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MULTIMODE CHARACTER OF DYNAMICAL SYSTEMS AS A CAUSE OF THEIR COMPLEX ("CHAOTIC") BEHAVIOR

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Abstract. *This study puts under consideration dynamic systems, whose visible complex ("chaotic") behavior is caused by transition from one motion mode to another. Such motion modes may be represented in particular by some stable in the small periodic motions with different periods. Two types of such systems are singled out. In the first case a number of motion regimes with closely located domains of attraction coexist in the phase space of the system at certain values of parameters. The transition from one stable regime of motion to another is due to the inaccuracy of computer calculations and variation of parameters in the corresponding physical system. In the second case a systematic transition from one regime of motion (which is not necessarily stable) to another occurs due to the internal properties of the system. As an example is the situation, when one of the phase variables may be considered as a relatively slowly varying parameter passing through the existence and steadiness domains of various regimes. The behavior of a particle over a horizontal vibrating plane and emergence of a turbulent surface layer in liquid placed in a vibrating vessel are considered in this paper as illustrative examples. The resemblance of this process to that of the thermoconvection is pointed out. The other example is an analogue of the Lorenz oscillator marked by a phase pattern projection similar to the well-known "butterfly" pattern. Some thoughts corroborating Landau turbulence theory are suggested. It is noted that the complex motion under consideration is characteristic for sufficiently wide scope of dynamic systems such as a pendulum with a vibrating axis, the self-synchronizing oscillating and rotating objects, the systems with period doubling, the parametrically excited distributed systems.*

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1 INTRODUCTION

A great many papers and monographs dealing with complex ("chaotic") behavior of dynamic systems followed the well-known Lorenz study [1-5]. Most of them ascribed the complex behavior to the instability of trajectories and to closeness to homoclinic structures. Accordingly, a great number of such rather complex "ways to chaos" was listed.

This paper means to discuss a simpler case of complex motions arising in simple systems. The case under study implies multimode behavior i.e. systematic transition of a system from one mode attraction domain to the other. These modes may represent e.g. some stable periodic motions and are not bound to be a totality of instability (diverging) motions.

After a brief general consideration of two such situation types the examples are given when the complexity of the system behavior is taken to be due to said simple circumstances. Among these instances is obtaining "the Lorenz butterfly" from a simple 3D system and emergence of a turbulent gas-saturated layer during water-filled vessel vibration. The attention is drawn to the similarity of the latter process to that of thermo-convection. It is stated that this variety of complexities (chaos) emergence is likely to occur more frequently than it is customary to believe. Above-described considerations are presumably related to L.D.Landau's theory of turbulence.

The term "chaos" is either omitted or accompanied by inverted commas in this paper. The reason is in that there are several definitions of the chaos which are strict and complex enough to be used. So we follow A.N. Kholmogorov's statement that a complex thing is undiscriminated from the accidental one. But definition of the complexity is even more difficult than the definition of the chaos. So when speaking of complexity we mean some speculative subjunctive notion. It was so do the authors of the majority of numerical investigation of dynamical systems.

2 COMPLEX SYSTEM BEHAVIOR DUE TO MULTIMODE PATTERN OF THE "FLAKY PASTRY" TYPE

Let us imagine that in some domain of the parameters space there exist a few or an infinitely great number of motion types such as stable motions in the sense of Lyapunov or motions of different character, e.g. periodic motions with various periods.

Fig. 1 illustrates schematically the polygonal ABCDE representing the existence and stability domain of three different motions 1, 2, 3 for two parameters a and b The space of parameters may have either one or many dimensions, the number of different simultaneously existing motions being other than three.

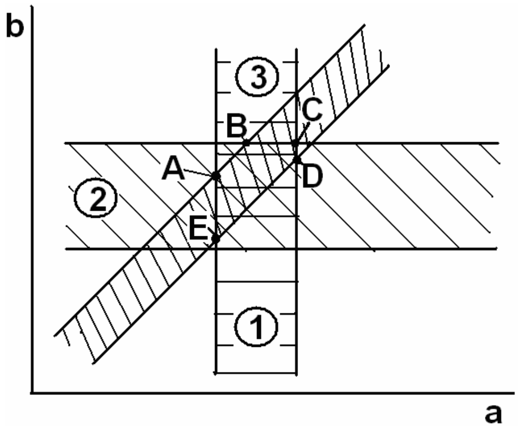

Figure 1: The existence domains of various type motions are superimposed (the case of "flaky pastry").

Let us further assume that in a phase space of the system the modes under study correlate to some attraction domains that are closely spaced or adjacent. Assuming the dimensions of these domains to be similar or less than the domains corresponding to the accuracy of calculations we find that the point reflecting the state of the system is prone to drift in the phase space passing from one attraction domain to the other. As a result of this the trajectories of this point can fill up some space which under numerical investigations may be perceived as a strange attractor.

The examples of such situation are given below.

3 COMPLEX SYSTEM BEHAVIOR DUE TO MULTIMODE PATTERN OF THE "ZEBRA" TYPE

In this case no minuteness of attraction domains or disposition of existence and stability domains in parameter space is presumed. Neither the limited calculation precision is taken into account.

The simplest case of the situation under study is as follows. Suppose there is a system containing some parameter p ,which when changing within a certain range entails the change of the phase pattern, so that the defining point occurs consistent within the attraction domain of various stable motions or even within the domain where all the trajectories either diverge or, if $t \to \infty$, travel to the infinity (see Fig 2).

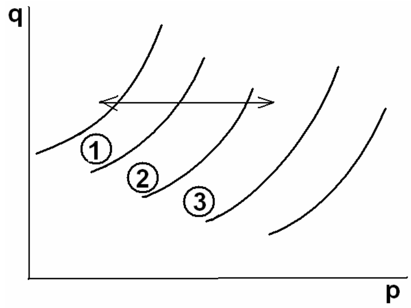

Figure 2: The parameter or the coordinate of a system within the parameter space periodically crosses the existence domains of various motions ("zebra" situation).

Suppose we have designed an additional controlling system where the parameter p is a coordinate varying comparatively slowly within above mentioned range. Then the combined system will eventually pass from one phase pattern to another, staying within for some finite time. It would be natural to expect that after a sufficiently long time interval the system might trace within the phase space a sufficiently complex trajectory, filling up some domain which under numerical investigation may be perceived as a strange attractor. It is safe to suppose that the complexity of this trajectory might increase when the accuracy of calculations, like in the first case, is limited.

The examples of such situation are given below.

Let us describe this situation as "zebra" type multimode behavior.

4 REMARKS

1) The above considerations about the multimode as the cause of the complexity of the movement close to the concepts of amplifiers stochastic in the book [1]

2) One can come across systems combining both of the multimode patterns considered above.

3) Following A.A. Andronov's terminology one can define multimode situation as a rough case of complexity ("chaos").

4) The trajectory of the system in the phase space might appear comparatively simple while its projection on the plane seems to be rather complicated ("chaotic"). By contrast, the trajectory might appear to be chaotic while its projection on some plane would be simple. The same relates to Poincare sections.

5 EXAMPLES

5.1 Lorenz oscillator and its simplest analogue. The "Butterfly effect"

Let us investigate the Lorenz equation system of the following form [1]

$$\ddot{\xi}+(\eta-1)\xi+\xi^3+\mu\dot{\xi}=0, \quad \dot{\eta}=-\frac{\mu}{\sigma+1}\left[b\eta-(2\sigma-b)\xi^2\right], \tag{1}$$

where μ,σ and b –are positive parameters, parameter μ being a small one. If $\eta=const$, then the first equation of the system (1) represents the Duffing equation. When $\eta-1>0$ it acquires one asymptotic stable equilibrium position $\xi_1=0$, while with $\eta-1<0$ –it acquires two such positions $\xi_{2-3}=\pm\sqrt{1-\eta}$.

Now what is to happen to the system when the quantity $\eta-1$ is slowly and periodically varying as e.g. following the order:

$$\eta-1=k^2(\sin\omega t-a), \quad 0<a<1, \tag{2}$$

where ω significantly less than k? It is evident that in this case the system with time intervals $t>>2\pi/\omega$ will intersect bifurcation point $\eta-1=0$ many times, passing from $\eta-1>0$ situation with only one asymptotic stable equilibrium position $\xi_1=0$ exist to the situation when $\eta-1<0$ with two such positions $\xi_{2-3}=\pm k\sqrt{1-\eta}$ occurs.

Hence, one may suppose that the motion described by the equation

$$\ddot{\xi}+k^2\xi(\sin\omega t-a)+\xi^3+\mu\dot{\xi}=0, \tag{3}$$

with ω significantly less than k? and changing time interval t sufficiently long, is likely to reveal complex behavior.

Fig.3 shows the trajectory corresponding in the equation (3) to the following value of the parameters:

$$k=10, \quad \omega=4, \quad \mu=0.1, \quad a=0.8$$

with initial conditions: $\xi(0)=5, \quad \dot{\xi}(0)=0$

One can see that the motion of this simple system is of a sufficiently complex character, identical to the trajectory of the well-known Lorenz oscillator (1), the projection of the phase trajectory being similar to the well-known "butterfly" pattern.

Similar properties are peculiar to the system where the function $\eta-1$ is a "control variable" defined by the following autonomous linear equation:

$$(\eta-1)^{\cdot\cdot}+\omega^2(\eta-1)=-k^2a$$

One can find similar considerations and examples in [7, 8].

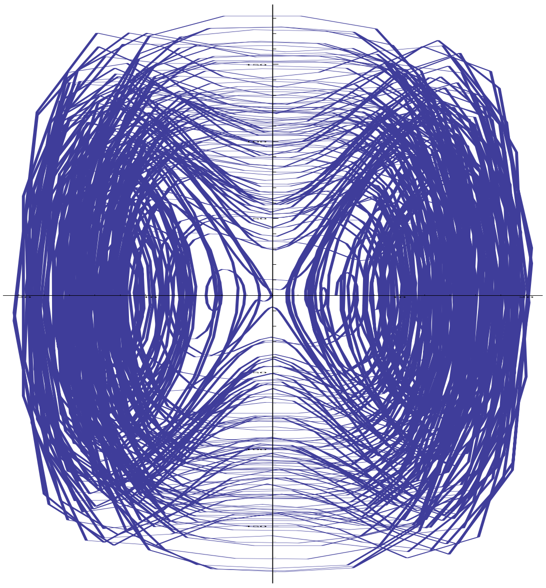

Figure 3: Phase pattern of the system.

5.2 Particle motion over a vertical vibrating plane

Let us consider the motion of a particle bounding over a horizontal plane that is vibrating vertically with amplitude A and frequency ω according to the law (Fig. 4)

$$\eta = A\sin \omega t \tag{4}$$

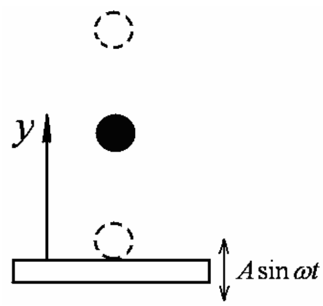

Figure 4: A particle bouncing over a horizontal vibrating plane.

The particle motion character is defined by two non-dimensional parameters, namely the coefficient of overload $w = A\omega^2 / g$ and coefficient of restitution R. It was shown by analytical investigation [9, 10] that on condition

$$\pi p \frac{1-R}{1+R} < w < \frac{\sqrt{\pi p^2 (1-R^2)(1+R^2) + 4(1+R^2)^2}}{(1+R^2)} \tag{5}$$

$$p = 1, 2, \ldots$$

there exist some stable in small (in Lyapunov's sense) periodic motions (regimes) of a particle when the latter keeps to be positioned over a vibrating plane and is colliding with the same with a period $T = pT_0 = 2\pi p/\omega$, that is a multiple in relation to the plane vibration period T_0. The domains of existence and stability for such regimes are shown in Fig. 5 (shaded).

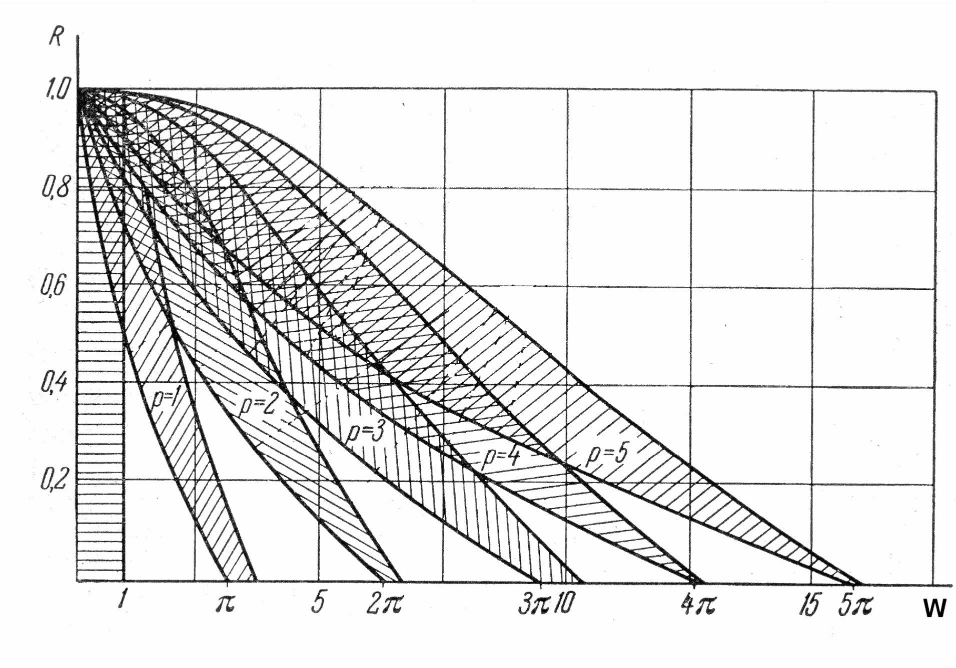

Figure 5: The domains of the existence and stability of periodic motions of a particle over vibrating plane.

As it is seen, with values R relatively close to 1 there exists a great (rather infinitely great) number of regimes to be considered while the domains of their existence and stability in parameter space are superposed. Each of periodic regimes corresponds to a certain sufficiently small domain of attraction within the phase space [9]. So we have the situation of “flaky pastry”.

In Fig. 6 phase diagram of a particle behavior on a plane $y, \dot{y}$ corresponding to different values of parameter R are represented

One can see that with $R = 0$ we have periodic motion of the particle, while with $R = 0.95$ – there exists a complex motion, the particle bouncing over the plane by various heights $y_{\max}$. In this case the complexity of the motion increases in accordance with the increase of overload parameter w.

It is to be noted that such multimode behavior and corresponding motion complexity with $R \approx 1$ has been found in other impact - vibration systems including the known Fermi oscillator [11].

The other feature of the system under study is systematic appearance of “fountains”, i.e. bouncing of the particle to rather great height (on Fig.6 *b* and 6 *c* conditions – to 1.7 m and 9.5 m accordingly). Similar splashes (fountains) can be observed in behavior of liquid in a vibrating vessel (see item 5.3 below). This is often referred to as intermittence in transition to chaos [1, 2, 5].

Let us emphasize that the above regularities characteristic not only of the mathematical models, but also to the physical systems.

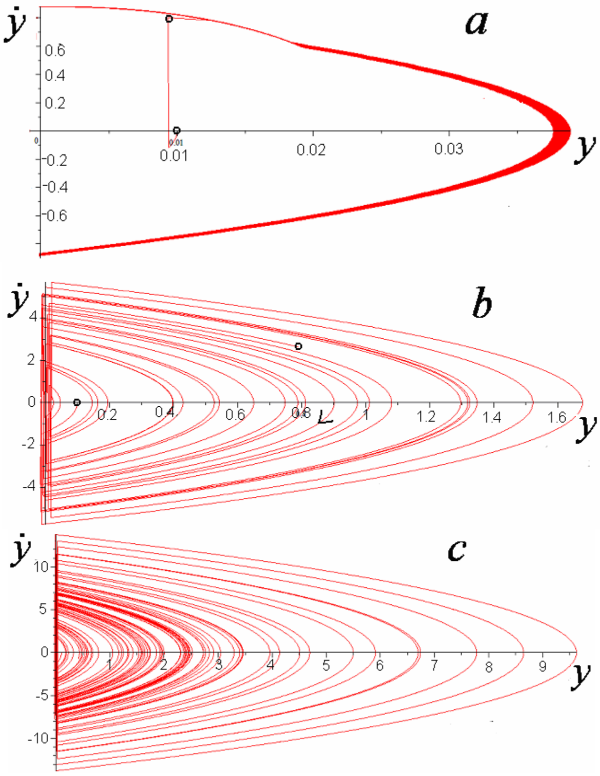

Figure 6: Particle over a vibrating plane. Periodic motion on a phase plane:
$a-\ R=0,\ A=0.025\,m,\ \omega=35\,s^{-1},\ w=\pi;\ \ b-\ R=0.95,\ A=0.016\,m,\ \omega=35\,s^{-1},\ w=2;$
$c-\ R=0.095,\ A=0.024\,m,\ \omega=35\,s^{-1},\ w=3.$

5.3 Liquid in a vibrating vessel. Cellular structures, fountain formation. Generation of turbulent gas-saturated layer. Resemblance to thermoconvection

In our experiments [12] a cylindrical vessel of 60 mm diameter and 450 mm height was filled with water to 20, 50 and 100 mm levels. Vertical vibrations with 1 mm amplitude were imparted to the vessel, the frequencies ω increased from 75 to 240 *rad/s*.

With relatively low vibration frequencies ($\approx$ up to 5 *rad/s*) the liquid surface remained horizontal. The further increase in frequencies resulted in emergence of standing waves (“cellular structures”). Cell dimensions tended to gradually decrease with the increase in ω so that at 146 *rad/s* frequency a gas-saturated turbulent layer of a gradually growing thickness was formed (for more details see [12]). The cells (standing waves) were found by theoretical analysis to correspond by their frequencies to a half of the liquid surface free oscillations in the gravity field λ, thus representing none other than Faraday ripple. As for cell dimensions, they were in correspondence with the dimensions of the free oscillation half waves and only in slight dependence on the volume of water in the vessel.

The other experiments were carried out with a rectangular box of 330 x 245 mm dimensions, shown in Fig.7. One can see clearly visible standing waves forming the cellular structure.

Figure 7: Box with fluid under vibration; cellular structures.

The frequencies $\lambda = \lambda_{n,m}$ of the liquid surface free oscillations in this box may be determined by the formula

$$\lambda_{n,m} = \sqrt{\pi g \sqrt{\left(\frac{n}{a}\right)^2 + \left(\frac{m}{b}\right)^2} \tan\left(\pi h_1 \sqrt{\left(\frac{n}{a}\right)^2 + \left(\frac{m}{b}\right)^2}\right)}, \quad (6)$$

$$n, m = 1, 2, \ldots$$

where *a* and *b* are the box dimensions, h_1 – liquid layer thickness, g - gravity acceleration. Here the number of the cells corresponding to the frequencies $\nu_{n,m} = \frac{1}{2}\lambda_{n,m}$ of the main parametric resonance was $N_{n,m} = \frac{1}{2}(m-1)(n-1)$, while the areas of the cells were equal to $F_{n,m} = 2ab/(m-1)(n-1)$. One can see that the cell dimensions tended to gradually decrease following the increase in the oscillation frequency ν.

The above table shows frequencies λ (*rad/s*) for the test box with water layer height $h_1 = 20$ mm. One can see that with greater n and m values the frequency values get denser. It is to be noted that the parametric resonance frequencies $\omega = 2\lambda_{n,m}$ are in correlation to some close values of *n, m*. Thus, for instance, at frequency ω= 50 *rad/s* and $\lambda = \omega/2 = 25$ *rad/s* the difference ± 1 *rad/s* has six frequencies which corresponds to the main parametric resonance.

m	n									
	1	2	3	4	5	6	7	8	9	10
1	4.98	7.13	9.65	12.26	14.86	17.39	19.83	22.16	24.38	26.49
2	8.46	9.84	11.74	13.91	16.18	18.47	20.73	22.91	25.02	27.03
3	12.10	13.05	14.47	16.20	18.10	20.08	22.09	24.08	26.02	27.90
4	15.66	16.35	17.43	18.80	20.37	22.05	23.79	25.56	27.30	29.02
5	19.05	19.58	20.42	21.51	22.79	24.21	25.70	27.24	28.79	30.33
6	22.23	22.65	23.32	24.20	25.25	26.44	27.71	29.04	30.40	31.77
7	25.21	25.54	26.08	26.80	27.67	28.66	29.75	30.90	32.09	33.30
8	27.97	28.24	28.68	29.28	30.00	30.84	31.76	32.75	33.79	34.86
9	30.53	30.75	31.12	31.61	32.22	32.93	33.72	34.58	35.49	36.43
10	32.90	33.09	33.39	33.81	34.33	34.93	35.61	36.36	37.16	37.99

Table: Frequencies (*rad/s*) of liquid free oscillations in the rectangular box

Fig 8. illustrates schematically the domains of parametric instability.

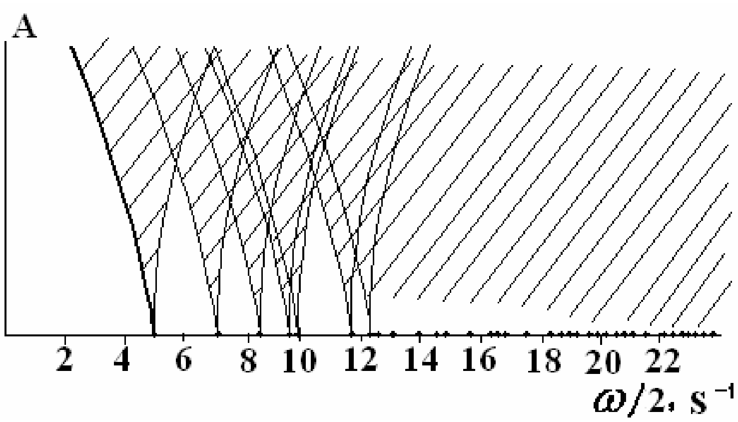

Figure 8: The accumulation and overlap of the instability domains corresponding to the main parametric resonance $\omega/2=\lambda_{n,m}$ (Faraday ripple) in the conditions of Table (shaded).

Each of these domains by presence with some nonlinearity and dissipation is in correlation with a certain steady periodical mode of oscillations, i.e. a certain cellular structure. With relatively great values of ω the domains of parametric resonance get denser and with sufficiently great amplitude *A* they overlap. Thus, the frequency range up to 10 *rad/s* contains 5 frequencies $\lambda_{n,m}$, the range within 10-20 *rad/s* contains 19 frequencies and the range within 20-30 *rad/s* includes 47 frequency values. This causes a complex (“chaotic”) motion representing the case of the “flaky pastry”.

One may pay attention to the resemblance of the above described picture to the phenomena taking place in thermoconvection when so called Benar’s cells are generated. In our case the part of temperature is played by the oscillation intensity. Such resemblance appears to be not casual. As for the theoretical presentation of the thermoconvection by means of Lorenz equations, which suggest quite another way to the chaos (“zebra”, see it.5.1), it is to be taken into account that these equations correspond to a double mode approach to the solution of corresponding partial differential equations. It is to be emphasized that the similarity of oscillatory and thermal influence on the mechanical systems was mentioned in a number of researches (e.g. [13]).

Generation of a gas-saturated turbulent layer during vibration is essential for development of a number of non-linear phenomena, in particular, the submergence of bubbles in a vessel. As far as we know no theoretical explanation of such phenomenon has been given up to now.

Further on it is to be noted that after generation of the turbulent layer some splashes (fountains) appear on its surface. With 23 Hz frequency the height of such splashes was 40-50 mm at about 5 s intervals. Close to the top of the splashes the liquid was broken into separate drops. At 25 Hz frequency the period of splash generation gets shorter – down to about 1 s while the height reached 70 mm. Further increase of oscillation frequency from 28.3 Hz to 38.3 Hz caused simultaneous emergence of a group of splashes reaching maximum height of 110 mm.

As mentioned above the generation of splashes whose height largely exceeds the oscillation amplitude is also characteristic for the behavior of particle over a vibrating plane. It may be of interest that in both cases the splashes are generated at close vibration acceleration amplitudes about $A\omega^2 \approx (3 \div 4)g$.

5.4 Other systems possessing the complex motion due to multimode behavior

Complex motion caused by the multimode situation has been observed to take place in a number of other dynamic systems. Some of these systems are listed below.

5.4.1 A pendulum (or an unbalanced rotor) with a vibrating suspension axis

Under certain conditions the pendulum or an unbalanced rotor reveal a complex ("chaotic") behavior stipulated by multimode situation. The pendulum (rotor) in the process of such motions may transit from vibrations to rotation in various directions [1, 15, 16]. In this case both "zebra" and "flaky pastry" situations are possible.

5.4.2 Synchronization of oscillations and rotations

Multimode situation is characteristic also for systems with self-synchronizing or external synchronized objects, in particular for unbalanced rotors, celestial bodies, pendulum clocks and multi-form oscillators. Such behavior is especially well pronounced in the case of similar objects numbering over three [15, 16].

In this case two kinds of multimode situation are also possible.

5.4.3 Dynamic planes

A flat surface plane formed by two systems of vibrating bars is implied. When vibrating , one system of bars penetrates into another. A particle sufficiently coarse during one part of vibration period will be lying on (over) one system of bars and during the next period part – on the other one. In this case two situations are also likely to occur [17].

5.4.4 Double period system

Systems with bifurcations of period redoubling are often taken into consideration as the models of a large group of dynamic systems marked by transition to chaos. (such as in [1-6]). In such systems any increase in the parameter leads to bifurcations resulting in a stable in the small periodic motion of double period. After a great number of such bifurcations even with the same parameter value there appears a large number of periodic motions with very small attraction domains, i.e. a “zebra’ type multimode situation takes place.

5.4.5 Arrhythmia of the cardiac systole

A presumption has been made [15, 18] that heart arrhythmia is likely to be explained by the multimode situation.

5.4.6 Parametrical excitation of distributed systems

In elastic bodies like plates and shells the domains of parametric resonance overlap (refer to [20]). Each mode of parametric resonance on condition of possessing non-linearity and dissipation is in correlation to a steady periodic motion. The existence and stability domains of such motions are overlapping (see also item 5.3). In this case “flaky pastry” situation takes place.

5.4.7 Turbulence

Now turning to the formation of turbulent layer in a vibrating water filled vessel (see 5.3) one can suggest that the turbulent motion of liquid should be understood as a simultaneous co-existence of a multitude of stable in the small periodic motions (in this case self-sustained oscillations) Such interpretation is in accordance with the renown Landau turbulence theory which has been put lately to some doubts [1, 2] (supposedly with no reason).

6 CONCLUSION

The present work deals with a relatively simple (“rough”) case study of arising complex motions in dynamic systems. This variety needs no necessity to seek other instability causes because the complex motions can be explained by the presence of several or many stable in the small motions (multimode situation).

Such multimode situation appears to be sufficient for generation of the complex motion in the dynamic system which is likely to be observed as a chaotic one.

Two varieties of such multimode situation referred to as “flaky pastry” and “zebra” have been studied. The formation of turbulent layer in a vibrating water filled vessel has been studied both experimentally and theoretically. The chaotic motion appeared to take place by way of cellular structures (Faraday ripple), the cells diminishing in size with the increase of vibration frequency – this is the “flaky pastry” type situation. It appears to be not accidental that this process is similar to thermoconvection.

It has been shown that the complexity of Lorenz double-mode thermoconvection model may be explained by the multimode situation of “zebra” type. A simple analogue of the Lorenz system has been constructed to prove the emergence of such multimode behavior , the phase pattern obtained resembling the well-known Lorenz butterfly.

The inferences set forth above are in agreement with L.D.Landau turbulence theory.

The instances considered above are thought to illustrate that such variety of complexity in dynamic system behavior (“chaos”) is likely to occur much more frequently than it is thought commonly.

REFERENCES

[1] Yu.I. Neimark, P.S. Landa, *Stochastic and chaotic oscillations, 2nd Edition*. LIBROCOM book house, 2009. (In Russian).

[2] D.I. Trubetskov, *Intruduction into synergetics. Chaos and structures. 4th Edition*. LIBROCOM book house, 2012. (In Russian).

[3] A.L. Fradkov, *Cybernetical physics: principles and practice*. St. Petersburg: Nauka, 2003. (In Russian).

[4] Y. Ueda, *The Road to Chaos – II*. Santa Cruz: Aerial Press, Inc., 2001.

[5] G.A. Leonov, *Strange attractors and classical theory of motion stability*. St. Peterburg: St. Peterburg State University, 2004. (In Russian).

[6] M.V. Zakrczevsky, R.S. Smirnova, I.T. Shchukin, et al, *Non-linear dynamics and chaos.* Riga: Riga Technical University, 2012. (In Russian).

[7] A.Yu. Loskutov, *Non-Feedback Controlling Complex Behavior: An Analytic Approach. Nonlinear Dynamics: New Theoretical and Applied Results*. Berlin: Akademic Verlag GmbH., 126-150, 1995.

[8] I. Kunin, B. Kunin, G. Chernykh. Lorenz – type controlled pendulum. *Inter. Journ. of Engineering Science*, 433-448, 2003.

[9] I.I. Blekhman., G.Yu. Dzhanelidze, *Vibrational Displacement*. Moscow: Nauka Press, 1964. (In Russian).

[10] I.I. Blekhman., K.S. Ivanov, On a vibroimpact system in wich chaos is caused by the rare attractor packing. *Proceedings of the XXXIX International Summer School Conference (APM 2011*), St. Petersburg (Repino), July 1-5, 2011. СПб., 95–100, 2011.

[11] Yu. Guo, A.C.J. Luo, Discontinuity and bifurcation analysis of motions in a Fermi oscillator under dual excitations. *Journ. of Vibroengineering*, **13**, Iss. 1, 66-101, 2011.

[12] I.I. Blekhman, L.I. Blekhman, V.S. Sorokin, V.B. Vasilkov, K.S. Yakimova, Surface and volumetric effects in a fluid subjected to high-frequency vibration. *Journal of Mechanical Engineering Science*, **226(8)**, 2028-2043, 2012.

[13] A.K. Belyaev, V.A. Palmov, Vibration conductance theory. *Dynamics and strength issues*, Riga: : Zinatne, **36**, 93–102, 1980. (In Russian).

[14] V.S. Batalova, G.V. Belyakova, Periodic motion stability diagrams of a pendulum with a vibrating axis. *Applied mathematics and mechanics*, **52**, Iss. 1, 41-48, 1988. (In Russian).

[15] I.I. Blekhman, *Synchronization in Science and Technology*. New York: ASME Press, 1988.

[16] I.I. Blekhman, *Vibrational Mechanics. (Nonlinear Dynamic Effects, General Approach, Applications).* Singapore et al: World Scientific Publishing Co., 2000.

[17] I.I. Blekhman, G.B. Bukaty, The motion of the body which is in contact in turn with two vibrating planes. *Izv. AN SSSR, MTT*, 2, 39–49, 1975. (In Russian).

[18] I.I. Blekhman, Non-linear oscillatory phenomena in functioning of cardiovascular system. *Proceedings of the 6-th ESGCO 2010, April 12-14*, Berlin, Germany, 2010.

[19] E. Surovyatkina, Nonlinear origin of sudden change of heart rhythm. *Proceedings of the 6-th ESGCO 2010, April 12-14*, Berlin, Germany, 2010.

[20] *Vibrations in engineering.* Reference book, **1**, Moscow: Machinostroyeniye, 1978. (In Russian).

[21] L.D. Landau. On Turbulence Problem. *USSR Academy of Sciences Transactions*, **XLIV**, 339-342, 1944. (In Russian).