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INFINTE PERIODIC STRUCTURE OF LIGHTWEIGHT ELEMENTS: REPRESENTATION OF WAVE PROPAGATION

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Abstract. *Lightweight wooden structures have become more popular as a sustainable, environmental-friendly and cost-effective alternative to concrete, steel and masonry buildings. However, there are certain drawbacks regarding noise and vibration due to the smaller weight and stiffness of wooden buildings. Furthermore, lightweight building elements are typically periodic structures that behave as filters for sound propagation within certain frequency ranges (stop bands), thus only allowing transmission within the pass bands. Hence, traditional methods based on statistical energy analysis cannot be used for proper dynamic assessment of lightweight buildings. Instead, this paper discusses and compares the use of finite element analysis and a wave approach based on Floquet theory.*

The present analysis has focus on the effect of periodicity on vibration transmission within semi-infinite beam structures. Two models of a semi-infinite Euler-Bernoulli and Timoshenko beam structure with periodic variation of the cross-sectional properties are analyzed. In case of the Euler-Bernoulli beam, vibrational behavior is studied in two dimensions by finite element analysis and Floquet theory. Wave propagation within the two-dimensional periodic Timoshenko beam structure is studied with a finite-element approach and compared with the periodic Euler-Bernoulli beam. The computations are carried out in frequency domain with the load acting as an impact load at the end of a semi-infinite beam. Results of various beam models and analytical approaches are compared and analyzed. A vibration-level distribution and propagation characteristics within the beam are presented for excitation frequencies up to 2 kHz.

1 INTRODUCTION

In recent years, simplifying the design of wooden building structures with adjusting the spacing between ribs and plates, research on formulate them into lightweight has increased. Infinite periodicity has been used as an advantage in process of analyzing the wave transmission phenomena of various panel structures. However, it is a challenge to predict the wave propagation in lightweight building systems due to their design and geometrical complexities.

Wave propagation within periodic structures deals with stop bands where no propagation occurs and pass bands where propagation occurs within certain frequency ranges. Several efforts to study and analyze periodic structures can be found [1, 2, 3, and 4]. Theoretical modelling of the sound transmission loss through double-leaf lightweight partitions stiffened with periodically placed studs was studied by Wang, Lu, Woodhouse, Langley, and Evans [5], who established analytical model for sound transmission through double-leaf partitions to understand the physics involved and allow future work like optimization of partition design for better sound insulation. Mace, Duhamel, Brennan and Hinke [6] also predicted wave transmission in one-dimensional structural waveguides using the finite-element method. The method is seen to yield accurate results for the wavenumbers and group velocities of both propagating and evanescent waves. Finite element analysis of the vibrations of waveguides and periodic structures was done by Duhamel, Mace and Brennan [7] where it was concluded that, point force responses for beams and plates showed the accuracy of the wave finite element approach when the size of the cell is small compared to a wavelength.

Floquet theory has been used by several researchers [8, 9, 10 and 11] to evaluate sound transmission behavior within periodic structures analytically. A periodic structure model using Timoshenko beam theory developed by Heckl could carry compressional waves, torsional waves, horizontal bending waves and vertical bending waves. It was shown that, if a single wave type was present on the beam, its dispersion relation spectrum showed a clear passing/stopping band behavior [12]. Vibrations of Timoshenko beams by variable order finite elements were studied by Houmet and concluded, advantage of this element is that highly accurate frequencies for Timoshenko beams can be obtained with a small number of system degrees of freedom [13].

Wave propagation within a one-dimensional periodic bar and two dimensional beam structures has been studied previously by the authors of the present paper, finding a good correlation between the results obtained by the FEM and Floquet theory [14]. Extensive efforts have been presented to study the problems of wave propagation in periodic structures. Concerning the calculation of wave propagation from one end to another end, three approaches are taken into account: Finite element method (FEM) for Timoshenko and Euler-Bernoulli beam and Floquet theory (Euler-Bernoulli beam).

The paper focuses on flexural wave transmission within a periodic beam structure which is made from Plexiglas and steel. The cross-sectional area is the same along the entire beam structure for the Timoshenko and Euler-Bernoulli beam. Transmitting boundary conditions [15] are introduced to mimic the behavior of a semi-infinite beam structure consisting of material 1, i.e. Plexiglas. The analyses concern the dynamic response of the beam structure to a harmonically varying concentrated force. Pass bands and stop bands that occur due to the periodic nature of the structures are identified in the frequency range 0 to 2 kHz using FEM for both of the beam structure and Floquet theory (Euler-Bernoulli beam). Results from the FE method and Floquet theory are compared and analyzed. Wave propagation behavior using FE method for Euler-Bernoulli and Timoshenko is also examined between 0 to 2 kHz.

2 PROBLEM OVERVIEW

Lightweight building structures are often constructed as panels with plates on beam (stud or joist frames). In the present case, an infinite periodic beam structure has been taken into consideration to study the occurrence of stop bands during various analytical methods like Euler-Bernoulli, Floquet theory, Timoshenko beam theory.

The structure consists of two different materials connected with each other in a periodic manner. The aim of the study is to investigate the wave propagation throughout the beam structure within various ranges of frequencies to predict the effect of periodicity on vibration transmission. Analyses are carried out in frequency domain using finite-element analysis (Euler-Bernoulli, Timoshenko beam theory and Floquet theory). Results from the methods are compared and investigated thoroughly.

2.1 Model geometry and material properties

The two-dimensional beam structure consists of a number of identical cells, each consisting of two materials: Plexiglas and steel. The Plexiglas part and steel part lengths, widths and heights are 0.3 × 0.03 × 0.012 m^3 and 0.05 × 0.03 × 0.012 m^3, respectively. Hence, the two material segments have identical cross-sectional areas throughout the beam structure. The material properties are:

- Material 1 (Plexiglas): Young's modulus, E_1 = 3.4 GPa; mass density, ρ_1 = 1190 kg/m^3;
- Material 2 (Steel): Young's modulus, E_2 = 200 GPa; mass density, ρ_2 = 7850 kg/m^3.

3 THEORTICAL FORMULATION

3.1 Euler-Bernoulli finite element formulation

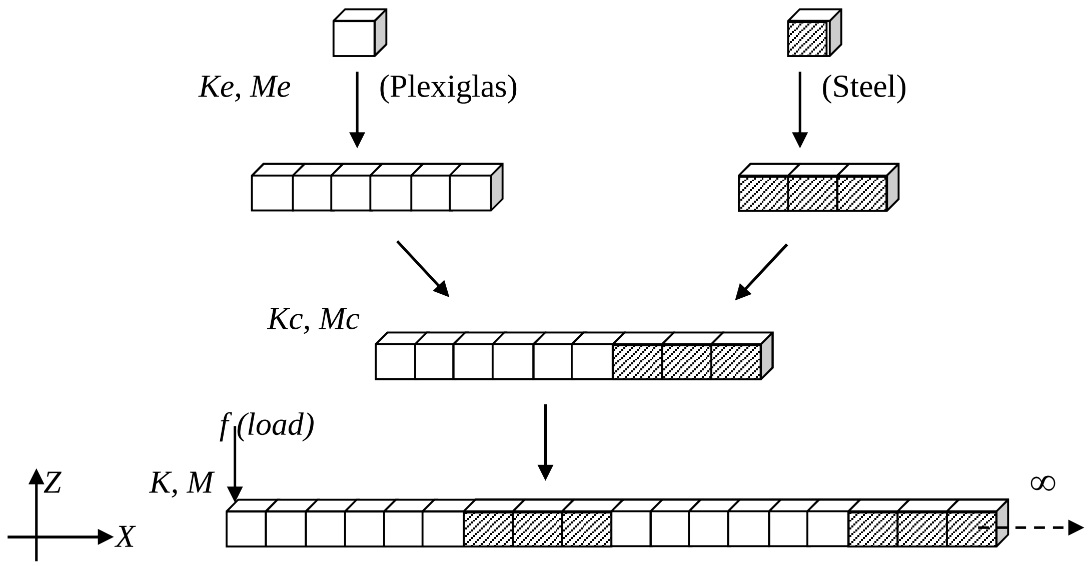

Figure 1: Formation of semi-infinite beam structure using the FEM approach;
Ke, Me = Stiffness and Mass element matrices;
Kc, Mc = Stiffness and Mass cell matrices;
K, M = Stiffness and Mass system matrices.

A semi-infinite beam structure is constructed from $Plexiglas_1$ and $Steel_2$ segments using a finite-element (FE) model. Each of the segments is specified by the following parameters: the segment length l_j, the bending stiffness E_jI_j, the mass density ρ_j, and the cross-sectional area A_j, j = 1, 2. Further, ω is introduced as the circular frequency. An FE code is generated based on the governing equations of motion for linear elastic wave propagation in an Euler-Bernoulli beam undergoing bending vibrations and finite elements with two nodes and two degrees of freedom at each node, i.e. displacement and rotation, and correspondingly two nodal forces, i.e. shear force and bending moment. No forces are applied over the infinite beam structure. The waves form a pair of propagating waves and a pair of evanescent waves at each element. The possible wavenumbers for Plexiglas and steel segments are $\pm k_{b(j)}$, $\pm ik_{b(j)}$, where,

$$k_{b(j)} = \sqrt[4]{\frac{\rho_j A_j \omega^2}{E_j I_j}}, \; j = 1, 2. \tag{1}$$

Figure 1 shows the complete formation of the semi-infinite beam structure. Firstly, the stiffness and mass matrices for the Plexiglas and steel elements are constructed:

$$K_{e(j)} = \frac{2E_j I_j}{l_{e(j)}^3}\begin{bmatrix} 6 & 3l_{e(j)} & -6 & 3l_{e(j)} \\ 3l_{e(j)} & 2l_{e(j)}^2 & -3l_{e(j)} & 2l_{e(j)}^2 \\ -12 & -6l_{e(j)} & 12 & -6l_{e(j)} \\ 6l_{e(j)} & 2l_{e(j)}^2 & -6l_{e(j)} & 4l_{e(j)}^2 \end{bmatrix}, \;\; j = 1, 2 \tag{2}$$

$$M_{e(j)} = \frac{\rho_j A_j l_{e(j)}}{420}\begin{bmatrix} 156 & 22l_{e(j)} & 54 & -13l_{e(j)} \\ 22l_{e(j)} & 4l_{e(j)}^2 & 13l_{e(j)} & -3l_{e(j)}^2 \\ 54 & 13l_{e(j)} & 156 & -22l_{e(j)} \\ -13l_{e(j)} & -3l_{e(j)}^2 & -22l_{e(j)} & 4l_{e(j)}^2 \end{bmatrix}, \;\; j = 1, 2 \tag{3}$$

Here $l_{e(j)}$ denotes the length of a finite element consisting of material j. The matrices defined by Eqns. (2) and (3) are coupled (i.e. assembled) into one single cell. Secondly, the system matrices are formulated by assembling a number of cell matrices.

3.2 Timoshenko finite element formulation

A semi-infinite Timoshenko beam structure is constructed from Plexiglas1 and steel2 segments using a finite-element (FE) model with the same parameters as the previous Euler-Bernoulli beam. Each of the segments is specified by the following parameters: the segment length $l_{e(j)}$, the bending stiffness E_jI_j, the mass density ρ_j, and the cross-sectional area A_j, shear cross sectional area A_s, phase velocity for longitudinal waves and shear waves c_j^l and c_j^f, frequency ω, shear modulus G_j, radius of gyration r_j, shear coefficient κ; j = 1, 2.

FE code is generated based on the governing equations of motion for linear elastic wave propagation in constant cross sectional beam where shear deformation and rotational inertia effects are taken into account. Each finite element has two nodes and two degrees of freedom at each node, i.e. displacement and rotation, and correspondingly two nodal forces, i.e. shear force and bending moment. The possible wavenumbers for Plexiglas and steel segments are:

$$k_{n(j)} = \pm\sqrt{\frac{1}{2}\left(\frac{\omega^2}{\left(c_j^l\right)^2}+\frac{\omega^2}{\left(c_j^f\right)^2}\right)\pm\frac{1}{2}\left(\frac{\omega^2}{\left(c_j^l\right)^2}-\frac{\omega^2}{\left(c_j^f\right)^2}\right)\sqrt{1+a_j}}\quad, j = 1, 2 \tag{4}$$

Where,

$$a_j = \frac{\left(4\,\left(c_j^l\right)^2\cdot\left(c_j^f\right)^4\right)}{\left(r_j\right)^2\cdot\omega^2\cdot\left(\left(c_j^l\right)^2-\left(c_j^f\right)^2\right)^2},\quad c_j^l = \sqrt{\frac{E_j}{\rho_j}},\quad c_j^f = \sqrt{\frac{A_{s_j}G_j}{\rho_j A_j}},\quad A_{s_j} = \left(\frac{6}{5}\right)A_j, j = 1, 2. \tag{5}$$

Stiffness matrices $K_{e(j)}$ are obtained by deliberating the exact static solution of an element with forces and moments applied at the two nodes and then consistent mass matrix $M_{e(j)}$ are derived. The stiffness matrix formulation is shown below,

$$K_{e(j)} = \frac{2E_jI_j}{l_{e(j)}^3\left(1+\phi_j\right)}\begin{bmatrix} 12 & 6\,l_{e(j)} & -12 & 6\,l_{e(j)} \\ 6\,l_{e(j)} & l_{e(j)}^2\left(4+\phi_j\right) & -6\,l_{e(j)} & l_{e(j)}^2\left(2-\phi_j\right) \\ -12 & -6\,l_{e(j)} & 12 & -6\,l_{e(j)} \\ 6\,l_{e(j)} & l_{e(j)}^2\left(2-\phi_j\right) & -6\,l_{e(j)} & l_{e(j)}^2\left(4+\phi_j\right)\end{bmatrix}, j = 1, 2 \tag{6}$$

where ϕ is the dimensionless number

$$\phi_{(j)} = \frac{12\,E_jI_j}{G_j\kappa_jA_jl_{e(j)}^2}\quad, j = 1, 2 \tag{7}$$

The mass matrix formulation is shown below,

$$M_{e(j)} = \frac{\rho_jA_jl_{e(j)}}{\left(1+\phi_j\right)^2}\begin{bmatrix} m_1 & m_2 & m_3 & m_4 \\ m_2 & m_5 & -m_4 & m_6 \\ m_3 & -m_4 & m_1 & -m_2 \\ m_4 & m_6 & -m_2 & m_5\end{bmatrix}_j + \frac{\rho_jA_jl_{e(j)}}{\left(1+\phi_j\right)^2}\left(\frac{r_j}{l_{e(j)}}\right)^2\begin{bmatrix} m_7 & m_8 & -m_7 & m_8 \\ m_8 & m_9 & -m_8 & m_{10} \\ -m_7 & -m_8 & m_7 & -m_8 \\ m_8 & m_{10} & -m_8 & m_9\end{bmatrix}_j, j = 1, 2 \tag{8}$$

$$m_{1(j)} = \frac{13}{35}+\frac{7\phi_j}{10}+\frac{\phi_j^2}{3},\qquad m_{2(j)} = \left(\frac{11}{210}+\frac{11\phi_j}{120}+\frac{\phi_j^2}{24}\right)l_e \tag{9a}$$

$$m_{3(j)} = \frac{9}{70}+\frac{3\phi_j}{10}+\frac{\phi_j^2}{6},\qquad m_{4(j)} = -\left(\frac{13}{420}+\frac{3\phi_j}{40}+\frac{\phi_j^2}{24}\right)l_e \tag{9b}$$

$$m_{5(j)} = \left(\frac{1}{105}+\frac{\phi_j}{60}+\frac{\phi_j^2}{120}\right)l_e^2,\qquad m_{6(j)} = \left(\frac{1}{140}+\frac{\phi_j}{60}+\frac{\phi_j^2}{120}\right)l_e^2 \tag{9c}$$

$$m_{7(j)} = \frac{6}{5},\qquad m_{8(j)} = \left(\frac{1}{10}-\frac{\phi_j}{2}\right)l_e \tag{9d}$$

$$m_{9(j)} = \left(\frac{2}{15}+\frac{\phi_j}{6}+\frac{\phi_j^2}{3}\right)l_e^2,\qquad m_{10(j)} = \left(-\frac{1}{30}-\frac{\phi_j}{6}+\frac{\phi_j^2}{6}\right)l_e^2 \tag{9e}$$

The matrices defined by Eqns. (6) and (8) are coupled (i.e. assembled) into one single cell and the system matrices are formulated by assembling a number of cell matrices.

To study the effects of periodicity in the both of the beam structures (i.e. Euler-Bernoulli beam and Timoshenko beam), 10 cells are coupled in periodic manner as shown in Figure 1. The problem is solved in the frequency domain at the discrete frequencies 0.1, 0.2, …, 2 kHz. At one end of the periodic structure, a nodal force with unit magnitude is applied. At the other end, a transmitting boundary condition is introduced to mimic the behavior of a semi-infinite beam structure consisting of material 1, i.e. Plexiglas.

The element length is adjusted to provide a minimum of ten elements per wavelength at the higher frequency, which provides a very high accuracy given that cubic interpolation of the displacement field is applied. The goal is to predict the characteristics of flanking noise transmission along the beam for flexural motion at each individual frequency. The Euler Bernoulli FE beam results are compared with the propagation characteristics predicted by Floquet theory for an infinite periodic structure. Comparisons between Euler-Bernoulli-beam and Timoshenko-beam FE results are presented.

3.3 Floquet theory for an Euler-Bernoulli beam

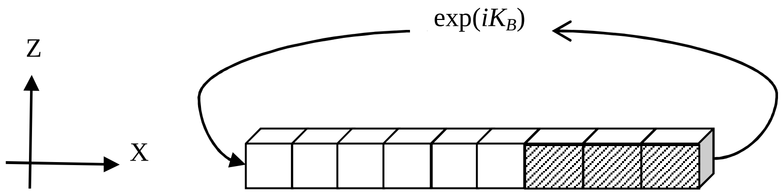

Figure 2: Formation of beam structure using Floquet theory.

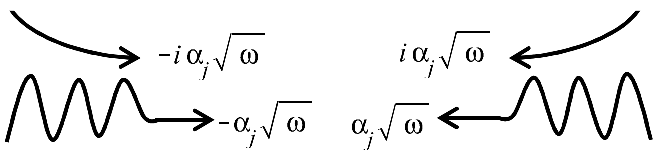

Figure 3: Illustrations of propagating and evanescent waves in a beam structure.

Prediction of wave motion within periodic structures can be done using Floquet theory for the Euler-Bernoulli beam, which eliminates the need to formulate the transfer matrices for the periodic elements. In the present case, the infinite periodic beam is constructed by a periodic repetition of two different segments composed of Plexiglas and Steel, respectively. Each of the segments is specified by the exact same parameters as used in the FEM Euler-Bernoulli model to compare the results from the two different approaches. Thus, the mass per unit length becomes $\mu_j=\rho_j A_j$, and the bending wave velocities are given as

$$c_j = \frac{\omega}{k_j} \Rightarrow c_j = \sqrt{\omega_j}\sqrt[4]{\frac{E_j I_j}{\rho_j A_j}},\ j = 1, 2. \tag{10}$$

Floquet theory is applied to the case of bending waves. In frequency domain, the governing equations for flexural waves are fourth order ordinary differential equations for the lateral displacements, $u_1(x)$ and $u_2(x)$, respectively, in the z direction, cf. Figure 2,

$$E_1 \mathrm{I}_1 \left(\frac{\mathrm{d}^4}{\mathrm{d}x^4} u_1(x) \right) - \mu_1 \omega^2 u_1 = 0 \,, \tag{11a}$$

$$E_2 \mathrm{I}_2 \left(\frac{\mathrm{d}^4}{\mathrm{d}x^4} u_2(x) \right) - \mu_2 \omega^2 u_2 = 0 \,, \tag{11b}$$

General solutions of these equations are displacements in the form:

$$u_1(\mathrm{x}) = \sum_{n=1}^{4} a_{n,1} \exp\left(ik_{n,1}\mathrm{x}\right) \tag{12a}$$

$$u_2(\mathrm{x}) = \sum_{n=1}^{4} a_{n,2} \exp\left(ik_{n,2}\mathrm{x}\right) \tag{12b}$$

The wavenumbers for propagating waves and evanescent waves are expressed via the frequency parameter. With reference to the illustration of wave directions in Figure 3,

- the term $-\alpha_j\sqrt{\omega}$ is for the waves propagating in the positive direction,
- the term $-i\alpha_j\sqrt{\omega}$ is for the evanescent waves going in the positive direction,
- the term $\alpha_j\sqrt{\omega}$ is for the waves propagating in the negative direction,
- the term $i\alpha_j\sqrt{\omega}$ is for the evanescent waves going in the negative direction.

The wavenumbers are:

$$\begin{aligned} k_{1,1} &= \alpha_1 \sqrt{\omega} \,, & k_{2,1} &= -\alpha_1 \sqrt{\omega} \,, \\ k_{3,1} &= i\,\alpha_1 \sqrt{\omega} \,, & k_{4,1} &= -i\,\alpha_1 \sqrt{\omega} \,, \\ k_{1,2} &= \alpha_2 \sqrt{\omega} \,, & k_{2,2} &= -\alpha_2 \sqrt{\omega} \,, \\ k_{3,2} &= i\,\alpha_2 \sqrt{\omega} \,, & k_{4,2} &= -i\,\alpha_2 \sqrt{\omega} \,, \end{aligned} \tag{13}$$

where,

$$\alpha_j = \sqrt{\frac{\rho_j A_j}{E_j I_j}} \,,\ j = 1, 2. \tag{14}$$

Matching conditions at the start of one cell in the periodic beam can be defined as:

$$u_1(0) = u_2(0) \,, \qquad u_1{}'(0) = u_2{}'(0) \,, \tag{15a}$$

$$E_1 \cdot I_1 \cdot u_1{}''(0) = E_2 \cdot I_2 \cdot u_2{}''(0) \,, \qquad E_1 \cdot I_1 \cdot u_1{}'''(0) = E_2 \cdot I_2 \cdot u_2{}'''(0) \,, \tag{15b}$$

$$u_1\left(l_2\right) = u_2\left(l_2\right) \,, \qquad u_1{}'\left(l_2\right) = u_2{}'\left(l_2\right) \,, \tag{15c}$$

$$E_1 \cdot I_1 \cdot u_1{}''\left(l_2\right) = E_2 \cdot I_2 \cdot u_2{}''\left(l_2\right) \,, \qquad E_1 \cdot I_1 \cdot u_1{}'''\left(l_2\right) = E_2 \cdot I_2 \cdot u_2{}'''\left(l_2\right) \,. \tag{15d}$$

Floquet theory is now used to formulate the periodicity conditions (see Fig. 2):

$$u_1\left(l_2\right) = \exp\left(i \cdot K_B\right) \cdot u_1\left(-l_1\right) \,, \tag{16a}$$

$$u_1{}'\left(l_2\right) = \exp\left(i \cdot K_B\right) \cdot u_1{}'\left(-l_1\right) \,, \tag{16b}$$

$$u_1''(l_2) = \exp(i \cdot K_B) \cdot u_1''(-l_1) \ , \tag{16c}$$

$$u_1'''(l_2) = \exp(i \cdot K_B) \cdot u_1'''(-l_1) \ , \tag{16d}$$

These periodicity conditions are substituted into Equation (15), and a system of eight homogeneous equations with respect to eight modal amplitudes grouped in two sets, $A_{n,j}$, $n = 1, 2, 3, 4$, and $j = 1, 2$, is derived. This set of equations is solved for each circular frequency, ω, with respect to the Bloch parameter K_B, which is a standard variable in the subject. K_B provides the phase shift in a periodic wave guide at a given frequency. As derived from Floquet theory, if all roots of the characteristic equation are complex numbered, then at a given frequency no propagating waves exist in the periodic beam. Hence, wave propagation in the periodic structure is possible only when the characteristic equation has at least one purely real root K_B for a given frequency, ω. Introducing $\lambda = \exp(-iK_B)$, it means when $|\lambda| \neq 1$ for all roots of the characteristic equation, the frequency falls into a stop band and vibration transmission within the beam is not possible. On the other hand, vibration transmission (i.e. a pass band) is characterized by $|\lambda| = 1$.

4 PREDICTION OF TRANSMISSION LOSS

The transmission loss (TL) for the infinite beam structure is evaluated from the nodal displacements within the frequency range 0 to 2 kHz using the finite-element method using Timoshenko and Euler-Bernoulli beam theory and Floquet theory for the Euler-Bernoulli beam. To get a better, more detailed visualization of the wave propagation properties within the frequency range, three plots are made—one for each 2 kHz frequency range (see Figure 4) where results between FE analyses for the Euler-Bernoulli beam and the Floquet approach are compared. In case of Figure 5, a detailed visualization of wave propagation within low frequency range till 100 Hz obtained by the FE method for Timoshenko and Euler-Bernoulli beams is represented, while results for higher range frequencies are visualized for an FE beam approach for both of the beam theories in Figure 6.

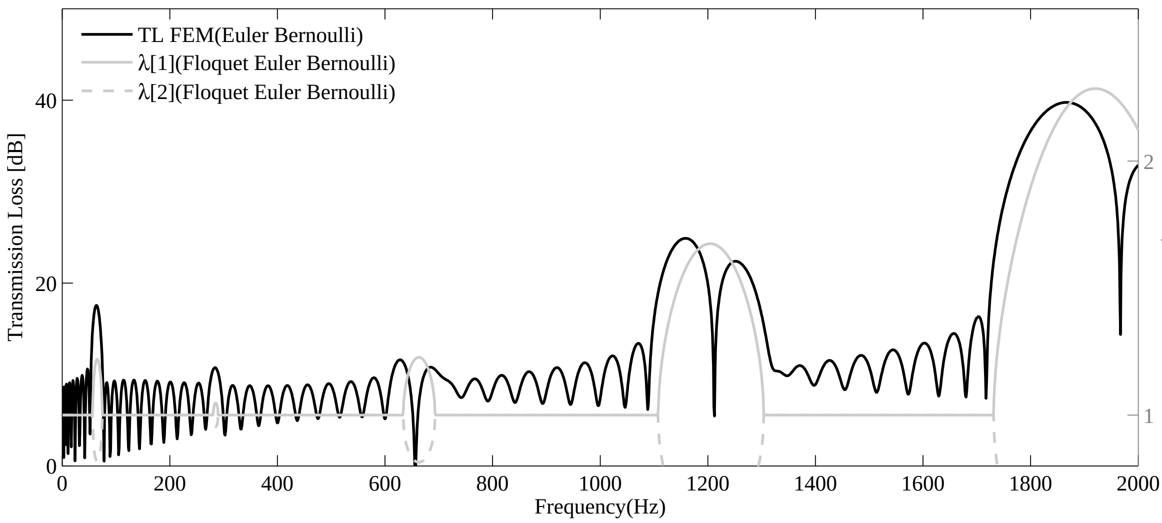

Figure 4: Transmission Loss (TL) in infinite Euler Bernoulli periodic beam structure (FEM) and Bloch parameter ($\lambda = \exp(iK_B)$ – Floquet theory) within the 0 to 2 kHz frequency range.
Note: Different scales on the left and right ordinate axis.

The frequency bands with high transmission loss (TL) broaden with increasing frequency as it is seen in Figure 4. Thus, peaks in the transmission loss are repeated periodically along the frequency axis, and the width of these peaks extends with increasing frequency.

It is furthermore observed that resonance within the pass bands does not lead to infinite (or nearly-infinite) response in the finite-element model. This is a result of the impedance condition introduced in order to mimic the behavior of a semi-infinite beam structure made of Material 1 (Plexiglas). No material damping is introduced in the finite-element model, but the transmitting boundary condition radiates energy into the semi-infinite beam at the end of 10th cell.

Vibration transmission within the infinite beam structure is also calculated using Floquet theory. As a results from this approach, a total of four parameters, $\lambda_n = \exp(-iK_B)$, n =1, 2, 3, 4, (two for bending waves and two for evanescent waves, K_B is the Bloch parameter) are deduced. Values of λ are very high for evanescent waves going in the positive x direction and very low for the corresponding components going in the negative x direction, which shows that the evanescent waves are highly damped. Therefore, these wave components will not provide any transmission of energy along the structure, whether or not it is periodic. Hence, the values of λ for the evanescent waves are not shown in the paper.

Figure 4 shows the results for bending wave propagation in terms of the parameter $\lambda = \exp(-iK_B)$ for the infinite beam in the frequency range from 0 to 2 kHz. Figure 4 shows that pass bands and stop bands can be easily observed in the low-frequency range—even below 100 Hz and the lengths of stop bands increase with increased frequency. It is noted that $|\lambda_1| = 1$ and $|\lambda_2| = 1$ within the pass bands, i.e. waves can propagate in the negative x direction as well as the positive x direction. Propagating waves are suppressed when all $|\lambda_n| \neq 1$, n =1, 2, 3, 4, and by inspection of Figures 4, it can be seen that the stop bands and pass bands equally cover the frequency spectrum from 0 to 2 kHz.

Figure 4 provides a comparison for the evaluation of vibration transmission within the beam structure using the finite-element method (10-cell structure) and the Floquet approach. The two approaches are highly correlated in the sense that stop bands and pass bands appear in the same range of frequencies for both methods. The range of stop band frequencies are 55 Hz to 75 Hz, 280 to 290 Hz, 630 Hz to 690 Hz, 1 kHz to 1.3 kHz and 1.7 kHz till 2 kHz within the prescribed range of frequencies.

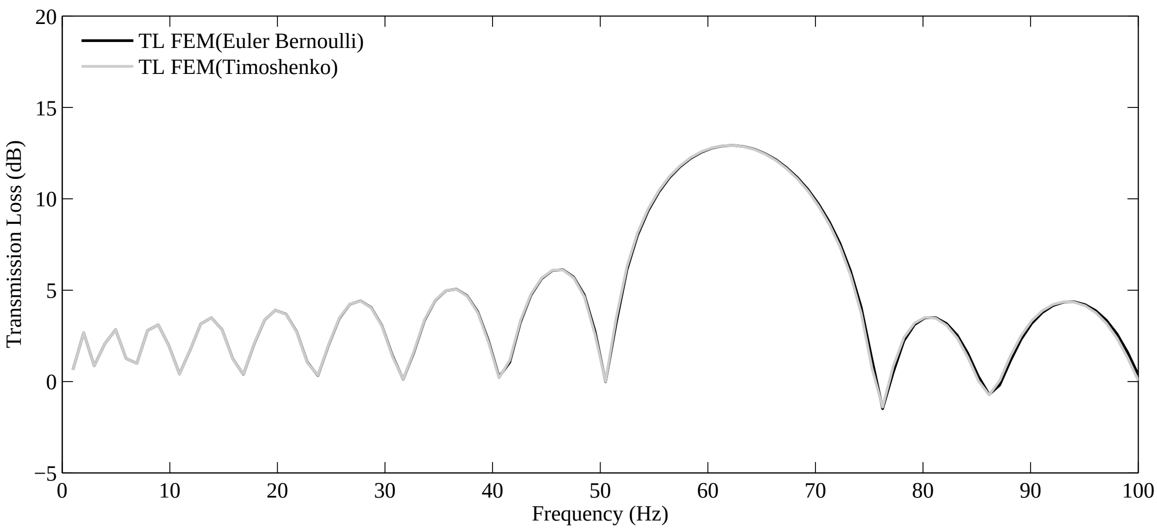

Figure 5: Transmission Loss (TL) in semi-infinite Euler Bernoulli and Timoshenko beam structure within 0 to 100 Hz.

Furthermore, in the case of Figure 5 where results for FE analysis for the various beam theories (Timoshenko and Euler-Bernoulli) are described within the 0 to 100 Hz frequency range. Stop band occurrences within Timoshenko beam show a good correlation with the results obtained by Euler-Bernoulli beam theory in the frequency range till 100 Hz.

Figure 6 provides a complete overview over representation of pass band and stop bands for Euler-Bernoulli and Timoshenko beam till 2 kHz of frequency range. At higher frequencies, a significant change in wave propagation is seen due to the effect of shear deformation and rotational inertia within the Timoshenko beam. Shear deformation effectively lowers the stiffness of the beam, which results into higher deflection and lower the prediction of eigenfrequencies for the semi-infinite beam structure. At higher frequencies within Timoshenko beam, when shear deformation and rotational inertia are present, it lowers the frequency of occurrence of stop bands, while length of the stop bands being close to the length of the stop bands in Euler-Bernoulli beam.

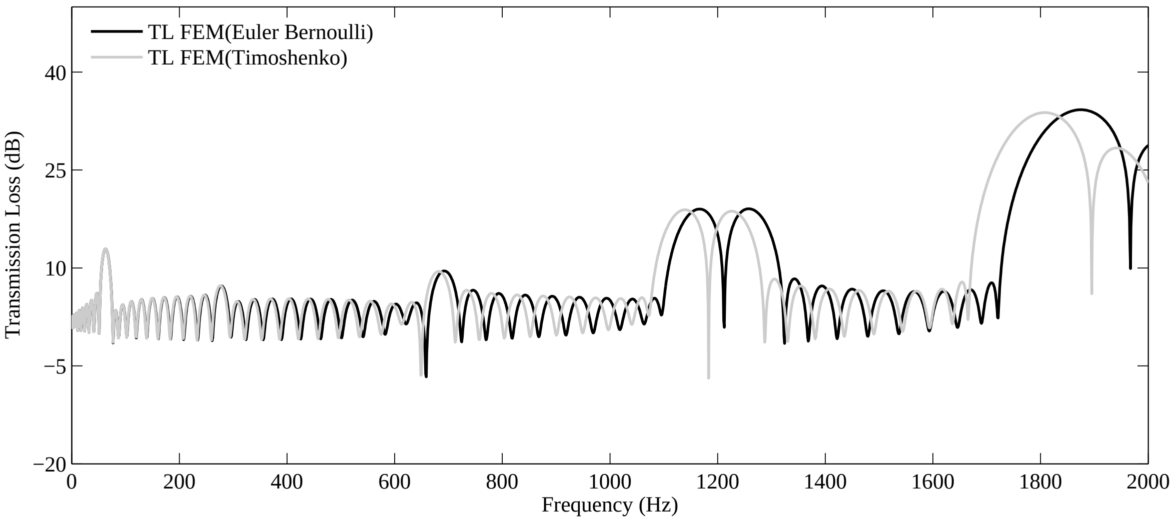

Figure 6: Transmission Loss (TL) in semi-infinite Euler Bernoulli and Timoshenko beam structure within 0 to 2 kHz.

5 CONCLUDING REMARKS

Flanking noise transmission within a periodically connected two-dimensional infinite beam structure made of Plexiglas and steel is analyzed using the finite-element method (FEM) for Timoshenko and Euler-Bernoulli beams and Floquet theory (Euler-Bernoulli beam). In case of the finite-element method, stiffness and mass matrices for Plexiglas and Steel elements were constructed and combined in a periodic manner to generate a beam structure with a finite number of cells, followed by a semi-infinite beam of Plexiglas modeled by means of an impedance condition at the end of the finite-element model.

The transmission loss (TL) is calculated using the FEM for 10 cells in the beam and impedance condition introduced in order to mimic the behavior of an infinite beam structure made of Plexiglas. Stop bands with no wave propagation are visible in certain frequency ranges. Prediction of wave propagation within an infinite two-dimensional beam structure is also evaluated through Floquet theory using Euler-Bernoulli beam, which eliminates the need to generate system matrices as in the FEM. Four parameters, λ_n = exp($-iK_B$), n =1, 2, 3, 4, (two for propagating waves and two for evanescent waves) are derived for frequencies in the range 0 to 2 kHz, where K_B is the Bloch parameter. In case of evanescent waves, the values of $|\lambda|$ are comparatively high in the positive direction and low in the negative direction along the beam, which shows that the structure is highly damped for transmission of evanescent waves. In case of bending wave propagation, pass bands are identified at certain frequency ranges when $|\lambda| = 1$ (bending wave propagation is visible), and stop bands identified by $|\lambda| \neq 1$ (no transmission of vibrations) are obtained in the remaining part of the frequency range.

A comparison between the FEM and the Floquet theory for the Euler-Bernoulli beam approach has been carried out. The methods have shown significant correlation in prediction of vibration transmission within the beam structure. Stop bands with no (or highly damped) wave propagation are surfaced at exactly the same frequency intervals with both approaches.

A comparison between the FEM for Timoshenko and Euler-Bernoulli beam has been carried out as well. It shows quite a significant correlation between results at lower frequencies like below 100 Hz. Results start to vary as the frequency increases, when shear deformation and rotational inertia effect give impact on wave propagation as expected. The FE model for the Timoshenko beam lowers the frequency for occurrence of stop bands in comparison with the Euler-Bernoulli beam, while the lengths of stop bands and pass bands being almost similar within the frequency range 200 Hz till 2 kHz.

Future tasks involve analysis of wave propagation within a thick beam (Timoshenko beam) structure using FEM and Floquet theory as well as an experimental approach, thus identifying the occurrence of stop bands regarding flexural waves within various frequency ranges. Results obtained by numerical and analytical methods will be compared with experimental results. The aim is to get a better understanding of vibration transmission within periodically connected lightweight structures.

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