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More versatility for an integration step size enlargement technique in time integration analysis

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Abstract. *A recent technique materializes time integration with steps larger than the steps by which the excitations are digitized. The ratio of integration steps to the digitization steps need to be an integer. By considering linear changes for excitations, between the excitation stations, this paper introduces a generalization, such that the technique becomes applicable when the above ratio can be stated, as a rational number, with a numerator arbitrary greater than the denominator. With a negligble additional computational cost, the proposed generalization entails more efficiency, and, hence, is beneficial in practical time history analysis of structural systems againt strong motions.*

1 INTRODUCTION

Time integration is a versatile tool to analyze semi-discretized equations of motion [1, 2]. In analysis of seismic structural behaviors, the governing equation is as noted below [1, 3-7]:

$$\begin{aligned}
&\mathbf{M}\ddot{\mathbf{u}}(t)+\mathbf{f}_{\mathbf{int}}(t)=\mathbf{f}(t) \qquad\qquad 0\le t<t_{\mathrm{end}}\\
&\text{Initial Conditions:}\ \left|\begin{array}{l}\mathbf{u}(t=t_0)=\mathbf{u}_0\\ \dot{\mathbf{u}}(t=t_0)=\dot{\mathbf{u}}_0\\ \mathbf{f}_{\mathrm{int}}(t=t_0)=\mathbf{f}_{\mathrm{int}_0}\end{array}\right.\\
&\text{Restraining conditions: } \mathbf{Q}\\
&\mathbf{f}(t)=-\mathbf{M}\begin{bmatrix}\alpha_1\\ \alpha_2\\ \vdots\\ \alpha_n\end{bmatrix}\ddot{u}_g
\end{aligned} \tag{1}$$

where, t and t_{end} imply the time and the duration of the dynamic behavior; $\mathbf{M}$ is the mass matrix; $\mathbf{f}_{\mathrm{int}}$ and $\mathbf{f}(t)$ stand for the vectors of internal force and excitation; $\mathbf{u}$(t), $\dot{\mathbf{u}}$(t), and $\ddot{\mathbf{u}}$(t) denote the vectors of displacement, velocity, and acceleration; $\mathbf{u}_0$, $\dot{\mathbf{u}}_0$ and $\mathbf{f}_{\mathrm{int}_0}$ define the initial status of the model (see [8]); $\mathbf{Q}$ represents some restricting conditions, e.g. additional constraints in problems involved in impact or elastic–plastic behavior [9, 10], and $\ddot{u}_g$ introduces the ground acceleration, available as a digitized record [1, 3, 4, 11], and generally

$$\alpha_1=\alpha_2=\ldots\ldots\alpha_n=1 \tag{2}$$

The size of the digitization, ${}_f\Delta t$, being dependent on the recording facilities [1, 3, 11], may differ from the size of integration steps needed for accuracy. Regarding the latter, the versatility of time integration is provided in the price of some computational error, and considerable computational cost [1, 3, 12]; both the error and the cost mainly depend on the algorithmic parameter [13], i.e. the integration step size, Δt [12]; however, the effects are adverse [1, 3, 12, 14]. This highlights the importance of assigning appropriate values to Δt, leading to:

$$\begin{aligned}
&\Delta t=\mathrm{Min}\ (\frac{T}{10},\ {}_s\Delta t) \qquad \text{for linear systems}\\
&\Delta t=\mathrm{Min}\ (\frac{T}{100},\ {}_s\Delta t) \qquad \text{for nonlinear systems}
\end{aligned} \tag{3}$$

as a broadly accepted comment for selecting the integration step size [3, 14, 15]. In seismic time integration analyses, the difference between ${}_f\Delta t$ and the right hand side of Eqs. (3) (addressed above), enforces consideration of ${}_f\Delta t$ as an additional term, after the ${}_s\Delta t$, in Eq. (3). Nevertheless such a selection of integration step size, specifically, when

$$\begin{aligned}
&{}_f\Delta t<\mathrm{Min}(\frac{T}{10},\ {}_s\Delta t) \qquad \text{for linear systems}\\
&{}_f\Delta t<\mathrm{Min}(\frac{T}{100},\ {}_s\Delta t) \qquad \text{for nonlinear systems}
\end{aligned} \tag{4}$$

results in additional computational cost. Recently a technique is proposed to reduce this additional cost, by replacing the excitation with an excitation, with n times larger digitization steps, where, n is a positive integer, greater than one [16]. The corresponding reduction of computational cost, A_c, is not more than

$$A_c \cong \frac{n-1}{n} \times 100 \quad \% \tag{5}$$

However, since n is a positive integer, the computational cost may be still considerably more than the computational cost of the corresponding conventional analysis (that with steps satisfying Eqs. (3), even when leading to integration steps larger than ${}_f\Delta t$). To say better, in view of the cost reduction values, stated in Table 1 (resulted from Eq. (5)), when, for an integer n

$$\begin{aligned} &n\ {}_f\Delta t < \mathrm{Min}\left(\frac{T}{10},\ {}_s\Delta t\right) < (n+1)\ {}_f\Delta t \qquad \text{for linear analyses} \\ &n\ {}_f\Delta t < \mathrm{Min}\left(\frac{T}{100},\ {}_s\Delta t\right) < (n+1)\ {}_f\Delta t \qquad \text{for linear analyses} \end{aligned} \tag{6}$$

$\frac{\Delta t}{{}_f\Delta t}$	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	...
A_c	50	66.6	75	80	83.3	85.7	87.5	89	90	91	92	92	93	93	94	94	94	...

Table1: Reduction of computation cost by considering different values of n in implementation of the recent technique (%).

we can analyze with step equal to $n\ {}_f\Delta t$, and save the computational cost according to Eq. (5). However, since n is an integer, some considerable additional computational cost cannot be eliminated (specifically $\cong 50\%$ when the right hand side of Eqs. (3) is almost equal but less than twice the digitization step size; see the minimum 50 % in Table 1). In view of the considerable computational cost of real seismic analysis, and hence the practical significance of the computational cost (specifically for smaller values of n, for which, according to Table 1, the steps of cost reductions are large), the objective in this paper is to overcome this shortcoming and arrive at more continuous changes of A_c in Table 1.

2 THEORY

Convergence is both the first essentiality in general numerical computation [17-19], and also the key basis in the recently proposed technique [16]. In numerical investigation of convergence and order of accuracy, for problems subjected to digitized excitations, e.g. equations of motion subjected to digitized earthquake records (Eqs. (1)), it is conventional to implement linear interpolation of the digitized records in analyses with smaller steps [12, 19-21]. In view of this idea, we can convert an earthquake record digitized at steps equal to Δt, to a record digitized at smaller steps, by linear interpolation, and expect no loss of accuracy in time integration analysis (compared to the exact responses); though in the price of more computational cost. Considering the new record, as an original record, we can implement the recent technique [16], to arrive at a record with the larger steps to be used in time integration, with reduced computational cost. Since in view of the existing experiences, the recent technique is successful [16, 22-33], the loss of accuracy is trivial. In other words, by linear interpolation, the original excitation (with digitization steps equal to ${}_f\Delta t$) changes to a new record, digitized at smaller steps equal to $\frac{{}_f\Delta t}{q}$ ($q = 2,3,4,\ldots$), and then, the resulting record can be converted to a record digitized at steps $\frac{{}_f\Delta t}{q}p$ ($p = 2,3,4,\ldots$, $q = 2,3,4,\ldots$), by the recent technique [16]. In consequence, while in implementation of the recent technique, the integration steps can be enlarged by a positive integer greater than one, the integration steps can now be enlarged by a positive rational number (ratio of two positive numbers) greater than one ($p > q$; see also Figure 1), and the reductions of computational cost, i.e.

$$A_c \cong< \frac{p-q}{p} \times 100\% \tag{7}$$

would neither change discontinuously nor would be lower-bounded by 50 % (see Table 1), e.g. consider the case $p = 3$, $q = 2$, leading to $A_c = 33\ \%$. It is also worth noting that the computational cost of implementing the procedure above is negligible considering the fact that while time integration and specifically implicit time integration analyses are involved in matrix computations depending on the number of degrees of freedom, the computation proposed above is simple and regardless of the number of degrees of freedom. Changing an earthquake record according to the explanation above implies a new technique, or indeed a simple generalization of the recent technique, introduced here, for the first time. Inheriting from the recent technique [16, 22-33], the generalization is independent of the structural system, and integration method, and considers earthquake records digitized at equal steps, as representative of originally smooth phenomena.

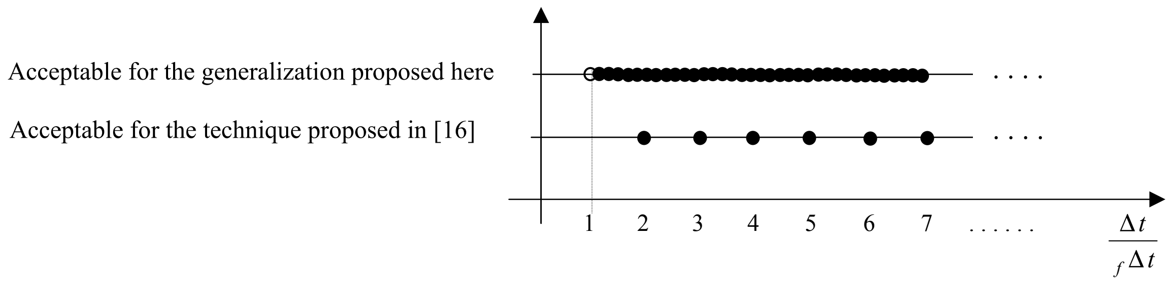


Figure 1: Acceptable values for the ratio of integration step size to the earthquake record digitization step size.

3 NUMERICAL ILLUSTRATION

Consider the single degree of freedom system below:

$$\ddot{u} + 0.2\dot{u} + 25u = -\ddot{u}_g \tag{8}$$

where, $\ddot{u}_g$ is stated in Fig. 2. Implementing the average acceleration method of Newmark [34] for time integration, Figs. 3 and 4, and Table 2, display the performance of the technique discussed in [16, 22-33] and the generalization proposed here for the problem introduced in Eq. (8) and Fig. 2. Clearly the results, and specifically the close resemblance of the responses in Figs. 3(b-d) and Figs. 4(a-d) to that in Fig. 3(a), evidence the adequacy of the provided capability. To explain better, while before the generalization, we could reduce the cost 50% or 67% or 75%, . . as apparent in Table 2 and Figs. 4, the intermediate reductions is now achievable. In this regard, specially, cost reductions less than 50 % is practically notable. A complicated example is studied next.

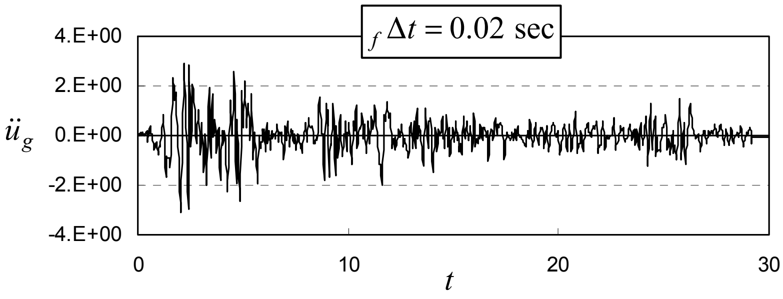


Figure 2: The ground acceleration applied to the structural system in the first example.

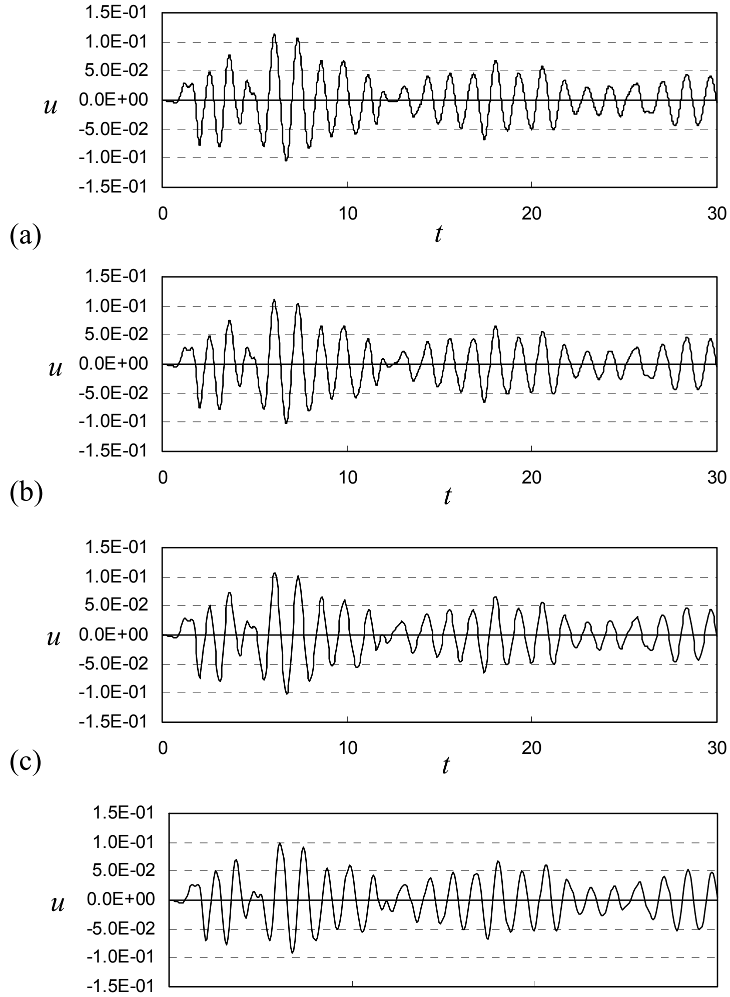


Figure 3: Responses obtained from average acceleration analysis of the problem introduced in Eq. (8) and Fig. 2: (a) ordinary analysis ($n=1$), (b) implementing the recent technique [16] with $n=2$, (c) implementing the recent technique [16] with $n=3$, (d) implementing the recent technique [16] with $n=4$.

Figure	3(a)	3(b)	3(c)	3(d)	4(a)	4(b)	4(c)	4(d)
Reduction	Basis	50	67	75	33	60	71	78

Table2: Reduction of computation cost (%) in different analyses of the problem introduced in Eq. (8) and Fig. 2, by the average acceleration method.

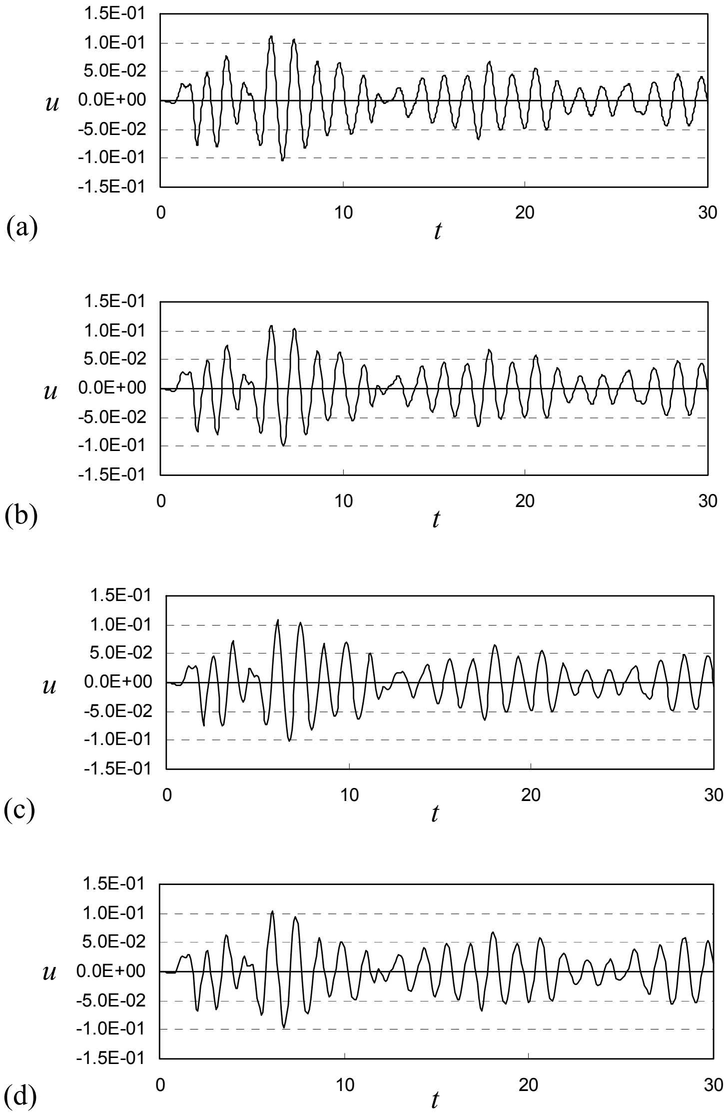
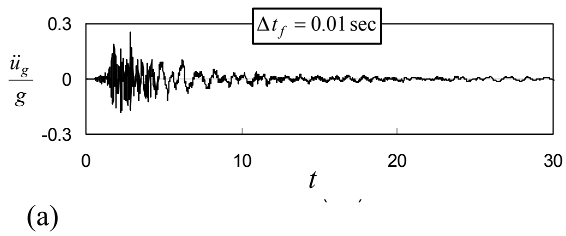


Figure 4: Responses obtained from average acceleration analysis of the problem introduced in Eq. (8) and Fig. 2, after implementing the recent technique [16] with the generalization proposed in this paper: (a) $p=3$ and $q=2$, (b) $p=5$ and $q=2$, (c) $p=7$ and $q=2$, (d) $p=9$ and $q=2$.

The second example is introduced in Fig. 5 and Table 3. The objective of the analysis is the mid-height velocity, of the structure, depicted in Fig. 6. The Wilson-θ method [35-37] is implemented



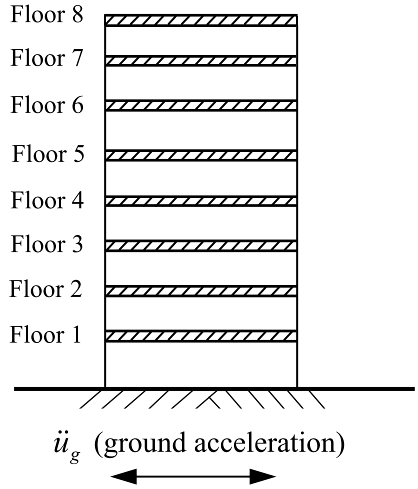


Figure 5: The structural system under consideration in the second example: (a) excitation, (b) shear building.

Floor	1	2	3	4	5	6	7	8
Mass $\times 10^{-3}$ (Kg)	1036	1034	1032	1030	1028	1026	1024	1022
Stiffness$\times 10^{-3}$ (N/m)	8600	8400	8200	7000	6800	6600	6400	6200
Damping (N sec/m)	Negligible (considered zero)							

Table 3: Numerical details of the shear building in Fig. 5(b).

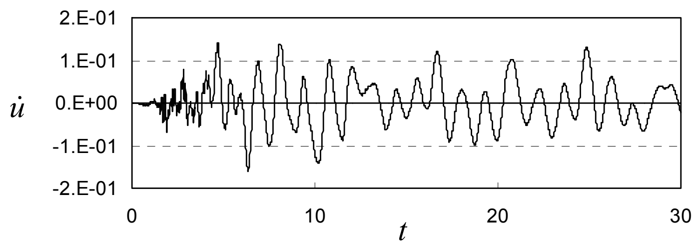


Figure 6: Exact mid-height velocity for the system introduced in Fig. 5 and Table 3.

for the analysis. In view of Fig. 7, when we are interested in high accuracies we would rather use at most $n = 2$ in implementation of the recent technique (see the last five seconds in the time histories). This implies 50% reduction of computational cost with a gap of 17% from the case $n = 3$, where, the reduction is about 67%. Even if the accuracy demand is between the accuracies provided at cases $n = 2$ and $n = 3$ (not as much as that corresponding to $n = 2$), according to the formulation of the recent technique there is no intermediate response. However, by implementing the generalization proposed in this paper, we can use the sets (9, 4), (10, 4), (11, 4), as (*p*, *q*), to materialize accuracies and computational cost reductions between the cases corresponding to $n = 2$ and $n = 3$ (provided by the recent technique); see Fig. 8 and Table 4. Consequently, this example once again demonstrates the additional versatility provided in this paper for the recently proposed technique, i.e. indeed the capability of reducing more computational cost without sacrificing the accuracy we are interested in.

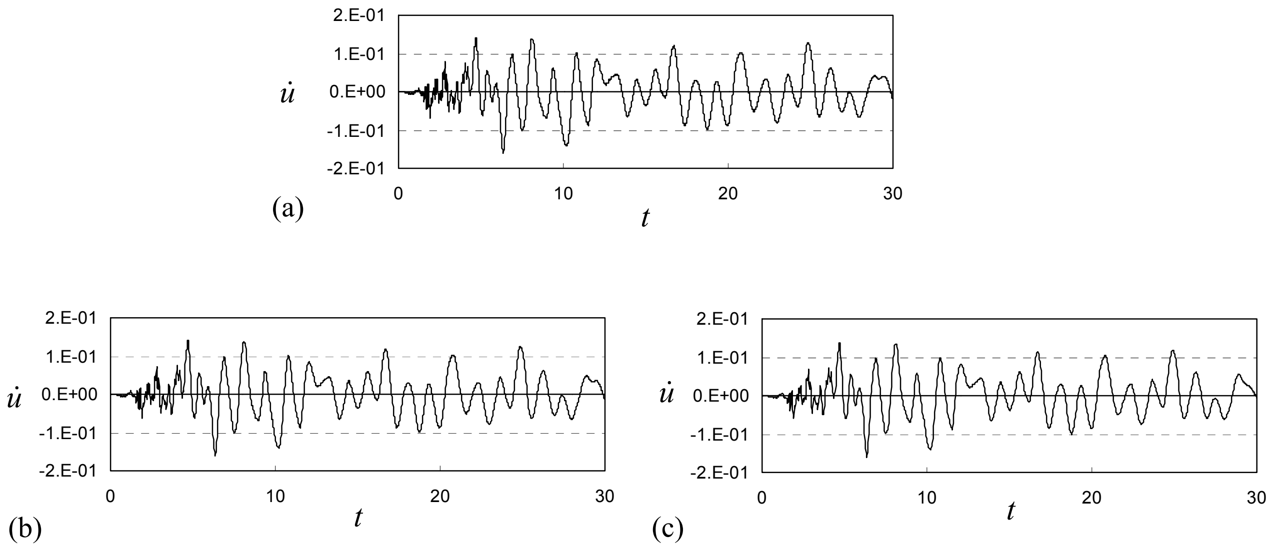


Figure 7: Mid-height velocity for the system in Fig. 5 and Table 3, obtained: (a) from ordinary analysis, (b) when implementing the recent technique [16] with $n = 2$, (c) when implementing the recent technique [16] with $n = 3$.

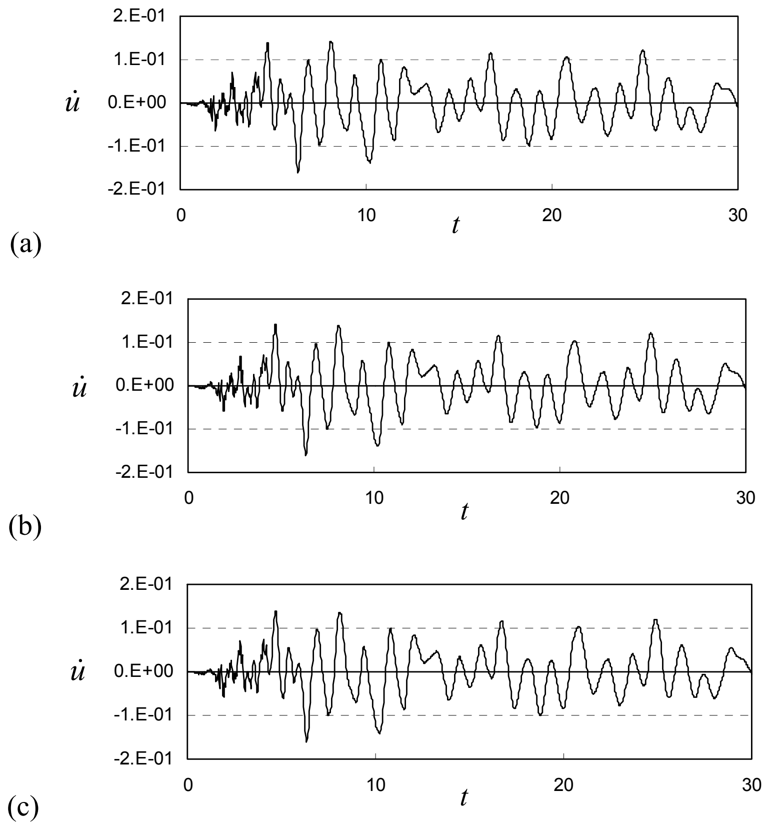


Figure 8: Mid-height velocity for the system in Fig. 5 and Table 3, obtained by implementing the recent technique [16] and the proposed generalization when: (a) $p = 9$ and $q = 4$, (b) $p = 10$ and $q = 4$, (c) $p = 11$ and $q = 4$.

Figure	7(a)	7(b)	8(a)	8(b)	8(c)	7(c)
Reduction	Basis of comparison	50	56	60	64	67

Table 4: Computational cost reduction, corresponding to the analyses reported in the second example (%).

4 CONCLUSION

With the objective of more efficient analysis of structural systems against earthquakes digitized records, attention in this paper is paid to a recent technique [16], proposed for implementing integration steps larger than the excitation steps. A restriction on this technique i.e. a positive integer as the ratio of integration steps to excitation steps, is eliminated, by implementing the practice in numerical analysis of the convergence for time integration analyses with digitized excitations. As the result, the ratio of integration steps to the steps of the earthquake records can now be a rational number, greater than one (with positive integers in the denominator and numerator), and practically the computational cost can be considered as a continuous function of the integration step.

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