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THE USE OF BAYESIAN MODEL UPDATING IN STOCHASTIC DESIGN PROBLEMS

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Abstract. *This work focuses on the effect of using updated reliability measures in the context of stochastic structural dynamical systems. A Bayesian probabilistic methodology for model updating is first implemented for the purpose of updating the structural model using dynamic data. The updated distribution of the system model parameters is then used to implement a strategy for updating the system reliability. The effect of the updated information on the reliability and design of dynamical systems under stochastic excitation is illustrated by an example problem.*

1 INTRODUCTION

During operation conditions structural systems may deteriorate from a number of reasons, such as fatigue, corrosion, damage induced in structural elements by strong wind loads or earthquakes, etc. These conditions may lead to significant reduction of the structural reliability. Therefore, the re-assessment of the reliability of a structure after it has been built by monitoring the dynamic response is of paramount importance. The updated reliability may be used to identify potentially unsafe structures, to schedule repairs, maintenances or inspection intervals, or to design retrofitting or control strategies [1, 2]. In this work a strategy for updating structural reliability using dynamic data is considered. For this purpose, a Bayesian probabilistic framework for model updating is integrated with advanced simulation tools [3]. In particular, a multi-level Markov chain Monte Carlo algorithm is adopted here [4]. In this framework, a revised information about the uncertainties in the system parameters is obtained, which is expressed by posterior probability density functions. Using the updated characterization of the system model, a strategy is implemented for computing the reliability of structural systems under future excitations characterized by means of stochastic processes. In this context, the probability that design conditions are satisfied within a particular reference period is used as reliability measure. Such measure is referred as the first excursion probability and quantifies the plausibility of the occurrence of unacceptable behavior of the structural system [5]. The strategy is based on subset simulation and it uses a Markov chain Monte Carlo technique based on the Metropolis algorithm for generating conditional samples [6]. Based on the proposed strategy for updating the structural reliability it is the objective of this work to study the effect of the updated information on the reliability and design of dynamical systems under stochastic excitation.

2 BAYESIAN UPDATING USING DYNAMIC DATA

2.1 General Considerations

Let M be the probabilistic model class of structural models parameterized by the vector of uncertain model parameters $\boldsymbol{\theta} \in \Theta \subset R^{n_p}$, and D the measured dynamic data from the system. The objective of Bayesian model updating is to define the posterior probability density function of $\boldsymbol{\theta}$ conditioned on D, $p(\boldsymbol{\theta}/M, D)$, as (Bayes' Theorem) [7]

$$p(\boldsymbol{\theta}/M, D) = \frac{p(D/M, \boldsymbol{\theta})\; p(\boldsymbol{\theta}/M)}{p(D/M)} \tag{1}$$

where $p(\boldsymbol{\theta}/M)$ is the initial (prior) probability density function of $\boldsymbol{\theta}$, $p(D/M, \boldsymbol{\theta})$ is the likelihood function, and $p(D/M)$ is the evidence of M. The prior probability distribution reflects the relative plausibility of each model in the model class M before utilizing the data D. The term $p(D/M, \boldsymbol{\theta})$ gives the probability of obtaining the data D based on a model specified by the model parameters $\boldsymbol{\theta}$. Finally, the normalizing constant $p(D/M)$ corresponds to the evidence for the model class M given by the data D. The likelihood function gives a measure of the agreement between the system response and the corresponding structural model output. This measure of the data fit of each model in the model class M, i.e. the value of the likelihood function for each parameter vector $\boldsymbol{\theta}$, is given by the probability model established for the system output. The probability model class for the prediction error $\mathbf{e}(t_n, \boldsymbol{\theta})$, that is, the difference between the measured response and the model response, considered in the present formulation is based on the maximum entropy principle which yields a multi-dimensional Gaussian distribution with zero mean and covariance matrix $\boldsymbol{\Sigma}_e$. In particular, the prediction error $\mathbf{e}(t_n, \boldsymbol{\theta})$ is modeled as a discrete zero mean Gaussian white noise process, that is,

$$E[\mathbf{e}(t_n, \boldsymbol{\theta})] = 0 \ , \ E[\mathbf{e}(t_n, \boldsymbol{\theta})\mathbf{e}(t_m, \boldsymbol{\theta})^T] = \boldsymbol{\Sigma}_e \ \delta_{nm} \tag{2}$$

where $E[\cdot]$ denotes expectation, δ_{nm} denotes the kronecker delta function, $\boldsymbol{\Sigma}_e$ denotes the $N_o \times N_o$ covariance matrix which is assumed to have the form $\boldsymbol{\Sigma}_e = \sigma_e^2 \mathbf{I}_o$, where $\mathbf{I}_o$ is the identity matrix, and N_o is the number of observed degrees of freedom of the structural model. The above assumption implies equal variances and stochastic independence of the prediction errors for different channels of measurements. Using the above probability model for the prediction error it can be shown that the likelihood function $p(D/M, \boldsymbol{\theta})$ can be expressed in terms of a measure-of-fit function $J(\boldsymbol{\theta}/M, D)$ between the measured response and the model response at the measured degrees of freedom. Such function is given by [8, 9]

$$J(\boldsymbol{\theta}/M, D) = \frac{1}{N_t N_o} \sum_{n=1}^{N_t} \parallel \mathbf{e}(t_n, \boldsymbol{\theta}) \parallel^2 \tag{3}$$

where $\parallel \cdot \parallel$ denotes the Euclidian norm of a vector and N_t is the number of available data in time.

2.2 Simulation-based Methods

For a large number of available data it has been found that the most probable model parameters $\hat{\boldsymbol{\theta}}$ are obtained by minimizing $J(\boldsymbol{\theta}/M, D)$ over all parameters in Θ that it depends on [10]. In this regard one of the difficulties of model updating is that the problem is potentially ill-posed, that is, there may be more than one optimal solution. The problem becomes even more challenging when only some degrees of freedom of the model are measured and when modeling errors are explicitly considered. Under certain conditions, ie. global or local identifiable cases, methods based on asymptotic approximations have been successful used in resolving the aforementioned difficulties [8, 9, 10]. For the general case more flexible methods such as simulation-based Bayesian model updating techniques should be used. In particular, an efficient method called transitional Markov chain Monte Carlo is implemented in this work [4]. Validation calculations have shown the effectiveness of this approach in a series of practical Bayesian model updating problems [11]. The method can be applied to a wide range of cases including high-dimensional posterior probability density functions, multimodal distributions, peaked probability density functions, and probability density functions with flat regions.

2.3 Transitional Markov Chain Monte Carlo Method

For completeness the basic ideas of the transitional Markov chain Monte Carlo method are presented in this section. The method iteratively proceeds from the prior to the posterior distribution. It starts with the generation of samples from the prior distribution in order to populate the space in which also the most probable region of the posterior distribution lies. For this purpose a number of intermediate distributions are defined as $p_j(\boldsymbol{\theta}/M, D) = c \ p(\boldsymbol{\theta}/M) p(D/M, \boldsymbol{\theta})^{\alpha_j}$ $j = 0, 1, ..., m,, \ 0 = \alpha_0 < \alpha_1 < ... < \alpha_m = 1$, where the index j denotes the step number, and c is a normalizing constant. The exponent α_j can be interpreted as the percentage of the total information provided by the dynamic data which is incorporated in the j^{th} iteration of the updating procedure. The first step ($j = 0$) corresponds to the prior distribution and in the last stage ($j = m$) the samples are generated from the posterior distribution. The idea is to choose the values of exponents α_j in such a way that the change of the shape between two adjacent intermediate distributions be small. This small change of the shape makes it possible to efficiently obtain samples from $p_{j+1}(\boldsymbol{\theta}/M, D)$ based on the samples from $p_j(\boldsymbol{\theta}/M, D)$. The samples are obtained by generating Markov chains where the lead samples are selected from the distribution $p_j(\boldsymbol{\theta}/M, D)$ by computing their plausibility weights with respect to $p_{j+1}(\boldsymbol{\theta}/M, D)$ which are given by

$$w(\boldsymbol{\theta}_j^k) = \frac{p(\boldsymbol{\theta}_j^k/M)\; p(D/M, \boldsymbol{\theta}_j^k)^{\alpha_{j+1}}}{p(\boldsymbol{\theta}_j^k/M)\; p(D/M, \boldsymbol{\theta}_j^k)^{\alpha_j}} = p(D/M, \boldsymbol{\theta}_j^k)^{\alpha_{j+1}-\alpha_j}$$
$$k = 0, 1, ..., N_j \tag{4}$$

where the upper index $k = 1, ..., N_j$ denotes the sample number in the j^{th} iteration step $(\boldsymbol{\theta}_j^k, k = 1, ..., N_j)$. Each sample of the current stage is generating using the Metropolis-Hastings algorithm [6, 12]. The starting point of the Markov chain is a sample from the previous step that is selected according to the probability equal to its normalized weight $\bar{w}(\boldsymbol{\theta}_j^k) = w(\boldsymbol{\theta}_j^k)/\sum_{l=1}^{N_j} w(\boldsymbol{\theta}_j^l)$, $k = 1, ..., N_j$. The proposal probability density function for the Metropolis-Hastings algorithm is a Gaussian distribution centered at the preceding sample of the chain and with a covariance matrix equal to the scaled version of the estimated covariance matrix of the current intermediate distribution. The previous steps are repeated until $\alpha_j = 1$ is reached (samples generated from the posterior distribution). At the last stage the samples $(\boldsymbol{\theta}_m^k, k = 1, ..., N_m)$ are asymptotically distributed as $p(\boldsymbol{\theta}/M, D)$. For a detailed implementation of the transitional Markov chain Monte Carlo method the reader is referred to [4].

3 SYSTEM RELIABILITY

The updating procedure presented in the previous section is now used to update the system reliability. As previously pointed out, the focus is in stochastic structural dynamical systems where the reliability is defined in terms of a first excursion probability, that is the probability that design conditions are satisfied within a particular reference period. In this context, a failure domain F can be defined as

$$F(M) = \{(\mathbf{z}, \boldsymbol{\theta}) \mid \max_{j=1,...,n_j} \max_{t\in[0,T]} \frac{\mid r_j(t, \mathbf{z}, \boldsymbol{\theta}) \mid}{r_j^*} > 1\} \tag{5}$$

where $\mathbf{z}$ represents the vector of random variables that specifies the stochastic excitation, $[0, T]$ is the time interval of analysis, $r_j(t, \mathbf{z}, \boldsymbol{\theta}), j = 1, ..., n_j$ are the response functions associated with the failure event that defines the failure domain F, and r_j^* is the corresponding critical threshold level. The response functions $r_j(t, \mathbf{z}, \boldsymbol{\theta}), j = 1, ..., n_j$ are obtained from the solution of the equation of motion that characterizes the structural model.

4 UPDATED RELIABILITY

To estimate the failure probability that takes into account the updated information a weighted integral of conditional failure probabilities over the whole parameter space must be evaluated. The weighting function in the integral is the posterior probability density function $p(\boldsymbol{\theta}/M, D)$. Thus the updated failure probability can be written as

$$P(F/M, D) = \int_{\Theta} P(F/M, \boldsymbol{\theta}) p(\boldsymbol{\theta}/M, D) d\boldsymbol{\theta} \tag{6}$$

where $P(F/M,\boldsymbol{\theta})$ is the conditional failure probability, that is the probability using known model parameters $\boldsymbol{\theta}$. Alternatively, Eq.(6) can be re-written in terms of the indicator function as

$$P(F/M,D) = \int_{\Theta}\int_{\Omega_z} \Pi_F(\mathbf{z},\boldsymbol{\theta}/M)\; f(\mathbf{z})\; p(\boldsymbol{\theta}/M,D)\; d\mathbf{z}\; d\boldsymbol{\theta} \tag{7}$$

where $f(\mathbf{z})$ is the probability density function that characterizes the vector of random variables $\mathbf{z}$, and $\Pi_F(\mathbf{z},\boldsymbol{\theta}/M)$ is the indicator function, that is, $\Pi_F(\mathbf{z},\boldsymbol{\theta}/M) = 1$ if $(\mathbf{z},\boldsymbol{\theta}/M) \in F$ and $\Pi_F(\mathbf{z},\boldsymbol{\theta}/M) = 0$ otherwise. The above probability integral usually involve a large number of random variables (hundreds or thousands) in the context of dynamical systems under stochastic excitation. Therefore, this integral represents a high-dimensional reliability problem which is difficult to evaluate. An approach based on the set of samples generated at the last stage of the transitional Markov chain Monte Carlo method is implemented in the present contribution [13].

5 APPLICATION

5.1 Description

The objective of this application problem is to evaluate the effect of the additional information gained about the structure from measured data on the reliability and design of a passive energy dissipation system. For this purpose, the two-story reinforced concrete structure under earthquake loading shown in Figures (1) and (2) is considered. Forty eight columns of square cross section support each floor. The dimension of the columns are equal to 0.57 m and 0.53 m for the first and second floor, respectively. Both floors have a constant height of 3.0 m. The mass of each floor is equal to 1260 ton. The behavior of the reinforced concrete structure is characterized by considering a Young's modulus equal to $E = 2.5\times10^{10}$ N/m^2 and Poisson ratio $\nu = 0.3$. Damping ratios equal to 3% are assuming for the vibrational modes that contribute significantly to the response. This model may be interpreted as the structural model obtained during the design phase. The structure is excited horizontally by a ground acceleration applied at 45° with respect to the axis x. The ground acceleration is modeled as a non-stationary stochastic process. In particular, a point-source model characterized by a moment magnitude and epicentral distance is considered here [15, 16]. The model is a simple, yet a powerful means for simulating ground motions which has been successfully applied in the context of earthquake engineering. The sampling interval and the duration of the excitation are taken equal to $\Delta t = 0.01$ (s) and $T = 20.0$ (s), respectively. Based on this information and according to the excitation model the generation of each ground motion involves more than 2000 random variables. For an improved earthquake performance the structure is reinforced with friction hysteretic devices at each floor. The devices follow the restoring force law $r(t) = k_d\Big(\delta(t) - \gamma^1(t) + \gamma^2(t)\Big)$, where k_d denotes the stiffness of the device, $\delta(t)$ is the relative displacement between floors, and $\gamma^1(t)$ and $\gamma^2(t)$ denote the plastic elongations of the friction device. Using the supplementary variable $s(t) = \delta(t) - \gamma^1(t) + \gamma^2(t)$, the plastic elongations are specified by the nonlinear differential equations [17]

$$\begin{aligned}
\dot{\gamma}^1(t) &= \dot{\delta}(t)H(\dot{\delta}(t))\Big[H(s(t)-s_y)\frac{s(t)-s_y}{s_p-s_y}H(s_p-s(t)) + H(s(t)-s_p)\Big]\ , \\
\dot{\gamma}^2(t) &= -\dot{\delta}(t)H(-\dot{\delta}(t))\Big[H(-s(t)-s_y)\frac{-s(t)-s_y}{s_p-s_y}H(s_p+s(t)) + H(-s(t)-s_p)\Big]
\end{aligned} \tag{8}$$

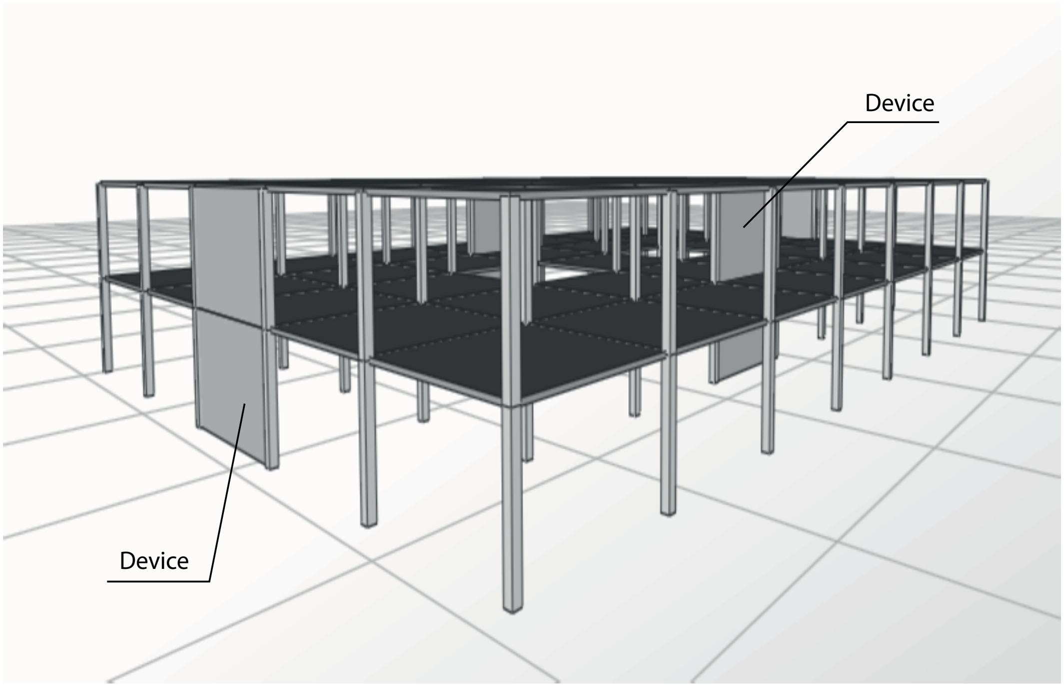

Figure 1: Isometric view of the structural model

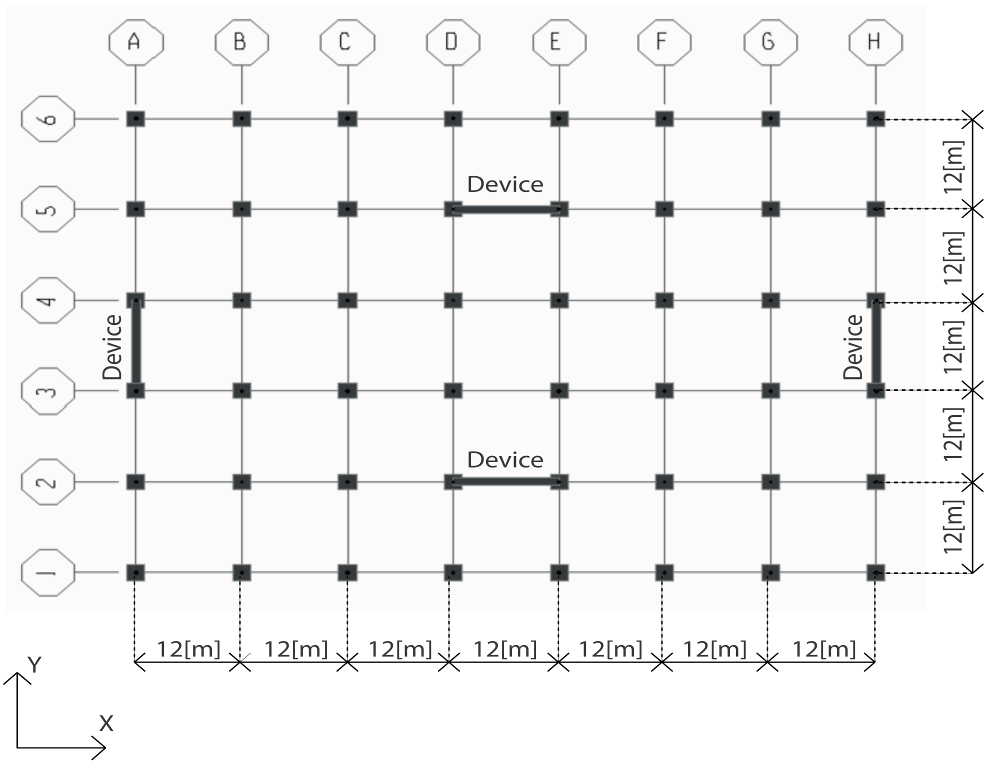

Figure 2: Plan view of the structural model

where $H(\cdot)$ denotes the Heaviside step function, s_y is a parameter specifying the onset of yielding, and $k_d\ s_p$ is the maximum restoring force of the friction device. The values $s_p = 0.006$

m and $s_y = 0.0042$ m are used in this case. Because of the yielding, energy dissipation due to hysteresis is introduced in the structural response.

5.2 Identification Process

For identification purposes, the following class of structural models M is considered. It is assumed that each floor may be represented as rigid within the $x - y$ plane when compared with the flexibility of the columns. Hence, each floor is represented by three degrees of freedom, i.e. two translatory displacements in the direction of the x and y axis, and a rotational displacement. Due to modeling errors the rigidities of the actual class of models are parameterized as: $EI_{1x} = \theta_1 E\bar{I}_{1x}$, $EI_{2x} = \theta_2 E\bar{I}_{2x}$, $EI_{1y} = \theta_3 E\bar{I}_{1y}$, and $EI_{2y} = \theta_4 E\bar{I}_{2y}$, where $\bar{I}_{ix}$ and $\bar{I}_{iy}, i = 1, 2$ represent the nominal moment of inertia of the columns of the i floor in the x and y direction, respectively. These nominal values correspond to the columns of the nominal system previously described. The model updating is based on measurements of the ground acceleration at the base and the absolute acceleration at the first and second floor of the structure. Simulated measured data are used in this application The actual structure used to generate the measured data corresponds to a finite element model with about 5000 degrees of freedom, which includes beam, column and shell elements. The moment of inertia of the columns are defined in terms of the nominal properties as: $I_{1x} = 0.8\bar{I}_{1x}$, $I_{2x} = 0.9\bar{I}_{2x}$, $I_{1y} = 0.9\bar{I}_{1y}$, and $I_{2y} = 0.95\bar{I}_{2y}$. Note that the actual structure does not correspond to any model in the class of models considered. This actual structure may correspond to the structural system already built where changes in the stiffness have occurred due to, for example, large response levels. The input ground acceleration history for model identification is taken as the N-S component of the 2012 Chilean earthquake, Concepcion record shown in Fig. (3) and applied at 45° with respect to the x axis. The measured response is simulated by first calculating the absolute acceleration response of the actual structure at the first and second floor (in the x and y direction) and then adding 15% *RMS* Gaussian white noise. The responses are computed at the center of mass of each floor.

Thirty seconds of data with sampling interval $\Delta t = 0.01$ s are used, given a total of $N_t = 3000$ data points. Independent uniform prior distributions are assumed for the parameters θ_1, θ_2, θ_3, and θ_4 over the range $[0.5, 1.5]$. The transitional Markov chain Monte Carlo method with $N_j = 1000, j = 1, ..., m$ is implemented for the identification process. The four components of the samples corresponding to the rigidity parameters are shown in Fig. (4) in two groups: θ_1 versus θ_2 and θ_3 versus θ_4. The values of the parameters of the nominal system are also indicated in the figure. The posterior samples are concentrated around the reference values $(0.8, 0.9)$ and $(0.9, 0.95)$, respectively, leading to a relatively peaked posterior probability density function. Recall that the reference values correspond to the rigidity of the model used to generate the measure data (actual structure). From Fig. (4) it is also observed that the data is strongly correlated along certain directions in the parameter space. Actually, there is a line of maximum likelihood estimates in the vicinity of the reference values in the $\theta_1 - \theta_2$ and $\theta_3 - \theta_4$ spaces. Note that the lines have negative slopes, which is reasonable since for example an increase in the stiffness of the first floor in the x direction is compensated by a decrease in the stiffness of the second floor in the x direction during the updating process. In other words, all points along that direction correspond to structural models that have almost the same response at the measured degrees of freedom.

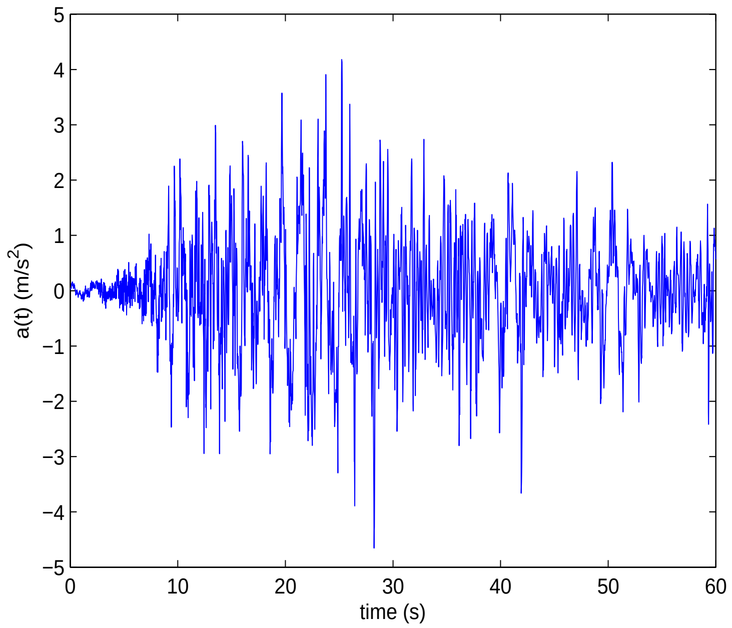

Figure 3: Ground acceleration time history for model identification

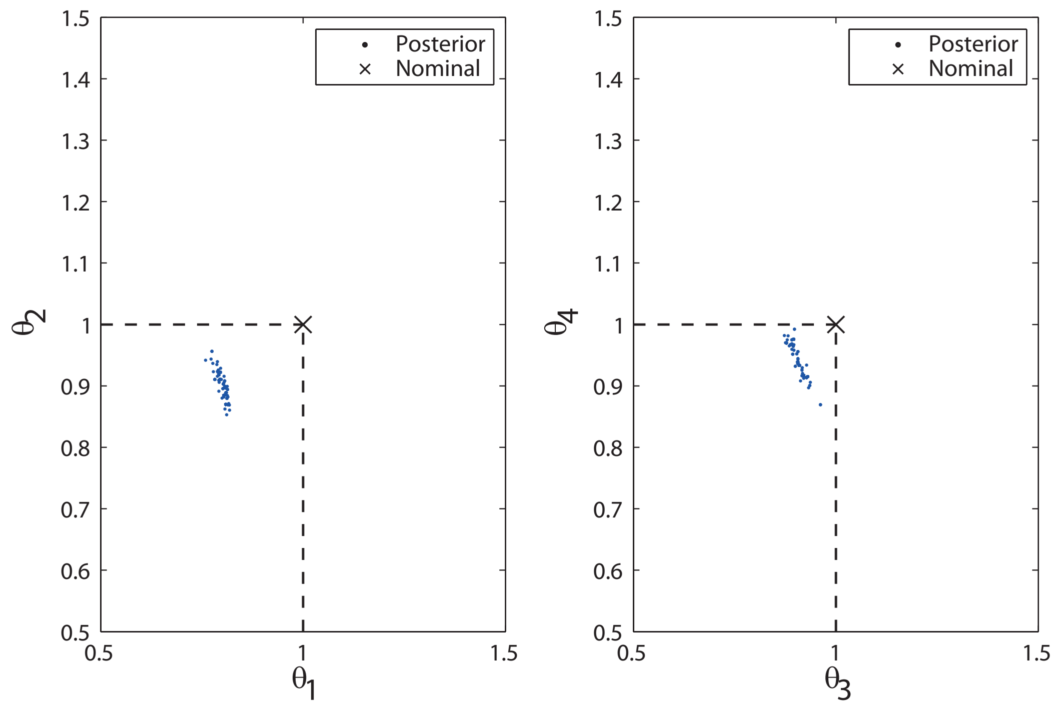

Figure 4: Samples of the rigidity parameters obtained by the transitional Markov chain Monte Carlo method

5.3 System Reliability

To evaluate the reliability of the structural system the following failure domain is considered

$$F(M) = \{(\mathbf{z}, \boldsymbol{\theta}) \mid \max_{j=1,2} \max_{t\in[0,T]}\{\frac{\mid r_{jx}(t,\mathbf{z},\boldsymbol{\theta}) \mid}{r_x^*}, \frac{\mid r_{jy}(t,\mathbf{z},\boldsymbol{\theta}) \mid}{r_y^*}\} > 1\} \tag{9}$$

where $r_{jx}(t,\mathbf{z},\boldsymbol{\theta})$ and $r_{jy}(t,\mathbf{z},\boldsymbol{\theta})$ represent the relative displacement between the $(j-1,j)th$ floor in the x and y direction, respectively, and r_x^* and r_y^* are the corresponding threshold levels. The critical response levels are calibrated such that the probability of failure of the reinforced nominal system is equal to 10^{-4}. The estimation of the failure probabilities represents a high-dimensional reliability problem since more than two thousand random variables are involved in the corresponding multidimensional probability integrals in this case. Figure (5) shows the failure probability of the nominal and updated models in terms of the stiffness of the nonlinear devices. The same properties are assumed for the devices placed in the first and second floor. A range between 0 and $20 \times \bar{k}_d$ is considered in the figure, where $\bar{k}_d$ is a percentage of the stiffness of the first floor in the x direction. In this case a 1% is considered. Note that zero stiffness corresponds to the linear model, i.e. the structural system without the hysteretic devices. It is observed that the probability of failure decreases as the stiffness of the nonlinear devices increases for both cases. This is reasonable since the structural model becomes stiffer when the stiffness increases and at the same time the devices introduce additional energy dissipation capacity to the structural system. In this regard, the use of friction hysteretic devices constitutes an effective control strategy.

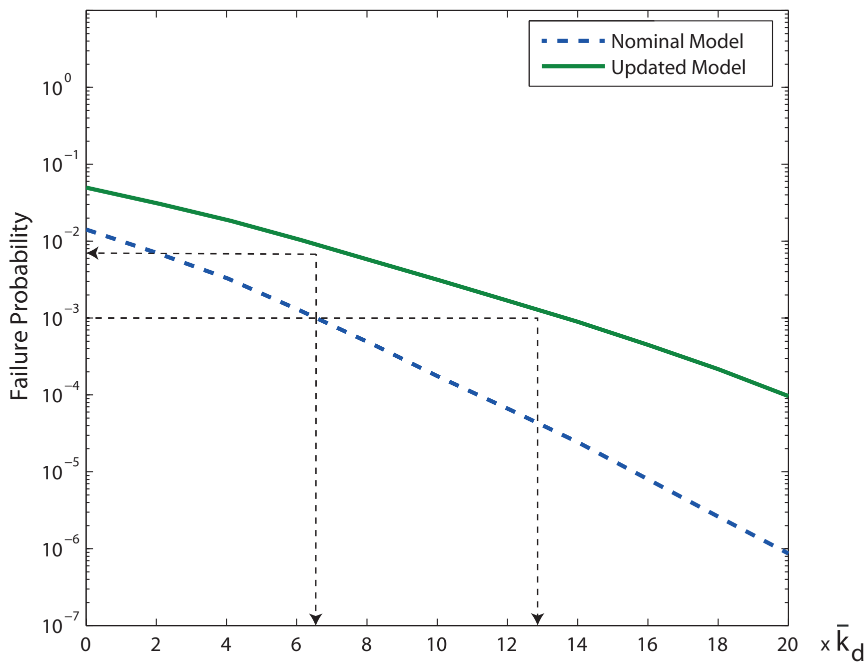

Figure 5: Failure probability of the nominal and updated models in terms of the stiffness of the hysteretic devices

Comparison of the curves obtained by the updated model and the nominal system demonstrates the effect of new information on the system failure probability. The failure probability of the updated model is significantly larger than the corresponding to the nominal system. For example, if a probability of failure equal to 10^{-3} is imposed as a design requirement the stiffness of the devices required in the nominal system is approximately $7 \times \bar{k}_d$ while devices with stiffness equal to $13 \times \bar{k}_d$ are required for the updated system. In other words, the updated structural model reinforced with the devices used in the nominal system is not feasible under the prescribed reliability requirement since the failure probability would be close to 10^{-2} (see Figure 5). Then, the reliability of the structural system computed before and after using dynamic data can differ significantly. Consequently, the effect of the additional information on the design of the hysteretic devices can be considerable.

6 Conclusions

The use of updated reliability measures in the context of stochastic structural dynamical systems has been explored. A simulation-based Bayesian framework for system identification is used to update the structural models using dynamic response data. The updated distribution of the system model parameters is then used to implement a methodology for estimating the system reliability which incorporates knowledge from the test data. Numerical results show that the structural reliability computed before and after using dynamic data can differ significantly. In fact, monitoring data makes an important difference in the design of structural systems. Therefore measured responses, whenever available, should be used.

Acknowledgments

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