A SIMPLIFIED METHODOLOGY TO OBTAIN RELIABILITY INDEXES FOR CALIBRATION OF DESIGN CODE FOR BUILDINGS

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Abstract. The aim of this work is to give support in the update of the Complementary Technical Standards for Actions for Structural Design of the buildings of the Federal District of Mexico. A methodology based on probability concepts was developed to obtain reliability indexes for some typical structural elements (columns, beams and walls). Structural element designs were performed using Mexican requirements. An interval of load factors values was analyzed including current values. Gravitational and gravitational plus accidental actions load combinations formats were analyzed in which variations of reliability index as a function of load factors were presented and discussed.
1 INTRODUCTION

Provisions are set out in building codes in order to furnish a protection from the view point of society, so that by complying with them there is assurance that the design will not drift excessively from what is optimum for society and at the same time it will exist an appropriate level of security against unwanted states. This in particular, requires proof that the structure meets those requirements, being capable of withstanding all load assumed to occur during the envisaged lifetime. The requirements are mainly comprised of design criteria such as design actions, factors and load combinations, limit states, resistance factors, as well as, general analysis and design procedures for a number of structural elements and materials.

Actually, a challenge faced by group of code writers it to keep regulations up to date. This process requires the constant development of basic research and incorporation of the new advanced techniques, technological progress and practical experience. Rational methodologies and judgment to assess safety levels are of paramount significant.

Quantifying the safety level can be carried out applying reliability theory. Early works like those performed by [1 and 2] have given us a guideline on reliability code formats and reliability indexes. Recently, groups from different countries have been conducting code calibrations using reliability techniques. Amongst the ones it could be highlighted are the following: the reference [3] for the American National Standard A58 standard in force at that year; [4, 5 and 6], for ASCE regulations; [7 and 8] for ACI regulations; [9] for Danish codes; and [10 and 11] for the Eurocodes [12 and 13].

Recognizing the need for evaluation safety levels in code building regulations, this paper aims to obtain measures of reliability indexes that can be achieve in keeping with the requirements of the Complementary Technical Standards for Criteria and Actions for the Structural Building Design at the Distrito Federal in México [14]. The developed methodology is applied to conventional structural elements. Quantitative measures of reliability for columns, beams and walls are obtained. Finally, a discussion of the findings of this study is presented.

1.1 Trend of load combinations formats and load factors

Building regulations have been improving and overcoming several past limitations that hindered the establishment of common criteria for an adequate level of safety of the structures, the assurance of consistency in similar structures, and in many cases, the experience extrapolation for materials to other building systems. This has been overcome mainly by recognizing the random nature of the variables involved (loads and resistance) and by providing procedures for dealing with uncertainty in the various stages of the design process in a rational and simple way.

Table 1 shows a summary of the factored load combinations and load factors on international codes. As it can be appreciated, all cases of load combination comprising dead load only, with no other action accompanying, a factor of 1.4 is applied. When using load combinations which include wind or earthquake loads, the load factor for dead load decreases from 1.4 to values between 0.9 and 1.25 because of the transient nature of wind and earthquake loads; that is based on the assumption that simultaneous occurrence of maximum value of each one it is not possible (this assumption is based on the fact that there is a very low probability of occurrence), and if they act at the same time, then it is possible that some load components may counteract other components.
Table 1: Summary of load factors and factored load combinations

<table>
<thead>
<tr>
<th>Code/Standard</th>
<th>Dead Load (D)</th>
<th>Wind Load Factor</th>
<th>Seismic Load Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>[15] ASCE/SEI 7-05</td>
<td>1.4</td>
<td>1.6</td>
<td>1.0</td>
</tr>
<tr>
<td>[16] UBC, Uniform Building Code</td>
<td>1.4</td>
<td>1.3</td>
<td>1.0</td>
</tr>
<tr>
<td>[17] IBC, International Building Code</td>
<td>1.4</td>
<td>1.6</td>
<td>1.0</td>
</tr>
<tr>
<td>[14] Normas Técnicas Complementarias</td>
<td>1.4</td>
<td>1.1</td>
<td>1.1</td>
</tr>
<tr>
<td>[18] NSR – 10</td>
<td>1.4</td>
<td>1.6</td>
<td>1.0</td>
</tr>
<tr>
<td>[19] SEAOC</td>
<td>1.4</td>
<td>1.3</td>
<td>1.0</td>
</tr>
<tr>
<td>[20] National Building Code of Canada</td>
<td>1.4</td>
<td>3.1</td>
<td>4.0</td>
</tr>
<tr>
<td>[21] ACI 318-08</td>
<td>1.4</td>
<td>5.1</td>
<td>5.0</td>
</tr>
</tbody>
</table>

1.2 Theoretical framework - Structural Reliability

Despite of the effort made to understand, quantify and reduce uncertainties, there is always a finite probability of failure.

Failure is often understood as the condition in which the loading effect (\( S \)) exceeds the resistance (\( R \)), it means that \( S > R \). In other words, there is a chance that a limit state will be reached or exceed – this conditions means that the behavior is no longer acceptable. Ultimate limit state and Serviceability limit state are the most well-known. The first limit state is used to restrict system or structural element total capacity of load-bearing, i.e., creep, brittle fracture, fatigue, instability, buckling and overturning. The serviceability limit state allows deformations with certain tolerances that do not exceed the overall system load-bearing capacity. Examples include cracking, corrosion, permanent deformation and vibration.

In structural design, component or system reliability is evaluated according to one or more failure modes. By means of a set of random variables grouped into a vector \( X \), the strength, stiffness, geometry and loading of the component is modeled. Variables are considered as stochastic in the sense that, according to its variability and other possible uncertainties, they can take random degrees of outcomes agreeing to a distribution function. For the failure mode been studied, the possible outcome of \( X \) can be separated into two groups: 1) the set of events in which the structural component is said to be safe (safe set) and 2) the set where it fails (failure set). The area of basic variables between the set of failure and safe assembly space is denoted as limit-state surface. Then reliability problem can conveniently be described through the limit state function \( g(X) \), defined as:

\[
g(X) = \begin{cases} 
> 0 & \text{for } X \text{ in the safe set} \\
= 0 & \text{for } X \text{ on failure surface} \\
< 0 & \text{for } X \text{ in the failure set}
\end{cases} \tag{1}
\]

The limit state function should be based on mathematical models that more closely reflect true mechanical behavior. The probability of failure can be defined as:

\[
P_f = P[g(X) \leq 0] = \int_{g(X)\leq0} f(x) dx \tag{2}
\]

Where \( f(x) \) is the joint probability density function of \( X \) and represents the uncertainty of variables \( X \).
The probability that a component or structural system does not fail, or more precisely, that it does not exceed a limit state is called reliability of the structure. The probability of failure $P_f$ is one of the means to measure the reliability of a structure.

For example, a case in which $X$ consists only of two variables, load, $S$, and resistance, $R$, the limit state function can be specified as $g(X) = R - S$. Assuming both variables are statistically independent, probability of failure becomes a convolution integral.

$$P_f = P[R - S < 0] = \int_{R-S\leq0} f_R(r)f_S(s)drds$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{s} f_R(r)f_S(s)drds$$

$$= \int_{-\infty}^{\infty} F_R(r)f_S(s)ds$$

Where $f_R$ and $f_S$ are the probability density functions of $R$ and $S$, respectively, and $f_S(r) = dF_S(r)/dr$, where $F_S$ is the cumulative distribution function $R$.

An alternative measure of reliability of a structure is obtained through the reliability index $\beta$, which can be related to the probability of failure by:

$$\beta = -\Phi^{-1}(P_f)$$

Where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. Reliability index also represents standard deviations times that the critical value of the variable is far away from the expectation.

The reliability index $\beta$ can be obtained through a structural reliability analysis either by numerical integration methods, first and second order analytical methods (FORM, SORM, and FOSM) or simulation methods. Some of them can lead to obtain approximate results of the reliability index. On one hand, the numerical integration is only recommended when $X$ consists of few stochastic variables. Solution with the methods of first and second order is often accurate enough and has advantage over simulation methods when failure probabilities are small. A complete discussion of these methods can be found in the following literature [22, 23, 24, and 25].

A rough estimate of the probability of failure corresponding to a value of $\beta$ is given by the expression proposed by [2]:

$$P_f = 460e^{-4.3\beta}$$

2 METHODOLOGY

The methodology proposed for this study is based on an approximation of first order and second moments. Random parameters influencing the design appear only through its expectation and covariance. It is assumed that probability density functions of the random variables can be described by a normal distribution function, because only the first moment are required to recover statistical moments. Subsequently, the variables are transformed into standardized normal distribution with zero expectation and unit variance. This transformation and approximation of random variables to the standard normal distribution integration simplifies the procedure for determining the probability of failure taking advantages of the normal distribution function properties. Although it is true that limit state function can be nonlinear, it can be linearized in order to carry out major simplifications. This technique is known as the method of
first order. These techniques of limit state approximation of first order and second moments of the random variables are known as First Order Second Moment (FOSM) reliability method and it is used in the present work.

To apply the first order approximation to the general design process, it should consider that variables may consist of one or more random variables, in the form:

\[ y = g(X_1, X_2, \ldots, X_n) \]  \hspace{1cm} (6)

Then, statistical moments of independent variables can be obtained through a development in Taylor series approach around the expectation of each variable, and then taking into account only terms of its first derivatives. If it is assumed a condition of independency between random variables, the expectation of the dependent variable will be obtained by applying the function to the expectation of each one of the independent variables. This is formulated as follows:

\[ m_y = g(m_{X_1}, m_{X_2}, \ldots, m_{X_n}) \]  \hspace{1cm} (7)

The variance is obtained as the sum of variances of the independent variables weighted by a factor whose value depends on its own influence on the variability of the dependent variable. This weighted factor is equal to the partial derivative of the limit state function with respect to each independent variable which, in turn, will be calculated by replacing the value of its expectation:

\[ \sigma_y^2 = \sum_{i=1}^{n} \left( \frac{\partial g}{\partial X_i} \right)_{m_{x_i}}^2 \sigma_{x_i}^2 \]  \hspace{1cm} (8)

With this methodology, main contribution is to determine the implicit reliability of structural elements in various design situations according to the regulations currently in force for the Federal District of Mexico [14].

First, it is necessary to obtain, from the design values, the mean value and the coefficient of variation of load actions and resistance. All these variables will also be a function of special considerations concerning accuracy.

Secondly, with above data, variation on the structural element reliability under certain loading conditions and design can be obtained, using the method described in the previous section. For this study the following structural elements were analyzed because they can be found among the most widely used elements type for building structures, usually built in the Federal District:

- Reinforced concrete beam
- Structural steel beam
- Reinforced concrete column
- Structural steel column
- Masonry wall

The methodology to determine reliability indexes is summarized in Table 2 and it will be described in the following paragraphs.

| I. Specification: member geometry and boundary conditions |
| II. Evaluation: load combinations, design and load effects |
| - Load combination: only gravitational loads |
| - Load combination: gravitational loads plus accidental loads |
| III. Design: member design for each load combination |
| IV. Determination: expected values of total load effect and resistance |
2.1 Specification of member geometry and boundary conditions

For each of the selected structural elements their geometry and support conditions as shown below in Figure 1. It is assumed that those structural elements are representative of typical they are at are characteristic of each element are frequently used in many structural designs.

![Figure 1: Selected structural elements for analysis: (a) R.C. column, (b) Steel column, (c) R.C. beam, (d) Steel beam, and (e) masonry wall.](image)

2.2 Evaluation of load combinations, design loads and load effects

Load combinations adopted are those stipulated at [14]. Accordingly to the requirements, design actions are defined as:
- Permanent actions
- Variable actions
- Accidental actions

In the present work, structural elements will be designed to comply with the following criteria of load combinations and the associated load factors:

<table>
<thead>
<tr>
<th>Load Combination</th>
<th>Load Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Permanent and variable actions</td>
<td>1.4</td>
</tr>
<tr>
<td>2. Permanent actions, variable and accidental</td>
<td>1.1</td>
</tr>
</tbody>
</table>

Table 3: NTC load factors.
In addition to the above load factors, structural elements are designed for a wide range of load factors for both prescribed load combinations that of course cover those values prescribed by the actual code and others. This sketch allows us to study the effects on reliability index as load factors are decreased or increased.

An overview to obtain load effects on members is provided in Table 4. Knowing the nominal load effects and the load factors the mechanical elements, such as axial and shear forces and moments are derived.

<table>
<thead>
<tr>
<th>Load Combination</th>
<th>Total Nominal Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravitational</td>
<td>$S = (S_m + S_v)$</td>
</tr>
<tr>
<td>Gravitational plus accidental</td>
<td>$S = (S_m + S_v + S_a)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Range of Load Factors</th>
<th>(14)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma \in [1.2 \ a \ 1.6] @ 0.05$</td>
<td>$\delta \in [0.9 \ a \ 1.2] @ 0.05$</td>
</tr>
<tr>
<td>($=1.4, [14]$)</td>
<td>($=1.1, [14]$)</td>
</tr>
</tbody>
</table>

**Action Effects in Terms of Mechanical Forces and Moments**

<table>
<thead>
<tr>
<th></th>
<th>$P_{d1} = \gamma (P_1)$</th>
<th>$P_{d2} = \delta (P_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V_{d1} = \gamma (V_1)$</td>
<td>$V_{d2} = \delta (V_2)$</td>
</tr>
<tr>
<td></td>
<td>$M_{d1} = \gamma (M_1)$</td>
<td>$M_{d2} = \delta (M_2)$</td>
</tr>
</tbody>
</table>

Table 4: Actions effects derived in terms of mechanical forces and moments.

Where: $S$ is the total design effect of the action pattern; $\gamma, \delta$ are load factors for gravitational and gravitational plus accidental load combinations, respectively. $\hat{S}$ represents the total nominal effect of the action, and the subscripts $m, v$ or $a$ indicate what kind of load is causing de action, permanent loads, variables or accidental, respectively. $P, V$ and $M$ represent the total action effects in terms of axial and shear forces and bending moment, respectively. The subscripts 1 and 2 indicate the load combination type. The subscript $d$ is used to denote design condition. Finally, the symbol $\wedge$ on $S, P, V$ and $M$, denotes the nominal effect of the corresponding action (i.e. denotes a value that has not yet been affected by load factors).

Gravity loads include dead loads such as the weight of the structural element and variable loads (live loads). In order to cover different load intensities and scenarios for both type of loads, representative values were adopted, being equal or proportional to the weight of the structural element.
2.3 Element design for each load combination

In real design situations, the actions effects on structural systems should be evaluated by a structural analysis. Usually, the response is obtained in terms of internal forces and deformations. In this work, for simplicity, instead of a structural system, a series of structural elements were chosen. That strategy omits the need for a structural analysis, so that to define action effects through defining representative ranges of the load intensity. That tries to cover many possible schemes, at which the structural element might be subjected during its lifetime into a structural system configuration.

For this study, only mechanical forces and moments will be taken into account as a result of any action effect on the structural element, coming from specific load combination and load factor. We are aware that certain situation may generate deformations and stresses, but now their incorporation within this methodology is out of scope of the present work.

The structural elements are designed for load combinations 1 and 2 (i.e. gravitational and gravitational loads plus accidental) and with respect to the failure modes listed in Table 6. It is pretended with this parametric work to a range of designs and to calculate the effect of the variation of the load factors on the reliability index values for each load combination.

2.4 Expected values of total load effects and resistance

2.4.1. Load bias factors

Fluctuations in loads constitute a significant proportion of the uncertainties that must be considered in the proposal of nominal load values for design. The estimation of the load intensities are affected by wide margins of uncertainty, lower for permanent actions and higher for accidental.

The current value adopted for a design action must be taken as the load action leading to the worst effects on the system; in general, this involves taking the maximum value that can be presented. However, uncertainties due to load fluctuations are only possible to determine if it is accepted a previously prescribed probability to be exceeded during a given time. The probability of this load must be small enough, but not zero. The assigned value will depend on how much conservatism it is desired to achieve, or it can be based on what is socially accepted for failure. Building Code [14] defines a nominal load value to that with a probability ranging from two to five percent of being exceeded during the lifetime of the structure (percentile 98).

The probability associated of a certain variable value to be exceeded or not achieved, may be formulated as function of the expected (or mean value), \( m_s \), and the standard deviation, \( \sigma_s \), or coefficient of variation, \( V_s \). For example, to calculate the nominal value, \( \bar{m}_s \), which has 2 percent probability of being exceeded, and if there are knowledge to assume that the random variable has a distribution function similar to normal distribution function, \( \bar{m}_s \) can be calculated with the following expression:

\[
\bar{m}_s = m_s + 2 \sigma_s = m_s \left( 1 + 2 V_s \right)
\]  

(9)

In this study it is assumed that all load variables can be described by a normal distribution, so that, the relations between the nominal values and the mean values (bias factors) of these loads are:
Permanente load

\[ S_m = S_m \cdot (1 + 2V_m) \]  

(10)

Variable load

\[ S_v = S_v \cdot [1 + 2V_v] \]  

(11)

Accidental load

\[ S_a = S_a \cdot (1 + 2V_a) \]  

(12)

where:

- \( S_m, S_m \) are nominal and expected permanent load value, respectively;
- \( S_v, S_v \) are nominal and expected variable load value, respectively;
- \( S_a, S_a \) are nominal and expected accidental load value, respectively;
- \( V_m, V_v \) and \( V_a \) are de coefficient of variation for the permanent, variable and accidental loads, with representatives values of 0.08, 0.3 and 0.3, respectively [26].

The adimensional coefficient resulting from the division of expected values between nominal ones will be named, in this work, as bias factor, and it will denoted with the Greek symbol \( \lambda \).

<table>
<thead>
<tr>
<th>Bias factor</th>
<th>Expected or mean load effect</th>
<th>Nominal load effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_m )</td>
<td>( \frac{S_m}{S_\text{nom}} ) ( \lambda )</td>
<td></td>
</tr>
</tbody>
</table>

### Load Combination

| Gravitational | \( \lambda_{S1} = \frac{S_m}{S_m + S_v} \)  
| Gravitational plus accidental | \( \lambda_{S2} = \frac{S_m + S_v + S_a}{S_m + S_v + S_a} \)  

(13)

(14)

### Mean total load

\[ S_m + S_v + S_a \]

Table 5: Load bias factors derivation for load combinations

The bias factors can also be expressed as functions of coefficients of variation:

\[ r = \frac{S_m}{S_m + S_v} \]  

(15)

And

\[ r' = \frac{S_v}{S_m + S_v + S_a} \]  

(16)

Where:

- \( r \) is a parameter that relates the effect of the permanent load with respect to the total effect (permanent load plus variable load). It can be noticed that \( r \) take values between 0 to 1 (when \( r \) tends to 0 means that \( S_m >> S_v \), and when \( r \) equals 1, \( S_v = 0 \)).

\[ \lambda_{S1} = \frac{1}{1 + 2V_m + 2(1 - r)V_v} \]  

(17)

\[ \lambda_{S2} = \frac{1}{1 + (1 - r)[2V_m + 2(1 - r)V_v] + 2rV_a} \]  

(18)
Equations (17) and (18) describe bias factors for gravitational and gravitational plus accidental load combinations as a function of the coefficients of variation (permanent load, variable and accidental), and the parameters \( r \) and \( r' \). It is noted that if there is no accidental load, the bias factor for the second combination will be the same as those prescribed by equation 17.

### 2.4.2. Coefficients of variation of total load effect

The coefficients of variation of the total loading effect is obtained by applying a first order approximation, using expressions (7) and (8) for the expected and the variance, respectively. The coefficient of variation is obtained by dividing variance between the square of the mean value.

For gravitational load combination of gravitational load, this coefficient of variation of total load effect is obtained as follows:

\[
V' = \sqrt{V_p^2 + 2\left[ V_p MV + V_m V_p + V_m^2 \right] + (r')^2 V_a^2}
\]  
(19)

For gravitational plus accidental load combination, the coefficient of variation of total load effect is:

\[
V' = \sqrt{V_p^2 + 2\left[ V_p MV + V_m V_p + V_m^2 \right] + (r')^2 V_a^2}
\]  
(20)

The values of the coefficients of variation for \( V_p \) is equal to 0.15. \( V_p \) is the coefficient of variation to take into account for the lack of precision in the analysis. The values of the parameters, \( V_p \), \( V_m \) and \( V_i \) were taken from [26].

### 2.4.3. Resistance bias factors

The design strength is obtained by multiplying nominal value by a factor which minor than unity, named resistance factor.

Nominal resistance value of a structural element must be a conservative value of its minimum capacity to withstand the actions effects. This value is also set with probabilistic basis. The determination of the resistance of a section or structural element may be analytically or experimentally, following procedures set forth in buildings regulations or codes of practice in association recognized techniques. In general, for nominal resistance values codes set up probabilities values between 0.02 to 0.05 that should be attained, i.e., the fact that a lower strength will be presented.

Analytically, the resistance should be expressed in terms of the internal forces or as the combination (axial forces, shear forces, bending moments and torsion), which are produced by the actions. This evaluation should be considered uncertainties in calculation formulas and variability of the factors involved in such expressions.

When resistance is obtained by testing, it is necessary to determine the nominal value or that with minimum probability from statistical information. If it only allowed to obtain the mean and the coefficient of variation of resistance, the nominal value can be approximated through the expression:
Where:

\[ \hat{m}_R = \frac{m_R}{1 + \zeta V_R} \]  \hspace{1cm} (21)

\[ R \] is the nominal strength value; \( m_R \) is the expected strength; \( V_R \) is the strength coefficient of variation; and \( \zeta \) is a factor in Mexican regulations equals 2.5 (corresponding to a probability of 0.02 of being reached).

The values adopted for coefficient of variations were adopted from the references [26, 27, 28, and 29]. A summary is presented in Table 6.

<table>
<thead>
<tr>
<th>Structural element</th>
<th>Failure mode</th>
<th>Bias Factor (Mean/nominal)</th>
<th>Coefficient of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>R.C. Beam</td>
<td>Bending*</td>
<td>1.34</td>
<td>12%</td>
</tr>
<tr>
<td></td>
<td>Bending**</td>
<td>1.34</td>
<td>12%</td>
</tr>
<tr>
<td>Steel Beam</td>
<td>Bending*</td>
<td>1.11</td>
<td>12%</td>
</tr>
<tr>
<td></td>
<td>Bending**</td>
<td>1.28</td>
<td>14%</td>
</tr>
<tr>
<td>R.C. Column</td>
<td>Axial Compression*</td>
<td>1.42</td>
<td>15%</td>
</tr>
<tr>
<td></td>
<td>Bending-axial compression**</td>
<td>1.49</td>
<td>15%</td>
</tr>
<tr>
<td>Steel Column</td>
<td>Compression*</td>
<td>1.20</td>
<td>15%</td>
</tr>
<tr>
<td></td>
<td>Bending-axial compression**</td>
<td>1.24</td>
<td>14%</td>
</tr>
<tr>
<td>Masonry wall</td>
<td>Axial Compression*</td>
<td>1.37</td>
<td>15%</td>
</tr>
<tr>
<td></td>
<td>Shear**</td>
<td>1.87</td>
<td>35%</td>
</tr>
</tbody>
</table>

* Gravitational load combination
**Gravitational plus accidental load combinación

Table 6: Strength bias factors and coefficients of variation

2.5 Reliability index

In the case of defining as failure criterion the relationship \( R - S = 0 \), where \( R \) and \( S \) are the elements (or critical section) strength and action effect of an element, respectively, the reliability index may be formulated as follows:

\[ \left( \frac{R}{S} \right) - 1 < 0 \]  \hspace{1cm} (20)

or \( \log R - \log S < 0 \)  \hspace{1cm} (21)

Taking as basic variables of \( R \) and \( S \), \( \log R \) and \( \log S \), failure criterion can be rewritten as in the form \( \log R - \log S < 0 \). The reliability index is calculated as:

\[ \beta = \frac{\log R - \log S}{\sqrt{\sigma^2(\log R) + \sigma^2(\log S)}} \]  \hspace{1cm} (22)

Applying a first order approximation to the mean values and variances, we obtain:
\[ R = S \exp \left( \beta \sqrt{V_R^2 + V_S^2} \right) \]  

\( V_R \) and \( V_S \) are the coefficients of variation of \( R \) and \( S \), respectively, and \( \beta \) is a reliability index.

### 3 RESULTS

Results show the measure of safety associated with particular design in terms of \( \beta \). In figures 2 to 9 it is illustrated the variation of the reliability index with respect to the load factor in accordance with a specific load combination. Contour plots are also included for each structural element (R.C. Columns, R.C. Beams, Steels Columns, Steel Beams and masonry walls). Through each contour line it is intended to represent the reliability index for different load factors values in combinations which may include either first load combination (gravitational only) or second load combination (gravitational plus accidental). A contour plot allows us to distinguish and to combine load factors in order to obtain constant reliability index on structural elements analyzed.

**Figure 2:** Reliability indexes, R.C. Column, (a) gravitational load combination, failure mode: axial compression; (b) gravitational plus accidental load combination, \( c = 0.08 \), failure mode bending and axial compression

**Figure 3:** Reliability indexes, Steel Column, (a) gravitational load combination, failure mode: axial compression (according to the maximum KL/r value); (b) gravitational plus accidental load combination, \( c = 0.33 \), failure mode bending and axial compression
Figure 4: Reliability indexes for various load factors in each load combination, (a) R.C. Column, \( \alpha = 0.66 \) and \( c = 0.08 \); (b) Steel Column, \( \alpha = 0.4 \) and \( c = 0.33 \).

Figure 5: Reliability indexes, R.C. Beams, (a) gravitational load combination, failure mode: bending; (b) gravitational plus accidental load combination, \( c = 0.50 \), failure mode: bending.

Figure 6: Reliability indexes, Steel Beams, (a) gravitational load combination, failure mode: bending; (b) gravitational plus accidental load combination, \( c = 0.5 \), failure mode: bending.
Figure 7: Reliability indexes for various load factors in each load combination, (a) R.C. Beams; (b) Steel Beams, $\alpha = 0.4$ and $c = 0.5$.

Figure 8: Reliability indexes, Masonry wall, (a) gravitational load combination, failure mode: axial compression; (b) gravitational plus accidental load combination, $\alpha = 0.4$, failure mode: shear.

Figure 9: Reliability indexes for various load factors in each load combination, masonry walls, $\alpha = 0.4$ and $c = 0.16$. 
A summary of some important $\beta$ values are show in Table 7 and Table 8. Maximum and minimum values are compared with the current values of the standard. The parameters $a$ and $c$ represent load ratios for variable load/ dead load and accidental load/ permanent load, respectively. It is showed in the last column of Table 7 the reliability indices for structural elements considering the load combination for gravity loads established in [14]. From the same Table, it should be noted that reliability indices for reinforced concrete elements are larger compared to those obtained for structural steel elements and masonry.

<table>
<thead>
<tr>
<th>Structural element / Failure mode</th>
<th>Reliability index</th>
<th>F.C.=1.4 (Currently)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Minimum (F.C.=1.2)</td>
<td>Maximum (F.C.=1.6)</td>
</tr>
<tr>
<td>R.C. Column/ Axial compression, $a = 0.66$</td>
<td>4.84</td>
<td>6.02</td>
</tr>
<tr>
<td>Steel Column/ Axial compression, $a = 0.4$</td>
<td>3.43</td>
<td>4.43</td>
</tr>
<tr>
<td>R.C. Beam, bending, $a = 0.4$</td>
<td>3.90</td>
<td>5.40</td>
</tr>
<tr>
<td>Steel Beam, bending, $a = 0.4$</td>
<td>2.96</td>
<td>4.25</td>
</tr>
<tr>
<td>Masonry wall, axial compression, $a = 0.4$</td>
<td>3.20</td>
<td>4.42</td>
</tr>
</tbody>
</table>

Table 7: Reliability indices for structural elements under gravitational load combination.

In Table 8 it is showed the reliability indices for the gravitational plus accidental load combination. In this case, the trend is almost the same as the previous table, but it can be noted an important decrease in reliability indices magnitudes compared to that presented in Table 7 for the gravitational load combination.

<table>
<thead>
<tr>
<th>Structural element / Failure mode</th>
<th>Reliability index</th>
<th>F.C.=1.1 (Currently)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Minimum (F.C.=0.9)</td>
<td>Maximum (F.C.=1.3)</td>
</tr>
<tr>
<td>R.C. Column/Bending- axial compression, $a = 0.66 , c = 0.20$</td>
<td>4.08</td>
<td>5.50</td>
</tr>
<tr>
<td>Steel Column/Bending- axial compression, $a = 0.4 , c = 0.33$</td>
<td>2.49</td>
<td>4.23</td>
</tr>
<tr>
<td>R.C. Beam, bending, $a = 0.4 , c = 0.33$</td>
<td>2.70</td>
<td>4.48</td>
</tr>
<tr>
<td>Steel beam, bending, $a = 0.4 , c = 0.33$</td>
<td>2.34</td>
<td>3.96</td>
</tr>
<tr>
<td>Masonry wall/ shear, $a = 0.4 , c = 0.16$ (F.C.=0.95)</td>
<td>3.06</td>
<td>3.75</td>
</tr>
</tbody>
</table>

(F.C.=1.25) 3.44

Table 2: Reliability indices for structural elements under the gravitational plus accidental load combination.
4 CONCLUSIONS

- Load factors and combinations of standards established in some national and international norms were reviewed. It was found that almost standards consider a factor of 1.4 in the case of load combination format that include only gravitational loads. For other combination like that including accidental loads, load factors change according to the nature of concerning load (seismic, wind, etc): for wind loads, factors can range from 1.3 to 1.6, for earthquake loads usually factors are unitary, but in both cases, the gravitational loads are affected by 1.1 or 1.2 with particularities. In the standard studied [14], it is considered a factor of 1.1 for the load combination format that includes gravity loads and accidental. It is thought that regarding the international scheme and results obtained, load factor for load combination format for facing accidental actions should be reviewed.

- A methodology to obtain quantitative measure of reliability for some typical structural elements where presented. The format provides a major benefit in permitting the individual contribution to uncertainty in structural performance to be separated, studies individually, and then incorporated in a consistent manner. The central idea is the use of only second order moments and associated approximations to express the influences of uncertainty. Coefficients of variation are introduced to express the dispersion of variation in these best estimates of measurable uncertainty and unmeasurable uncertainty.

- It was observed that there is a strong correlation between load factors and reliability indices, in both load combinations formats, i.e. gravitational loads and gravitational loads plus accidental load, in which of course, the reliability indices increase with increasing load factors.

- Reliability indices were obtained for some ratios of variable load / dead load and accidental load / dead load. From this, it was noted that all the curves follow the same trend, regardless of the ratio values. The largest values of reliability indices were observed in reinforced concrete columns, while the opposite were observed in structural steel beams, for both load combinations.

- The convenience of adopting a factored load combination format in terms of reliability index can for a specific structural element be evaluated through the use of contour plots.

REFERENCES


[18] AIS (Colombian Association for Earthquake Engineering), NSR - 10, Rules Colombian Earthquake Resistant Construction, Colombia, 2010.


[21] ACI (American Concrete Institute), ACI 318-08 Building Code Requirements for Structural Concrete and Commentary.USA, 2008.


