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EFFECT OF POUNDING ON THE SEISMIC RESPONSE OF TWO ADJACENT ASYMMETRIC BUILDINGS UNDER SINGLE UNI-DIRECTIONAL GROUND MOTION

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Keywords: lateral-torsional pounding; two adjacent buildings; asymmetric buildings; non-linear behaviour; uni-directional seismic action; numerical analysis.

Abstract. *The aim of this paper is to present a detailed analytical study able to simulate the earthquake induced lateral-torsional pounding between two multi-storey adjacent buildings with different dynamic properties.*

Coupled lateral-torsional pounding between asymmetric structures has been investigated only in the last decades, but always with reference to impact between two single-storey structures, while most of the existing buildings are of course multi-storey. Moreover in many cases contact is modelled by linear impact elements, while contrarily pounding models utilizing the non-linear Hertz contact law appear to be more realistic, earthquake poundings being highly non-linear phenomena.

For this reason in the proposed model both a non-linear viscoelastic model to simulate impact and a non-linear behaviour of the storey shear forces and torques are considered, differently from many commercial codes which admit just one non-linearity. Torsional effects are taken into account by implementing all the possible impact scenarios, depending on the sign of the rotations of two adjacent floor-diaphragms. Moreover it is assumed that, at each storey-level, pounding occurs when a corner of one building impacts onto an arbitrary point of the other building. Just single uni-directional earthquake ground motion is incorporated in the analysis. As a consequence pounding-induced response is evaluated exclusively in the seismic direction, also due to the small values of rotations involved in the analysis.

The results show the significance of coupled lateral-torsional pounding responses on adjacent structures. Torsional vibrations of both structures are important components of the overall structural response and mainly influence the number of impacts.

1 INTRODUCTION

Pounding between adjacent buildings or between parts of the same building is one of the most frequent source of severe damage during earthquakes. This problem is particularly frequent in many cities located in seismically active regions, where, under the influence of socioeconomic factors and land usage requirements, building codes in the long run permitted contact between adjacent buildings. Moreover real structures are almost always irregular since perfect regularity is an idealization which rarely occurs. Focusing on plan irregularity, assessments of structural performances during past earthquakes demonstrate that this type of irregularity, which is due to asymmetric distributions of mass, stiffness and strength, significantly affect structural damage during earthquakes, since it results in floor torsional rotations in addition to floor translations [1-5].

Several researchers have studied seismic pounding problem. Nevertheless most of these studies focus only on translational pounding, often between two adjacent single or multi-storey symmetric buildings, neglecting torsional effects [6-16]. Among all the adopted approaches, pounding models utilizing the non-linear Hertz contact law appear to be more realistic, since earthquake poundings are highly non-linear phenomena [17-20]. On this topic, Jankowski [11] proposed a detailed investigation on pounding-involved response of two three-storey buildings with different dynamic properties, by adopting the Hertz contact law. Jankowski also showed the results of experimental studies in order to verify the validity of this impact law [21].

Coupled lateral-torsional pounding between asymmetric structures has been investigated only in the last decades, but always with reference to impact between two single-storey structures [22-25], while most of the existing buildings are of course multi-storey. Hao and Shen [23] employed the random vibration method to calculate the relative displacement of adjacent asymmetric single-storey structures. Gong and Hao [22] proposed a parametric study on coupled lateral-torsional pounding responses of adjacent symmetric and asymmetric single-storey structures, modelling contact by linear impact elements. Wang et al. [25] adopted the Hertz contact law to simulate lateral-torsional pounding between two single-storey towers, but in the analysis just few impact scenarios were taken into account.

In this framework a clear analytical model simulating lateral-torsional pounding, including all the most probable impact scenarios and easily usable for practical purposes, is not available in literature. The aim of the present study is to propose a reliable analytical model to simulate coupled lateral-torsional pounding between two adjacent multi-storey buildings, able to reproduce the highly nonlinear and complex pounding phenomenon. As explained in the following sections, a macro-model approach is adopted [26].

2 DYNAMIC EQUATION OF MOTION

The study herein presented is focused on earthquake-induced lateral-torsional pounding between two adjacent three-storey buildings. Without loss of generality, only ground excitation in the *x*-direction and eccentricity in the *y*-direction are considered, as shown in Fig. 1; CM and CS indicate mass center and stiffness center of each storey, while *d* is the initial separation gap. The motion of each building is described by 6 degrees of freedom: three translational displacements x_i along the *x* direction and three floor rotations θ_i.

An elastic-perfectly plastic approximation of storey shear forces and torques is enforced in the model. The dynamic equation of motion for such a structural model, including pounding between buildings at each floor level, can be written in matrix form as:

$$\boldsymbol{\Lambda} \cdot \ddot{\boldsymbol{\eta}}(t) + \boldsymbol{C} \cdot \dot{\boldsymbol{\eta}}(t) + \boldsymbol{\Gamma}^S(t) + \boldsymbol{\Gamma}\ (t) = -\boldsymbol{\Lambda} \cdot \ddot{\boldsymbol{\eta}}_g(t) \tag{1}$$

with:

$$\boldsymbol{\Lambda} = \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_6 \end{bmatrix} \quad \ddot{\boldsymbol{\eta}}(t) = \begin{bmatrix} \ddot{x}_1(t) \\ \ddot{x}_2(t) \\ \ddot{x}_3(t) \\ \ddot{\theta}_1(t) \\ \ddot{\theta}_2(t) \\ \ddot{\theta}_3(t) \\ \ddot{x}_4(t) \\ \ddot{x}_5(t) \\ \ddot{x}_6(t) \\ \ddot{\theta}_4(t) \\ \ddot{\theta}_5(t) \\ \ddot{\theta}_6(t) \end{bmatrix} \quad \dot{\boldsymbol{\eta}}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{\theta}_1(t) \\ \dot{\theta}_2(t) \\ \dot{\theta}_3(t) \\ \dot{x}_4(t) \\ \dot{x}_5(t) \\ \dot{x}_6(t) \\ \dot{\theta}_4(t) \\ \dot{\theta}_5(t) \\ \dot{\theta}_6(t) \end{bmatrix} \quad \ddot{\boldsymbol{\eta}}_g(t) = \begin{bmatrix} \ddot{x}_g(t) \\ \ddot{x}_g(t) \\ \ddot{x}_g(t) \\ 0 \\ 0 \\ 0 \\ \ddot{x}_g(t) \\ \ddot{x}_g(t) \\ \ddot{x}_g(t) \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \boldsymbol{\Gamma}(t) = \begin{bmatrix} F_{x14}(t) \\ F_{x25}(t) \\ F_{x36}(t) \\ T_{\theta 14}(t) \\ T_{\theta 25}(t) \\ T_{\theta 36}(t) \\ -F_{x14}(t) \\ -F_{x25}(t) \\ -F_{x36}(t) \\ T_{\theta 41}(t) \\ T_{\theta 52}(t) \\ T_{\theta 63}(t) \end{bmatrix}$$

$$\boldsymbol{C} = \begin{bmatrix} c_{x1,2} & -c_{x2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -c_{x2} & c_{x2,3} & -c_{x3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -c_{x3} & c_{x3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{\theta 1,2} & -c_{\theta 2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c_{\theta 2} & c_{\theta 2,3} & -c_{\theta 3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -c_{\theta 3} & c_{\theta 3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & c_{x4,5} & -c_{x5} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -c_{x5} & c_{x5,6} & -c_{x6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_{x6} & c_{x6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_{\theta 4,5} & -c_{\theta 5} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_{\theta 5} & c_{\theta 5,6} & -c_{\theta 6} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_{\theta 6} & c_{\theta 6} \end{bmatrix} \quad \boldsymbol{\Gamma}^S(t) = \begin{bmatrix} F_{x1}^S(t) - F_{x2}^S(t) \\ F_{x2}^S(t) - F_{x3}^S(t) \\ F_{x3}^S(t) \\ T_{\theta 1}^S(t) - T_{\theta 2}^S(t) \\ T_{\theta 2}^S(t) - T_{\theta 3}^S(t) \\ T_{\theta 3}^S(t) \\ F_{x4}^S(t) - F_{x5}^S(t) \\ F_{x5}^S(t) - F_{x6}^S(t) \\ F_{x6}^S(t) \\ T_{\theta 4}^S(t) - T_{\theta 5}^S(t) \\ T_{\theta 5}^S(t) - T_{\theta 6}^S(t) \\ T_{\theta 6}^S(t) \end{bmatrix}$$

where m_i and I_i (i=1,...,6) are the mass and the polar moment of inertia about a vertical axis through the CM of a single storey; $\ddot{x}_i$, $\dot{x}_i$, $\ddot{\theta}_i$, $\dot{\theta}_i$ (i=1,...,6) are the acceleration and velocity of a single storey in the longitudinal and torsional directions; $F_{xi}^S(t)$ (i=1,…,6) and $T_{\theta i}^S(t)$ (i=1,..,6) are the inelastic storey shear forces and torques; c_{xi} (i=1,…,6) and $c_{\theta i}$ (i=1,…,6) are the elastic damping coefficients, with $c_{xi,j} = c_{xi} + c_{xj}$ and $c_{\theta i,j} = c_{\theta i} + c_{\theta j}$; $F_{xij}(t)$ and $T_{\theta ij}(t)$ are the pounding forces and torques between storeys with masses m_i, m_j; $\ddot{x}_g(t)$ is the longitudinal acceleration component of the input ground motion. Since the proposed formulation concerns pounding between two adjacent buildings, it results $F_{xji}(t) = -F_{xij}(t)$ (i=1,...,3; j=4,...,6).

The storey shear forces $F_{xi}^S(t)$ and torques $T_{\theta i}^S(t)$, according to an elastic-perfectly plastic behaviour, are expressed for the elastic range by:

$$F_{xi}^S(t) = K_{xi}\left[x_i(t) - e_{yi}\theta_i(t)\right] \ (i = 1,4); \quad F_{xi}^S(t) = K_{xi}\left[\left(x_i(t) - e_{yi}\theta_i(t)\right) - \left(x_{i-1}(t) - e_{y(i-1)}\theta_{i-1}(t)\right)\right] \ (i = 2,3,5,6); \tag{2}$$

$$T_{\theta i}^{S}(t)=\left(\sum_{j=1}^{n}k_{xj,i}\left(x'^{2}_{j,i}+y'^{2}_{j,i}\right)\right)\theta_i(t)-K_{xi}e_{yi}x_i(t)\quad (i=1,4); \tag{3}$$

$$T_{\theta i}^{S}(t)=\left[\sum_{j=1}^{n}k_{xj,i}\left(x'^{2}_{j,i}+y'^{2}_{j,i}\right)\right]\left[\theta_i(t)-\theta_{i-1}(t)\right]-K_{xi}\left[e_{yi}x_i(t)-e_{y(i-1)}x_{i-1}(t)\right];\quad (i=2,3,5,6); \tag{4}$$

and for the plastic range by:

$$F_{xi}^{S}(t)=F_{xi}^{Y}\qquad (i=1,...,6); \tag{5}$$

$$T_{\theta i}^{S}(t)=T_{\theta i}^{Y}\qquad (i=1,...,6); \tag{6}$$

where x_i and θ_i are the displacement in the x direction and the rotation of a single storey; $K_{xi}=\sum_{j=1}^{n}k_{xj,i}$ is the elastic structural stiffness in the x direction of storey i; $k_{xj,i}$ and n are respectively the j^{th}-column lateral stiffness and the number of columns of the concerned storey; $x'_{j,i}$, $y'_{j,i}$ are the co-ordinates of the j^{th} column with the origin at CM of the concerned storey; e_{yi} is the static eccentricity at storey i; F_{xi}^{Y} and $T_{\theta i}^{Y}$ are the storey lateral and torsional yield strengths. Figure 1 shows the positive signs of displacements and rotations.

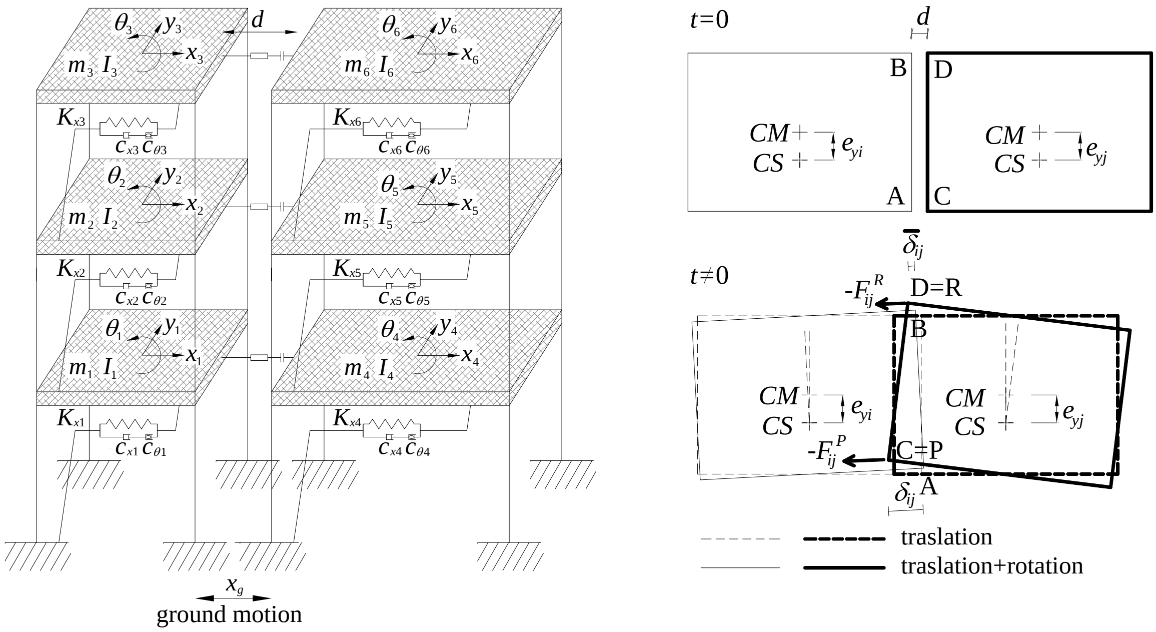

Figure 1: Analytical model.

2.1 Contact forces

In this formulation it is assumed that, at each storey-level, pounding occurs when a corner of one building impacts onto an arbitrary point of the other building. Actually if, at a generic time-step, the whole opposite edges AB and CD overlap due to floor rotation and translation, pounding forces are distributed along the entire contact segment. For the seek of simplicity, the resultant pounding forces are considered concentrated in correspondence of the impact point and of the second corner of the colliding edge. So the main pounding force $F_{ij}^{P}(t)$ is localized at the impact corner, while the pounding force at the second corner $F_{ij}^{R}(t)$ is different from zero only when a full overlapping occurs (Fig. 1). Pounding-involved response is evaluated in the hypothesis of frictionless contact [25]. According to this assumption, the pounding force $F_{ij}^{P}(t)$ at each storey-level is always perpendicular to the edges AB or CD when they are impacted by a corner of the other building (Fig. 1). The same direction is assumed for the pounding force $F_{ij}^{R}(t)$. In the analysis pounding forces are simulated according to the non-linear viscoelastic model based on the Hertz's contact law [11, 17-20]; the force $F_{ij}^{P}(t)$ assumes the following form:

$$\begin{aligned} &F_{ij}^{P}(t)=0 \quad \text{for } \delta_{ij}(t)\le 0\,; \\ &F_{ij}^{P}(t)=\beta\delta_{ij}^{\frac{3}{2}}(t)+\bar{c}_{ij}(t)\dot{\delta}_{ij}(t) \quad \text{for } \delta_{ij}(t)>0\,;\ \dot{\delta}_{ij}(t)>0\,; \\ &F_{ij}^{P}(t)=\beta\delta_{ij}^{\frac{3}{2}}(t) \quad \text{for } \delta_{ij}(t)>0\,;\ \dot{\delta}_{ij}(t)\le 0\,; \end{aligned} \tag{7}$$

with:

$$\bar{c}_{ij}(t)=2\xi\sqrt{\beta\sqrt{\delta_{ij}(t)}\frac{m_i m_j}{m_i+m_j}} \tag{8}$$

where $\delta_{ij}(t)$ is the penetration depth at the pounding point, β is the impact stiffness parameter simulating the local stiffness at the contact point, d is the initial separation gap and ξ denotes the impact-damping ratio. The term ξ is estimated using Jankowski's formula [19]:

$$\xi=\frac{9\sqrt{5}}{2}\frac{1-e^2}{e[e(9\pi-16)+16]} \tag{9}$$

where e is the coefficient of restitution accounting for the energy dissipation during impact.

The aliquot $F_{ij}^{R}(t)$ of pounding force can be obtained by Eqs. (7) replacing the quantity $\delta_{ij}(t)$ with the penetration depth $\bar{\delta}_{ij}(t)$ at the second corner of the impacting edge.

The resultant pounding force is given by:

$$F_{ij}(t)=F_{ij}^{P}(t)+F_{ij}^{R}(t). \tag{10}$$

The pounding torques can be expressed as:

$$\begin{aligned} T_{\theta ij}(t)=&-F_{xij}^{P}(t)\cdot\left(y_{Pij}(t)-y_{CMi}\right)+F_{yij}^{P}(t)\cdot\left(x_{Pij}(t)-x_{CMi}\right)+ \\ &F_{xij}^{R}(t)\cdot\left(y_{Rij}(t)-y_{CMi}\right)-F_{yij}^{R}(t)\cdot\left(x_{Rij}(t)-x_{CMi}\right) \\ &(i=1,2,3;\ j=4,5,6),\quad (i=4,5,6;\ j=1,2,3) \end{aligned} \tag{11}$$

where $F_{xij}^{P}(t)$, $F_{yij}^{P}(t)$ and $F_{xij}^{R}(t)$, $F_{yij}^{R}(t)$ are the x, y components of the pounding forces $F_{ij}^{P}(t)$ and $F_{ij}^{R}(t)$ respectively, while $x_{Pij}(t), y_{Pij}(t)$ $x_{Rij}(t), y_{Rij}(t)$ and x_{CMi}, y_{CMi} are the x, y coordinates of the pounding impact point P, of the second corner R and of the CM in the reference system chosen for the analysis. During the analysis the relative locations of the two buildings at each storey-level are checked at each time-step, to verify whether pounding occurs or not. In order to evaluate pounding forces and torques, it is necessary to identify all the possible pounding scenarios. Eight pounding scenarios are herein considered, depending on the sign of rotations of two adjacent floor-diaphragms (Fig. 2). The scenarios so defined include also the cases in which one or both buildings do not rotate at the concerned level. On the basis of the above scenarios, the penetration depth $\delta_{ij}(t)$ at the pounding point, including also the torsional effect, can be evaluated by the following expression:

$$\delta_{ij}(t) = x_i(t) - x_j(t) + \delta_{ij,torq}(t) \tag{12}$$

where the term $\delta_{ij,torq}(t)$ denotes the overlapping aliquot of two adjacent storeys *i*, *j* due to torsional rotations; observing Fig. 2, it represents the distance between the impacting corner and the opposite edge of the other building and is calculated for each scenario with respect to the fixed reference system defined in Fig. 2 itself.

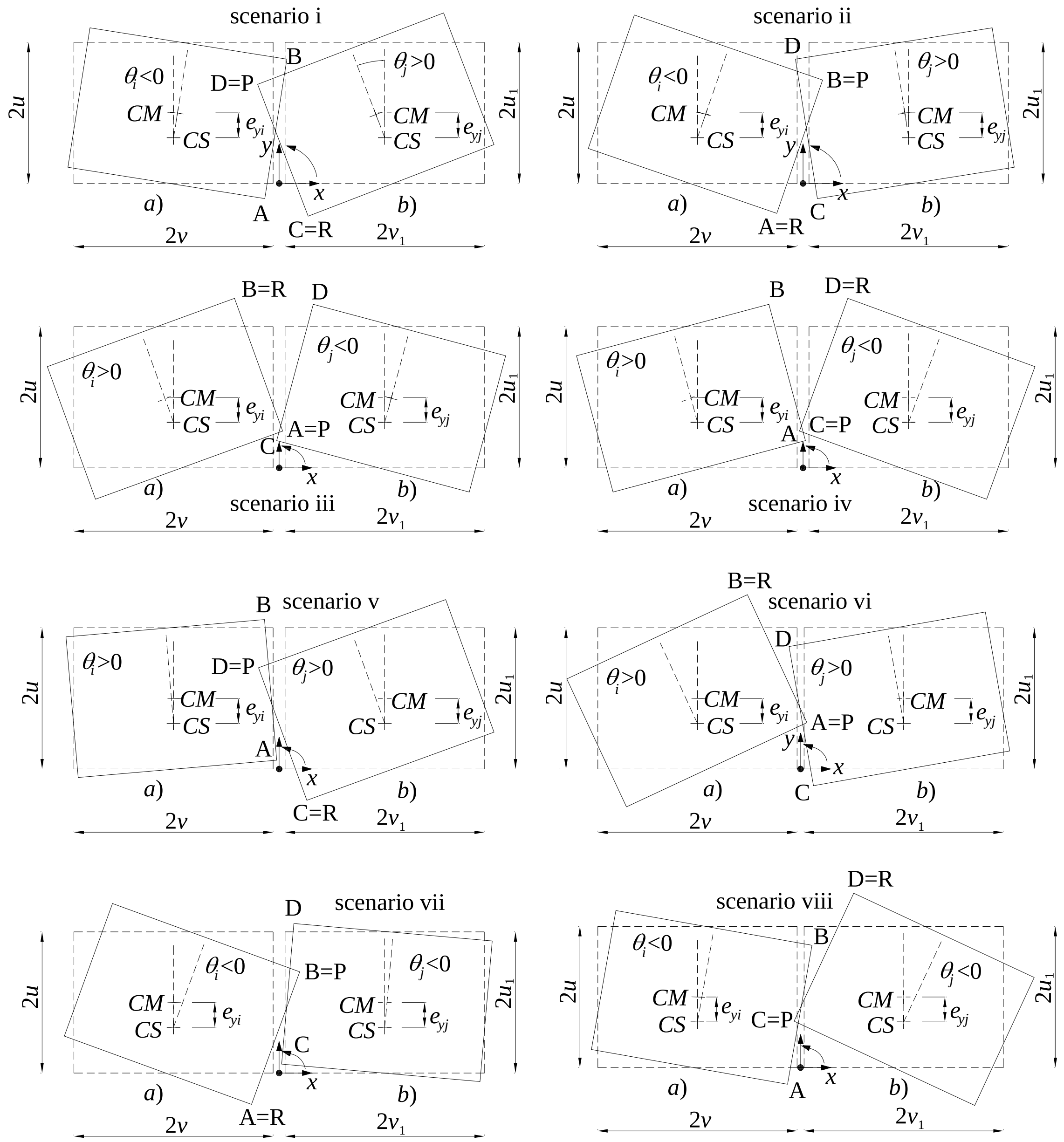

Figure 2: Impact scenarios.

3 RESPONSE ANALYSIS

The results of the numerical lateral-torsional pounding simulations are described with reference to two three-storey buildings having the following geometric and structural characteristics:

- left building:
  u=3 m, v=3 m, $e_{y1} = e_{y2} = e_{y3} = 2$ m,
  $m_1 = m_2 = m_3 = 2.85 \times 10^4$ kg,
  $I_1 = I_2 = I_3 = 5.079 \times 10^5$ kg·m^2,
  $K_{x1} = K_{x2} = K_{x3} = 3.44 \times 10^6$ N/m (ω_{lx}=11 rad/sec),
  $c_{x1} = c_{x2} = c_{x3} = 3.135 \times 10^4$ kg/s (ξ_{lx}=0.05),
  $c_{\theta 1} = c_{\theta 2} = c_{\theta 3} = 7.41 \times 10^5$ kg·m^2/s ($\xi_{l\theta}$=0.05, $\omega_{l\theta}$=14.6 rad/sec),
  $F^Y_{x1} = F^Y_{x2} = F^Y_{x3} = 1.2 \times 10^5$ N, $T^Y_{\theta 1} = T^Y_{\theta 2} = T^Y_{\theta 3} = 3.5 \times 10^5$ N·m.
- right building: u_1=2.5 m, v_1=2.5 m,
  $e_{y4} = e_{y5} = e_{y6} = 1.5$ m,
  $m_4 = m_5 = m_6 = 1.3 \times 10^6$ kg,
  $I_4 = I_5 = I_6 = 1.625 \times 10^7$ kg·m^2,
  $K_{x4} = K_{x5} = K_{x6} = 2.34 \times 10^9$ N/m (ω_{rx}=42.45 rad/sec),
  $c_{x4} = c_{x5} = c_{x6} = 5.518 \times 10^6$ kg/s (ξ_{rx}=0.05),
  $c_{\theta 4} = c_{\theta 5} = c_{\theta 6} = 8.5 \times 10^7$ kg·m^2/s ($\xi_{r\theta}$=0.05, $\omega_{r\theta}$=52.35 rad/sec),
  $F^Y_{x4} = F^Y_{x5} = F^Y_{x6} = 1.2 \times 10^7$ N, $T^Y_{\theta 4} = T^Y_{\theta 5} = T^Y_{\theta 6} = 4 \times 10^7$ N·m.

The above data concern two equal height structures with different dynamic properties. The initial separation gap d between buildings is set equal to 0.02 m. For the non-linear viscoelastic impact model's parameters, the following values are assumed: β=2.75x10^9 N/m$^{3/2}$, ξ=0.35 (e=0.66). As to the seismic input, the El Centro earthquake accelerogram has been incorporated in the analysis. In order to point out the torsional effects, the displacement time histories for the three storeys of both left and right building are shown in Fig. 3 with reference to the four corners A, B, C, D on the adjacent edges of the two structures (Fig. 2).

The pounding force and torque time histories are reported in Fig. 4.

Observing Figs. 3 and 4 it emerges that under the seismic action the response of the lighter and more flexible left building is significant, while the displacements of the heavier and stiffer right building are nearly negligible. Pounding force and torque reach their maximum values at the third storeys, while at the second and first storeys their values drop to about 80% and 50% respectively, but are anyway meaningful.

Figure 5 finally shows the displacement and pounding force time histories at the third level obtained neglecting the torsional effects, that is setting e_{yi}=0 (i=1,…,6). From Fig. 5 it is finally evident that plan eccentricity substantially changes the structural response, leading to an increasing of collisions and an overall reduction of peak displacements.

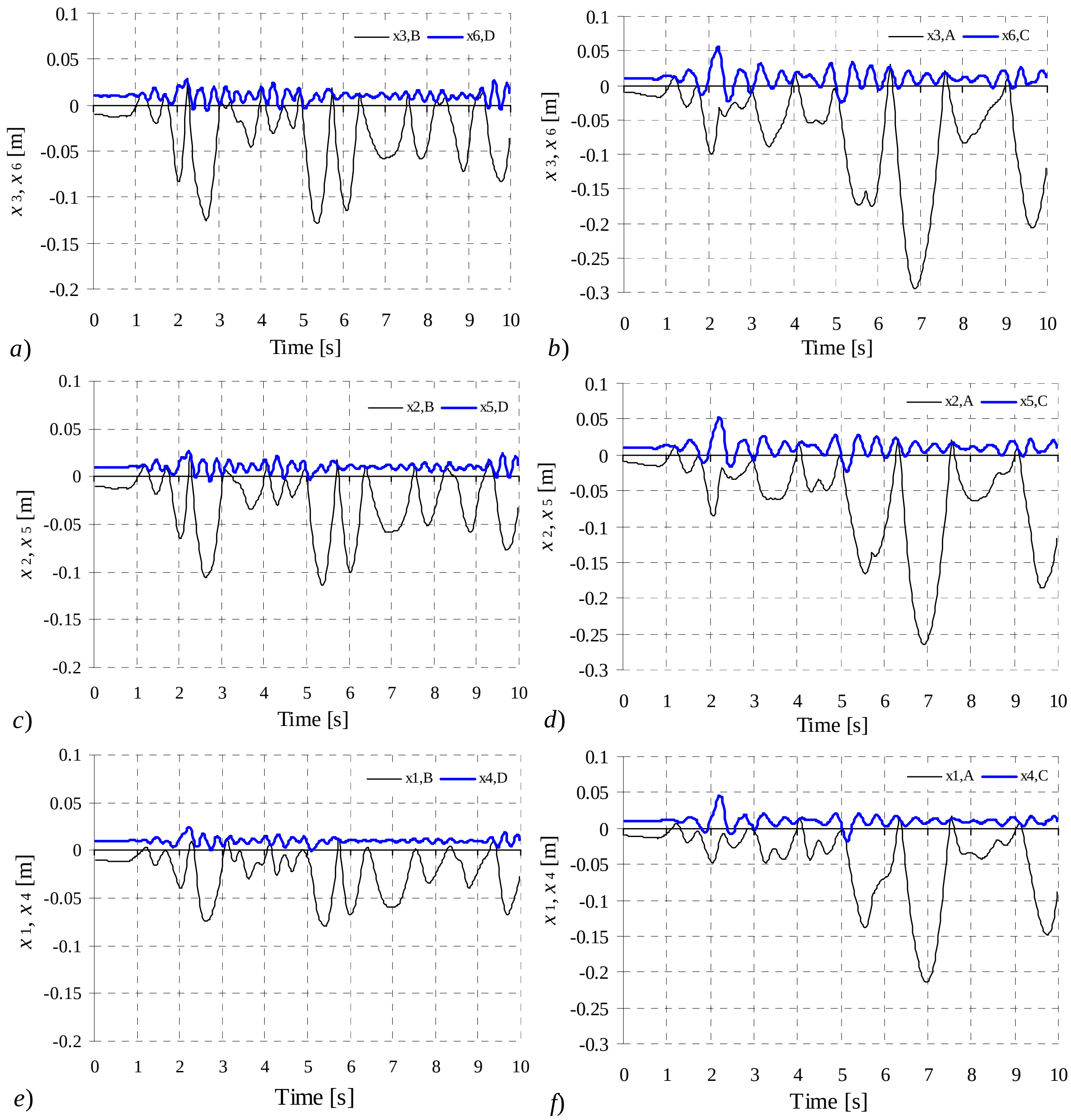

Figure 3: Displacement time histories of buildings at corners A, B, C, D, for each storey.

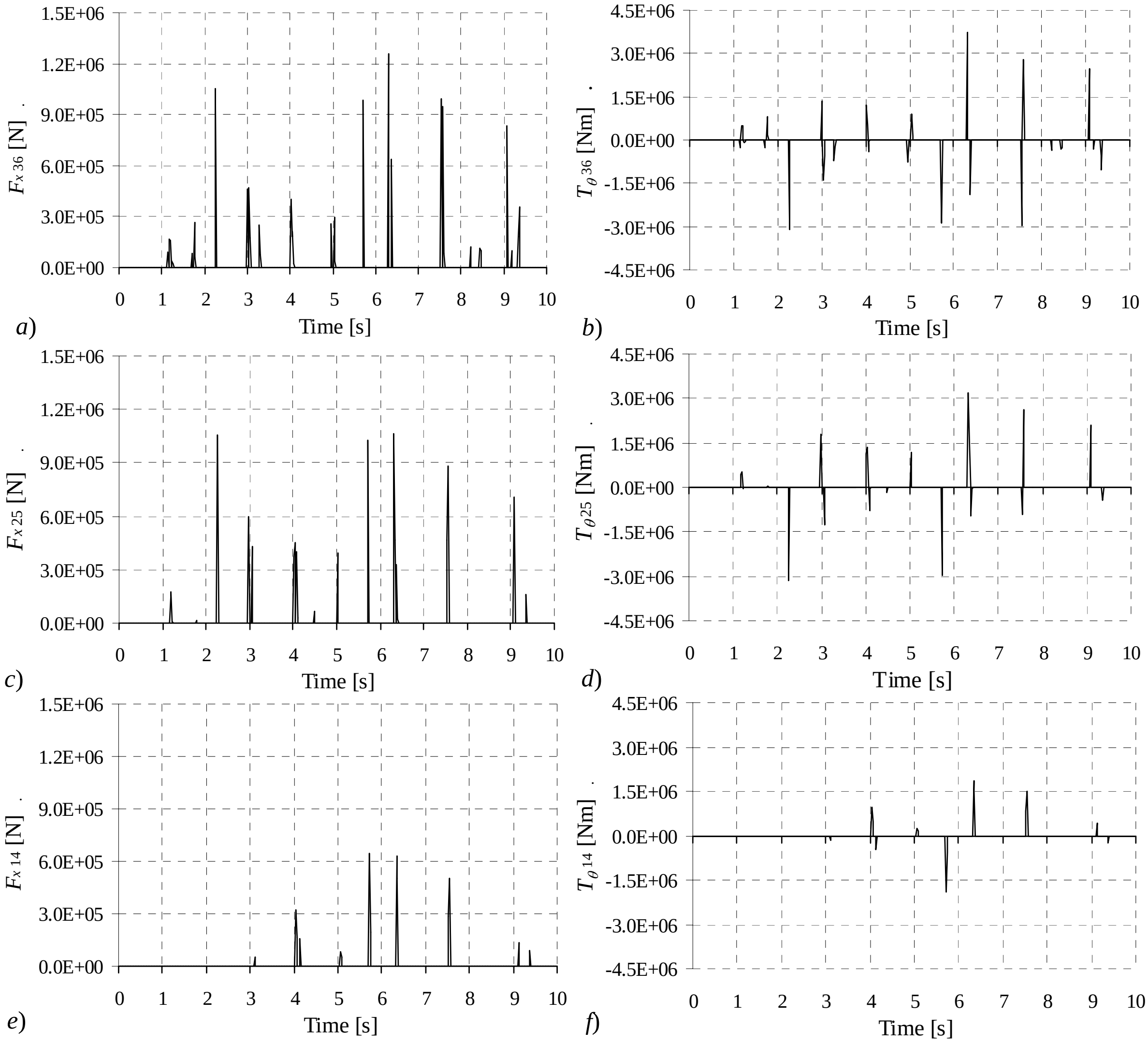

Figure 4: Pounding force and torque time histories of left building at each storey.

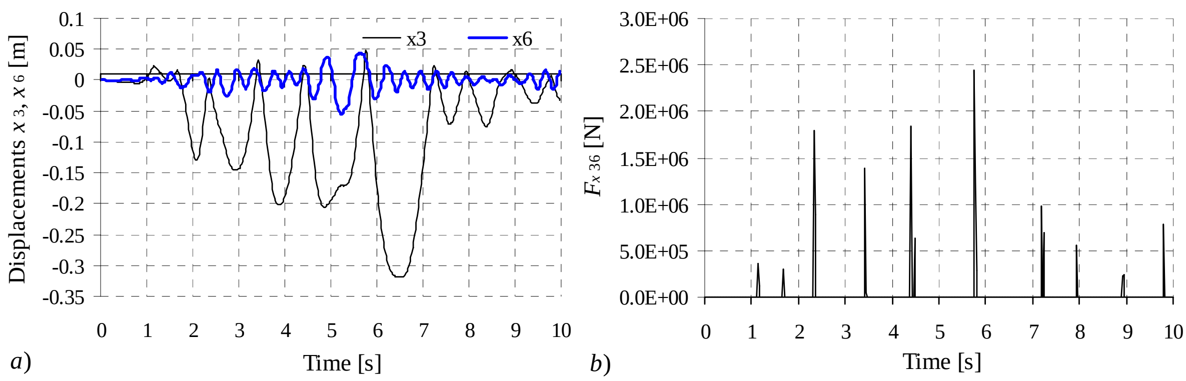

Figure 5: Displacement (a) and pounding force (b) time histories at the third storey neglecting torsional effects.

4 CONCLUSIONS

In this paper a theoretical and analytical lateral-torsional pounding-induced response analysis between two three-storey buildings under seismic action has been presented.

All the possible pounding scenarios have been implemented in the analysis, depending on the sign of rotations of two adjacent floor-diaphragms. Both a non-linear viscoelastic model to simulate impact and a non-linear behaviour of the storey shear forces and torques have been considered, differently from many commercial softwares which admit just one non-linearity. A single uni-directional seismic action, represented by El-Centro earthquake, has been incorporated in the analysis. Just response in the seismic direction has been calculated.

The results of the study show that torsional vibrations of both structures are important components of the overall structural response, affecting both the number of collisions and the entity of overall displacements. Results also indicate that pounding force and torque reach their maximum values at the third storeys, while at the first and second storeys their values drop to about 80% and 50%, but are anyway meaningful.

Finally the authors underline that in prospective the application of this specific formulation to the case of three or more buildings in series could furnish useful informations in order clarify the phenomenon of pounding between existing buildings, especially under the awareness that suitable mechanical models are not available in technical codes.

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