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QUANTIFICATION OF THE STRUCTURAL RESPONSE OF A STAIRCASE DAMPED BY THE PRESENCE OF PEOPLE: NUMERICAL MODEL AND EXPERIMENTAL TESTS

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Abstract. *People acting on a structure such as a footbridge or a staircase are a source of excitation but can also affect the dynamic behavior of the structure they occupy. Particularly, considerable damping ratio value changes are often experienced. If this phenomenon occurs, considering the dynamic properties of the structure when empty for estimating the structural response can lead to significant overestimation of the amplitudes of vibration.*
In this paper this problem is faced considering the case of a lightly damped steel staircase. The structure was tested with and without the presence of people. A model, able to describe people effect on a structure, is used to predict the changes of the modal properties due to the presence of people. It is finally proposed a comparison between the amplitudes of vibration measured experimentally and those predicted starting from the model of the structure when empty and the model of the structure with people respectively. Results highlight how the correctly estimated damping values leads to a significant increase in the overall in service vibration level prediction accuracy.

1 INTRODUCTION

People acting on a pedestrian structure, such as a footbridge or a staircase, behave as dynamical systems capable of modifying the dynamics of the structure itself as well as of introducing a load. At present, however, the knowledge of this phenomenon, known as Human Structure Interaction (HSI), is still limited. Therefore, although the experimental evidence suggests that appropriate dynamic models of human occupants should be used in order to obtain an accurate description of human-structure interaction [1], at the design stage, it is common practice to consider people interacting with a structure as a source of force only [2], [3], [4]. This approach, however, may lead to an erroneous estimation of the vibration levels at the design stage as evidenced in some striking cases, such as the Millennium Bridge [5]. This work focuses on the analysis of vertical vibrations of a slender structure. As regards vertical vibrations, the experimental evidence suggests that people interacting on a structure are a source of added damping [6], [7], [8]. If considerable damping ratio value changes are experienced, considering the dynamic properties of the structure when empty for estimating the structural response can lead to significant overestimation of the amplitudes of vibration. To the purpose of obtaining an accurate prediction of the structural response, a model capable of accounting for the phenomenon of HSI is used. This model [9] was proved to be effective to predict the effect of passive people on the modal properties of a slender structure. In Section 2 the model and a possible extension to the case of moving people are presented. To the purpose of a first verification of the possibility of extending the proposed model to the case of moving people, the simple case of a subject marching on a defined point of the structure is considered. In Section 3 the experimental tests performed to verify the effectiveness of the model are presented. A comparison between the test results and the prediction obtained with the model are reported in Section 4. The performed tests allowed to show the suitability of the proposed model and to show how to treat people as a source of forcing only can lead to significant errors in the estimation of the structural response.

2 NUMERICAL MODEL

To obtain an accurate prediction of the structural response, a method to account for the phenomenon of HSI is proposed. This method is based on a model [9] that was proved to be effective to predict the effect of passive people on the modal properties of a slender structure. Using the method proposed in [9], starting from the transfer functions of the empty structure and introducing each passive subject separately, it is possible to obtain the transfer functions of the Human+Structure system with high accuracy. The model is rigorous for the case of passive people. For the case of moving people, experimental evidence suggested that their action could be modeled through a set of static positions, as explained in the next section. Therefore, the proposed method to extend the static model proposed in [9] to the case of moving people consists in a two steps approach:

1. obtaining an equivalent static model capable of accounting for the average effect of moving people
2. introducing, on the modified model, the load induced by people on the structure.

This approach allows obtaining an approximate description of the HSI phenomenon that can provide useful information on the behaviour of the Human+Structure system.

In this section the method used to extend the results to the case of moving people is presented while the fundamentals of the model proposed in [9] are recalled.

2.1 Equivalent dynamic model for a moving subject

In the proposed approach each subject is introduced on the empty structure through a bio-mechanical model. In particular, subjects were modelled with their so called apparent mass. To this purpose the work of Mastumoto and Griffin [10], who proposed mathematical models to represent the dynamic behaviour of standing subjects, was employed.

In their work the authors proposed the results of several tests conducted with different subjects in different postures. In particular, tests on 12 subjects were performed and the experimental data were fitted using lumped parameter models. The model considered in this work is 2 degrees of freedom and is referred to as model 2a in [10]. Figure 1 shows the model used in this study, while the corresponding analytical expression of the apparent mass is reported in Equation (1).

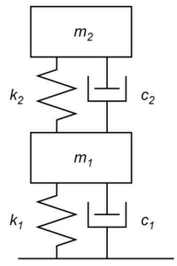

Figure 1: Mechanical model (2a) of a standing person proposed by Matsumoto and Griffin

$$M_{2\mathrm{a}}(\omega) = \frac{(\mathrm{i}c_1\omega + k_1)\{m_1(-m_2\omega^2 + \mathrm{i}c_2\omega + k_2) + m_2(\mathrm{i}c_2\omega + k_2)\}}{\{-m_1\omega^2 + \mathrm{i}(c_1 + c_2) + (k_1 + k_2)\}(-m_2\omega^2 + \mathrm{i}c_2\omega + k_2) - (\mathrm{i}c_2\omega + k_2)^2} \quad (1)$$

The authors propose the optimized model parameters (m_1, m_2, c_1, c_2, k_1, $k_{2)}$ obtained using the mean normalized apparent mass (m_{2a}=M_{2a}/m, where m is the static weight of the subject) of 12 male subjects involved in the tests. Different postures (standing, one leg, bent legs) were investigated.

In this work the case of a subject marching on a fixed point of the structure was considered. Experimental tests were performed to get information on the posture of some subjects while performing this action. If T is the time elapsing between two touches on the ground of the same foot (1 cycle), a subject, during 1 cycle, was seen to stand on two legs for a time of about 5-15% of T and on one leg (left or right) for the remaining time.

Therefore, in order to get en equivalent model of a marching subject, the apparent masses proposed by Matsumoto and Griffin for the cases of standing and on one leg subjects were considered.

Figure 2 shows the mean normalized apparent mass of standing and on one leg subjects according to model 2a while the corresponding parameters are reported in Table 1.

	Stiffness (Nm-1kg-1)		Damping (Nsm-1kg-1)		Mass (no unit)	
Case	k1	k2	c1	c2	m1	m2
Model 2a (standing)	4.39·103	5.53·102	3.71·101	1.18·101	5.74·10-1	3.94·10-1
Model 2a (one leg)	6.66·102	6.07·102	2.90·101	8.81·100	5.13·10-1	4.22·10-1

Table 1: Modal parameters for the mean normalized apparent mass of standing subjects

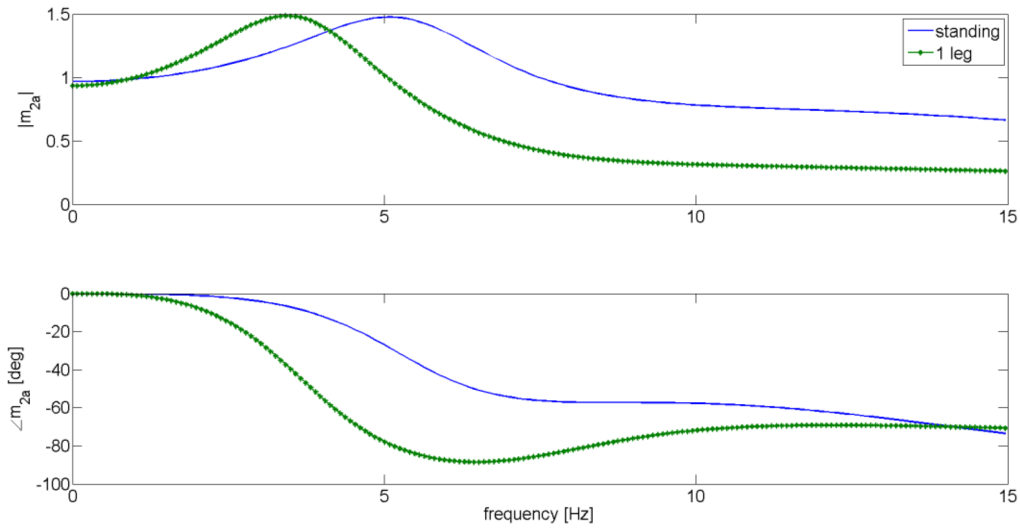

Figure 2: Mean normalized apparent mass, model 2a

The equivalent apparent mass for a marching subject was then obtained as in Equation (2).

$$m_{2a(marching)} = 0.1 \cdot m_{2a(standing)} + 0.9 \cdot m_{2a(one\ leg)} \qquad (2)$$

The so determined apparent mass was used to assess the equivalent effect of a subject marching on a fixed point of the structure.

2.2 Model of the Human-Structure system

The model to account for the presence of people relies on a modal model of the empty structure. The model proposed in [9] was employed. This model is based on the approach proposed by Krenk [11]. The same model was then used to introduce people. One of the advantages of this model is the possibility to introduce each person individually.

The model starts from the frequency response function $\mathbf{G}(\omega)$ of the empty structure, expressed in Equation (3)

$$\mathbf{G}(\omega) = \sum_{\mathbf{j=1}}^{\mathbf{n}} \frac{\boldsymbol{\phi}_{\mathbf{j}} \boldsymbol{\phi}_{\mathbf{j}}^{\mathbf{T}}}{\omega_j^2 - \omega^2 + 2j\zeta_j\omega\omega_j} \qquad (3)$$

where $\boldsymbol{\phi}_{\mathbf{j}}$ is the jth unit modal mass scaled mode shape vector, ω_{j} is the natural frequency of the jth mode and ζ_{j} is the jth non-dimensional damping ratio. The dynamic behaviour of the structure is expressed as in Equation (4)

$$\mathbf{x}(\boldsymbol{\omega}) = \mathbf{G}(\omega)\mathbf{f}(\omega) \qquad (4)$$

being $\mathbf{x}(\omega)$ the displacement vector and $\mathbf{f}(\omega)$ the generic force vector

As detailed in [9], the force exerted by each static person is modelled through its apparent mass, as in Equation (5)

$$\boldsymbol{f}_{\mathbf{i}}^{\mathbf{Human}}(\omega) = -\boldsymbol{M}_{\mathbf{2a}}(\omega)\boldsymbol{\omega}^{\mathbf{2}}\boldsymbol{x}_{\mathbf{i}}(\omega) = \boldsymbol{H}(\omega)\boldsymbol{x}_{\mathbf{i}}(\omega) \tag{5}$$

The model allows then to obtain an estimate of the frequency response function of the subjects+structure system, as expressed in Equation (6), (7) and (8)

$$\mathbf{G}^{-\mathbf{1}}(\omega)\mathbf{x}(\omega) = \mathbf{f}(\omega) - \mathbf{f}^{\mathbf{Human}}(\omega) = \mathbf{f}(\omega) - \mathbf{W}\mathbf{H}(\omega)\mathbf{W}^{\mathbf{T}}\mathbf{x} \tag{6}$$

$$\mathbf{G}_{\mathbf{H}}(\omega) = [\mathbf{G}(\omega)^{-\mathbf{1}} + \mathbf{W}\mathbf{H}(\omega)\mathbf{W}^{\mathbf{T}}]^{-\mathbf{1}} \tag{7}$$

$$\mathbf{G}_{\mathbf{H}}(\omega)^{-\mathbf{1}}\mathbf{x}(\omega) = \mathbf{f}(\omega) \tag{8}$$

where $\mathbf{W} = [\mathbf{w}_{\mathbf{1}}, \ldots, \mathbf{w}_{\mathbf{m}}]$ represents the connection of m elements, $\mathbf{H}$ is the (diagonal) transfer function matrix, containing the mechanical impedance of all the subjects, and $\mathbf{G}_{\mathbf{H}}$ represents the new transfer function of the people-structure system.

The new frequency response function can be expressed explicitly in terms of the frequency response function $\mathbf{G}(\omega)$ by the Woodbury matrix identity.

$$(\mathbf{A} + \mathbf{UCV})^{-\mathbf{1}} = \mathbf{A}^{-\mathbf{1}} - \mathbf{A}^{-\mathbf{1}}\mathbf{U}(\mathbf{C}^{-\mathbf{1}} + \mathbf{V}\mathbf{A}^{-\mathbf{1}}\mathbf{U})^{-\mathbf{1}}\mathbf{V}\mathbf{A}^{-\mathbf{1}} \tag{9}$$

as

$$\begin{aligned}\mathbf{G}_{\mathbf{H}}(\omega) &= [\mathbf{G}(\omega)^{-\mathbf{1}} + \mathbf{W}\mathbf{H}(\omega)\mathbf{W}^{\mathbf{T}}]^{-\mathbf{1}} \\ &= \mathbf{G}(\omega) - \mathbf{G}(\omega)\mathbf{W}(\mathbf{H}(\omega)^{-\mathbf{1}} + \mathbf{W}^{\mathbf{T}}\mathbf{G}(\omega)\mathbf{W})^{-\mathbf{1}}\mathbf{W}^{\mathbf{T}}\mathbf{G}(\omega)\end{aligned} \tag{10}$$

This simple equation allows calculating the transfer function of the new human+structure system. This model allows evaluating the local effect due to the presence of each subject separately. This effect is a function of the subject characteristics, posture and the point where the subject is located (i.e. mode shape components relating to that point). In the next section the tests performed to verify the effectiveness of the proposed model are presented.

3 EXPERIMENTAL TESTS

A slender staircase was used to verify the effectiveness of the proposed model. The staircase was instrumented with 12 accelerometers in the vertical direction. At first an experimental modal analysis was performed. To this purpose an electrodynamic shaker was used to force the staircase in order to obtain a modal model of the empty structure. Once estimated the experimental FRFs of the empty structure, a dynamometric plate was added in order to measure the force induced by one subject in a fixed point. Figure 3 shows the considered staircase and a scheme of the used sensors.

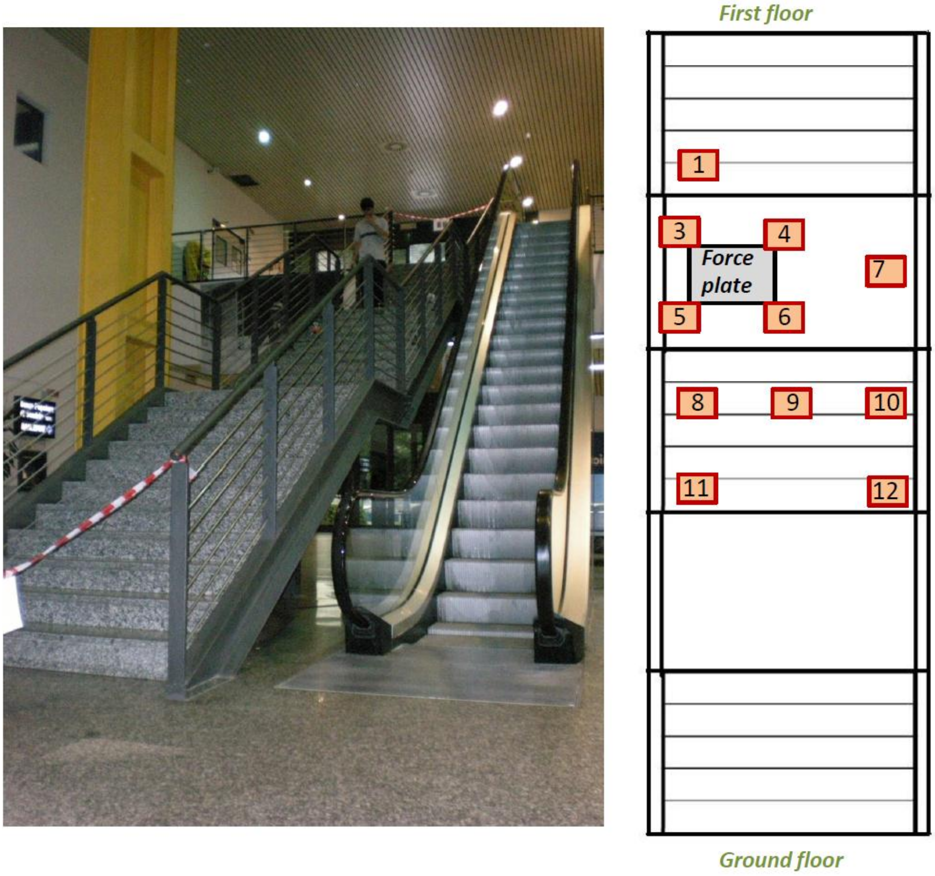

Figure 3: Structure under test and experimental setup

The considered structure has two dominant modes around 8 Hz and 9 Hz. The modes at a higher frequency, being little forced by human walking, are not considered in this study.

Figure 4 shows an example of FRF of the empty structure while Table 2 shows the associated modal parameters.

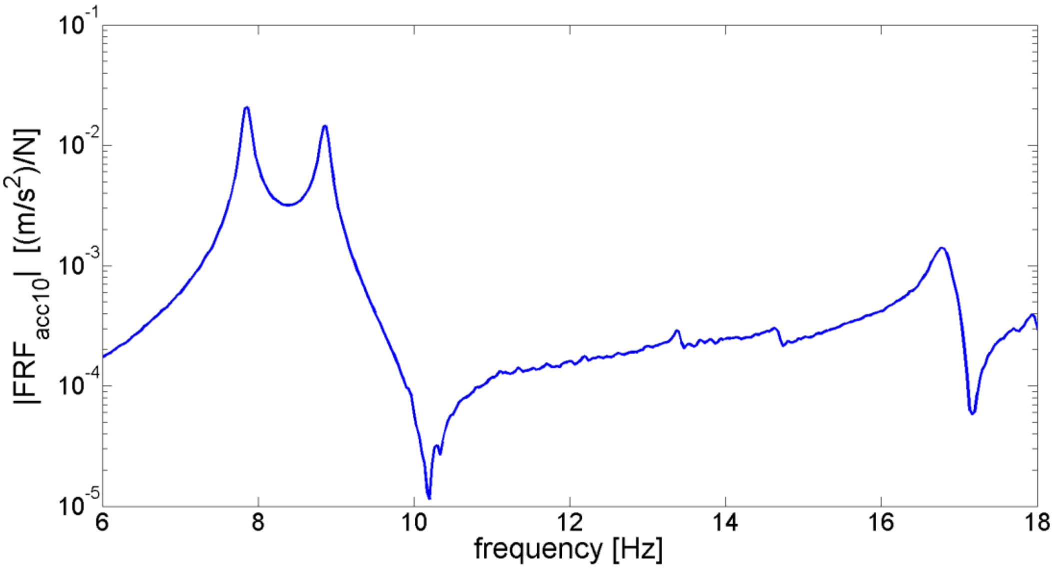

Figure 4: Experimental FRF – accelerometer 10

f_n [Hz]	ζ [%]
7.85	0.48
8.86	0.50

Table 2: Modal parameters of the empty staircase

Once obtained a modal model of the empty staircase, the structure was tested in different conditions. In particular two different kinds of tests were performed:

1. A single subject marching on the force plate
2. A subject marching on the force plate plus an additional subject standing on a different point of the structure

The second case was chosen to increase the damping introduced by people on the structure

For what concern the type of forcing, two cases were considered:

1. Subject free to march at his own frequency
2. Subject forced to march at a frequency of 1.95 Hz

The last frequency was chosen to concentrate the introduced energy in correspondence of the first natural frequencies of the structure. The fourth harmonic of the forcing thus introduced indeed falls in the vicinity of the first resonance peak

In the following section the experimental results of two tests, one corresponding to case 1 and one corresponding to case 2, are reported. In addition a comparison among these results and those obtained starting from the model of the empty structure and the model of the human+structure system are presented. For the last two cases the accelerations were obtained using the transfer functions of the empty structure $G(\omega)$ and of the Human+ Structure system $\mathbf{G_H}(\omega)$**,** as in Equation (4) and (8), and the force $f(\omega)$ introduced by the subject marching on the force plate.

4 EXPERIMENTAL AND NUMERICAL RESULTS

In this section the results of two significant tests are proposed. In the first test one subject was asked to march at his own frequency on the force plate. This condition is more disadvantageous for our purposes since the damping introduced by a single subject is not very high and an average model was used to model the human body. The modal parameters predicted for the case of a single subject (75 kg) marching on the force plate are reported in Table 3.

f_n [Hz]	ζ [%]
7.84	0.95
8.85	0.85

Table 3: Modal parameters predicted with 1 subject marching on the force plate (test 1)

Figure 5 shows, for the case of test 1, accelerometer 10, a comparison among the power-spectrum of the measured acceleration and the power-spectrum of the acceleration predicted using the force measured with the dynamometric plate (power spectrum in Figure 6) and the model of the empty structure and subject+structure respectively. As shown in Figure 6 and Figure 5**,** the force induced by the subject on the structure is capable of introducing energy at the first two resonances of the structure. The RMSs of all the accelerations in a frequency band of 0-10 Hz are reported in Table 4 with the corresponding relative errors estimated as in Eq. (11) and Eq. (12).

$$\boldsymbol{Error}\ \%\ \boldsymbol{empty} = \frac{(\boldsymbol{RMS\ empty}) - (\boldsymbol{RMS\ experimental})}{(\boldsymbol{RMS\ experimental})} \qquad (11)$$

$$\boldsymbol{Error}\ \%\ \boldsymbol{H+S} = \frac{(\boldsymbol{RMS\ H+S}) - (\boldsymbol{RMS\ experimental})}{(\boldsymbol{RMS\ experimental})} \qquad (12)$$

where (RMS empty) is the RMS of the acceleration estimated using the model of the empty structure, (RMS H+S) is the RMS of the acceleration estimated using the model of the Human+Structure system and (RMS experimental) is the RMS of the measure acceleration.

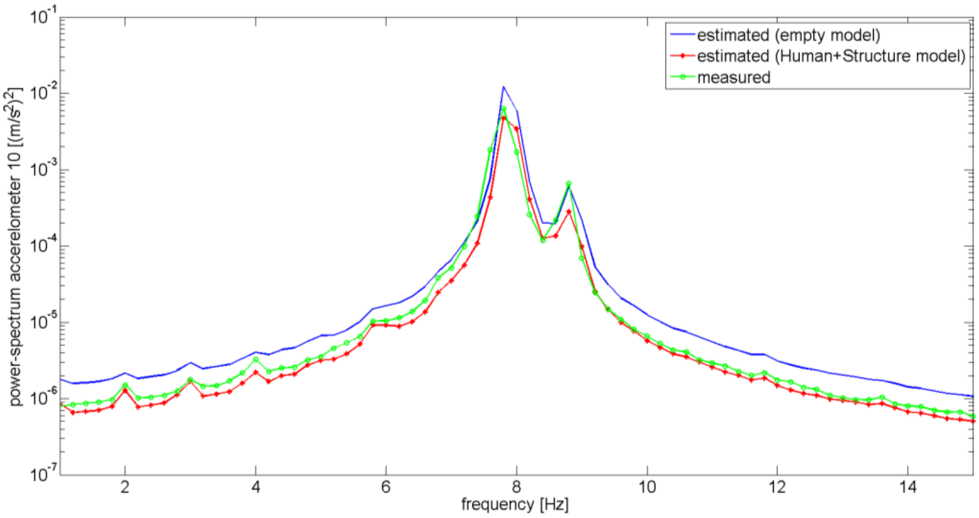

Figure 5: Test 1 – power-spectrum accelerometer 10

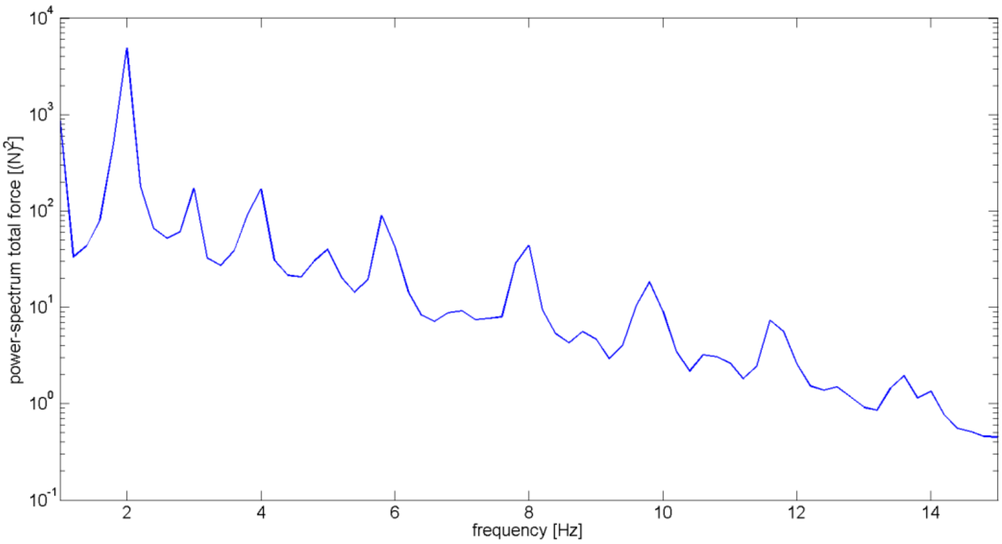

Figure 6: Test 2 – power-spectrum total force

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12
RMS meas (m/s^2)	0.065	0.091	0.075	0.078	0.077	0.081	0.112	0.0790	0.084	0.109	0.057	0.081
RMS empty (m/s^2)	0.087	0.125	0.101	0.111	0.105	0.117	0.153	0.105	0.120	0.147	0.074	0.108
Error % Empty	35.1	37.2	34.9	42.6	36.7	43.5	35.9	33.4	43.1	35.4	29.0	33.3
RMS H+S (m/s^2)	0.061	0.085	0.070	0.075	0.073	0.080	0.104	0.074	0.081	0.100	0.0520	0.074
Error % H+S	-6.1	-6.7	-6.4	-2.9	-5.1	-2.2	-7.5	-6.7	-2.7	-7.8	-8.9	-9.1

Table 4: Test 1 – RMS and relative errors

As might be expected, the use of the model of the empty structure led to an overestimation of the results. In this particular case, using a model of the subject+structure a light underestimation of the results was obtained. The agreement between the experimental data and the predicted is, however, much better in the latter case.

In the second test a subject was asked to march at a frequency of 1.95 Hz on the force plate while another subject was standing still in correspondence of point 9. This condition is more advantageous for the purpose since the damping introduced in this case is much higher, as reported in Table 5.

f_n [Hz]	ζ [%]
7.77	2.34
8.84	0.90

Table 5: Modal parameters predicted with 1 subject marching on the force plate + 1 subject in point 9 (test2)

Figure 7 and Table 6 show the results obtained for the case of test 2.

In this case the results obtained using the model of the empty structure greatly overestimate the structural response while there is a very good agreement between the experimental results and those predicted with the model of the human+structure system.

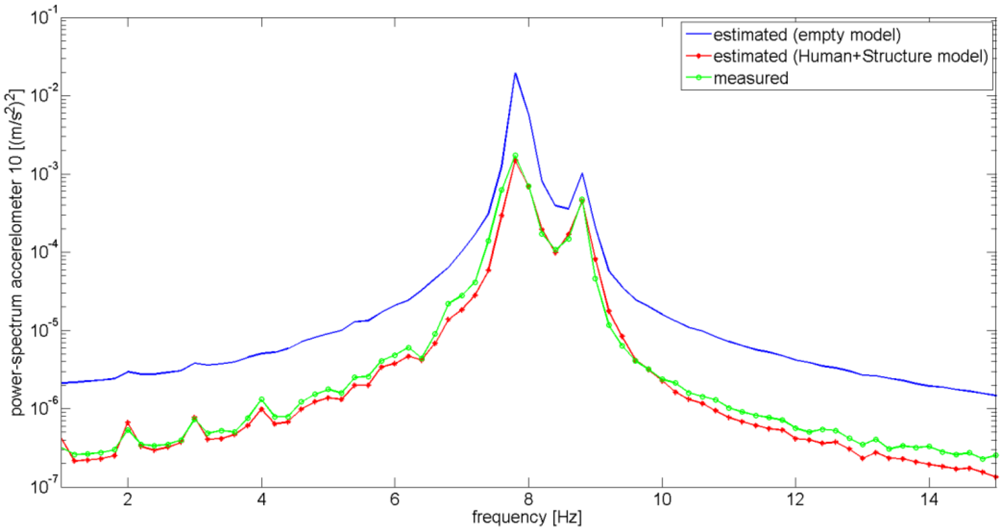

Figure 7: Test 2 – power-spectrum accelerometer 10

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12
RMS meas (m/s^2)	0.043	0.054	0.049	0.045	0.051	0.047	0.067	0.055	0.048	0.066	0.041	0.050
RMS empty (m/s^2)	0.105	0.148	0.121	0.132	0.127	0.140	0.181	0.127	0.143	0.174	0.088	0.128
Error % empty	145.4	174.5	147.9	194.1	148.0	194.6	169.4	132.0	195.0	165.6	115.4	155.4
RMS H+S (m/s^2)	0.044	0.050	0.049	0.044	0.052	0.046	0.062	0.056	0.046	0.060	0.043	0.046
Error % H+S	1.9	-7.5	1.1	-2.9	2.9	-2.3	-8.0	3.2	-5.0	-8.9	4.6	-7.5

Table 6: Test 2 – RMS and relative errors

In addition, another important consideration regards the mode shapes of the considered structure. The empty structure has nearly real mode shapes. Due to people presence the mode shapes become increasingly complex. In the case of just one or two people on the structure the complexity of the mode shapes is still low and so, in this particular case, the use of real or complex mode shapes to simulate the structural response leads to similar results. With an increasing number of people acting on the structure these differences could become important. Therefore all the modifications in the dynamic properties should be considered when predicting the response of a structure occupied by people.

5 CONCLUSIONS

This work aimed at presenting the results of a model capable of accounting for the effect of people on the modal properties of a structure they are interacting with. The idea behind this model is that of finding and equivalent model capable of accounting for the effects of moving people in terms of changing of modal parameters. Starting from the transfer functions of the empty structure, a new set of transfer functions accounting for people is obtained. The load induced by people on the structure is then introduced on this modified model. To the purpose of verifying the effectiveness of the proposed approach, a slender staircase was used. Tests were performed by asking a subject to march on a fixed point and measuring the corresponding load on the structure and the structural response. The experimental results, in terms of amplitudes of vibration of the structure, were compared with those obtained starting from the measured force and using the model of the empty structure and the model of the subjects+structure respectively. In all the considered cases the use of the model of the empty structure to simulate the structural response caused an overestimation of the amplitudes of vibration, while using the proposed model the results were always compatible with the experimental ones. The results are even more satisfactory since people were introduced using average model and the tests involved a maximum of two subjects on the structure.

The proposed approach was verified for the case of subjects marching on a fixed point. The performed tests were useful to verify the possibility of applying the proposed model but require additional development in order to be used in practice. Further work aims at extending the proposed model to the case of people walking freely on a generic structure.

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