COMPDYN 2013
4th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, V. Papadopoulos, V. Plevris (eds.)
Kos Island, Greece, 12–14 June 2013

UKF ESTIMATION OF SP-TAR MODELS FOR THE IDENTIFICATION OF TIME-VARYING STRUCTURES

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Keywords: Time-varying structures, Autoregressive, Smoothness Priors, Unscented Kalman Filter.

Abstract. *An important class of methods for the effective identification of time-varying structures based on random vibration response data records is that of stochastic parameter evolution methods. Methods of this class rely on parametric time-varying models with a stochastic structure imposed on the time evolution of their parameters. The latter are considered as random variables allowed to vary in time, with their evolution being subject to stochastic smoothness constraints (smoothness priors constraints). In the present study, Smoothness Priors Time-dependent (SP-TAR) models characterized by stochastic smoothness constraint equations with unknown a-priory coefficients are considered. The SP coefficients of the model along with the time-varying AR coefficients have to be estimated based on the measured response of the structure. This is achieved by expressing the generalized SP-TAR model in a nonlinear state-space form and employing the Unscented Kalman Filter (UKF) algorithm. The introduced method is validated through its application for the identification of a simulated gantry crane system.*

1 INTRODUCTION

The increasing interest among engineers for lightweight configurable mechanisms and structures such as the ones based on the principle of tensegrity, brings forth the need for more efficient identification methods able to handle inherent structural variability. Structures characterized by properties, either physical or geometrical, that vary with time are commonly referred to as time-varying or non-stationary structures, and are encountered in numerous applications such as heavy vehicle-bridges systems, cranes, robotic devices and others.

The available time-varying identification methods may be distinguished into three distinct classes (outlined in order of increasing structural complexity; [1, 2]): (i) unstructured parameter evolution methods , (ii) stochastic parameter evolution methods, and (iii) deterministic parameter evolution methods.

Methods of the first class are based on the assumption that the time-varying characteristics of the system to be identified evolve slowly with time. Thus, they employ stationary (short-time or recursive) methods in windowed versions of the signal, within which the stationarity assumption is approximately valid. These methods impose no-structure on their model parameters which are thus free to vary with time. Typical methods of this class include stationary models applied on short segments of the non-stationary signal, and recursive methods that employ exponential windows in order to estimate their parameters based only on recent values of the non-stationary signal [3, Ch. 11].

On the other hand, deterministic parameter evolution methods rely on functional series time-varying models, that is models with coefficients expanded on a time-dependent functional basis [4]. These types of models have been widely used over the last years and are particularly effective for the identification of systems with properties that evolve in a deterministic way [1, 5].

Stochastic parameter evolution methods stand between the aforementioned extreme cases. They are based on time-varying models with parameters that are considered as random variables, allowed to vary with time, with their evolution being subject to stochastic smoothness constraints [6]. These constraints are often referred to as *smoothness priors constraints* and they may be tuned in order to track a wide range of evolutions: from random walk processes to deterministic evolutions described by low order polynomial functions [7].

Although in the vast majority of engineering applications the time-variability of structural properties is designed to be deterministic, disturbances, control errors and random effects lead to deviation from the predetermined variability thus introducing stochasticity. In this way, stochastic parameter evolution methods may be deemed as appropriate for the identification of time-varying structures.

However, the main limitation of the stochastic parameter evolution methods is that they commonly use pre-determined coefficients for the stochastic equation of their parameters. The aim of the present study is to overcome this drawback by using generalized smoothness priors constraints for the time-varying parameters with initially unknown or rather uncertain coefficients. This is achieved by expressing a general Smoothness Prior Time-dependent AutoRegressive (SP-TAR) model in a nonlinear state-space form and estimating its unknown parameters by means of the Unscented Kalman filter (UKF) method. The introduced method is validated through its application on a numerical case study.

2 SMOOTHNESS PRIORS TIME-DEPENDENT AR MODELS

A Time-dependent AutoRegressive (TAR) model of order n is given by the following equation:

$$y[t] + a_1[t]y[t-1] + a_2[t]y[t-2] + \ldots + a_n[t]y[t-n] = e[t], \qquad e[t] \sim \text{NID}(0, \sigma_e^2[t]) \quad (1a)$$

where t designates discrete time (with $t = 1, 2, \ldots, N$), $y[t]$ the observed nonstationary signal, $e[t]$ the residual sequence, that is the unmodeled part of the signal, which is assumed to be normally identically distributed with zero mean and time-varying variance $\sigma_e^2[t]$, and $a_i[t]$ the time-varying AR parameters.

In the case of smoothness priors models, stochastic difference equations of the following general form govern each of the AR parameters.

$$a_i[t] + \delta_1 a_i[t-1] + \delta_2 a_i[t-2] + \ldots \delta_p a_i[t-p] = w_i[t], \qquad w_i[t] \sim \text{NID}(0, \sigma_w^2[t]) \quad (1b)$$

In the equation above p designates the difference equation order, and $w_i[t]$ zero-mean, uncorrelated, mutually uncorrelated and also uncrosscorrelated with $e[t]$, Gaussian sequences with time-dependent variance $\sigma_w^2[t]$. The smoothness of evolution of the AR parameters is controlled by this variance, where a small variance indicates smooth evolution of the corresponding time-varying parameter and vice-versa. This fact may be easily demonstrated by considering the two parts of the solution of Eq. (1b): the solution of the corresponding homogeneous equation $a_i[t] + \delta_1 a_i[t-1] + \ldots \delta_p a_i[t-p] = 0$ and the particular solution for a given driving input noise $w_i[t]$. In the limiting case of $w_i[t] = 0$, the difference equation is deterministic and its solution is an initial values problem, as long as the values $a_i[-p+1], \ldots, a_i[-1], a_i[0]$ are sufficient to solve recursively for any $a_i[t]$ with $t = 1, \ldots, N$. On the other hand, a driving input noise $w_i[t]$ with large variance would manifest the solution of the inhomogeneous stochastic difference equation and the AR parameters would vary almost randomly.

The significant difference of the general SP-TAR model defined in Eq. (1) compared to the conventional approach [6], is that the coefficients of the stochastic difference equations are not predetermined, but assumed unknown. Thus, they have to be estimated from the observed non-stationary signal along with the time-varying AR parameters. In this way, the smoothness priors constraint equation is able to adapt to a specific identification problem under study.

It should be mentioned that for the conventional approach the smoothness priors constraints are typically of the form $\Delta^p a_i[t] = w_i[t]$ where Δ^p is the p order difference operator ($\Delta^p = (1-\mathcal{B})^p$; where $\mathcal{B}$ the backshift operator $\mathcal{B}^i y[t] = y[t-i]$).

The model parameter estimation problem of the general SP-TAR model of Eq. (1) is considered in the following section.

2.1 Parameter estimation

The parameter estimation problem for the SP-TAR model of Eq. (1) involves estimation of both the parameter vector $\boldsymbol{\delta} = [\delta_1 \ \delta_2 \ \ldots \ \delta_p]^T$ and the time-varying AR parameters $a_1[t]$, $a_2[t]$, ..., $a_n[t]$ $\forall\ t$. This problem may be treated by expressing the SP-TAR model in a non-linear state space form. Toward this end, the model parameters are concatenated into an augmented joint state vector

$$\boldsymbol{z}[t] = \left[\ a_1[t] \ldots a_n[t] \vdots a_1[t-1] \ldots a_n[t-1] \vdots \ \ldots a_1[t-p+1] \ldots a_n[t-p+1] \vdots \delta_1 \ldots \delta_p \ \right]^T_{p(n+1)\times 1}$$

and the SP-TAR$(n)_p$ model may be expressed as:

$$\boldsymbol{z}[t] = \boldsymbol{F}(\boldsymbol{z}[t-1]) \cdot \boldsymbol{z}[t-1] + \boldsymbol{G} \cdot \boldsymbol{w}[t] \quad (2a)$$
$$y[t] = \boldsymbol{h}[t]^T \cdot \boldsymbol{z}[t] + e[t] \quad (2b)$$

with:

$$F(z[t-1]) = \left[\begin{array}{cccccc|c} -\delta_1 \cdot \boldsymbol{I}_n & -\delta_2 \cdot \boldsymbol{I}_n & \cdots & -\delta_{p-1} \cdot \boldsymbol{I}_n & -\delta_p \cdot \boldsymbol{I}_n & & \mathbf{0}_{n\times p} \\ \boldsymbol{I}_n & \mathbf{0}_{n\times n} & \cdots & \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} & & \mathbf{0}_{n\times p} \\ \mathbf{0}_{n\times n} & \boldsymbol{I}_n & \cdots & \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} & & \mathbf{0}_{n\times p} \\ \vdots & \vdots & \vdots & \ddots & \vdots & & \vdots \\ \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} & \cdots & \boldsymbol{I}_n & \mathbf{0}_{n\times n} & & \mathbf{0}_{n\times p} \\ \hline \mathbf{0}_{p\times n} & \mathbf{0}_{p\times n} & \mathbf{0}_{p\times n} & \cdots & \mathbf{0}_{p\times n} & & \boldsymbol{I}_p \end{array}\right]_{p(n+1)\times p(n+1)}$$

$$\boldsymbol{G} = \left[\ \boldsymbol{I}_n \ \ \mathbf{0}_{n\times n(p-1)+p}\ \right]^T_{p(n+1)\times n}$$

$$\boldsymbol{w}[t] = \left[\ w_1[t]\ w_2[t]\ \ldots\ w_n[t]\ \right]^T_{n\times 1}, \quad \boldsymbol{w}[t] \sim \text{NID}\left(\mathbf{0}_{n\times 1}, \boldsymbol{Q}[t]\right), \text{ with } \boldsymbol{Q}[t] = \sigma_w^2[t] \cdot \boldsymbol{I}_n$$

$$\boldsymbol{h}[t] = \left[\ -x[t-1]\ \ -x[t-2]\ \ldots\ -x[t-n] \ \vdots\ 0\ \ldots\ 0\ \right]^T_{p(n+1)\times 1}$$

where $\boldsymbol{I}_{\text{dim}}$ and $\mathbf{0}_{\text{dim}}$ designate identity and zero matrices, respectively, of the indicated dimension. This state space model is non-linear since $\boldsymbol{F}$ is a function of the joint state vector $\boldsymbol{z}[t-1]$ and more specifically of the parameter vector $\boldsymbol{\delta}$.

Several techniques have been proposed for nonlinear identification applications in Civil Engineering, including the Least Squares Estimation (LSE) [8], the Extended Kalman Filter (EKF) [9], the sequential Monte Carlo methods (particle filters, PF) [10] and the Unscented Kalman Filter (UKF) [11]. Presently, the UKF algorithm is employed for the recursive estimation of the joint state vector $\boldsymbol{z}[t]$. The UKF algorithm is known to offer a number of advantages compared to the Extended Kalman Filter (EKF) algorithm, which has been the standard approach for nonlinear state and/or parameter estimation problems [12]. This is mainly due to the first-order linearization approach adopted by the EKF algorithm that may introduce large errors in the posterior mean and covariance of the estimated random variables. In addition, the EKF algorithm necessitates the analytic differentiation of the nonlinear functions for the derivation of the Jacobian, in contrast to the UKF algorithm which derives this information implicitly.

The UKF algorithm is based on the propagation of a minimal set of carefully chosen sample points through the true non-linear system. These sample points, refereed to as sigma points, completely capture the true mean and covariance of the Gaussian random variables to be estimated, while they also capture the posterior mean and covariance by propagating these sigma points through the nonlinear system [12]. The versatility of the filter has been demonstrated in joint-state and parameter identification problems relating to nonlinear hysteretic structural systems in [13], [14].

A summary of a normalized version of the estimation method, based on the use of the UKF algorithm, is provided in Table 1. For this version, the time-varying state and measurement noise processes are normalized by $\sigma_e^2[t]$. In this way the problem of introducing arbitrary values for the normally a-priori unknown $\sigma_w^2[t]$ and $\sigma_e^2[t]$ values is overcome.

Finally, the structure of a specific SP-TAR model is completely defined by the AR model order n and the smoothness constraints order p, which may be selected based on a suitable criterion such as the Mean Square Error (MSE) or the Akaike Information Criterion (AIC) [6].

Table 1: Unscented Kalman filter for the estimation of generalized SP-TAR models.

Calculate sigma points:

$$\mathcal{Z}_{t-1} = \left[\hat{\boldsymbol{z}}[t-1|t-1] \quad \hat{\boldsymbol{z}}[t-1|t-1] + \gamma\sqrt{\widetilde{\boldsymbol{P}}[t-1|t-1]} \quad \hat{\boldsymbol{z}}[t-1|t-1] - \gamma\sqrt{\widetilde{\boldsymbol{P}}[t-1|t-1]} \right]$$

Time update (prediction):

$$\mathcal{Z}^*_{t|t-1} = \boldsymbol{F}\big(\mathcal{Z}_{t-1}\big)$$

State prediction $\hat{\boldsymbol{z}}[t|t-1] = \sum_{i=0}^{2L} W_i^{(m)} \mathcal{Z}^*_{i,t|t-1}$

"Covariance" update $\widetilde{\boldsymbol{P}}[t|t-1] = \sum_{i=0}^{2L} W_i^{(c)} \Big\{ \mathcal{Z}^*_{i,t|t-1} - \hat{\boldsymbol{z}}[t|t-1] \Big\} \Big\{ \mathcal{Z}^*_{i,t|t-1} - \hat{\boldsymbol{z}}[t|t-1] \Big\}^T + G\widetilde{\boldsymbol{Q}}[t]G^T$

Redraw sigma points $\mathcal{Z}_{t|t-1} = \left[\hat{\boldsymbol{z}}[t|t-1] \quad \hat{\boldsymbol{z}}[t|t-1] + \gamma\sqrt{\widetilde{\boldsymbol{P}}[t|t-1]} \quad \hat{\boldsymbol{z}}[t|t-1] - \gamma\sqrt{\widetilde{\boldsymbol{P}}[t|t-1]} \right]$

$$\mathcal{Y}_{t|t-1} = \boldsymbol{h}[t]^T \mathcal{Z}_{t|t-1}$$

$$\hat{y}[t|t-1] = \sum_{i=0}^{2L} W_i^{(m)} \mathcal{Y}_{i,t|t-1}$$

Observation update (filtering):

$$\boldsymbol{P}_{\tilde{y}_t\tilde{y}_t} = \sum_{i=0}^{2L} W_i^{(c)} \left\{ \mathcal{Y}_{i,t|t-1} - \hat{y}[t|t-1] \right\} \left\{ \mathcal{Y}_{i,t|t-1} - \hat{y}[t|t-1] \right\}^T + 1$$

$$\boldsymbol{P}_{z_t y_t} = \sum_{i=0}^{2L} W_i^{(c)} \left\{ \mathcal{Z}_{i,t|t-1} - \hat{\boldsymbol{z}}[t|t-1] \right\} \left\{ \mathcal{Y}_{i,t|t-1} - \hat{y}[t|t-1] \right\}^T$$

Gain $\mathcal{K}_t = \boldsymbol{P}_{z_t y_t} \boldsymbol{P}^{-1}_{\tilde{y}_t\tilde{y}_t}$

State update $\hat{\boldsymbol{z}}[t|t] = \hat{\boldsymbol{z}}[t|t-1] + \mathcal{K}_t(y[t] - \hat{y}[t|t-1])$

"Covariance" update $\widetilde{\boldsymbol{P}}[t|t] = \widetilde{\boldsymbol{P}}[t|t-1] - \mathcal{K}_t \boldsymbol{P}_{\tilde{y}_t\tilde{y}_t} \mathcal{K}_t^T$

$\gamma = \sqrt{L+\lambda}$, where λ is the composite scaling parameter and $L = p(n+1)$ the dimension of the joint state.

Weights: $W_0^{(m)} = \frac{\lambda}{L+\lambda}, \quad W_0^{(c)} = \frac{\lambda}{L+\lambda} + (1-\alpha^2+\beta), \quad W_i^{(m)} = W_i^{(c)} = \frac{1}{2(L+\lambda)}, \quad i = 1,\ldots,2L.$

Initialization: $\hat{\boldsymbol{z}}[0|0] = \boldsymbol{0}$, $\widetilde{\boldsymbol{P}}[0|0] = \boldsymbol{I}$

Normalization: $\widetilde{\boldsymbol{P}}[t|t] = \frac{\widetilde{\boldsymbol{P}}[t|t]}{\sigma_e^2[t]}, \quad \widetilde{\boldsymbol{P}}[t|t-1] = \frac{\boldsymbol{P}[t|t-1]}{\sigma_e^2[t]}, \quad \widetilde{\boldsymbol{Q}}[t] = \frac{\boldsymbol{Q}[t]}{\sigma_e^2[t]} = \frac{\sigma_w^2[t]}{\sigma_e^2[t]}\boldsymbol{I}_n$

Ratio $\nu = \frac{\sigma_w^2[t]}{\sigma_e^2[t]}$ assumed to be constant over time.

3 NUMERICAL CASE STUDY

In order to validate the introduced method, a numerical case study is presently considered. The setup is based on the simplified model of a planar gantry crane system consisting of a trolley moving over a girder (Fig. 1).

The crane lifts a payload of mass m from point A and deposits it at point D following the route A-D as shown in Fig. 1(a). The variability of the system stems from the varying length of the steel wire rope $\ell(t)$ and the corresponding stiffness coefficient $k_2(t) = \frac{EA}{\ell(t)}$ (see Fig. 2). The simulation study is based on the simplified 2-DOF mass-spring-damper system depicted in Fig. 1(b). The analysis focuses on the vertical vibration response of the trolley mass M and more specifically on the acceleration $\ddot{y}_1(t)$. The system is excited by the random unobservable force $F_2(t)$ generated from the disturbances of the payload. Assuming $\theta \approx 0$ the equations of motion

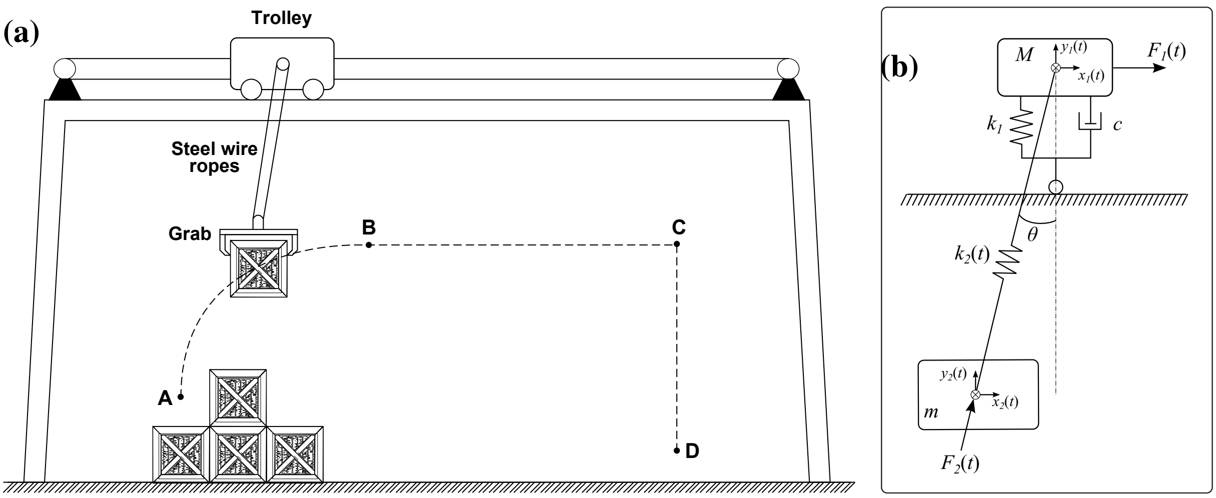
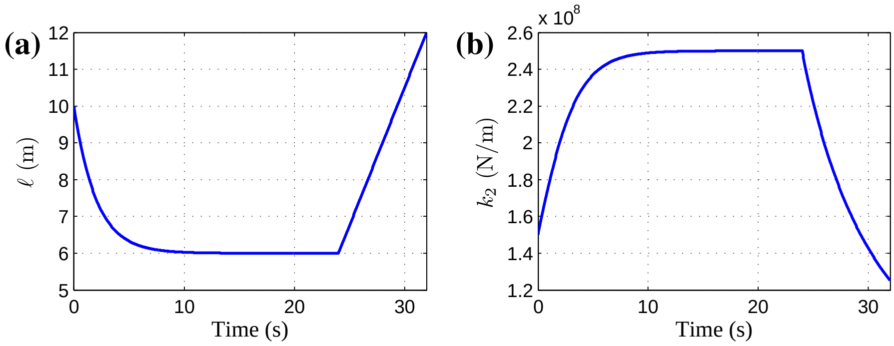


Figure 1: (a) The gantry crane, and (b) the equivalent mass-spring-damper system.



(c)

M	1000 (kg)
m	3000 (kg)
k_1	1 $\times 10^7$ (N/m)
c_1	1 $\times 10^3$ (Ns/m)

Figure 2: (a) The steel wire rope length, (b) the corresponding stiffness coefficient, and (c) the time-invariant properties of the gantry crane system.

may be written as follows:

$$M\ddot{y}_1(t) + c\dot{y}_1(t) + k_1 y_1(t) + k_2(t)[y_1(t) - y_2(t)] = 0 \tag{3a}$$

$$m\ddot{y}_2(t) + mg + k_2(t)[y_2(t) - y_1(t)] = F_2(t) \tag{3b}$$

Given a realization of the excitation force $F_2(t)$ which is assumed to follow a normal distribution with zero mean value and standard deviation equal to 100 N, the vibration response of the gantry crane model is simulated by solving the equations of motion by utilizing the Runge-Kutta 4/5 method (MATLAB *ode45* function). Simulation time is defined as $t \in [-16s, 32s]$, while a constant integration step equal to $T_s = 0.004$ s is selected. In order to minimize initialization effects, the first 4000 samples (16 s) of the response signal are discarded, resulting in a 8000 samples long response realization. The latter along with its instantaneous power spectral density estimate based on the Short-Time Fourier Transform (STFT) is depicted in Fig. 3.

The identification of the simulated system is based on a SP-TAR model of AR order equal to four estimated through the UKF method described in the previous section (parameter $\alpha = 1, \beta = 2, \kappa = 0$) [15]. The algorithm is applied in three sequential passes over the data – forward, backward and a final forward pass – in order to minimize the effects of arbitrary initial conditions. The Rauch-Rung-Striebel smoother is also applied as a last step [15].

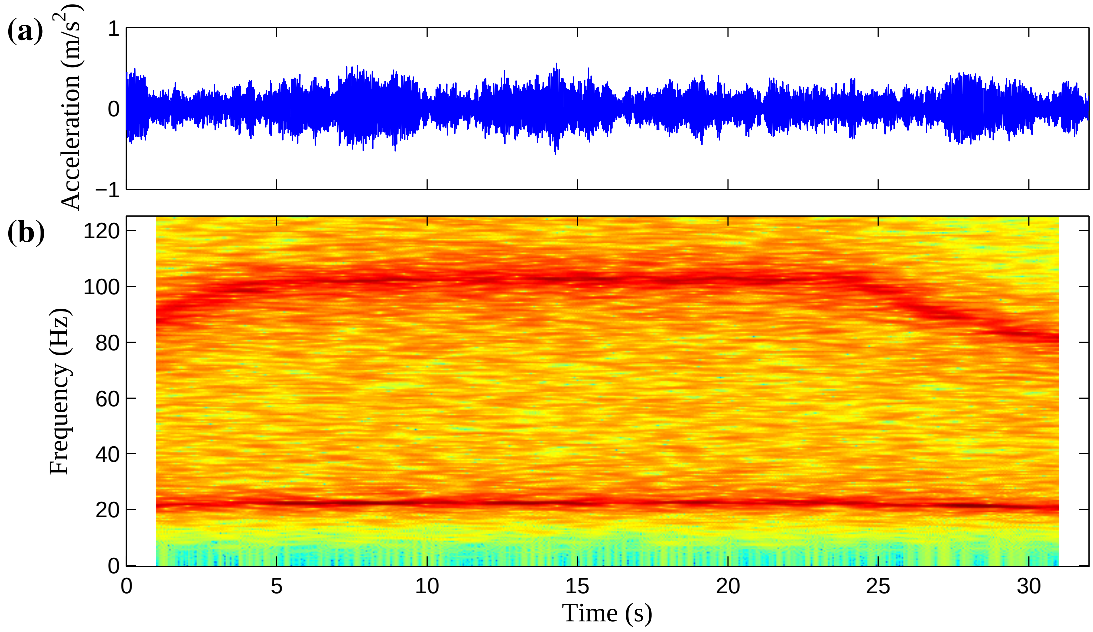


Figure 3: (a) The simulated vibration response signal and (b) the non-parametrically obtained instantaneous power spectral density estimate (STFT method).

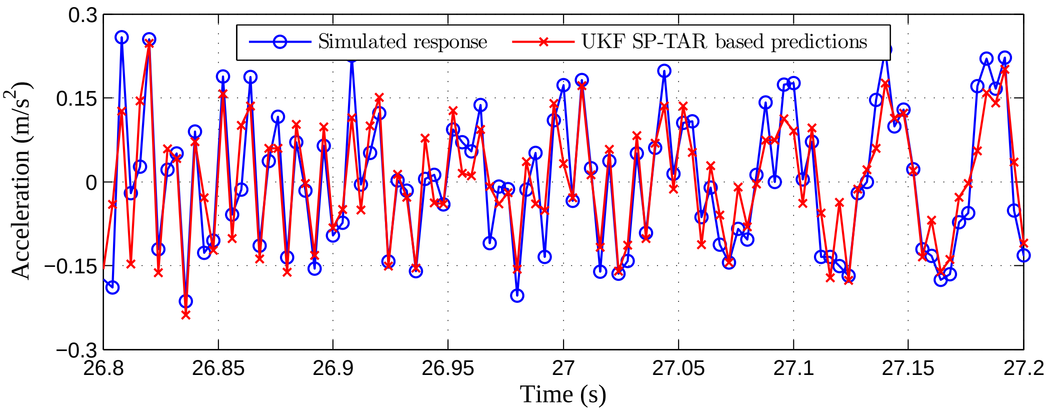


Figure 4: Segment of the simulated response signal and SP-TAR$(4)_6$ based predictions.

The mean squared error (MSE) given by the following expression

$$\text{MSE} = \frac{1}{N}\sum_{t=1}^{N}(y[t] - \hat{y}[t|t-1])^2 \tag{4}$$

is used for selecting the appropriate SP equation order p (search space $[1, 2, \ldots, 6]$) and the variance ratio ν (search space $[10^{-1}, 10^{-2}, \ldots, 10^{-5}]$). Based on the MSE values obtained by the various estimated models an SP-TAR$(4)_6$ model with ratio $\nu = 10^{-2}$ is finally selected as appropriate for the representation of the time-varying structural dynamics. Indicative signal predictions ($\hat{y}[t|t-1]$) obtained by this model are compared to the actual simulated response $y[t]$ in Fig. 4, while the estimated time-varying AR parameters and the coefficients of the SP equation are shown in Fig. 5.

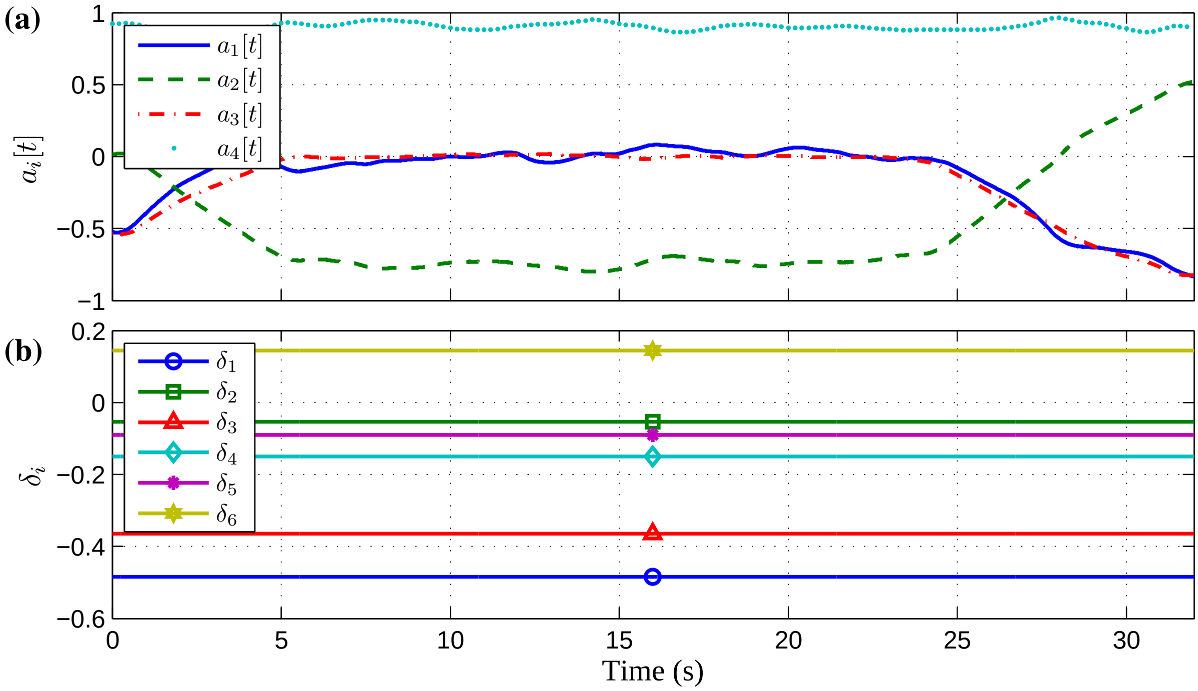


Figure 5: UKF estimated SP-TAR$(4)_6$ parameters: (a) The time-varying AR parameters, and (b) the smoothness priors equation coefficients.

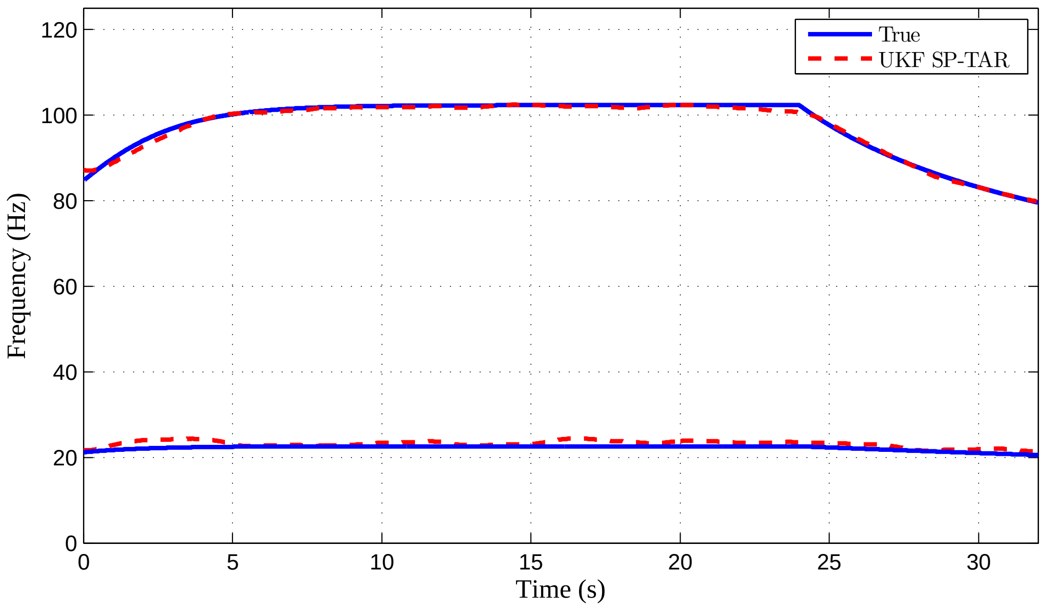


Figure 6: SP-TAR$(4)_6$ based instantaneous natural frequency estimates contrasted to those of the true simulated gantry crane system.

Finally, the SP-TAR$(4)_6$ based instantaneous natural frequency estimates are contrasted to those of the true system in Fig. 6. It should be noted that the tracking errors of the instantaneous natural frequencies achieved by the estimated SP-TAR$(4)_6$ model are below $1.5\ \%$ for both frequencies and all time instants t.

4 CONCLUSIONS

A time-varying identification method based on generalized SP-TAR models and UKF-based parameter estimation was presently proposed. The introduced method uses a smoothness priors constraint equation with unknown a-priori coefficients in contrast to the conventional approach. In this way, the potential of the SP-TAR method for accurate tracking of the time-varying dynamics of a structural system is enhanced. A primary investigation of the method's effectiveness has been carried out via its successful implementation on the identification of a simulated planar gantry crane system.

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