COMPDYN 2013
4th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, V. Papadopoulos, V. Plevris (eds.)
Kos Island, Greece, 12–14 June 2013

GRADIENT-BASED STUDY ON SEISMIC PERFORMANCE REDUCTION OF STEEL BRACED FRAMES DUE TO IRREGULAR BRACE OVER-STRENGTH DISTRIBUTIONS

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Keywords: Braced Frames, Buckling-Retrained Braces, Dissipative Braces, Gradient Methods, Sensitivity Analysis, Steel Frames.

Abstract. *In this paper a general sensitivity-based gradient methodology developed to investigate the relation between seismic performance reduction and regularity of brace over-strength distributions is used for performance-based design and assessment of braced steel frames. A range of variation of the brace over-strength is defined and the brace over-strength patterns that are the most unfavorable for each response parameter are identified. Then, the relationship between the brace over-strength used in the seismic design and the maximum seismic performance reduction is obtained. Results for a case study considering a realistic buckling-restrained braced frame are presented and discussed showing the practical use of the developed sensitivity-based gradient methodology.*

1 INTRODUCTION

Seismic design of structures and relevant codes of practice are often based on the reduction factor method. The success of this approach is undoubtedly due to its simplicity as it permits approaching the seismic problem by a linear structural analysis, even though some drawbacks in its application have been highlighted, especially in the retrofit of existing structures. The reduction factor method aims at establishing a relation between the linear elastic behaviour of a structure and its ultimate post-elastic capacity under seismic actions, assuming that all necessary provisions are taken to provide a pre-defined yielding path in the structure (global ductility) and to ensure a sufficient ductility of critical yielded zones (local ductility). While it is possible to control the local ductility by means of specific design and detailing provisions, it is usually more difficult to ensure the desired global ductility that is influenced by two main issues. The first issue concerns the ratio between the seismic demand and the strength capacity in the yielding portions of the structure, such a ratio should be as uniform as possible in order to attain yielding in all the expected yielding parts. The second issue concerns the structural elements that must remain elastic, target commonly accomplished by making these elements more resistant than the yielding elements. The former issue is a quite critical point in the practical applications of the reduction factor design procedure, as it is often very difficult to obtain a satisfactory uniformity in the design of real structures and irregularity has significant negative effects on the seismic performance, in particular for low redundant structures. Steel braced frames are very sensitive to this problem and all the most diffused typologies show a significant tendency to soft story formation during seismic events in consequence of a not regular distribution of over-strength in bracings, as shown for concentrically braced frames (CBFs), e.g. [1][2][3], eccentrically braced frames (EBFs), e.g. [4][5], and more recently buckling-restrained braced frames (BRBFs), e.g. [7][8][9][10]. This significant influence is mainly due to the low redundancy of these systems and becomes particularly high when the frames coupled with the bracings are very deformable, as in the case of pinned beam-column connections. Overall, the studies available in the technical literature delineate the need to further examine the influence of the brace over-strength on the structural seismic performance as well as the effectiveness of current code recommendations.

This study aims at establishing a rational and consistent relation between the global reduction factor and the over-strength regularity, completing the approach presented in [11] with information that can be directly applied in performance-based engineering. This should be a first step towards a more safe use of this popular design method and towards the definition of new rules capable to overcome some drawbacks of current provisions that are often difficult to be applied in practice or may be ineffective and lead to excessively expensive solutions. The first objective of this paper consists of defining a general methodology to investigate the relation between seismic reduction factor and over-strength distributions. The method is based on sensitivity analysis of dynamic response and linearization of functional operators. A specific formulation to study the influence of brace over-strength distributions on the reduction factor of steel braced structures is presented, including the random nature of the seismic input. The actual nonlinear variations of specific demand measures of interest in seismic response, i.e., engineering demand parameters (EDPs), as well as their linearly approximated variations, are derived. A second objective, more practically oriented, concerns the definition of the maximum expected decrement of the reduction factor within an accepted range of variability in the over-strength coefficients. To this extent, it is assumed an over-strength domain as suggested by Eurocode 8 (hyper-cube) and the most unfavourable patterns are identified together with the corresponding decrement of the reduction factor. The solution is determined by constrained optimization tools supported by the problem gradient defined in the sensitivity analysis. Selected results for a BRBF used as case study are presented and discussed in order to illustrate the practical application of the proposed methodology.

2 RELATION BETWEEN SEISMIC PERFORMANCE AND BRACE OVER-STRENGTH DISTRIBUTIONS: METHODOLOGY

2.1 Seismic demand sensitivity analysis of nonlinear structures

It is assumed that the motion of the structural system is described by the following differential equations and relevant initial conditions:

$$\dot{\boldsymbol{\chi}}(\boldsymbol{\theta};t)=\mathbf{a}(\boldsymbol{\chi}(\boldsymbol{\theta};t),\boldsymbol{\theta})+\mathbf{p}(t) \qquad \boldsymbol{\chi}(\boldsymbol{\theta};0)=0 \tag{1}$$

where $\boldsymbol{\theta}\in\Re^m$ is a vector collecting m parameters defining material and geometric properties of the structural system, $\boldsymbol{\chi}:\Re^m\times[0,T]\mapsto\Re^s$ is a vector-valued function describing the evolution of the s state variables, e.g., displacement, velocity and state variables of history-dependent constitutive material models, $t\in[0,T]$ is the time and T the duration of the dynamic analysis, a superposed dot represents one derivative with respect to time, $\mathbf{p}:[0,T]\mapsto\Re^s$ is a function describing the seismic input, $\mathbf{a}:\Re^m\times\Re^s\mapsto\Re^s$ a vector-valued nonlinear function describing the response of the structure. It is assumed that **a** and **p** are continuous functions and that **a** is not an explicit function of time. Given a reference motion $\boldsymbol{\chi}_0:[0,T]\mapsto\Re^s$ corresponding to assigned values of the material and geometric parameters collected in $\boldsymbol{\theta}_0$, the variation of the motion in consequence of a variation $\hat{\boldsymbol{\theta}}$ of the parameters can be linearized in the neighbourhood of the reference motion, i.e., for $\|\hat{\boldsymbol{\theta}}\|\to 0$, by means of the following series expansion:

$$\boldsymbol{\chi}\left(\boldsymbol{\theta}_0+\hat{\boldsymbol{\theta}};t\right)=\boldsymbol{\chi}_0(t)+\mathbf{S}_0(t)\hat{\boldsymbol{\theta}}+\mathrm{o}\left(\|\hat{\boldsymbol{\theta}}\|\right) \quad \forall t\in[0,T] \tag{2}$$

where $\mathbf{S}_0:[0,T]\mapsto\Re^s\times\Re^m$ is the sensitivity matrix at $\boldsymbol{\chi}_0$ defined by the condition

$$\mathbf{S}_0(t)\mathbf{e}=\lim_{\lambda\to 0}\frac{\boldsymbol{\chi}(\boldsymbol{\theta}_0+\lambda\mathbf{e};t)-\boldsymbol{\chi}_0(t)}{\lambda} \qquad \forall\mathbf{e}:\|\mathbf{e}\|=1 \tag{3}$$

and whose components $S_{0ij}(t)=\partial\chi_i(\boldsymbol{\theta};t)/\partial\theta_j\big|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0}$ describe the ratio between the variation of the i-th component $\chi_i(t)$ of $\boldsymbol{\chi}$ due to the variation of the j-th component θ_j of $\boldsymbol{\theta}$ when $\boldsymbol{\theta}=\boldsymbol{\theta}_0$ and at the considered time instant t, o( ) is the "little-o" Landau symbol, i.e. given a function $f(x)$ and a positive function $\phi(x)$, then $f=\mathrm{o}(\phi)$ means that $f/\phi\to 0$. Similarly, the response of the perturbed structural system in the neighbourhood of the reference motion $\boldsymbol{\chi}_0$ may be written as (time dependence omitted for the sake of brevity):

$$\mathbf{a}\left(\boldsymbol{\chi}\left(\boldsymbol{\theta}_0+\hat{\boldsymbol{\theta}}\right),\boldsymbol{\theta}_0+\hat{\boldsymbol{\theta}}\right)=\mathbf{a}\big|_0+\nabla_{\chi}\mathbf{a}\big|_0\mathbf{S}_0\hat{\boldsymbol{\theta}}+\nabla_{\vartheta}\mathbf{a}\big|_0\hat{\boldsymbol{\theta}}+\mathrm{o}\left(\|\hat{\boldsymbol{\theta}}\|\right) \quad \forall\mathbf{e}:\|\mathbf{e}\|=1 \tag{4}$$

where the symbol $\big|_0$ attached after a function means that that function is evaluated at the reference motion $\boldsymbol{\chi}_0$ corresponding to $\boldsymbol{\theta}=\boldsymbol{\theta}_0$. The sensitivity matrix can be deduced from the differential equation:

$$\dot{\mathbf{S}}_0 = \nabla_{\chi}\mathbf{a}\big|_0 \mathbf{S}_0 + \nabla_{\vartheta}\mathbf{a}\big|_0 \tag{5}$$

obtained differentiating Equation (2) with respect to time, using Equation (1) and then comparing the result to Equation (4). Equation (5) is solved and $\mathbf{S}_0$ determined, the motion variation $\hat{\boldsymbol{\chi}}$ directly and linearly related to the small variations $\hat{\boldsymbol{\theta}}$ of the structural parameters:

$$\hat{\boldsymbol{\chi}} \cong \mathbf{S}_0\hat{\boldsymbol{\theta}} \tag{6}$$

Commonly, in seismic analysis the structural performance is evaluated by one or more EDPs. An EDP is a positive scalar d giving a measure of the structural damage occurring during the motion produced by the earthquake and it is a derived quantity of $\boldsymbol{\chi}$:

$$d = \mathsf{D}(\boldsymbol{\chi}) \tag{7}$$

where $\mathsf{D}: U_{\chi} \to \Re$ is a functional operator acting on the space of motions U_{χ}. Consequently, the previous sensitivity analysis must be extended to evaluate the EDP variations $\hat{d}$ due to the small variations $\hat{\boldsymbol{\theta}}$ of the model parameters through the difference:

$$\hat{d} = \mathsf{D}\left(\boldsymbol{\chi}_0 + \mathbf{S}_0\hat{\boldsymbol{\theta}}\right) - \mathsf{D}(\boldsymbol{\chi}_0) \tag{8}$$

The relationship between d and the motion $\boldsymbol{\chi}$ is usually nonlinear. Coherently with the sensitivity approach, oriented to investigate the neighbourhood of the reference motion, the relationship between d and $\boldsymbol{\chi}$ may be linearized:

$$\hat{d} = \mathsf{L}_0\mathbf{S}_0\hat{\boldsymbol{\theta}} + o\left(\left\|\hat{\boldsymbol{\theta}}\right\|\right) \tag{9}$$

introducing the operator L_0 defined as the derivative of D:

$$\mathsf{L}_0\hat{\boldsymbol{\chi}} = \lim_{\lambda \to 0} \frac{\mathsf{D}(\boldsymbol{\chi}_0 + \lambda\mathbf{c}) - \mathsf{D}(\boldsymbol{\chi}_0)}{\lambda} \qquad \forall \mathbf{c} : \|\mathbf{c}\| = 1 \tag{10}$$

In this way a complete overview of the effects on the seismic demand due to any combination of the variations of the structural parameters $\boldsymbol{\theta}$ is obtained. This is a qualitative and quantitative information that is quite important in understanding the structural behaviour as well as in studying the propagation of uncertainties of $\boldsymbol{\theta}$ to the structural response. In addition, structural design requires precise information on the entity of seismic performance reduction, measured by the EDPs, due to possible deviations from the reference design solution. It is thus crucial for the sake of safety: (i) to establish the range of the potential deviations from the design solution, and (ii) to assess the largest performance reduction to be expected within the set of admissible deviations. To this end, it is essential to complete the sensitivity analysis with a constrained extreme problem where the EDP is the objective function:

$$\begin{aligned} &\mathsf{D}\left(\boldsymbol{\chi}_0 + \mathbf{S}_0\hat{\boldsymbol{\theta}}^*\right) - \mathsf{D}(\boldsymbol{\chi}_0) = \max \\ &\mathbf{g}\left(\hat{\boldsymbol{\theta}}^*\right) \geq \mathbf{0} \\ &\mathbf{n}\left(\hat{\boldsymbol{\theta}}^*\right) \leq \mathbf{0} \end{aligned} \tag{11}$$

and $\hat{\boldsymbol{\theta}}^*$ is the most dangerous variation of $\boldsymbol{\theta}$ which must be found within the set defined by the constraints $\mathbf{g}$ and $\mathbf{n}$. Clearly, the problem solution becomes simpler when both the EDP expression and the constraint equations are linear.

The formulation presented above assumes a deterministic seismic input **p** while it is well-known that the seismic input is affected by an high level of randomness that cannot be neglected in the design. Various code provisions, e.g., Eurocode 8, suggest the description of the seismic input uncertainty with the use of response results averaged over the response results obtained from a sufficiently large number of accelerograms. According to this approach, the EDPs variations is evaluated by a mean sensitivity function $\mathbf{S}_0$ obtained from the analyses performed using an adequately large set of accelerograms. Different and more advanced treatments of the seismic input randomness are beyond the objectives of this study.

The seismic response analysis of nonlinear structures is commonly performed through the finite element (FE) method where the structure motion is reduced to the time histories of a set of discrete variables, i.e., the generalized displacements and velocities of the nodes of the FE model and the required state variables of the inelastic material models. The computation of the response sensitivities within the FE framework has been investigated in the past and several methods are available, such as the Direct Differentiation Method (DDM) and the Finite Difference Method (FDM) [12][13][14].

2.2 Sensitivity of seismic capacity and seismic reduction factor

Let $\mathbf{p}_0(t)$ be a reference seismic input, $\alpha \in \Re$ a multiplying factor, and the dynamic problem be posed in the form:

$$\dot{\boldsymbol{\chi}} = \mathbf{a}(\boldsymbol{\chi}, \boldsymbol{\theta}) + \alpha \mathbf{p}_0(t) \tag{12}$$

The value $\alpha_0(\boldsymbol{\theta}_0)$ corresponds to the multiplier providing the maximum value of acceptable damage d_0 and can be interpreted as a measure of the seismic capacity of the structural system for the assigned values $\boldsymbol{\theta}_0$ of the system parameters.

It is also assumed that the dynamic system shows a linear elastic behaviour for motions sufficiently small to produce a damage measure smaller than d_y. In other words, if $\boldsymbol{\chi}$ is such that $\mathsf{D}(\boldsymbol{\chi}) \le d_y$ then a linear operator $\mathbf{A}(\boldsymbol{\theta}_0)$ such that $\mathbf{a}(\boldsymbol{\chi}, \boldsymbol{\theta}_0) = \mathbf{A}(\boldsymbol{\theta}_0)\boldsymbol{\chi}$, exists. The attention is now focused on two particular values of the multiplying parameter: the value $\alpha_y(\boldsymbol{\theta}_0)$ providing the damage value d_y bounding the system linear range, and the smallest value $\alpha_0(\boldsymbol{\theta}_0)$ providing the maximum value d_0 acceptable for the damage.

The seismic reduction factor is defined as the following ratio

$$R_0 = \frac{\alpha_0(\boldsymbol{\theta}_0)}{\alpha_y(\boldsymbol{\theta}_0)} \tag{13}$$

The nonlinear problem at the ultimate condition

$$\dot{\boldsymbol{\chi}}_0 = \mathbf{a}(\boldsymbol{\chi}_0, \boldsymbol{\theta}_0) + \alpha_0 \mathbf{p}_0 \tag{14}$$

is the reference solution of the sensitivity analysis and its behavior in the neighborhood of the solution $\boldsymbol{\chi}_0$ were already studied in the previous section. By a little change in this perspective, the perturbed (linear) motion $\boldsymbol{\chi}_0 + \overline{\boldsymbol{\chi}}$ deriving both from small variations of system parameters $\overline{\boldsymbol{\theta}}$ and a small variation of the input scaling factor $\overline{\alpha}$ has the expression

$$\dot{\boldsymbol{\chi}}_0 + \dot{\overline{\boldsymbol{\chi}}} = \mathbf{a}_0 + \nabla_{\chi}\mathbf{a}\big|_0 \overline{\boldsymbol{\chi}} + \nabla_{\vartheta}\mathbf{a}\big|_0 \overline{\boldsymbol{\theta}} + (\alpha_0 + \overline{\alpha})\mathbf{p}_0 \tag{15}$$

that can be simplified as

$$\dot{\bar{\boldsymbol{\chi}}} - \nabla_{\chi}\mathbf{a}\big|_0 \bar{\boldsymbol{\chi}} \cong \nabla_{\vartheta}\mathbf{a}\big|_0 \bar{\boldsymbol{\theta}} + \bar{\alpha}\mathbf{p}_0 \tag{16}$$

The solution $\bar{\boldsymbol{\chi}}$ of the linear problem is the sum of a contribution due to parameter variations and a contribution due to data variation. The former one may be denoted as $\bar{\boldsymbol{\chi}}^*$ and it is the result of the sensitivity analysis. In particular, the perturbation directly due to the worst parameter variation $\hat{\boldsymbol{\theta}}^*$, previously defined, is

$$\bar{\boldsymbol{\chi}}^* = \mathbf{S}_0 \hat{\boldsymbol{\theta}}^* \tag{17}$$

The latter contribution, denoted as $\bar{\alpha}\bar{\boldsymbol{\chi}}_p$, can be expressed as the product of the scaling factor $\bar{\alpha}$ and the solution of the linear problem

$$\dot{\bar{\boldsymbol{\chi}}}_p - \nabla_{\chi}\mathbf{a}\big|_0 \bar{\boldsymbol{\chi}}_p = \mathbf{p}_0 \tag{18}$$

Within a linear approximation, $\bar{\boldsymbol{\chi}} = \bar{\boldsymbol{\chi}}^* + \bar{\alpha}\bar{\boldsymbol{\chi}}_p$ describes the relation between motion variation and seismic intensity, when the variation of the parameters is the worst one. Also for what concerns the damage, the linearization results of the previous section, Equation (9), can be used to evaluate the change close to the reference solution

$$\hat{d} \cong \mathsf{L}_0 \bar{\boldsymbol{\chi}}^* + \bar{\alpha}\mathsf{L}_0 \bar{\boldsymbol{\chi}}_p \tag{19}$$

so that it is possible to found the seismic intensity variation providing the ultimate value of damage d_0 in the parameter-varied system, simply by posing a null damage in Equation (19). This leads to

$$\bar{\alpha}\left(\hat{\boldsymbol{\theta}}^*\right) \cong -\frac{\mathsf{L}_0 \mathbf{S}_0 \hat{\boldsymbol{\theta}}^*}{\mathsf{L}_0 \bar{\boldsymbol{\chi}}_p} \tag{20}$$

that is usually a negative value expressing a reduction of the seismic capacity performance.

A new reduction factor can consequently be determined for the varied parameters, by evaluating the elastic limit $\alpha_y\left(\boldsymbol{\theta}_0 + \hat{\boldsymbol{\theta}}^*\right)$ corresponding to the linear operator $\mathbf{A}\left(\boldsymbol{\theta}_0 + \hat{\boldsymbol{\theta}}^*\right)$; the variation in the reduction factor has the form

$$\hat{R} \cong \frac{\alpha_0(\boldsymbol{\theta}_0) + \bar{\alpha}\left(\hat{\boldsymbol{\theta}}^*\right)}{\alpha_y\left(\boldsymbol{\theta}_0 + \hat{\boldsymbol{\theta}}^*\right)} - R_0 \,. \tag{21}$$

3 APPLICATION EXAMPLE

3.1 Geometry and data

A realistic 4-storey steel frame is considered as case study structure. Interstorey height h = 3.4 m is constant between adjacent floors; columns are continuous with pinned beam-to-column connections and hinge restraints at the base. Four V-bracing systems (BRBs) for each direction are the only seismic resistant components (Figure 1a), their preliminary design was based on the procedure described in [15]. Masses from vertical live and superimposed dead loads are 1200 $\mathrm{kNs^2/m}$ for each floor. The yield length of BRBs (L_y) is one third of the overall length (L_d) of diagonal braces (Figure 1b). The design yield stress of the BRBs is 275 MPa, columns and beams are made of steel with design yield stress equal to 355 MPa. Seismic design was based on spectrum type 1 of Eurocode 8 for ground type B with design ground acceleration $a_g = 0.35g$). A simplified model was used to reduce the dimensions of the finite element model (Figure 1b). Details on geometric properties are reported in Table 1 while additional data not given here can be found in [11].

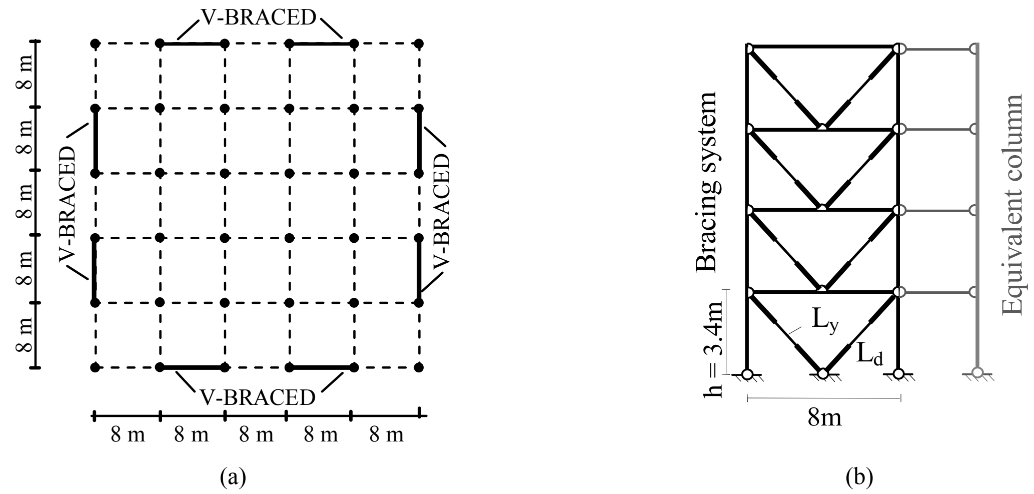

Figure 1: Case study: (a) floor configuration with arrangement of bracing system; (b) simplified planar model of the bracing system (bracing system and equivalent column).

Storey #	Bracing system columns Area (cm^2)	Bracing system columns Inertia (cm^4)	BRBs Area (cm^2)	Equivalent column Area (cm^2)	Equivalent column Inertia (cm^4)
4	58	4099.3	15.55	406	17637.3
3	101	12733.4	24.47	587	29457.9
2	141	35151.8	30.44	787	70325.6
1	192	45760.0	35.17	1069	91876.7

Table 1: Case study: geometric data of the 4-storey bracing system and equivalent column.

Beams and columns were modelled using geometric nonlinear (moderate rotations theory) Euler-Bernoulli frame FEs with linear elastic steel (elastic modulus E = 210 GPa). The BRBs were modelled with truss elements having rigid links to represent the unrestrained non yielding segments in 2/3 of the brace total length. An elastoplastic constitutive model based on a simple rheological scheme, specifically developed for steel BRBs [16], was used in this study for the yielding segments of trusses representing the BRBs. Such BRB model has only one internal variable, i.e., plastic strain, and was formulated in order to replicate the experimental behaviour (isotropic hardening and tension-compression asymmetry), as well as to include some highly desired requirements (explicit computation of the plastic component of the deformation as required in BRB capacity models, smoothness of the elastic-to-plastic transition to improve convergence rate, limited number of parameters to facilitate its implementation and use in response sensitivity analysis). In addition to the damping provided by the dissipative elastoplastic braces, a global damping for the structure was included using the Rayleigh model, with the damping matrix proportional to the mass matrix and updated stiffness matrix, and 5% of the critical damping assigned to the first and second vibration modes. The constant average acceleration method with constant time step $\Delta t = 0.01$ s, in conjunction to the Newton-Raphson iterative procedure, was used in all the dynamic analyses performed.

3.2 Seismic response results

The behaviour of the case study was studied through time history analyses having as seismic input 28 natural ground motions selected from the PEER strong motion database and scaled so that the elastic response spectra for the records matched the Eurocode 8 elastic spectrum at the first natural period T_1 = 0.746 s. More details on the selected ground motions can be found in [11]. Incremental dynamic analyses (IDA) were performed to assess the performance of the bracing systems. The multiplier of the scaled selected ground motions leading to a maximum interstorey drift, averaged over the 28 ground motions, equal to the design value (1% of the story height) is 0.94 corresponding to the elastic spectrum with $a_g = 0.33g$. This acceleration is very close to the design one, notwithstanding the record to record variability, the possible concentration of inelastic deformation at some story levels, and the effects of the geometric nonlinearity included in the FE model. Response results, i.e., interstorey drifts and BRB cumulative ductility at each story are reported in Table 2 in terms of mean values (averaged over the 28 ground motions) and coefficients of variations (COVs) of the absolute peak values. For each nonlinear time history analysis it was verified that beams and columns remained within their elastic range.

	Interstorey drift		BRB cumulative ductility	
Storey #	Mean	COV	Mean	COV
4	0.78%	22.18%	66.23	47.34%
3	0.69%	23.25%	50.10	37.47%
2	0.88%	35.18%	59.15	41.59%
1	0.99%	35.22%	76.28	43.62%

Table 2: Response results at a_g = 0.33g for the considered set of 28 ground motions.

3.3 Seismic response sensitivity results

Response sensitivities of the selected EDPs, i.e., ζ_i (interstorey drift at the i-th floor normalized with respect the interstorey height h) and μ_i^c (cumulative plastic strain at the i-th floor normalized with respect to the BRB yield strain ε_y) were computed with respect to the independent sensitivity parameters $\theta_k = A_k / A_{0k}$, being A_k the actual core area and A_{0k} the core area of the reference design solution of each of the two BRBs at the k-th floor. For ζ_i attention is limited to the sensitivities of each EDP when its peak value is attained, whereas for μ_i^c the value attained at the end of the ground motion is considered. The local response sensitivity results could be questioned being derivatives of the peak response and thus representative of the local effect of small variations of the sensitivity parameters θ_k. In order to clarify this issue, the global sensitivities of the peak values of the considered EDPs, computed with respect to θ_k using the FDM for finite variations $\Delta\theta_k$ = 0.125 and 0.250, are also shown in this study. In addition, the FDM with $\Delta\theta_k$ = 0.1, 0.01, and 0.001 was used to approximate local sensitivities, showing that the convergence of the FDM approximation is achieved with $\Delta\theta_k$ = 0.01 without incurring in the step-size dilemma. When FDM is used, the sensitivities are computed from the maximum values of the EDPs in the reference and perturbed motion, even if the maximum values are attained in different time instants. Despite the inevitable differences in structural response obtained from the various seismic inputs, similar qualitative trends were observed for the sensitivity results, regardless of the ground motion considered.

Response sensitivity results (averaged over the set of 28 ground motions) with the normalized interstorey drifts as EDPs of interest are summarized in Figure 2. The local response sensitivities, i.e., $\partial\zeta_i/\partial\theta_k$, are depicted in Figure 2a, the global response sensitivities, i.e.,

$\Delta\zeta_i/\Delta\theta_k$, in Figures 2b,c. Comparisons between local and global sensitivities, i.e., between Figures 2a and 2b,c, reveals some (typically minor) numerical differences. However, it is important to observe that the qualitative results are basically the same. Thus the response sensitivities (local derivatives) of the interstorey drifts with respect to the sensitivity parameters θ_k, are to some extent representative of the effects of finite variations of θ_k, and thus of finite variations of the BRB core areas. A positive (negative) value of the sensitivity means an increment (decrement) of the relevant EDP due to the increment of the sensitivity parameter considered. Thus, the results shown in Figure 2 allow the quantification of the increments and decrements of peak interstorey drifts due to the sensitivity parameters. In addition to such quantification, qualitative considerations can be deduced. For example, it is observed that the increment of the internal core area of the BRBs of a given floor causes a notable reduction of the peak interstorey drift of that same floor and a smaller increment of the peak drift values of all the other floors.

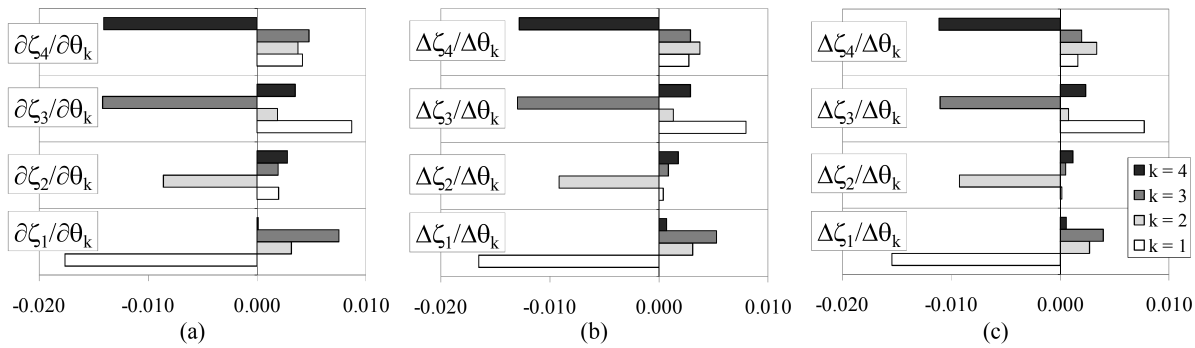

Figure 2: (a) local normalized sensitivities of the maximum interstorey drifts; (b) global (ΔA_k / A_k = 12.5%) normalized sensitivities of the maximum interstorey drifts; (c) global (ΔA_k / A_k = 25.0%) normalized sensitivities of the maximum interstorey drifts.

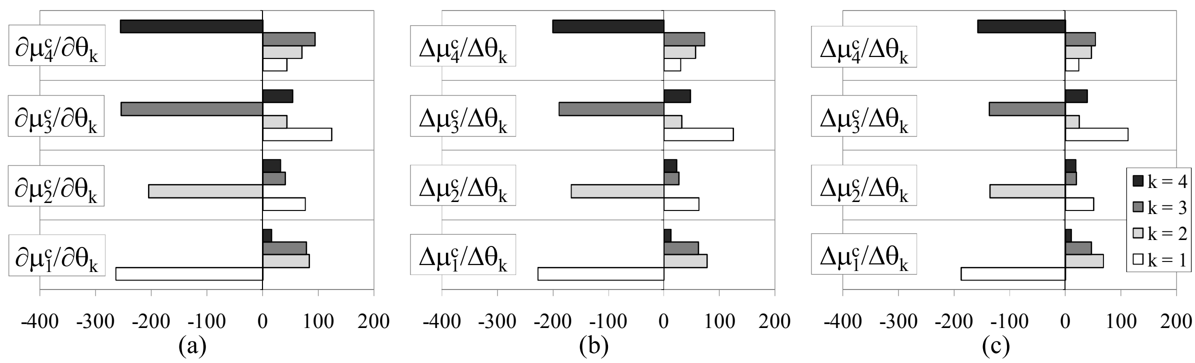

Figure 3: (a) local normalized sensitivities of the BRB cumulative ductility; (b) global (ΔA_k / A_k = 12.5%) normalized sensitivities of the BRB cumulative ductility; (c) global (ΔA_k / A_k = 25.0%) normalized sensitivities of the BRB cumulative ductility.

Similar trends of the local and global sensitivities (averaged over the set of 28 ground motions) are observed when the EDPs related to the seismic demand on the BRBs are considered, i.e., local ($\partial\mu_i^c/\partial\theta_k$) and global ($\Delta\mu_i^c/\Delta\theta_k$) response sensitivities of the cumulative ductility (Figure 3). It is observed that the increment of the core area of the BRBs at a given floor decreases the seismic demand on the BRBs of that same floor and typically increases the seismic demand on the other BRBs, with increment amplitude usually smaller than the amplitude of the decrement.

3.4 Effects of over-strength distributions and relevant seismic performance reduction

Response sensitivity diagrams shown in Figures 2 and 3 can be used to identify the variation of the sensitivity parameters (BRB core areas) resulting in largest increment in the EDPs considered. The most unfavourable combination of sensitivity parameters is found according to the linearized formulation and set of variations described section 2. In this way, the response sensitivities assume a role similar to influence lines: whereas influence lines permit to locate moving loads in bridge analysis so to attain the largest effects, response sensitivity diagrams permit to estimate the worst combination of the variation of the BRB core areas to attain the largest EDP increment. For example, if the goal is to evaluate the maximum interstorey drift at the first floor of the considered case study, then the brace areas of the second, third and fourth floors must be increased (positive sensitivity) and the brace areas at the first floor left unchanged (negative sensitivity), as depicted in Figure 4.

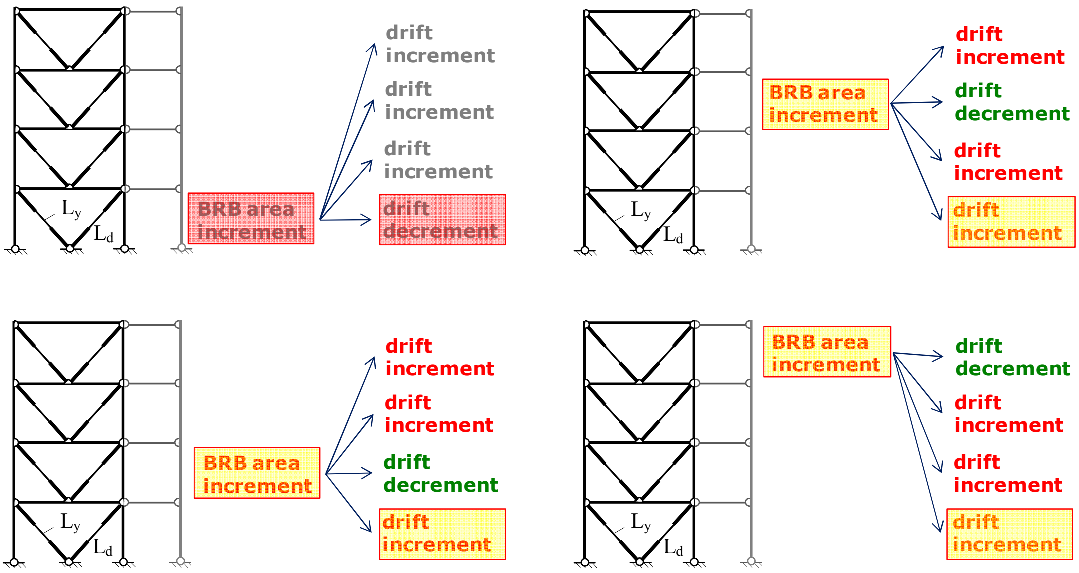

Figure 4: Use of response sensitivity results to identify the worst distributions of brace over-strength.

Results derived from the set of variations with amplitudes spanning from 0 to 25%, are henceforth reported and commented. Results (averaged over the set of 28 ground motions) for the interstorey drift as EDP are given in Figure 5a. For comparison purposes, results obtained with an homogeneous amplification of all sensitivity parameters θ_k are reported in Figure 5b. Additionally, in order to verify the validity of the linear approximations based on the local response sensitivity calculations, the same Figure 5 also show the actual EDP variations obtained from the nonlinear analysis with finite increments of BRB areas. It is observed that the linear approximations based on the local response sensitivity results give fairly accurate predictions of the actual variations, in particular when the amplifications in the BRB areas are below 12.5%. The results in Figure 5 show significant differences between the maximum interstorey drifts derived from the most adverse combinations of BRB over-strengths and those derived from uniform BRB over-strengths. The BRB uniform over-strength reduces the interstorey drifts. On the other hand, the worst combination of BRB over-strength for the *i*-th interstorey drift significantly increases the *i*-th interstorey drift, e.g., the minimum interstorey increment is about 10% in floor #2 for 25% parameter increment. In addition, the worst combination of BRB over-strength for the *i*-th interstorey drift has the effect of reducing the other interstorey drifts (results not shown in the presented figures for the sake of brevity) with reductions in the range from 1% to 30%, always smaller than the increments in the *i*-th interstorey drift. Results similar to those of the interstorey drift are observed for the BRB cumulative ductility (Figure 6). The increment of BRB cumulative ductility under its most adverse combinations of BRB over-strength is more important that those noted for the interstorey drift.

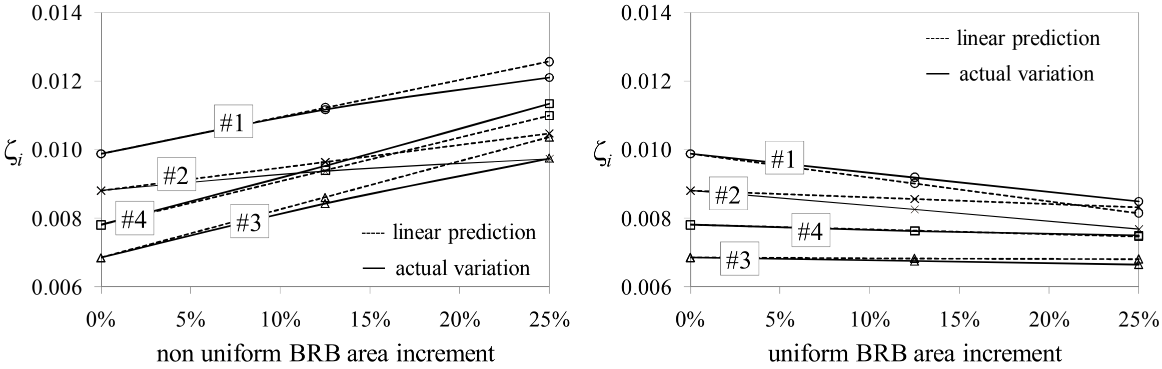

Figure 5: (a) variation of the maximum interstorey drift from the worst combination of the increment of BRB areas; (b) variation of the maximum interstorey drift from the uniform increment of BRB areas.

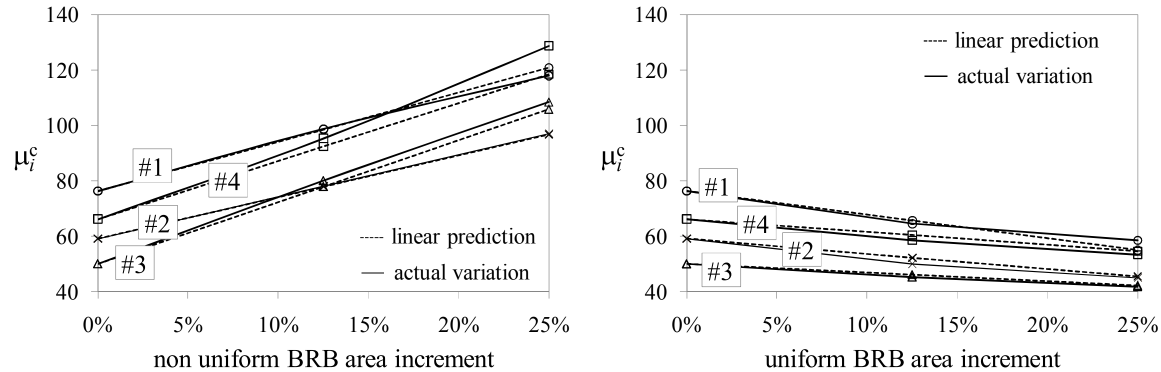

Figure 6: (a) variation of the BRB cumulative ductility from the worst combination of the increment of BRB areas; (b) variation of the BRB cumulative ductility from the uniform increment of BRB areas.

The variation of the reduction factor of the reference design solution (R_0 = 4.86) due to the worst combination of the increment of BRB areas as determined based on sensitivity results is quite significant. Using the described methodology, the obtained values are R = 3.62 for the 12.5% worst variation amplitude of the BRB area increment and R = 2.37 for the 25% worst variation amplitude of the BRB area increment.

4 CONCLUSIONS

In this paper a general sensitivity-based gradient methodology developed to investigate the relation between seismic performance reduction and regularity of brace over-strength distributions was presented for performance-based design and assessment of braced steel frames. Seismic response and response sensitivity analyses were performed using as input a given set of natural accelerograms. The sensitivity analysis results averaged over the considered set of accelerograms were used to identify the most unfavourable brace over-strength patters for each EDP, and the relevant worst possible increments of each EDP with respect to the reference design solution were predicted using a linear approximation based on local sensitivities and compared to the actual increments computed from nonlinear analysis. Afterwards, the maximum expected decrement of the reduction factor was determined within an accepted range of variability in the over-strength amplitude. In this way it was highlighted how response sensitivity results are a powerful and relatively simple strategy to better understand the structural seismic response, and an efficient tool for more effective seismic designs.

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