COMPDYN 2013
4th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, V. Papadopoulos, V. Plevris (eds.)
Kos Island, Greece, 12–14 June 2013

CONTROL OF THE EIGENSOLUTIONS OF A HARMONICALLY DRIVEN COMPLIANT STRUCTURE

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Keywords: Compliant Structures, Eigensolutions, Systematic Control, Sensitivities, Local Structural Modifications

Abstract. *The motion amplitude of a harmonically driven compliant structure is maximized if the driving frequency equals one of the structural resonance frequencies. For practical use of resonating compliant structures control of the eigensolutions, i.e. eigenfrequencies and eigenmodes, is often required. In this work, a systematic way to control the eigensolutions of harmonically driven structures is presented while maintaining the advantages of a resonating structure. This control is realized using local structural modifications. Eigensolution sensitivities for these local structural modifications are used to indicate the locations for the most effective structural modifications, minimizing the required control power. This method uses the modal basis of the structure as the preferred basis. A simple harmonically driven compliant structure is used to show how local structural modifications are selected to obtain a desired change in the eigensolutions. The proposed method is found to be a convenient tool to determine effective local structural modifications to control the eigensolutions of resonating compliant structures. During the design phase it provides valuable insights in the possibilities and limitations of controlling specific eigensolutions.*

1 INTRODUCTION

The motion amplitude of a harmonically driven compliant structure is maximized if the driving frequency equals one of the structural resonance frequencies. Exciting a structure in its resonance frequency enables large output amplitudes at relatively low power consumption [1]. Typical examples of mechanical applications which use resonance to their advantage are ultrasonic and microlinear actuators [2, 3], resonant silicon sensors [4] and Micro-Electro-Mechanical Systems (MEMS) in general [5]. Ideally, the actuation power to excite a structure in its resonance frequency is determined by the dissipated energy, *e.g.* due to damping.

An example of intentional use of resonance within nature is found in insects. Insects use a resonating thorax structure to reduce the work to accelerate and decelerate their wings during flapping flight [6]. This limits the size of the flight muscles since the required actuation power is, ideally speaking, determined by the dissipated energy only, consisting of aerodynamic drag and damping. Nowadays, a number of Flapping Wing Micro Air Vehicle (FWMAV) designs mimics the resonating thorax structure of insects [7, 8]. These designs use the resonance of the structure to achieve large flapping amplitudes with low power consumption.

For practical use of resonating (compliant) structures control of the eigensolutions, *i.e.* eigenfrequencies and eigenmodes, is often required. For instance, flight control of a FWMAV design requires an asymmetric flapping motion, which implies a controlled change of the initially symmetric eigenmode. Controlling the eigensolutions of resonating FWMAV designs is particularly complicated by constraints on weight and power consumption. In literature, optimization methods are used to design vibrating structures that target desired eigensolutions [9, 10] and piezoelectric material is distributed over a structure to achieve a desired dynamic behavior [11]. However, systematic ways to control the eigensolutions of resonating structures during operation, while maintaining the advantages of a resonating structure, are not well established in literature.

The eigensolutions of a resonating structure are determined by its structural properties, *e.g.* mass, damping, stiffness and their spatial distribution. Local changes of these properties will result in changes of the eigensolutions. Local structural modifications, such as stiffness changes, can be obtained using actuators such as piezoelectric patches [12]. Besides being used as high frequency actuators, they may also alter the stiffness or change the stiffness indirectly by altering the shape of the structure. Additionally, they may be used as dampers.

The changes in eigensolutions due to local structural modifications can be estimated using sensitivity analysis. Although the calculation of the eigenfrequency sensitivity is straightforward, the eigenmode sensitivity is more involved. Fox and Kapoor [13] showed that the eigenmode sensitivity can be represented by a linear combination of the structural eigenmodes. Nelson [14] presented a method where only the eigenmode of interest was required to determine the eigenmode sensitivity. These methods have been extended to allow for eigensolution sensitivities of asymmetric non-conservative systems [15] and systems with repeated eigenfrequencies [16]. In our present work which is to find a systematic way to control the eigensolutions, conservative systems will be considered as a first step. Thus, damping is ignored for now. The eigenfrequencies are, besides the possible presence of rigid-body modes, assumed to be distinct. Moreover, a modal basis is used as a preferred basis, which allows for a more intuitive understanding and for maximum insight in the eigensolution control.

The well-known discretized eigenproblem for free vibrations and the eigenproblem sensitivities to structural modifications, using a modal basis, are presented in Section 2. The obtained expressions are interpreted to understand the consequences for systematic eigensolution control.

In Section 3, a method is presented to analyze the influence of local structural modifications on the eigenmode sensitivities. This method provides valuable insights in the possibilities and limitations of controlling specific eigensolutions. A simple 2D example of a compliant structure is used in Section 4 to demonstrate the usefulness of the method. In Section 5, conclusions and future research are presented.

2 EIGENSOLUTION SENSITIVITIES

This section starts with the well-known discretized eigenproblem for free vibrations and corresponding orthogonality relationships. Next, the forced harmonic response using mode superposition is summarized [17]. Subsequently, the sensitivity of the eigensolutions using the modal basis is shown for self containment [13]. Interpretations of the mathematical expressions are given to understand the consequences of this modal representation for systematic eigensolution control.

2.1 Eigenproblem formulation

The eigenfrequencies and corresponding eigenmodes of a conservative, N degrees of freedom system are determined by the general eigenproblem,

$$\left(\mathbf{K} - \omega_k^2 \mathbf{M}\right) \mathbf{v}_k = \mathbf{0} \quad \text{where} \quad k = 1 \ldots N. \tag{1}$$

In Eq. (1), $\mathbf{M}$ and $\mathbf{K} \in \mathbb{R}^{N \times N}$ are the mass and stiffness matrix, respectively, ω_k^2 is the k^{th} eigenfrequency and $\mathbf{v}_k \in \mathbb{R}^N$ is the k^{th} eigenmode. The mass matrix, $\mathbf{M}$, is symmetric, positive definite and the stiffness matrix, $\mathbf{K}$, is symmetric, semi-positive definite. $\mathbf{M}$ and $\mathbf{K}$ both depend on the structural design parameters. The number of possible rigid-body modes is m and it is assumed that the remaining eigenvalues, ω_k^2, are distinct and ordered conform $\omega_k^2 < \omega_{k+1}^2$ where $k = m+1 \ldots N$. Mass normalization of the eigenmodes is assumed such that the orthogonality relationships become,

$$\begin{aligned} \mathbf{v}_l^T \mathbf{M} \mathbf{v}_k &= \delta_{kl}, \\ \mathbf{v}_l^T \mathbf{K} \mathbf{v}_k &= \omega_k^2 \delta_{kl}, \end{aligned} \tag{2}$$

where $k, l = 1 \ldots N$ and δ_{kl} represents the Kronecker delta.

2.2 Forced harmonic response

The transient response of a conservative, N degree of freedom system, submitted to a harmonic force with constant amplitude $\mathbf{s}$, is governed by equation,

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{s}\cos(\omega t), \tag{3}$$

where $\mathbf{q}$ represents the response and ω is the excitation frequency. The forced response is the part of the response, synchronous to the excitation,

$$\mathbf{q} = \mathbf{x}\cos(\omega t). \tag{4}$$

By substituting Eq. (4) into the equations of motion, Eq. (3), and by using an eigenmode series expansion, including the possible presence of m rigid-body modes, the forced response amplitude $\mathbf{x}$ is given by,

$$\mathbf{x} = \left\{ -\frac{1}{\omega^2} \sum_{i=1}^{m} \mathbf{v}_i \mathbf{v}_i^T + \sum_{s=m+1}^{N} \frac{\mathbf{v}_s \mathbf{v}_s^T}{\left(\omega_s^2 - \omega^2\right)} \right\} \mathbf{s}, \tag{5}$$

where eigenmode orthogonality is extensively used. This solution indicates, that the amplitude of the response $\mathbf{x}$ goes to infinity when the excitation frequency ω approaches an eigenfrequency, ω_s. Provided that the eigenmode $\mathbf{v}_s$ is not orthogonal to the constant amplitude $\mathbf{s}$, the harmonic oscillation of the structure is, in that case, determined by the shape of eigenmode $\mathbf{v}_s$. Controlling the response of a structure which is harmonically excited at a frequency close to a structural eigenfrequency ω_s is, thus, equivalent to controlling eigenmode $\mathbf{v}_s$.

2.3 Eigenproblem sensitivity

Suppose a number of design parameters, $\boldsymbol{\mu} \in \mathbb{R}^p$, where p represents the number of design parameters. The sensitivities of the rigid-body eigensolutions to a design parameter, μ_i, are zero in this study. The eigenfrequencies of the rigid-body modes remain zero and the rigid-body eigenmodes remain the same since the nodal position of the discretized elements is not influenced by the design parameters [18]. The sensitivity, or derivative, of Eq. (1) to a design parameter, μ_i, yields,

$$\left(\mathbf{K}' - \omega_k^2\mathbf{M}'\right)\mathbf{v}_k - \left(\omega_k^2\right)'\mathbf{M}\mathbf{v}_k + \left(\mathbf{K} - \omega_k^2\mathbf{M}\right)\mathbf{v}_k' = \mathbf{0}, \tag{6}$$

where $(..)' = {}^{\partial ..}/_{\partial \mu_i}$. Since the sensitivity of the rigid-body eigensolutions is zero, the subindex k excludes the rigid-body eigensolutions, *i.e.*, $k = m+1 \ldots N$ in the remainder of this work. Premultiplying Eq. (6) by eigenmode $\mathbf{v}_k$ and using Eq. (1) and Eq. (2), the eigenfrequency sensitivity is given by,

$$\left(\omega_k^2\right)' = \mathbf{v}_k^T\left(\mathbf{K}' - \omega_k^2\mathbf{M}'\right)\mathbf{v}_k, \tag{7}$$

which shows that the eigenfrequency sensitivity of the k^{th} eigenfrequency requires information of the k^{th} eigensolution only. Since the N structural eigenmodes form a complete and orthogonal basis, the eigenmode sensitivities, as present in Eq. (6), can be written as a linear combination of the structural eigenmodes,

$$\mathbf{v}_k' = \sum_{l=1}^{N}\alpha_{ki}^l\mathbf{v}_l, \tag{8}$$

where α_{ki}^l determines the contribution of each eigenmode $\mathbf{v}_l$ to the sensitivity of eigenmode $\mathbf{v}_k$ with respect to μ_i. Substitution of Eq. (8) into the sensitivity of the mass orthogonality relationship of Eq. (2) with respect to μ_i, yields,

$$\alpha_{ki}^l = -\frac{1}{2}\mathbf{v}_k^T\mathbf{M}'\mathbf{v}_k \quad \text{for} \quad l = k. \tag{9}$$

Premultiplying Eq. (6) by eigenmode $\mathbf{v}_n$, substituting Eq. (8) for $\mathbf{v}_k'$ and using the orthogonality relationships of Eq. (2) yields,

$$\alpha_{ki}^l = -\frac{\mathbf{v}_l^T\left(\mathbf{K}' - \omega_k^2\mathbf{M}'\right)\mathbf{v}_k}{\omega_l^2 - \omega_k^2} \quad \text{for} \quad l \neq k. \tag{10}$$

The coefficients α_{ki}^l, as given by Eqs. (9) and (10), arise from the definition of the eigenmode sensitivity as given by Eq. (8). This modal representation gives a convenient starting point for optimal eigensolution control since it uses the natural basis of the structure.

2.4 Consequences for systematic eigensolution control

To calculate the eigenfrequency sensitivity, Eq. (7), and the eigenmode sensitivity, Eq. (8) with Eqs. (9) and (10), only the sensitivity of the mass and stiffness matrix to a design variable μ_i needs to be determined. The sensitivity of the structural matrices is given by $\mathbf{M}'$ and $\mathbf{K}'$, respectively. In this work, changing a design variable μ_i implies a local structural modification. Thus, provided that the design variable, μ_i, influences the structural properties, the matrices $\mathbf{M}'$ and $\mathbf{K}'$ contain non-zero entries which correspond to the locations where the design variable is modified. Although the eigenfrequency sensitivity requires information on the eigensolution of interest only, the eigenmode sensitivity requires information on all N eigensolutions. Based on the expressions for the coefficients α_{ki}^{l} in Eqs. (9) and (10) it can be observed that:

- The magnitude of the numerator of Eq. (10) depends on the size and spatial location of the applied local structural modification. This dependency on the size and location is caused by the interplay between the segments of the eigenmodes $\mathbf{v}_k$ and $\mathbf{v}_l$ and the sensitivity of the structural matrices, $\mathbf{M}'$ and $\mathbf{K}'$, corresponding to the locations where the structural modifications are applied. The contribution of the eigenmodes $\mathbf{v}_k$ and $\mathbf{v}_l$ in the regions not affected by the structural modifications is irrelevant since the sensitivity of the structural matrices, $\mathbf{M}'$ and $\mathbf{K}'$, is zero corresponding to those locations.

- Since the eigenmodes are normalized using the mass matrix, Eq. (2), eigenmode $\mathbf{v}_k$ contributes to the sensitivity of eigenmode $\mathbf{v}_k$ if the sensitivity of the structural mass matrix, $\mathbf{M}'$, is non-zero, see Eq. (9). However, the actual shape change of eigenmode $\mathbf{v}_k$ will be determined by the contribution of the modes $\mathbf{v}_l$, where $l \neq k$.

- The term $(\mathbf{K}' - \omega_k^2 \mathbf{M}')\,\mathbf{v}_k$ in Eq. (10) can be seen as a pseudo load which acts locally on the regions where the structural modifications are applied. Maximum gain is scored if the inner product between the pseudo load and the eigenmode $\mathbf{v}_l$ is maximized. For a specific l, this inner product can be maximized by adjusting the size and location of specific structural modifications.

- Due to the denominator in Eq. (10), $(\omega_l^2 - \omega_k^2)$, the contribution of eigenmode $\mathbf{v}_l$ to the sensitivity of eigenmode $\mathbf{v}_k'$, Eq. (8), is reduced if the corresponding eigenvalue ω_l deviate more from eigenvalue ω_k.

These observations will contribute to a better understanding and more insights into the control of a specific eigenmode using local structural modifications.

3 SYSTEMATIC EIGENSOLUTION CONTROL

This section shows how the eigenproblem sensitivity, as described with respect to a modal basis in Section 2.3, can be used to identify the most effective locations to apply structural modifications, to control a specific eigenmode. First, a projection is defined to indicate the effectiveness of a certain eigenmode change. Secondly, the consequences of this measure for the design and optimization of structural modifications with the aim of structural eigensolution control are investigated.

3.1 Measure effectiveness of eigenmode change

Our goal is to modify a certain vibration in a desired fashion. Prescribing the entire desired shape of the eigenmode of interest is rather complex and this level of detail is usually not required. For controlling the eigenmode shape, spatial portions of the eigenmode shape are specified for which the desired dynamic response should be maximized or minimized and the remaining portion is left unspecified.

In former research on the design of vibrating structures that targets desired eigenmode shapes, unit loads are defined at specific spatial locations where modifications of the dynamic response are required [9, 11]. By projecting the eigenmode of interest onto these unit loads, a quantitative measure of the dynamic response was determined for each structural design. The value of this projection is used to determine the optimal structural design to achieve the desired eigenmode shape.

In this work, a unit projection vector $\mathbf{b} \in \mathbb{R}^N$ is defined, which has non-zero entries for the locations which correspond to the spatial locations where the desired dynamic response should be modified. A quantitative measure of the current dynamic response at the locations of interest is obtained by projection of the eigenmode of interest, $\mathbf{v}_k$, onto this projection vector, $\mathbf{b}$, while including the structural mass matrix, $\mathbf{M}$, as a weighting matrix. The structural mass matrix is included to simplify the intuitive understanding of the projection as shown in the remainder of this work. The projection is given by,

$$r_k = \mathbf{b}^T \mathbf{M} \mathbf{v}_k. \tag{11}$$

In this work, the projection value, r_k, can be changed using local structural modifications. Sec. 2.3 describes the change of eigenmode $\mathbf{v}_k$ due to structural modifications. Accordingly, the change of the projection of Eq. (11) is given by,

$$r'_k = \mathbf{b}^T \mathbf{M} \mathbf{v}'_k, \tag{12}$$

where the weighting matrix, the structural mass matrix $\mathbf{M}$, remains constant. The change of projection r'_k, Eq. (12), depends on the size and spatial location of the structural modifications. Depending on the desired dynamic response at the spatial locations of interest, the change of the projection value, Eq. (12), should be

- maximized if a specific control state requires that the dynamic response of eigenmode $\mathbf{v}_k$ is maximized at the locations of interest,
- minimized if a specific control state requires that the dynamic response of eigenmode $\mathbf{v}_k$ is minimized at the locations of interest.

The change of r'_k is maximized or minimized by determining the optimal location and dimension of structural modifications. Using the modal basis, the projection vector $\mathbf{b}$ can be written as,

$$\mathbf{b} = \sum_{m=1}^{N} \beta_m \mathbf{v}_m. \tag{13}$$

By combining Eq. (13) and the modal representation of the eigenmode sensitivity, Eq. (8), with the expressions for the coefficients α_{ki}^l, Eqs. (9) and (10), the change of the projection, Eq. (12), is rewritten to,

$$r'_k = \sum_{\substack{l=1 \\ l \neq k}}^{N} \frac{1}{\omega_k^2} \beta_l \frac{1}{1 - \omega_l^2/\omega_k^2} \left(\mathbf{v}_l^T \left[\mathbf{K}' - \omega_k^2 \mathbf{M}' \right] \mathbf{v}_k \right) - \frac{1}{2} \beta_k \mathbf{v}_k^T \mathbf{M}' \mathbf{v}_k, \tag{14}$$

where the mass orthogonality relationship of Eq. (2) is extensively used. The usefulness of Eq. (14) is investigated by examining each term.

This analysis shows that:

- the first term, $(1/\omega_k^2)$, depends on the eigensolution of interest. This term is, therefore, fixed and given in advance.
- the second term, (β_l), determines the modal contribution of each eigenmode $\mathbf{v}_l$ to the projection vector $\mathbf{b}$. The magnitude of this term depends on the alignment of eigenmode $\mathbf{v}_l$ with respect to the projection vector $\mathbf{b}$ resulting in zero if $\mathbf{v}_l$ is orthogonal to $\mathbf{b}$.
- the third term, $(1/(1-\omega_l^2/\omega_k^2))$, will drop rapidly if eigenfrequency ω_l starts to deviate from eigenfrequency ω_k. For a maximization of r_k', contributions of eigensolutions for which ω_l deviates significantly from ω_k become marginally important.
- the fourth term, $\left(\mathbf{v}_l^T\left[\mathbf{K}'-\omega_k^2\mathbf{M}'\right]\mathbf{v}_k\right)$, is a scalar that depends on the size and spatial location of the applied structural modifications. The magnitude of this term is determined by the interplay between the segments of the eigenmodes $\mathbf{v}_k$ and $\mathbf{v}_l$ and the sensitivity of the structural matrices, $\mathbf{M}'$ and $\mathbf{K}'$, corresponding to the locations where the structural modifications are applied. The magnitude of this scalar product can be maximized if eigenmode $\mathbf{v}_l$ is as much as possible aligned to the pseudo load, $(\left[\mathbf{K}'-\omega_k^2\mathbf{M}'\right]\mathbf{v}_k)$, at the locations where the modifications are applied. However, if the second or third term of Eq. (14) are approximately zero for a specific l, a maximization of this term will not add much to the change of the projection, r_k'.
- the fifth term, $\left(\frac{1}{2}\beta_k\mathbf{v}_k^T\mathbf{M}'\mathbf{v}_k\right)$, originates from the normalization conditions, Eq.(2). Since there is only a contribution of eigenmode $\mathbf{v}_k$, this term is less interesting while considering the change of the response due to a shape change of eigenmode $\mathbf{v}_k$. However, this term is important to provide the exact magnitude of r_k'

The change of the projection, Eq. (14), together with the above described systematic breakdown, gives valuable insights in the systematic control of eigenmodes. Since the modal basis is used as a preferred basis, the influence of the individual eigensolutions becomes transparent.

3.2 Design and optimization of structural modifications

To control the eigensolution of interest of a resonating structure during operation, one would like to have a quick estimation and an understanding of the most effective spatial locations to apply structural modifications. The change of the proposed projection, r_k', as introduced in Sec. 3.1, is found to be an effective measure to understand the possibilities and limitations of the potential achievable changes of the eigenmode of interest. Since the modal basis is used as the preferred basis in the entire formulation, an intuitive understanding of the eigenmode change due to local structural modifications is obtained.

The different terms of Eq. (14), as described in Sec. 3.1, show in an early phase the dominance of specific eigenmodes $\mathbf{v}_l$ to the change of the projection r_k' and, thus, to the change of eigenmode $\mathbf{v}_k$. The change of the projection, r_k', is maximized if the local structural modifications are chosen in such a way that the fourth term of Eq. (14), $\left(\mathbf{v}_l^T\left[\mathbf{K}'-\omega_k^2\mathbf{M}'\right]\mathbf{v}_k\right)$, is maximized for these specific, dominant eigenmodes, $\mathbf{v}_l$. The structural sensitivity matrices, $\mathbf{M}'$ and $\mathbf{K}'$, contain only non-zero entries corresponding to the location where the modifications are applied. The fourth term of Eq. (14) is therefore maximized for a specific l, if the eigenmodes $\mathbf{v}_k$ and $\mathbf{v}_l$ are well aligned at these spatial locations. This term results in zero if the segments of eigenmode $\mathbf{v}_k$ are orthogonal to the eigenmode $\mathbf{v}_l$ at the locations where the modifications are applied.

At some point, optimization techniques can be used to find the optimal locations to apply local structural modifications for the control of a specific eigensolution more precisely. In that case, topology optimization techniques can be utilized to determine the locations where the actuator patches should be located. The presented projection r_k is attractive to use since the number of modes included in the analysis can be very limited. The eigenfrequency sensitivity, Eq. (7), depends also on the locations of the structural modifications. Thus, the shift of the eigenfrequency, due to the modifications, can be added to the optimization. This allows for a more comprehensive control of the eigensolutions.

4 NUMERICAL EXAMPLE

In this section, the method for the systematic control of eigensolutions, as described in Section 3, is applied to a simple 2D compliant structure. The numerical model of the compliant structure is discussed first, followed by the free vibration solutions of this structure. Thereafter, the projection, as introduced in Section 3, is used to demonstrate the usefulness of the proposed method for systematic eigensolution control.

4.1 Numerical model

Figure 1 shows the model of the 2D compliant structure used in this example. The model is composed using 2D beam elements which allow stretching and bending. Shear deformation is neglected by assuming slender elements. The global mass and stiffness matrix are composed by assembling consistent mass and stiffness element matrices for 2D beam elements [19]. The beam elements have a circular cross section with a diameter $d_e = 0.0035$ m. The ring of the structure has a diameter $d_r = 0.05$ m, and the length of the wing-type cantilevers $l_w = 0.05$ m, see Figure 1. The structure has a Young's modulus $E = 1.5$ GPa, and a density $\rho = 1175$ kg/m^3, which corresponds to the mechanical properties of plastic, *i.e.* polypropylene. The total mass of the structure is 4 g.

The 2D compliant structure of Figure 1 might be compared with a simplified FWMAV design. In that case, the ring of the structure represents the main body and the four cantilevers

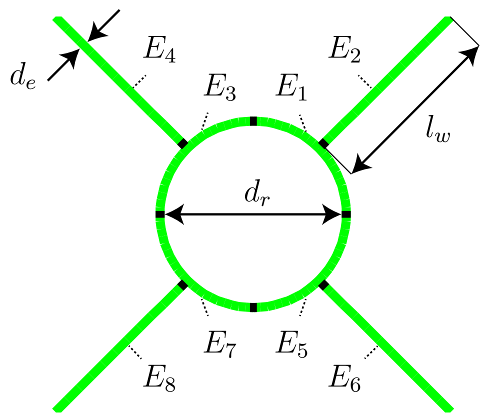
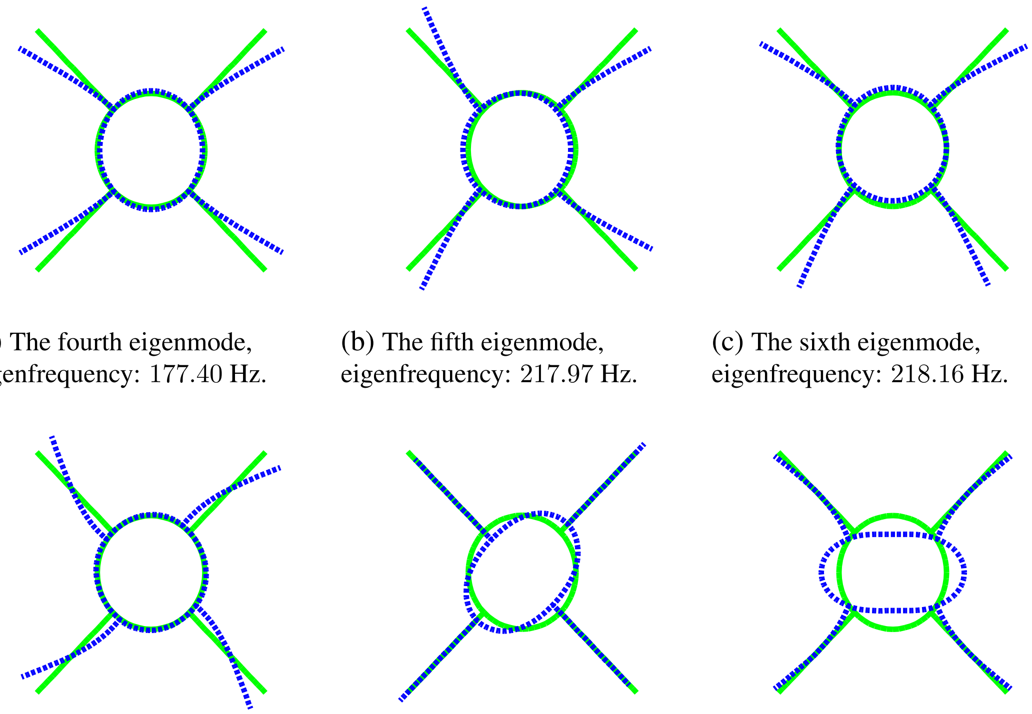


Figure 1: Compliant 2D model showing the circular beam diameter, d_e, the diameter of the ring, d_r, the length of the wing, l_w, and the eight different Young's moduli, E.

Young's modulus	[Gpa]
E_1	1.43
E_2	1.45
E_3	1.47
E_4	1.49
E_5	1.51
E_6	1.53
E_7	1.55
E_8	1.57

Table 1: The eight different Young's moduli required for asymmetry.



(a) The fourth eigenmode, eigenfrequency: 177.40 Hz.

(b) The fifth eigenmode, eigenfrequency: 217.97 Hz.

(c) The sixth eigenmode, eigenfrequency: 218.16 Hz.

(d) The seventh eigenmode, eigenfrequency: 384.05 Hz.

(e) The eighth eigenmode, eigenfrequency: 385.22 Hz.

(f) The nineth eigenmode, eigenfrequency: 767.58 Hz.

Figure 2: The first six eigensolutions with a non-zero eigenfrequency. In these figures, the green, solid structure represents the reference, static structure and the blue, dotted structure represents the eigenmode.

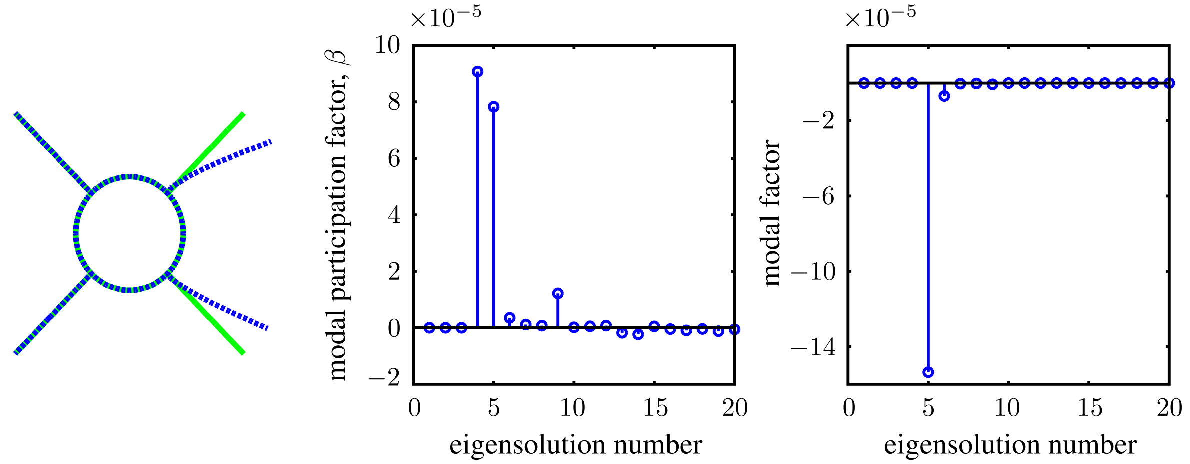
represent the wings. The geometrical model of Figure 1 is highly symmetric. This symmetry will result in free vibrations with repeated nonzero eigenfrequencies. To circumvent the particularities of repeated eigenfrequencies, the structure is divided in eight parts as shown in Figure 1. By slightly varying the Young's modulus of each of these parts, an asymmetric structure is obtained, see Table 1.

4.2 Eigensolutions

Since the 2D model as shown in Figure 1 is not constrained, the structure is floating. Therefore, the structure exhibit three orthogonal rigid-body eigensolutions, $i.e.$, translation in x- and y-direction and rotation about the z-axis. Figure 2 shows the first six eigensolutions with a non-zero eigenfrequency. Although the eigensolutions appear to have a high degree of symmetry, all eigenfrequencies are distinct due to the variations in Young's moduli, Table 1. Practically symmetric looking eigenmodes are called symmetric in the remainder of this work.

4.3 Amplification of the dynamic response in a specific region

An interesting oscillation is obtained if the structure of Figure 1 is harmonically excited with an excitation frequency close to the fourth eigenfrequency. Provided that the excitation is not orthogonal to the fourth eigenmode, the structure will vibrate according to the fourth eigenmode, (see Section 2.2). For FWMAV designs, this vibration mode is of particular interest since it corresponds to a symmetric flapping of the wings, required for hovering flight. The eigenfrequency associated to the flapping mode is somewhat to high for practical FWMAV.



(a) The unit projection vector, $\mathbf{b}$, in dotted blue.

(b) The modal participation factors, β, to the projection vector, $\mathbf{b}$.

(c) Modal factor showing the significant eigensolutions ($l \neq k$).

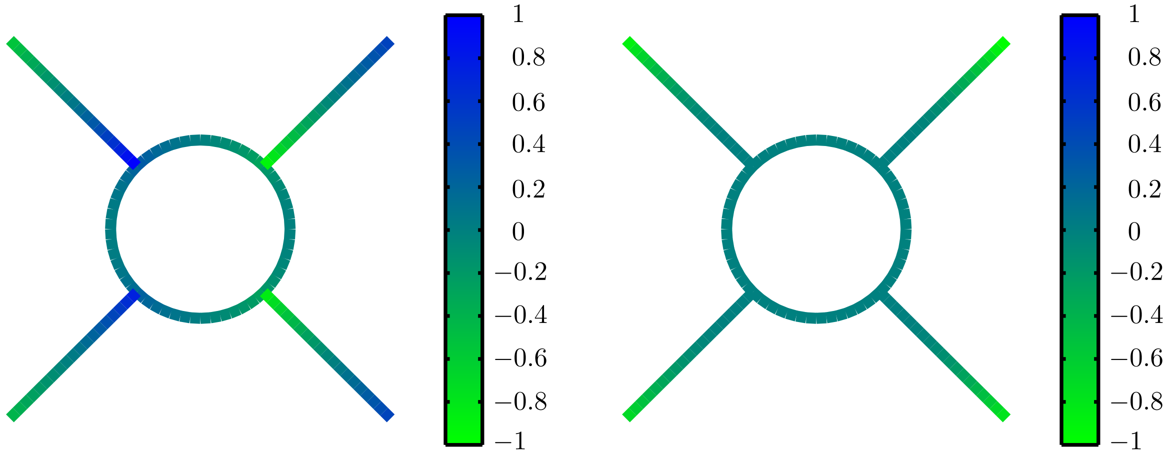
Figure 3: Consideration of dominant eigenmodes. (a) the unit projection vector, $\mathbf{b}$, represented by the dotted, blue line, (b) the modal participation factors of the projection vector, (c) and the multiplication factor between the modal participation factors and the eigenfrequency dependent factor for $l \neq k$.

However, the concept is nicely illustrated.

To study the locations at which structural modifications need to be applied to obtain an asymmetric flapping, a projection vector, $\mathbf{b}$, is defined. To modify the dynamic response of the wing-type cantilevers on the right side, $\mathbf{b}$ contains only non-zero entries on the right side, see Figure 3a. By maximizing or minimizing the change of the projection, r'_k, where $k = 4$, using structural modifications, the dynamic response of the wing-type cantilevers is modified.

In order to determine which eigenmodes $\mathbf{v}_l$ are most dominantly contributing to the change of the projection, r'_4 , the modal participation factors, β, (see Eq. (13)), are determined as a first step. Figure 3b shows that the the modal participation factor, β, is only significant for a very few eigenmodes, *i.e.* eigenmode 4, 5 and 9. For the eigensolutions $l \neq k$, where $k = 4$, the number of significant eigensolutions is even more convincingly reduced by multiplying the modal participation factor, β, with the eigenfrequency dependent factor, the third term in the change of the projection of Eq. (14), $\left(1/(1 - \omega_l^2/\omega_k^2)\right)$. Figure 3c shows the modal factors after this multiplication. The Figures 3b and 3c show clearly that eigensolutions 4, 5 and 6 (to a less degree) are most dominant in determining the change of the projection, r'_4.

Finally, to determine the total change of the projection, r'_4, the spatial location at which the local structural modifications are applied is required. Using these locations the sensitivity of the structural matrices, $\mathbf{M}'$ and $\mathbf{K}'$, to the design variable is determined. In this work, the circular beam diameter, d_e, is the design variable used which influences both the mass and stiffness matrix. Due to the high modal factor of eigensolution 5, (see Figure 3c), the magnitude of the fourth term of Eq. (14), $\left(\mathbf{v}_l^T \left[\mathbf{K}' - \omega_k^2 \mathbf{M}'\right] \mathbf{v}_k\right)$, for $l = 5$ will have a large influence on the final change of the response, r'_4. Figure 4a shows how the magnitude of this term depends on the location at which the circular beam diameter, d_e, is influenced. It is clearly shown, that the influence of a circular beam diameter change is highest at the root of the wing-tip cantilevers and around zero if the modification is applied at the central ring structure. The modal participation term of the fourth eigensolution, β_4, plays only a role in the fifth term of Eq. (14), $\left(\frac{1}{2}\beta_k \mathbf{v}_k^T \mathbf{M}' \mathbf{v}_k\right)$. Figure 4b shows how the magnitude of this term depends on the location at which the circular beam diameter is modified. Since this term depend on the fourth



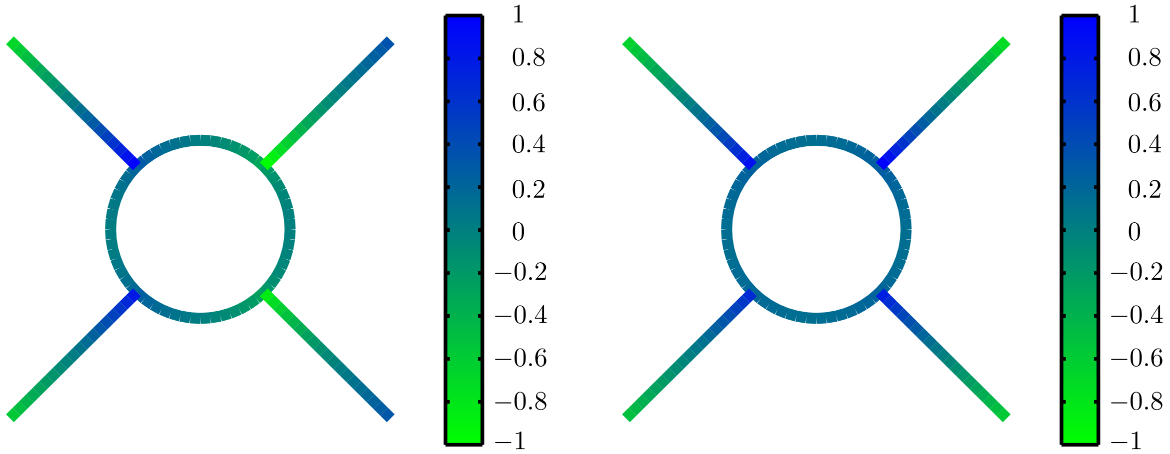
(a) Location dependent, normalized measure of the term $\left(\mathbf{v}_l^T \left[\mathbf{K}' - \omega_k^2 \mathbf{M}'\right] \mathbf{v}_k\right)$ for $k = 4$ and $l = 5$.

(b) Location dependent, normalized measure of the term $\left(\mathbf{v}_k^T \mathbf{M}' \mathbf{v}_k\right)$ for $k = 4$.

Figure 4: Location dependent, normalized plots of significant terms present in the change of the projection value. (a) shows the interplay between the eigenmodes $\mathbf{v}_4$ and $\mathbf{v}_5$ and the sensitivity of the structural matrices, (b) shows the interplay of eigenmode $\mathbf{v}_k$ and the sensitivity of the structural mass matrix.

eigensolution only, the shape of eigenmode $\mathbf{v}_4$, see Figure 2a, is clearly visible.

Based on Figure (4), the circular beam diameter should be modified at the root of the wing-type cantilevers to maximize the change of the projection, r_4'. Figure 5a shows the change of the projection r_4', as a function of the location where the circular beam diameter, d_e, is modified, while taking all N eigensolutions into account. Figure 5a confirms that the change of the projection is maximized if the circular beam diameter is modified at the root of the wing-type cantilevers. The dominance of eigensolutions 4 and 5 in the total change of the response is clearly visible while comparing Figure 4 with Figure 5a. This simple example shows, that the most effective spatial locations to apply structural modifications to maximize or minimize the change of the response, r_k', can be identified using by a very limited number of eigensolutions.



(a) Location dependent, normalized measure of the change of the total projection, r_4'.

(b) Location dependent, normalized measure of the change of the fourth eigenfrequency, ω_4^2.

Figure 5: Location dependent, normalized plots. (a) shows the change of the response, r_4' while taking all N eigensolutions into account, (b) shows the change of the fourth eigenfrequency depending on the location at which the circular beam diameter is modified.

A close look to the structural eigensolutions, (see Figure 2a), and the modal representation of the eigenmode sensitivity, (see Eq. (8)), shows already that a high contribution of the fifth eigenmode leads to a maximization of the change of the projection, r_k'. Detailed observation and knowledge of the eigensolution of interest and the neighboring eigensolutions will therefore provide insights into the possibilities and limitations of systematic eigensolution control.

Figure 5b shows how the change of the fourth eigenfrequency, ω_4^2, depends on the location at which the circular beam diameter d_e is modified. This figure can be used to limit the shift in eigenfrequency due to the applied modifications. Figure 5b shows, as an example, that the shift in eigenfrequency can be neglected by applying local structural modifications at the left wingroot and at the right wingtip. This means, while looking to Figure 5a, that the change of the response can be maximized while the eigenfrequency of the excited eigenmode is not varied.

5 CONCLUSIONS

In this work, a method is presented to increase the understanding of systematic eigensolution control via local modifications. The changes in the eigensolutions, due to local structural modifications, are determined using sensitivity analysis. The proposed method uses the modal basis of the structure as the preferred basis, which increases the insights into the possibilities and limitations during systematic eigensolution control. A projection is proposed to study the effectiveness of specific local structural modifications. With this projection, the initially present N-dimensional modal space is largely decreased towards a limited number of dominant eigensolutions. This leads to a more intuitive understanding of the eigensolution control. Subsequently, these most dominant eigensolutions can be used to obtain significant changes in the eigensolution of interest using minimal structural modifications.

Because of constraints on weight and power consumption, high effectiveness is of great interest in controlling the eigensolutions of resonating FWMAV designs. For active flight control, one often requires a controlled change of the initially symmetric flight mode towards an asymmetric flapping motion. In this work, a simple 2D model of a compliant structure is used to show the locations at which structural modifications should be applied to change a symmetric eigenmode into an asymmetric one. This simple study shows that a proper understanding of the neighboring eigensolutions contributes to the understanding of the required location of the structural modifications to obtain this asymmetry.

The proposed projection method will decrease computational costs for optimization methods, such as topology optimization. Optimization techniques might be used to determine the optimal size, number and spatial location of actuator more precisely. Since one is able to identify the most significant eigenmodes, the computation time will be decreased. The shift of the eigenfrequency, due to structural modifications, is easily taken into account such that constraints on the eigenfrequency shift can be incorporated.

Harmonically driven compliant structures, such as FWMAV designs, are generally non-conservative, due to aerodynamic damping. In future research, the systematic eigensolution control, using the modal basis, will be investigated for non-conservative systems. Subsequently, the potential of repeated eigenvalues for the rapid control of eigensolutions will be investigated. This is motivated by the reasoning, as presented in this work, that neighboring eigenmodes become more significant if their corresponding eigenfrequencies are closely spaced. Additionally, the presence of multiple control states will be investigated to allow for more advanced control of the eigensolution of interest.

REFERENCES

[1] S. Calmes, S. Schweizer, Ph. Renaud, Resonating Large Angle And Aow Consumption Micromachined Optical Scanner. M.E. Motamedi, R. Goring. eds. *Proc SPIE Int Soc Opt Eng, Conference of Miniaturized Systems with Micro-Optics and Micromechanics III*, San Jose, CA, January 26-27, **3276**, 96–102, 1998.

[2] P. Hagedorn, J. Wallaschek, Travelling wave ultrasonic motors, Part I: Working principle and mathematical modelling of the stator. *Journal of Sound and Vibration*, **155**(2), 31–46, 1992.

[3] K. Saitou, D.-A. Wang, S.J. Wou, Externally Resonated Linear Microvibromotor for Microassembly. *Journal of Microelectromechanical Systems*, **9**(3), 336–346, 2000.

[4] G. Stemme, Resonant silicon sensors. *Journal of Micromechanics and Microengineering*, **1**(2), 113–125, 1991.

[5] C.T.-C. Nguyen, Frequency-Selective MEMS for Miniaturized Low-Power Communication Devices. *IEEE Transactions on Microwave Theory and Techniques*, **47**(8), 1486–1503, 1999.

[6] C.H. Greenewalt, The Wings of Insects and Birds as Mechanical Oscillators. *Proceedings of the American Philosophical Society*, **104**(6), 605–611, 1960.

[7] R.J. Wood, The First Takeoff of a Biologically Inspired At-Scale Robotic Insect. *IEEE Transactions on Robotics*, **24**(2), 341–347, 2008.

[8] C.T. Bolsman, J.F.L. Goosen, F. van Keulen, Design Overview of a Resonant Wing Actuation Mechanism for Application in Flapping Wing MAVs. *International Journal of Micro Air Vehicles*, **1**(4), 263–272, 2009.

[9] Y. Maeda, S. Nishiwaki, K. Izui, M. Yoshimura, K. Matsui, K. Terada, Structural topology optimization of vibrating structures with specified eigenfrequencies and eigenmode shapes. *International Journal for Numerical Methods in Engineering*, **67**(5), 597–628, 2006.

[10] A. Takezawa, M. Kitamura, Sensitivity analysis and optimization of vibration modes in continuum systems. *Journal of Sound and Vibration*, **332**(6), 1553–1566, 2013.

[11] P.H. Nakasone, E.C.N. Silva, Dynamic Design of Piezoelectric Laminated Sensors and Actuators using Topology Optimization. *Journal of Intelligent Material Systems and Structures*, **21**(16), 1627–1652, 2010.

[12] E.F. Crawley, J. de Luis, Use of Piezoelectric Actuators as Elements of Intelligent Structures. *AIAA Journal*, **25**(10), 1373–1385, 1987.

[13] R.L. Fox, M.P. Kapoor, Rates of Change of Eigenvalues and Eigenvectors. *AIAA Journal*, **6**(12), 2426–2429, 1968.

[14] R.B. Nelson, Simplified Calculation of Eigenvector Derivatives. *AIAA Journal*, **14**(9), 1201–1205, 1976.

[15] S. Adhikari, M.I. Friswell, Eigenderivative analysis of asymmetric non-conservative systems. *International Journal for Numerical Methods in Engineering*, **51**(6), 709–733, 2001.

[16] I.U. Ojalvo, Efficient Computation of Modal Sensitivities for Systems with Repeated Frequencies. *AIAA Journal*, **26**(3), 361–366, 1988.

[17] M. Géradin, D. Rixen, *Mechanical Vibrations: Theory and Application to Structural Dynamics, 2nd Edition*. John Wiley & Sons, 1999.

[18] F. van Keulen, H. de Boer, Rigorous Improvement of Semi-Analytical Design Sensitivities by Exact Differentiation of Rigid Body Modes. *International Journal for Numerical Methods in Engineering*, **42**(1), 71–91, 1998.

[19] R.D. Cook, D.S. Malkus, M.E. Plesha, R.J. Witt, *Concepts and Applications of Finite Element Analysis, 4th Edition*. John Wiley & Sons, 2001.