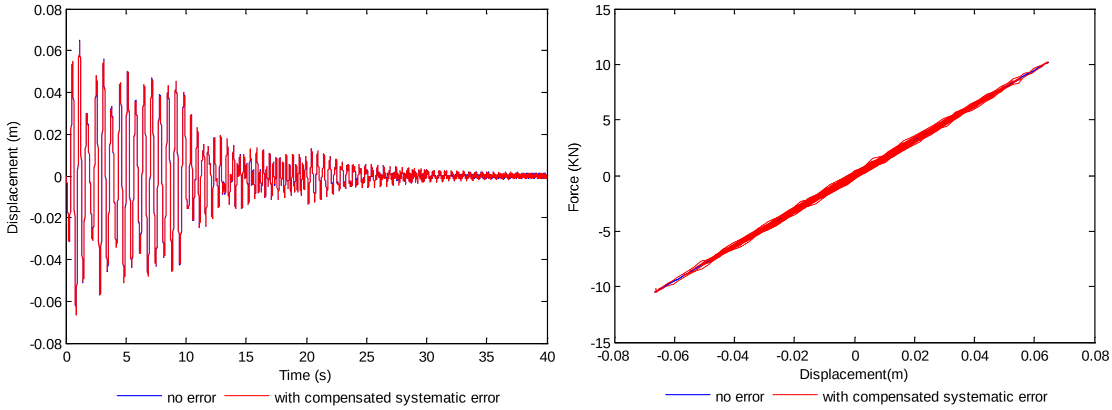
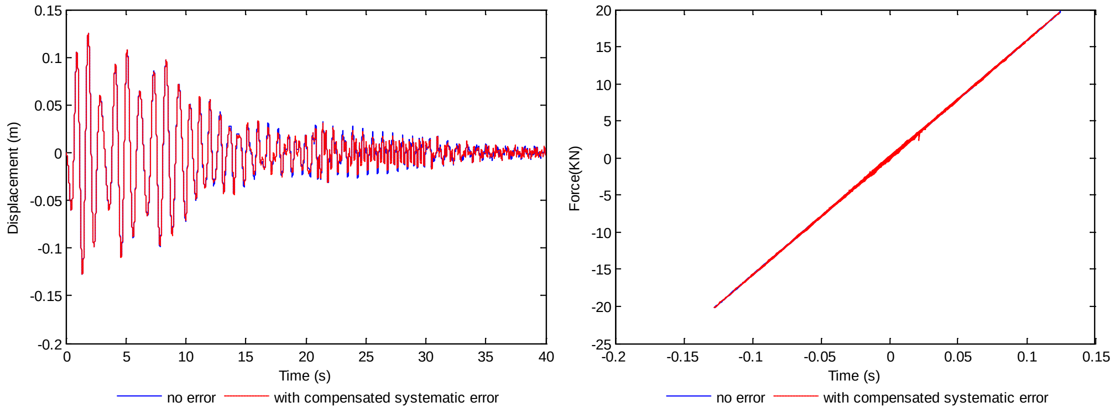
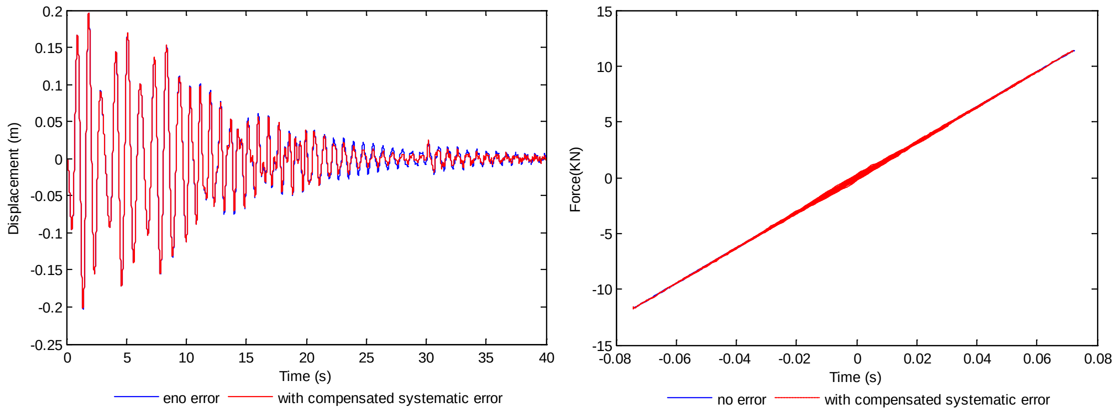


Figure 5: Displacement response and force-deformation relationship of the SDOF structure without displacement error and compensated systematic displacement control error



a



b

Figure 6: Displacement response and force-deformation relationship of the two DOF structure without displacement error and compensated systematic displacement error; a) lower storey and b) upper storey

4 CONCLUSION

This paper presents a new intuitive scheme to compensate for actuator delay in a fast PSD test. Compensation of actuator delay is essential to ensure a stable and accurate result of the tests. The proposed compensation scheme dissipates the additional energy erroneously introduced into the system through a variable viscous damping matrix that is proportional to the energy error at every time step. Although some residual errors can still be observed, numerical simulations show that the scheme can accurately capture the exact response of the structures. The simplicity of the method can also be seen as it only requires data that is readily available during a fast PSD test at every time step, as in the case of HSEM scheme.

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