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R_y-μ-T_n RELATIONS FOR SEISMICALLY ISOLATED STRUCTURES

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Abstract. *The relations between the strength reduction factor R_y, the displacement ductility μ and the vibration period of the structure T_n have been extensively studied for fixed-base structures by numerous researchers in the past. This project aims at identifying similar relations for base-isolated structures. The investigation is conducted using a two-degree-of-freedom model of a base-isolated structure. The hysteretic behavior of the base isolation devices and the isolated superstructure is simulated in Matlab and OpenSees using a Bouc-Wen model. The results of the observed response are verified through the excitation of the isolated structure by a large number of recorded ground motions. These motions cover a wide range of ground motion types, magnitudes and distances. The effects of base isolation and superstructure design parameters, such as stiffness and strength, are quantified through parametric studies. The resulting R_y-μ-T_n relationship for inelastic seismically isolated structures is based on the statistical processing of the inelastic response data of the isolated superstructure.*

1 INTRODUCTION

The current codes for seismic design of structures focus on preventing their collapse. However, this design approach does not address the short and long-term effects due to the loss or the disruption of the function of the built infrastructure after an earthquake event.

Seismic response modification technologies are used to modify the dynamic response of structures to mitigate their damage and guarantee their post-earthquake functionality. Seismic isolation of structures is a response modification technology which has been widely used in the past for the design of new structures or the seismic retrofit of existing structures [6-9].

Seismic isolation is defined as a system of flexible or sliding structural elements that decouple a structure from the horizontal components of ground excitation. The orthogonality of the structure vibration mode to the seismic isolation vibration mode results in low transmission of energy from the ground motion to the structure [1]. This lower transmission of energy results in the reduction of the earthquake-induced damage to the structure. Furthermore, the fundamental vibration period of the isolated structure is lengthened comparing to a conventional fixed-based structure. This lengthening of the vibration period leads to a significant decrease of the seismic base shear acting on the structure.

However, as the design codes worldwide prohibit extensive yielding of the isolated superstructure, the initial construction cost of an isolated structure is significant. This cost could be reduced if the design base shear for the isolated structure is reduced, thus allowing it to develop ductility demands similar to those permitted for fixed-base structures. The reduction of cost due to a lighter superstructure could then offset the high bearing installation cost. From this point of view, a further insight into the behavior ranges of the inelastic seismically isolated structures is needed for two reasons:

First, to identify the possibility of yielding for a more economical design of new seismically isolated structures. Constantinou and Quarshie [2], Ordonez et al. [3], Kikuchi et al. [4], Thiravechyan et al. [5] have investigated the response of yielding seismically isolated structures and agreed that allowing seismically isolated structures to yield needs careful consideration.

Second, to account for the case in which the applied forces exceed the design forces due to an extreme earthquake event or reduced structural strength in existing seismically isolated structures. This analysis could lead to the identification of the existing base-isolated structures whose strength is not sufficient to keep their response in the elastic range.

This study focuses on the determination of the relations between the force reduction factor R_y, the displacement ductility demand μ and the vibration period of the superstructure T_n, for base-isolated structures. Numerous previous studies have investigated relationships between R_y, μ and T_n for fixed-base structures. Newmark and Hall [10], Lai and Biggs [11] and Riddel and Newmark [12] have proposed piece-wise linear R_y-μ-T_n relations for fixed-base structures. Riddel, Hidalgo and Cruz [13] and Vidic, Fajfar and Fischinger [14] have presented bilinear approximations for R_y-μ-T_n relations. Elgadamsi and Mohraz [15], Arias and Hidalgo [16], Nassar and Krawinkler [17], Miranda [18], and Miranda and Bertero [19] have suggested the use of nonlinear curves for R_y-μ-T_n relations. According to these studies, the seismic response of structures is categorized in three regions: 1) an "Elastic" or "Acceleration sensitive region", which governs the behavior of very stiff structures; 2) a "Hysteretic energy conservation region"; and 3) a "Displacement conservation region", which is observed for very soft structures. The goal of this study is to identify the vibration period delineated behavior ranges for base-isolated structures analogous to those for fixed-base structures.

1.1 Dynamic modeling

The dynamics of a seismically isolated structure, according to the work of Naeim and Kelly [1] can be investigated by using a two-degree-of-freedom system, as presented in Figure 1.1. Masses m_s *and* m_b represent the mass of the superstructure and the mass of the base above the isolation system, respectively. The stiffness and damping are expressed as k_s, c_s, when referring to the superstructure and k_b, c_b when referring to the base. The deformation u_s is the deformation of the superstructure with respect to the base mat, and u_b is the deformation of the bearings with respect to the ground.

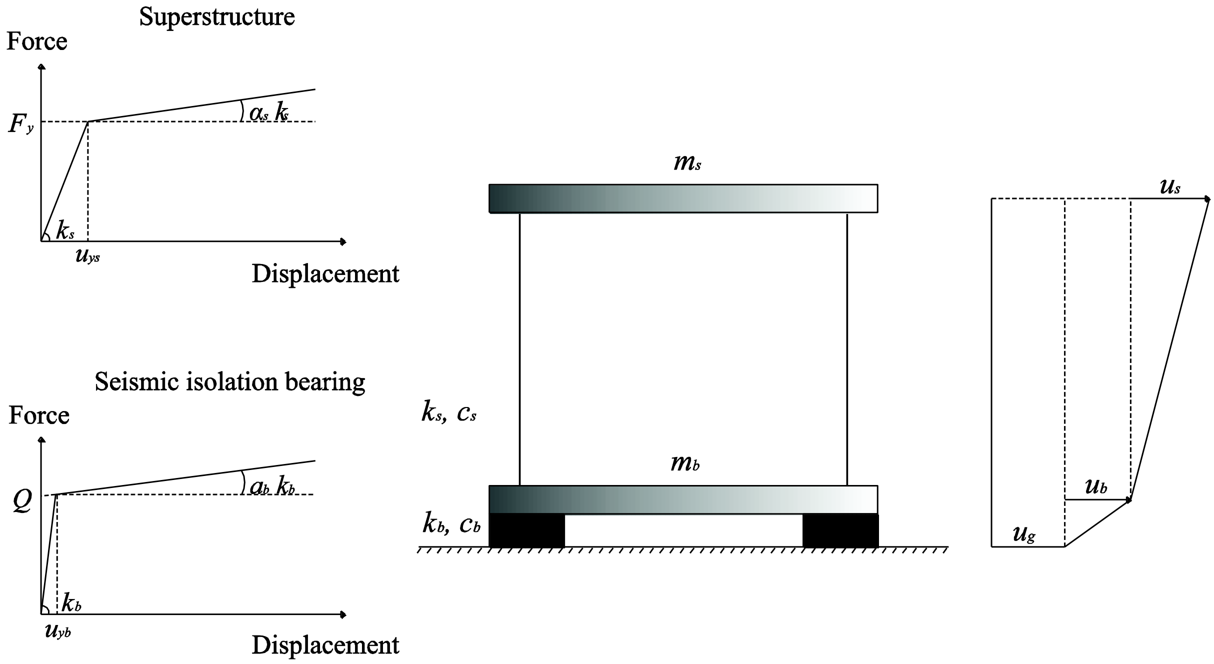

Figure 1.1: Parameters of the 2-DOF model of a base isolated structure.

The following quantities are defined:
1. Fixed-base period and cyclic frequency:

$$T_s = 2\pi\sqrt{\frac{m_s}{k_s}}\ ,\ \omega_s = \sqrt{\frac{k_s}{m_s}} \tag{1}$$

2. Isolation period and cyclic frequency:

$$T_b = 2\pi\sqrt{\frac{m_s + m_b}{\alpha_b k_b}}\ ,\ \omega_b = \sqrt{\frac{\alpha_b k_b}{m_s + m_b}} \tag{2}$$

3. Non-hysteretic structural and isolation damping ratio:

$$\xi_s = \frac{c_s}{2 m_s \omega_s}, \qquad \xi_b = \frac{c_b}{2\left(m_s + m_b\right)\omega_b} \tag{3}$$

4. Mass ratio:

$$\gamma_m = \frac{m_s}{m_s + m_b} \tag{4}$$

A Bouc-Wen [20, 21] model is used to simulate the bilinear hysteretic behavior of the isolation system. The restoring force of the isolation system is modeled as:

$$F_b(t) = -\alpha_b k_b u_b(t) - Q \cdot z_b(t) - c_b \dot{u}_b(t) \tag{5}$$

where α_b is the hardening ratio of post-yield to pre-yield stiffness of the isolation system (Figure 1.1), Q is the strength of the system (force at zero displacement), and $z_b(t)$ is a dimensionless parameter of the Bouc-Wen model.

The strength of the isolation system is determined for friction pendulum bearings using the following equation:

$$Q = (m_s + m_b)\mu_f g \tag{6}$$

where μ_f is the bearing coefficient of friction.

A Bouc-Wen model in parallel with a viscous damper is used to model the bilinear hysteretic behavior of the isolated structure. The restoring force of the isolated structure is given by:

$$F(t) = -\alpha_s k_s u_s(t) - (1-\alpha_s) k_s u_{y_s} z_s(t) - c_s \dot{u}_s(t) \tag{7}$$

where α_s is the hardening ratio of post-yield to pre-yield stiffness of the isolated structure (Figure 1.1), u_{ys} is the yield displacement of the isolated structure and $z_s(t)$ is a dimensionless parameter of the Bouc-Wen model.

The yield strength of the isolated structure is:

$$F_y = k_s u_{ys} \tag{8}$$

Dynamic equilibrium of the isolated structure and the base isolation system gives:

$$(m_s + m_b)\ddot{u}_b + m_s \ddot{u}_s + \alpha_b k_b u_b + Q z_b(t) + c_b \dot{u}_b(t) = -(m_s + m_b)\ddot{u}_g \tag{9}$$

Dynamic equilibrium of the isolated structure gives:

$$m_s \ddot{u}_s + m_s \ddot{u}_b + \alpha_s k_s u_s + (1-a_s) k_s u_{y_s} z_s + c_s \dot{u}_s = -m_s \ddot{u}_g \tag{10}$$

Consequently, equations (9) and (10) become equations of motion of the combined structure-isolation system. When dividing equations (9) and (10) by (m_s+m_b), equations (11) and (12) are derived: The results presented in this study were obtained by solving the derived equations (11) and (12) using Matlab and Opensees:

$$\ddot{u}_b + \gamma_m \ddot{u}_s + \omega_b^2 u_b + \frac{Q}{m_s + m_b} z_b + 2\xi_b \omega_b \dot{u}_b = -\ddot{u}_g \tag{11}$$

$$\ddot{u}_s + \ddot{u}_b + \alpha_s \omega_s^2 u_s + (1-\alpha_s)\omega_s^2 u_{y_s} z_s + 2\xi_s \omega_s \dot{u}_s = -\ddot{u}_g \tag{12}$$

2 ANALYSIS

2.1 Ground motion data

The 160 motions used in this study cover a wide range of ground motion types, magnitudes (5.5 to 7.7) and distances (10 to 60 km). They were taken from the Pacific Earthquake Engineering Research (PEER) Center next generation attenuation (NGA) strong motion database [22, 23].

2.2 Methodology

The analysis method used in this study involves the determination of strength reduction factors R_y for a certain predetermined displacement ductility ratio μ. The structural model shown in Figure 1.1 was subjected to the ground motion ensemble presented in 2.1. The vibration period of the isolation bearings T_b was selected first. The strength of the isolation system was determined by the choice of the bearing friction coefficient μ_f=0.05. Then, for each ground motion record, an elastic analysis is conducted for the calculation of the maximum elastic force and displacement demands of the isolated structure with vibration period $T_n \in \{0.1, 0.5, 1.0, 1.5, 2.0, 3.0, 4.0, 5.0\}$. After the determination of these values, an iterative procedure is performed to compute the maximum strength needed to keep the displacement ductility demand to the predefined level μ within a range of 1%. The strength reduction factors that result in this displacement ductility value can then be easily determined.

2.3 Statistical fit

Based on the analysis presented above, the results obtained for T_b=2 and μ=2 are shown in Figure 1.2. These results (labeled "actual") represent the mean value and the standard deviation of the response of the model subjected to the applied ground motion ensemble. Bilinear curve fitting is carried out to the mean of the data to establish a first approximation of R_y-μ-T_n relation. The proposed bilinear curve representing the statistical fit is also presented in Figure 1.2.

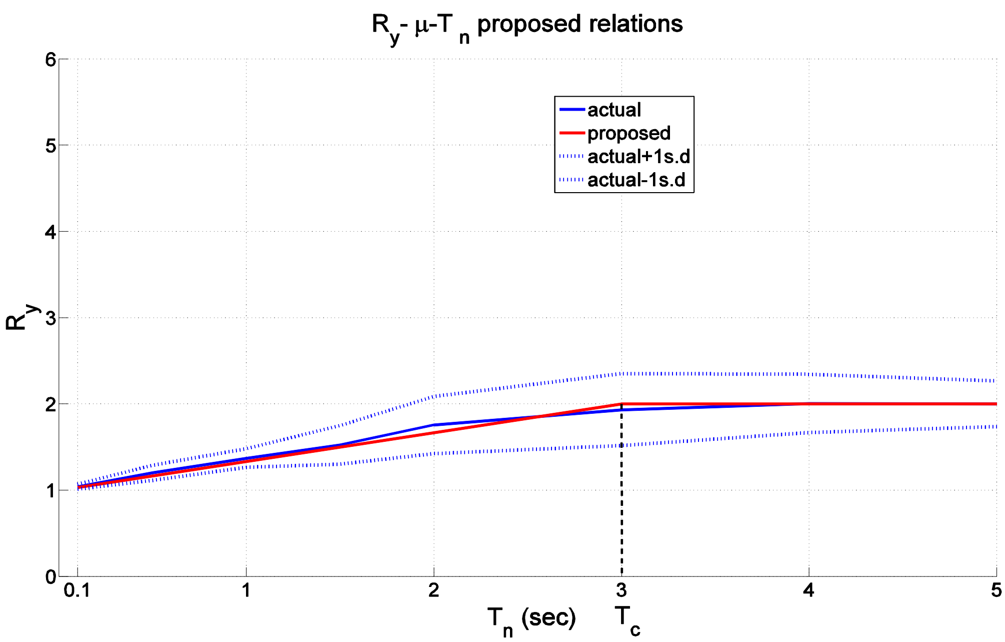

Figure 1.2: R_y-μ-T_n relation for T_b=2 and μ=2 (actual and proposed)

Similar simulations have been performed for a wide variety of values of the isolation period T_b and the ductility demand μ. As the deviation of the results varies with respect to the vibration period of the structure T_n, the maximum standard deviation $\sigma_{max}(T)$ was computed to quantify the dispersion of the results (Table 1.1).

$T_b \backslash \mu$	2	3	4
2 seconds	0.42	0.86	0.98
3 seconds	0.31	0.75	0.97
4 seconds	0.28	0.80	0.86

Table 1.1: Standard deviation $\sigma_{max}(T)$ for alternate values of T_b and μ

3 PARAMETRIC STUDY

As the relation between R_y and μ is non-linear, the period range for which the equal displacement rule applies ($R_y=\mu$) cannot be easily identified. From this point of view, the use of a bilinear curve is more appropriate, it can lead to the determination of the vibration period T_c as an intersection of 2 lines (Figure 1.1). Among other issues, this study attempts to quantify the range of this displacement-sensitive region by taking into account the most crucial parameters that influence this relation:

a) The isolation period T_b.
b) The displacement ductility ratio μ
c) The hardening ratio α_s of post-yield to pre-yield stiffness of the isolated structure

3.1 Proposed R_y-μ-T_n relations for isolated structures

The form of the proposed R_y-μ-T_n relation for isolated structures was adapted from T. Vidic, P. Fajfar and M. Fischinger [14].

$$R_y = 1,\ T < T_a \tag{12}$$

$$R_y = c_\alpha \left(\mu - 1\right)^{c_\mu} \frac{T}{T_b + 1} + 1,\ T < T_c \tag{13}$$

$$R_y = \mu,\ T > T_c \tag{14}$$

The coefficients c_a and c_μ were chosen to quantify the influence of the hardening ratio α_s and the displacement ductility ratio μ on the strength reduction factor R_y. Their values, determined by statistical fitting vary as shown in the following Table 1.2:

α_s	c_a		μ	c_μ
0%	1.00		2	0.85
5%	1.10		3	0.90
10%	1.15		4	0.95

Table 1.2: Coefficients c_a and c_μ for alternate values of α_s and μ

Then, the vibration period, after which the equal displacement rule applies can be determined as an intersection ($R_y=\mu$) between two lines:

$$T_c = \frac{1}{c_\alpha} \left(\mu - 1\right)^{1-c_\mu} \left(T_b + 1\right) \tag{15}$$

3.2 Influence of essential parameters

The essential parameters that influence the proposed R_y-μ-T_n relation are: The isolation period T_b, the displacement ductility ratio μ of the isolated structure and the hardening ratio α_s of post-yield to pre-yield stiffness of the isolated structure. As presented in Figures 1.3 and 1.4, the use of softer isolators giving longer isolation period values T_b significantly increases T_c (moves it to the right), thus reducing the range of the displacement conservation region. Similar, but less pronounced, is the influence of isolated structure post-yield hardening (Figures 1.5 and 1.6): The displacement conservation region decreases for structures with less hardening. Finally, the effect of the displacement ductility ratio μ of the isolated structure on the location of the displacement conservation range is rather negligible and becomes less important for larger displacement ductility values (Figures 1.7 and 1.8).

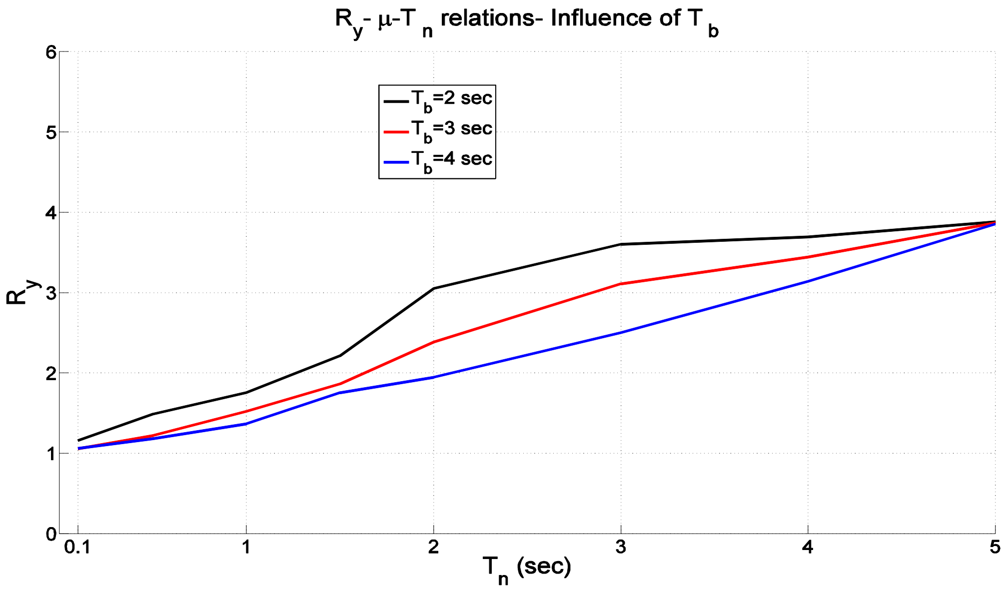

Figure 1.3: R_y-μ-T_n relation for T_b=2,3,4 sec and μ=4 (actual mean values)

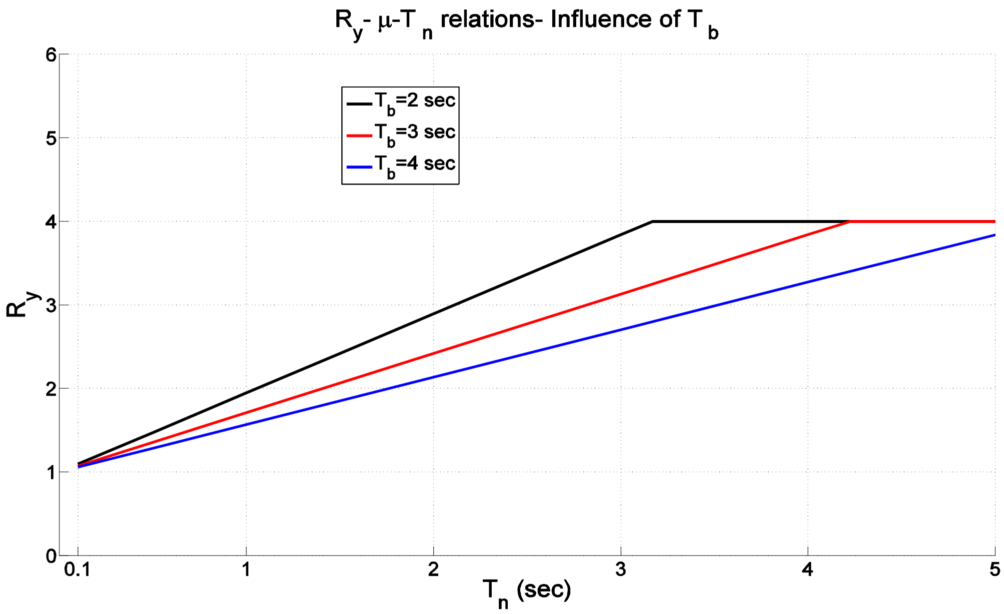

Figure 1.4 R_y-μ-T_n relation for T_b=*2,3,4 sec* and μ=*4* (proposed values)

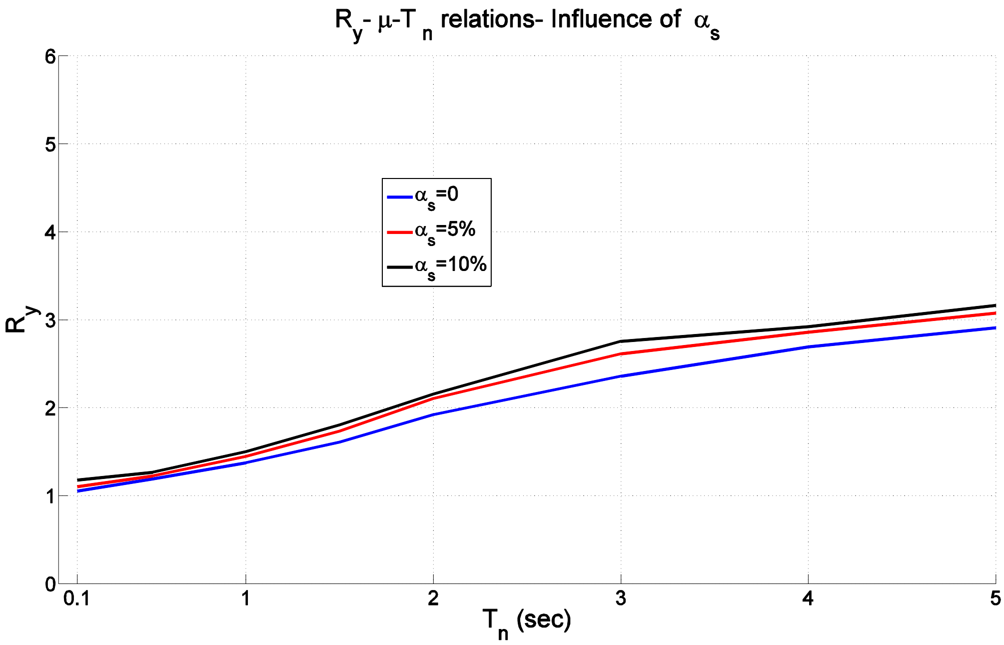

Figure 1.5*:* R_y-μ-T_n relation for a_s=0, 5%, 10%, μ=3 and T_b=3 sec (actual mean values)

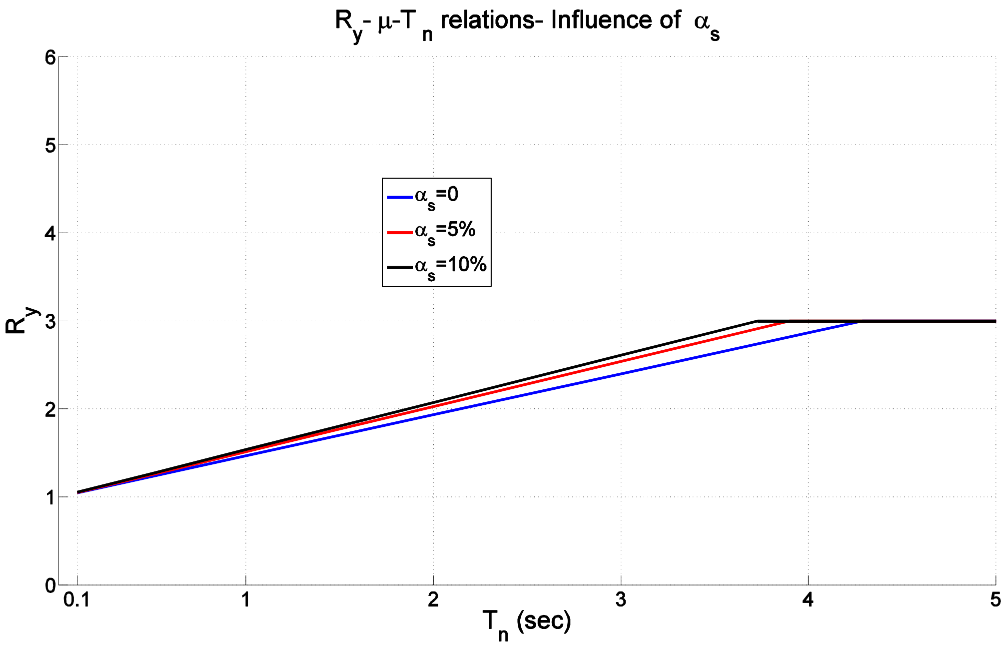

Figure 1.6: R_y-μ-T_n relation for a_s=0, 5%, 10%, μ=3 and T_b=3 sec (proposed values)

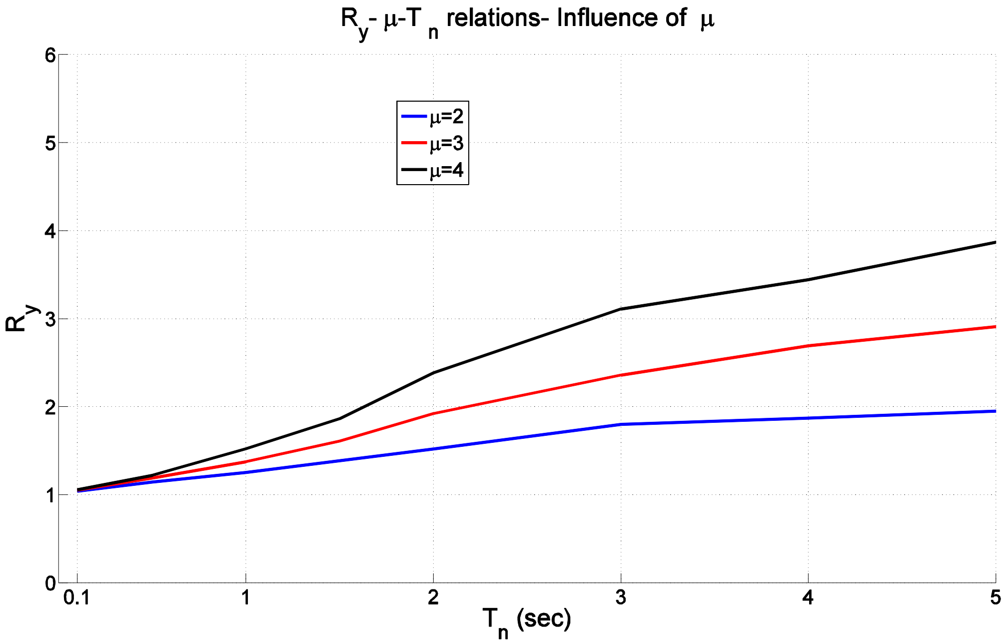

Figure 1.7: R_y-μ-T_n relation for μ=2,3,4 and T_b=3 sec (actual mean values)

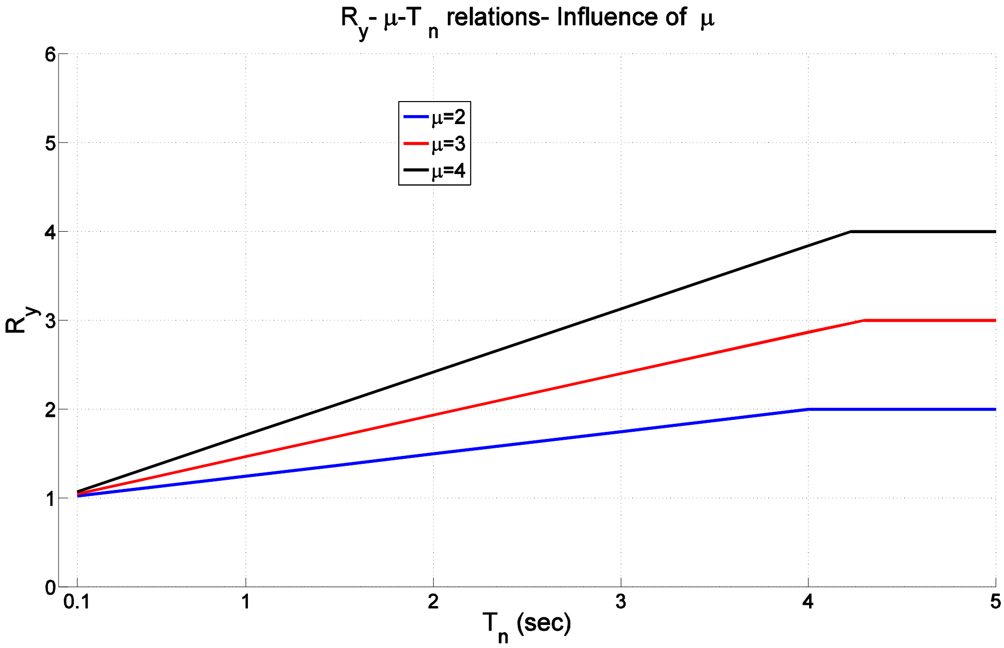

Figure 1.8: R_y-μ-T_n relation for μ=2,3,4 and T_b=3 *sec* (proposed values)

4 CONCLUSIONS

The results obtained in the parametric analysis conducted in this study indicate the nature of the relation between the force reduction factor R_y, the displacement ductility demand μ and the vibration period of the superstructure T_n, for base-isolated structures. These relations are non-linear and show significant similarity to the ones proposed for fixed-base structures [10-19]. Compared to fixed-base structures, the acceleration sensitive region of the response of inelastic base-isolated structures extends towards longer periods. Moreover, the hysteretic energy and the displacement conservation regions shrink and translate along the period axis.

However, the determination of the exact vibration period T_c, after which the equal displacement rule applies ($R_y=\mu$) is difficult without using linear approximations of the obtained R_y-μ-T_n relations. Therefore, simple bilinear expressions are proposed in this study to quantify the sensitivity of the relations to the selected parameters (3.1).

The isolation period T_b has the largest effect on the proposed R_y-μ-T_n relation for isolated structures. Increasing the isolation period (by using softer isolation bearings) moves the equal displacement range for the isolated structures towards longer period values (T_c moves to the right). The effects of post-yield hardening and displacement ductility of the isolated structure on the proposed R_y-μ-T_n relation are not significant.

The ultimate goal of the R_y-μ-T_n relations proposed in this study is the quantification of the inelastic performance of base-isolated structures for a wide range of earthquake records. The proposed R_y-μ-T_n relation indicates that if the strength of typical seismically isolated structures is selected such that they are allowed to enter their inelastic response range, the structures will develop displacement ductility demands significantly larger than those predicted by the equal displacement rule. The accuracy of the proposed R_y-μ-T_n relation can be improved by using a larger sample of more diverse ground motion records and by widening and denser sampling of the design parameter space.

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