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# ESTIMATING ROOF-LEVEL ACCELERATION SPECTRA FOR SINGLE STOREY BUILDINGS

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**Abstract.** *The performance of both structural and non-structural elements should clearly be considered when assessing the seismic safety and performance of a building. The performance of non-structural elements tends to be dependent on storey drift demands, acceleration demands or both. However, a simplified reliable means of estimating the acceleration demands on non-structural elements appears to be lacking, particularly for non-structural elements characterized by an elastic damping ratio that is not equal to 5% critical damping. As such, this paper presents a simplified means of predicting roof level acceleration spectra for single storey buildings. Such spectra should be useful in assessing the response of parapets or relatively lightweight appendages such as antennae, ceilings or mechanical services attached to the roof of single-storey buildings. As the procedure is formulated using concepts of mechanics and dynamics it is applicable to single-storey buildings that respond either in the linear or non-linear range, and is able to be adjusted for a wide range of elastic damping values. To gauge the performance of the procedure, spectra obtained from the approach are compared to those obtained from non-linear time-history analyses of a single storey building subject to ten ground motions scaled to various levels of intensity. The results demonstrate that the simplified approach for estimating roof level acceleration spectra works well and the possibility of extending it to multi-storey buildings should be explored as part of future research.*

# 1 INTRODUCTION

Past earthquakes have highlighted the need to consider both structural and non-structural elements when assessing seismic risk, both in terms of likely financial losses and safety of human lives. Events such as the Northridge earthquake [1] and the recent Darfield (New Zealand) earthquake of September 2010 [2] have indicated that financial losses from damage to non-structural components can far exceeded losses from structural damage. Moreover, the failure of non-structural building components can become a safety hazard and hamper the safe movement of occupants as they evacuate or rescuers as they enter the building [3], [4]. Furthermore, non-structural damage can severely limit the functionality of critical facilities such as hospitals, as demonstrated in the 1994 Northridge earthquake [5] and the 2006 Kiholo Bay and Hawi, Hawaii earthquakes [6].

Clearly, in order to reliably assess the safety of a building system, one should take steps to assess the risk posed by both structural and non-structural elements. Tools for the assessment of risk to structural elements are fairly well developed, as is illustrated by the numerous provisions provided in building codes. Indications for the assessment of non-structural elements are, however, somewhat limited. The critical type of loading for non-structural elements varies from one type of non-structural element to another. Lightweight partition walls are likely to be drift-sensitive whereas ceilings are expected to be acceleration sensitive [4],[7]. Figure 1 illustrates some of the damage to an acceleration-sensitive ceiling observed in the recent Darfield (N.Z.) earthquake [2].

Figure 1: Elevation view of case study building (reproduced from [2])

In order to permit reliable assessment of ceiling systems and other acceleration sensitive elements, one requires a means of estimating the acceleration demands on the element. This is not a trivial task since, aside from the uncertainty related to the ground motion arriving at the base of the building, it is expected that the acceleration demands vary up the height of a structure, being filtered and amplified according to the dynamic characteristics of the main structural system. This implies that an acceleration spectrum recorded at the roof of a building is likely to be very different from that at the base of the building, as illustrated in Figure 2. This paper demonstrates how a recently proposed procedure for the estimation of floor spectra [8] can be used to provide improved estimates of roof-level acceleration demands in single-storey buildings.

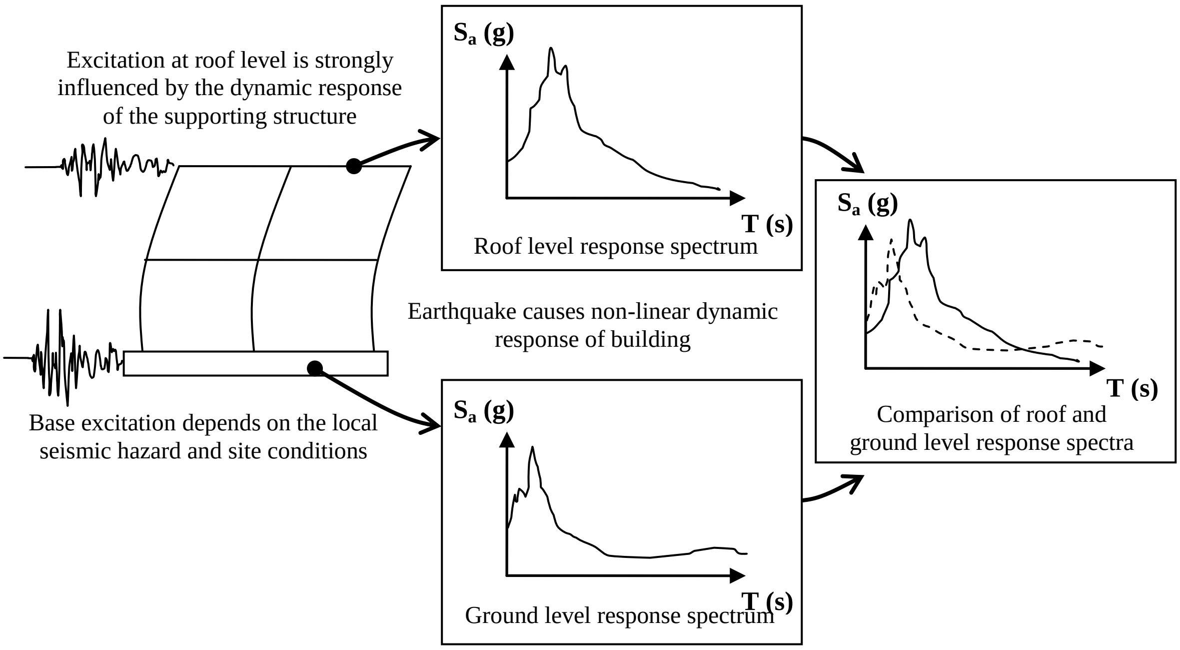


Figure 2: Elevation view of case study building (adapted from [8])

## 2 CODE METHODS FOR THE PREDICTION OF ACCELERATION DEMANDS

Current code approaches for the prediction of acceleration demands on non-structural elements up the height of buildings vary quite significantly, suggesting that there is considerable uncertainty as to the best methodology. In Europe [9], the acceleration demand, $S_a$, acting on a non-structural element of a building can be estimate as:

$$S_a = a_g.S.\left(\frac{3(1+z/H)}{1+(1-T_a/T_n)^2} - 0.5\right) \geq a_g.S \qquad (1)$$

where $z$ is the height of the non-structural element above the ground level, $H$ is the total height of the building, $a_g$ is the design ground acceleration (units of g) for a rock site, $S$ is a modification factor to account for other soil site conditions, $T_a$ is the period of the non-structural element and $T_n$ (shown as $T_1$ in EC8 [9]) is the natural (first-mode) period of the building in the relevant direction of excitation.

Equation 1 suggests that at roof level, when the period of the non-structural element corresponds to that of the structural system, the element could be subjected to an acceleration 5.5 times the peak ground acceleration (PGA) at the site. In contrast, using the ASCE7-05 [10] approach in the U.S. one could predict a maximum acceleration demand of 7.5 times the PGA but the code does limit the maximum design force for non-structural elements such that the peak acceleration demand is anticipated to be approximately 4.0 times the PGA. In New Zealand [11] the maximum acceleration on non-structural elements at roof level could be predicted as 6.0 times the PGA, which is not far from the EC8 estimate. However, the New Zealand approach only depends on the period of the non-structural element (and adopts maximum values for periods up to 0.75s) and does not depend on the period of the supporting structure.

In order to illustrate the potential of the different code approaches, Figure 3 compares the acceleration spectra recorded at the roof level of an 8-storey RC wall structure subject to non-linear time-history analyses by Sullivan et al. [8] using accelerograms spectrum-compatible with the EC8 spectrum constructed at a PGA of 0.2g. The spectra are constructed for 2% and 5% elastic damping in order to illustrate how acceleration demands can be affected by the damping of the supported element. Note that none of the code approaches appear to modify acceleration demands to account for differences in element damping even though it should certainly be expected that some non-structural elements will possess considerably different damping values than others [8].

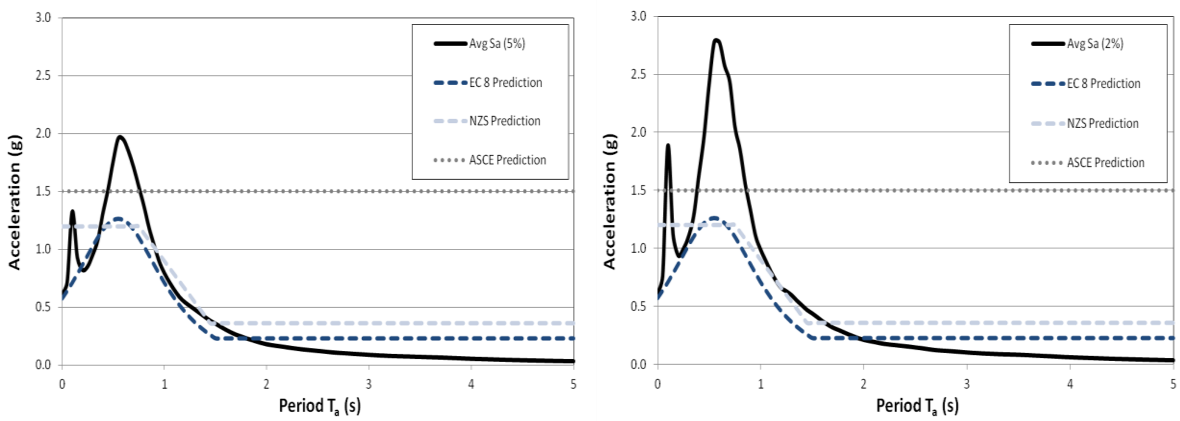


Figure 3: Comparison of roof level acceleration spectra at 5% damping (left) and 2% damping (right) as predicted via different code approaches and from non-linear time-history analyses of an 8-storey case study building (adapted from [8]).

From the comparison provided in Figure 3 it is apparent that the elastic damping of the non-structural element has a very significant influence on the peak acceleration demands and should be considered in seismic assessment. In addition, note that all of the code methods underestimate the peak acceleration demands. One can also observe the presence of two peaks in the floor spectra; the main peak is associated with amplification of demands at the fundamental period of the structure whereas the peak at short periods is related to amplification of demands due to resonance with the 2$^{nd}$ mode of vibration of the building. Such higher mode amplification is not considered at all in the Eurocode approach and merits further investigation but this is outside the scope of this paper. Instead, the next section presents an alternative means of predicting the acceleration that takes account of the elastic damping characteristic of the supported element as well as the possible effects of non-linear response of the supporting structure.

## 3 ALTERNATIVE PROCEDURE FOR THE ESTIMATION OF ROOF LEVEL ACCELEARTION SPECTRA

### 3.1 Conceptual Considerations

Any alternative means of predicting acceleration spectra at various levels of a structure should recognize the following points (discussed in Sullivan et al. [8]):

- Acceleration demands are affected by the ratio $\beta$ of the period of the forcing function to the period of the structure (i.e. the period of the supporting element divided by the period of the supported element).
- Acceleration demands are affected by the elastic damping of the supported element.

- The forcing function period cannot typically be considered as a single constant value during an earthquake because of the participation of different modes of vibration in structural response and because of the period lengthening that occurs as the supporting structure yields and responds non-linearly.
- The peak acceleration demands at the roof of a SDOF structure are physically limited by the lateral resistance of the structure itself (provided that the roof behaves like a rigid diaphragm).

Considering the first of these points, many texts on structural dynamics (e.g. [12]) have shown that the dynamic amplification, *DAF*, (i.e. the ratio of the peak acceleration to the acceleration of the forcing function) of a system subject to harmonic excitation can be estimated from Equation 2:

$$DAF = \frac{\ddot{u}_{\max}}{\ddot{u}_0} = \frac{\beta}{\sqrt{(1-\beta^2)^2 + (2\beta\xi)^2}} \tag{2}$$

Where $\beta$ is the ratio of the forcing function period to the structure's period of vibration and $\xi$ is the elastic damping of the (supported) structure.

This expression would imply that the peak dynamic amplification, $DAF_{\max,harm}$, that could be expected (for $\beta = 1.0$) is a function of the elastic damping, and is given by:

$$DAF_{\max,harm} = \frac{1}{2\xi} \tag{3}$$

Unfortunately, however, as pointed out in [8] this equation cannot strictly be applied to earthquake engineering because real earthquakes do not impose harmonic excitation. Instead, Sullivan et al. [8] established the following empirical expression for the peak dynamic amplification (expected at $\beta = 1.0$) that is again only a function of the elastic damping on the supported element:

$$DAF_{\max} = \frac{1}{\sqrt{\xi}} \tag{4}$$

The difference between Equations 3 and 4 could be considered a reflection of the difference is the number of significant cycles of excitation imposed around the peak response. This would in turn suggest that the expression could be strongly dependent on the characteristics of the earthquake motion, possibly varying greatly for pulse-type or long duration earthquakes. However, Sullivan et al. [8] found that this may not be the case and that the significant number of cycles for dynamic amplification may in fact be rather constant for a wide range of ground motions.

The third bullet point above relates to the period of the forcing function. As demonstrated in the previous section, for multi-degree-of-freedom systems higher modes of vibration could provoke significant amplification of acceleration demands on non-structural elements. The third bullet point additionally states though, that period lengthening may occur as the supporting structure yields and responds non-linearly. This concept is illustrated in Figure 4 for a SDOF system. The figure is annotated to illustrate how the stiffness, K, reduces with the development of yielding and inelastic response and it also includes equations showing how an effective period can be related to the effective stiffness at any point of the non-linear force-displacement curve.

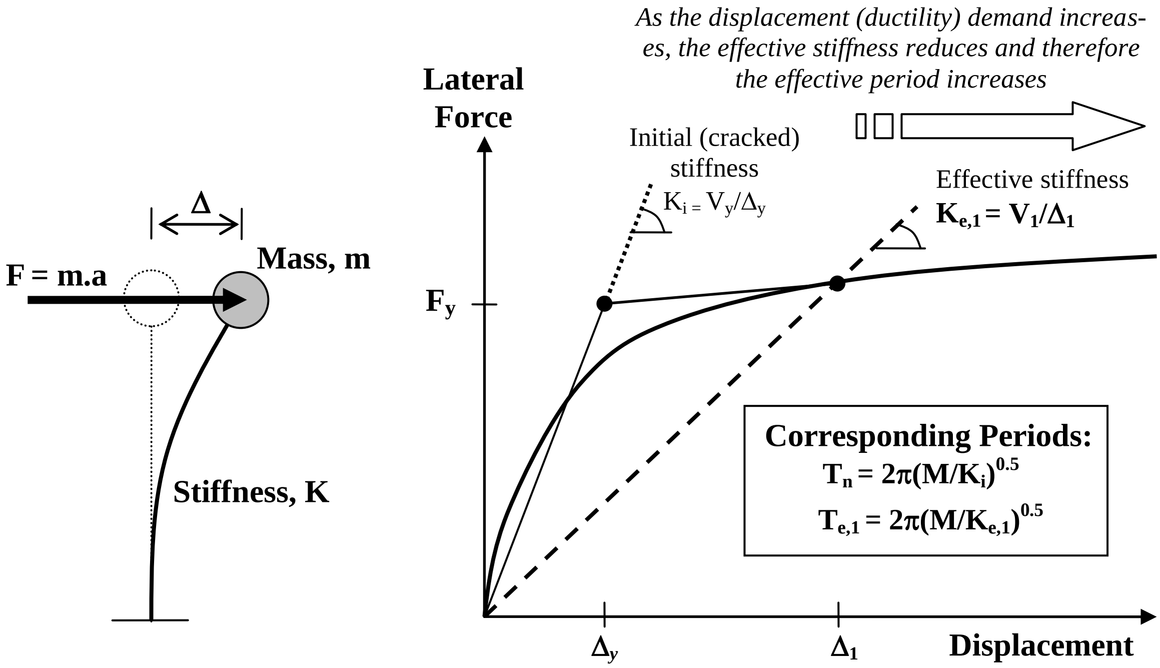


Figure 4: Force-displacement response of a SDOF structure, annotated to illustrate concept of the effective stiffness and effective period at a displacements of $\Delta_1$ (adapted from [8]).

The fourth bullet point above states that the peak acceleration demands at the roof of a SDOF structure are physically limited by the lateral resistance of the structure itself. This concept is also illustrated in Figure 4 since it is shown that, according to Newton's second law, force is equal to mass times acceleration and given that the maximum force in the SDOF system is limited by the strength (resistance) of the system itself, so too will be the maximum acceleration of the mass. This implies that the maximum acceleration of the SDOF supporting system is given by:

$$a_{\max} = \frac{V_b}{M} = \frac{V_y\left[1 + r(\mu - 1)\right]}{M} \tag{5}$$

where $V_b$ is the base shear resistance, $M$ is the mass of the supporting structure, $V_y$ is the base shear at yield, $r$ is the post-yield strain hardening factor (typically 0.05 for RC structures) and $\mu$ is the displacement ductility demand.

Reflecting for a moment on current code methods, it is apparent that none of the existing approaches account for all the points raised above. The Eurocode accounts for the period ratio but not the other points. The New Zealand approach could arguably be interpreted as recognizing period lengthening and multiple forcing frequencies by rendering the amplification independent of the period ratio. However, even this approach risks underestimating the peak demands, particularly if the elastic damping of the non-structural element is low.

### 3.2 Proposed Methodology

As explained in the introduction, Sullivan et al. [8] have recently proposed a method for the prediction of floor spectra in SDOF systems that accounts for all the conceptual issues identified in the previous subsection. In the approach, the acceleration spectrum at the roof of a SDOF supporting system can be computed using the following equations:

$$a_m = \frac{T_a}{T_n}.\left[a_{\max}\left(DAF_{\max} - 1\right)\right] + a_{\max} \qquad \text{for} \quad T_a < T_n$$

$$a_m = a_{\max} DAF_{\max} \qquad \text{for} \quad T_n < T_a < T_e \tag{6}$$

$$a_m = \frac{a_{\max}}{\sqrt{\left(1 - \frac{1}{\beta}\right)^2 + \xi}} \qquad \text{for} \quad T_a > T_e$$

where $a_m$ is the acceleration spectral coordinate for a supported element of period $T_a$, $a_{\max}$ is the maximum acceleration of the mass of the supporting structure (given by Eq.5), $T_n$ is the natural (initial) period of the supporting structure, $T_e$ is the effective period of the supporting structure (refer Figure 4) and $DAF_{\max}$ is the maximum expected dynamic amplification obtained from Eq.(4) as a function of the elastic damping, $\xi$.

### 3.3 Performance of the method

In order to illustrate the performance of this method, Equation 6 is used to estimate acceleration spectra at the roof level of a single-storey case study building. The case study building taken into consideration here is an existing precast reinforced concrete frame industrial building, with a lateral load resisting system provided by cantilever columns and with a (cracked) fundamental period of $T_n$=1.33s. The structure has a ratio of yield strength to building weight of 15%. The long period is a reflection of the low stiffness provided to such existing structures, similar to those recently damaged in the Emilia (Italy) earthquake. A non-linear model of the structure is developed for non-linear time-history (NLTH) analyses in Ruaumoko (refer [13]) and is provided with Takeda-thin hysteretic properties following recommendations in [14]. The structure is subject to a series of real accelerograms (taken from [15]) that were scaled to be spectrum compatible with the EC8 type 1 spectrum on soil type A. The analyses are run at three different intensity levels corresponding to a peak ground acceleration demand of 0.2g, 0.4g and 0.8g in order to illustrate the effect of intensity on roof level acceleration spectra. The ductility demand on the case study building at each of these intensities levels could have been estimated a variety of ways, including traditional structural assessment techniques in current codes or the displacement-based assessment method of [14]. However, in this case study the mean ductility demands on the system are computed directly from the results of the non-linear time history analyses as 1.20 for 0.2g, 2.35 for 0.4g and 5.52 for the 0.6g intensity level. The roof level acceleration spectra are then computed using Equation 6 and are compared to the mean of the spectra obtained for the case study building in Figure 5. Comparisons are made for acceleration spectra developed at 2%, 5% and 10% elastic damping. Note that the roof level acceleration spectra are predicted quite well (being on slightly conservative in some instances) in all cases.

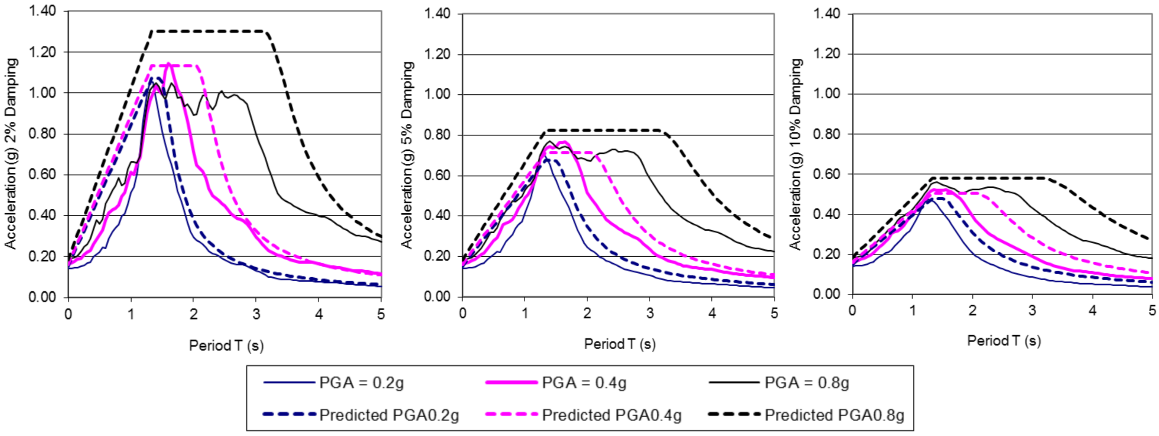


Figure 5: Mean roof level acceleration spectra obtained for the case study structure via NLTH analyses at three different intensity levels compared with those predicted using Equation 6, for 2% (left), 5% (middle) and 10% (right) elastic damping.

## 4 CONCLUSIONS

Previous earthquakes have illustrated that seismic safety depends on the adequate performance of both structural and non-structural elements. Non-structural elements can typically be grouped into drift-sensitive or acceleration sensitive elements, with suspended ceilings being a good example of an acceleration-sensitive non-structural element. In order to adequate assess the risk posed by acceleration-sensitive non-structural elements it has been shown that improvements methods of predicting acceleration spectra are required, particularly for non-structural elements characterized by an elastic damping ratio that is not equal to 5% critical damping. As such, this paper has presented a simplified means of predicting roof level acceleration spectra for single storey buildings. As the procedure is formulated using concepts of mechanics and dynamics it is applicable to single-storey buildings that respond either in the linear or non-linear range, and is able to be adjusted for a wide range of elastic damping values. To gauge the performance of the procedure, spectra obtained from the approach have been compared to those obtained from non-linear time-history analyses of a single storey building subject to ten ground motions scaled to various levels of intensity. The results have demonstrated that the simplified approach for estimating roof level acceleration spectra works well and the possibility of extending it to multi-storey buildings should be explored as part of future research.

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