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INCLUSION OF GEOMETRIC NONLINEARITY IN CALCULATING SEISMIC BUILDING FRAMES

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Abstract. *In calculating the seismic response of a building, the Spanish Instructions NCSE-02 and CTE, paragraph 3.7.7(also EUROCODE 8 paragraph 1.2 part 1-1), establish that if for all storeys the interstory drift sensitivity coefficient, ξ, satisfies:*

$$\xi = \frac{P \cdot d}{F \cdot h} \leq 0.1$$

then it will not be necessary to consider the effects of the 2nd order ($P-\Delta$ effects).
In this discussion we review this claim:

- *Because even for $\xi \leq 0.1$, increases of the bending moment at the ends of the columns due to the inclusion of second order effects can account for between 15% and 34% of its value for static service loads.*

- *Because most adverse effects are shown in the lower height buildings (up to 5 floors) which is precisely the range in which most of the housing stock of Spain is located.*

- *Also in paragraph 6 we delimit the coefficient for buildings of lesser height (up to 5 floors), proposing to lower it generally to $\xi \leq 0.06$.*

1 DESCRIPTION OF THE PROBLEM AND THE BUILDING USED AS AN EXAMPLE

The NCSE-02 [1] and CTE [2] Spanish Instructions are instructions that allow us to study dynamic effects in different structural elements. Spain is a country that has different provinces in which dynamic studies are important and mandatory. In this article, we analyze the interstory drift sensitivity coefficient, and consider whether or not the 2nd order effects are important in building structures to dynamic stresses.

This coefficient is defined in Eurocode 8 [3], but one of the conditions of whether or not to consider the effects of the 2nd order of coefficient, is formulated identically in Spanish Instructions NCSE 02 and the CTE.

The interstory drift sensitivity coefficient depends on the total gravity load at and above the story considered in the seismic design situation *P*, and the design interstory drift, evaluated as the difference of the average lateral displacements d_s at the top and bottom of the story under consideration *d*, which is directly proportional, and the total seismic story shear *F* and interstory height *h*, which is inversely proportional. To analyze the problem, we take a building type, as shown in Figure 1, and study the influence of 2nd order effects using the corresponding theoretical model we develop in the body of the article.

The example in Figure 1 is a regular rectangular concrete building of dimension 25 × 25 m with 25 columns arranged to give rise to 5 frames in each direction with lights of 5.50 m. For the analysis, we adopt the hypotheses that it might have 2, 4, 7, and 10 floors,the loadings considered are for medium duty use, and that the location of the building is in an average Spanish earthquake zone.

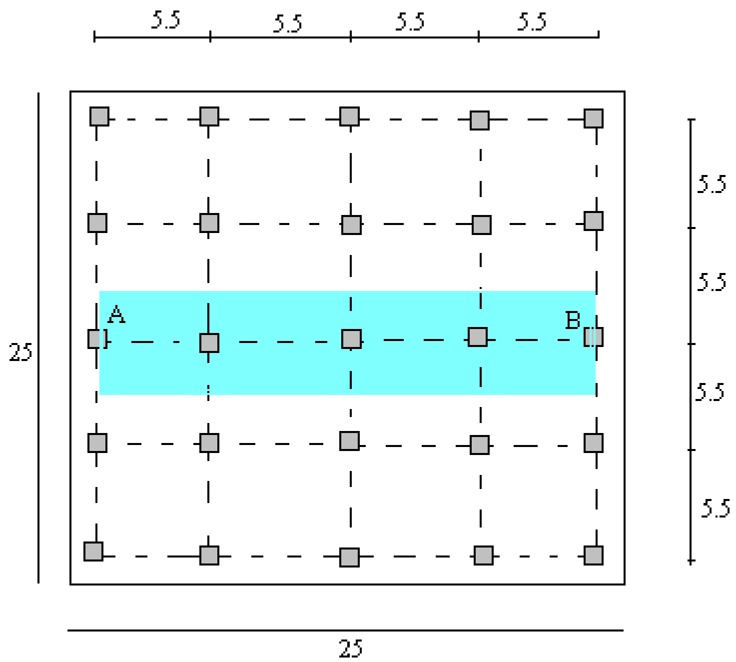

Figure 1:Rectangular Plant of 25 × 25 m with the situation of the columns.

The section of the beams of the floor is 0.30 × 0.40 m (Figure 2). The dead load is the corresponding estimated forged 0.28 m thick, flooring and ceiling. Overloading is the appropriate use of public spaces with chairs and tables. The basic seismic acceleration is 0.102g and the coefficients of terrain features and contribution are C = 1.4 and K = 1.

With these dimensions, loads and frequently used materials (concrete HA-20 or 25 and reinforcement steels B-500) the obtained mechanical loading amounts of a permanent reinforcement are quite normal (e.g., 4–6 ϕ 16–20 for columns of 25 × 25 cm on two floors, or 10–12 ϕ 20 for columns, 40–45 × 40–45 cm in seven storys).

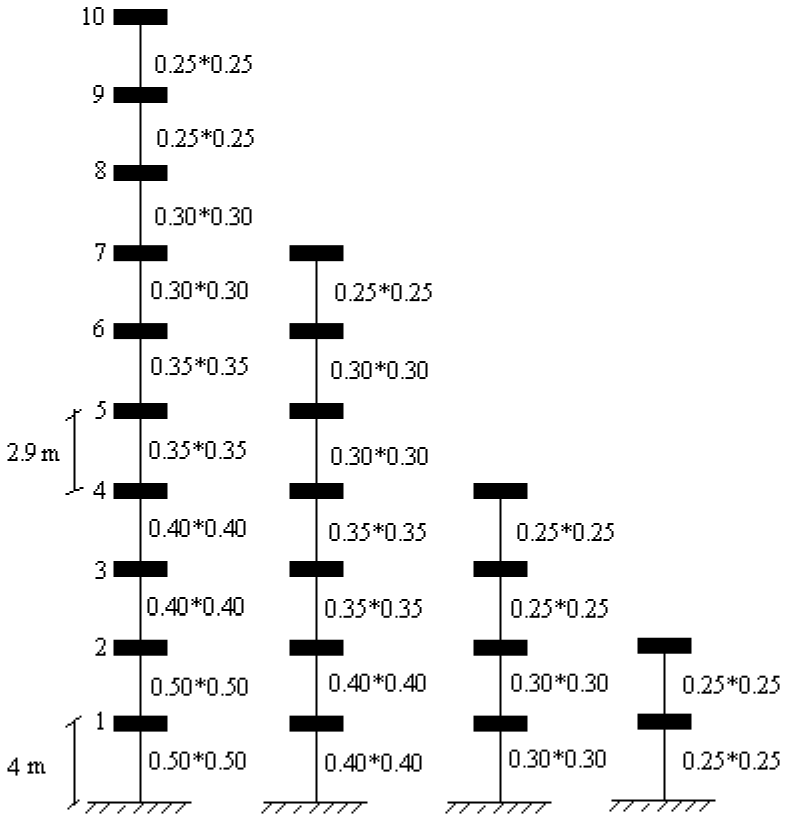

Figure 2: Simple Shear Model for the frame A-B
under the assumptions of 2, 4, 7, and 10 storys

2 GEOMETRIC STIFFNESS MATRIX OF FIRST ORDER OF BEAMS OF 2D FRAMES

In order to consider the 2nd order effects $(P-\Delta$ effects$)$, we must know the stiffness matrix, including the axial force in local coordinates of a beam belonging to a planar structure of rigid nodes, connected rigidly at both ends. A very appropriate methodology for this deduction is based on the consideration of the equilibrium of the slice in the deformed geometry. Alternatively, we can consider nonlinear expressions of deformations of a beam in bending (valid for small deformations and moderate or large displacements) [4].In this paper, we choose the former for its better description of the problem.

Let us consider a beam whose initial neutral axis and deformed neutral axis initial imperfections are represented as in Fig.3a. As a consequence of geometric initial imperfections, the centerline of the beam is deflected by the amount $\hat{v}$ The transverse sections are rotated at an angle $\hat{\vartheta}$.

Consider a slice of this beam of length dx, unstressed axial, bending and shear, and its geometry deformed as in Fig.3b, where the slope of the centerline is given by the shift v along the axis relative to one side, and on the other is given by

$$OO_1 = \vartheta\cdot dx + \hat{\vartheta}\cdot dx = \frac{dv}{dx}\cdot dx + \hat{\vartheta}\cdot dx = dv + \hat{\vartheta}\cdot dx \quad . \qquad (1)$$

We can express the sum $\hat{\vartheta}\cdot dx$ as a fraction loading α of the displacement v and consequently, we obtain

$$OO_1 = \vartheta\cdot dx + \hat{\vartheta}\cdot dx = \frac{dv}{dx}\cdot dx + \hat{\vartheta}\cdot dx = (1+\alpha)\cdot dv \qquad (2)$$

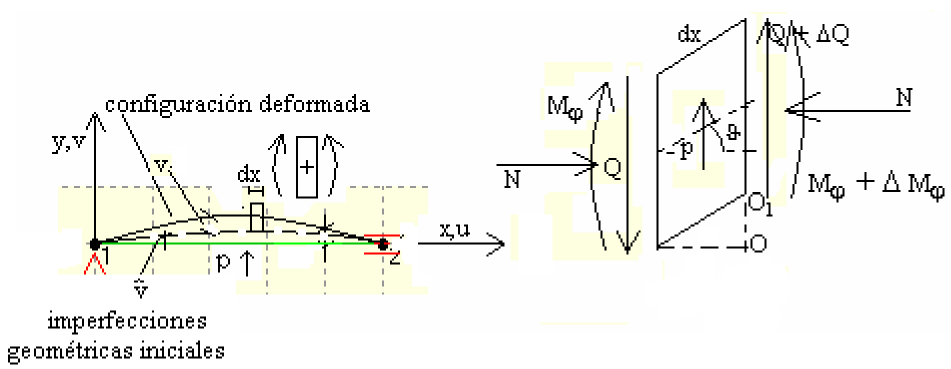

Figure 3a): Deformed configuration
b): Equilibrium of a slice of the beam in its deformed configuration.

If we use the bending theory of thin beams, where $M_\varphi = E\cdot I\ v''$ and establish the equilibrium of the slice into the deformed geometry, we obtain

$$\Sigma Fy = 0 \Rightarrow P\cdot dx - Q + Q + \Delta Q = 0 \qquad (3)$$
$$\Sigma M = 0 \Rightarrow M_\varphi + \Delta M_\varphi - M_\phi + Q\cdot dx + N(1+\alpha)\cdot dv = 0 \qquad .(4)$$

The coefficient α gives us an idea, as a percentage, of the level of imperfections on the deformed configuration. Solving for Q in the 2nd equation, and deriving and substituting in the 1st equation, we reach the following differential equation

$$P(x) = E\cdot I\frac{d^4v}{dx^4} + (1+\alpha)\cdot N\frac{d^2v}{dx^2} \quad . \qquad (5)$$

In studying free transverse oscillations P = 0, such that the equation reduces to

$$E\cdot I\frac{d^4v}{dx^4} + (1+\alpha)\cdot N\frac{d^2v}{dx^2} = 0 \quad . \qquad (6)$$

This is usually written as $\omega^2 = (1+\alpha)\cdot N/(E\cdot I)$, as $\frac{d^4v}{dx^4} + \omega^2\frac{d^2v}{dx^2}$, whose general solution is given by the following expression:

$$v = -\frac{A}{\omega^2}\cdot \cos(\omega\cdot x) - \frac{B}{\omega^2}\cdot \mathrm{sen}(\omega\cdot x) + C\cdot x + D \qquad (7)$$

Integration constants A, B, C, and D are provided by our boundary conditions:

$$v_{x=0} = v_1 \,; \quad v_{x=L} = v_2 \,; \quad v'_{x=0} = \vartheta_1 \quad ; v'_{x=L} = \vartheta_2 \tag{8}$$

where v_i and ϑ_i, are respectively the deflection and rotation of the end i. If we particularize equation (7) on the values x = 0 and x = L, we obtain:

$$\begin{gathered} v_1 = -\frac{A}{\omega^2} + D \\ v_2 = -\frac{A}{\omega^2}\cdot \cos(\omega\cdot L) - \frac{B}{\omega^2}\cdot \operatorname{sen}(\omega\cdot L) + C\cdot L + D \\ \vartheta_1 = -\frac{B}{\omega} + C \\ \vartheta_2 = \frac{A}{\omega}\cdot \operatorname{sen}(\omega\cdot L) - \frac{B}{\omega}\cdot \cos(\omega\cdot L) + C \end{gathered} \tag{9}$$

which we can write in matrix form, as follows:

$$\begin{pmatrix} v_1 \\ \vartheta_1 \\ v_2 \\ \vartheta_2 \end{pmatrix} = \begin{pmatrix} -\frac{1}{\omega^2} & 0 & 0 & 1 \\ 0 & -\frac{1}{\omega} & 1 & 0 \\ -\frac{\cos(\omega\cdot L)}{\omega^2} & -\frac{\operatorname{sen}(\omega\cdot L)}{\omega^2} & L & 1 \\ \frac{\operatorname{sen}(\omega\cdot L)}{\omega} & -\frac{\cos(\omega\cdot L)}{\omega} & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \Im \cdot \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} \tag{10}$$

If we suppose that the inverse matrix $\Im^{-1}$ is partitioned in the form

$$\Im^{-1} = \begin{pmatrix} \Im^{-1}{}_{11} & \Im^{-1}{}_{12} \\ \Im^{-1}{}_{21} & \Im^{-1}{}_{22} \end{pmatrix} \tag{11}$$

then, if we incorporate the horizontal displacements u_1 and u_2 from the beam ends, and we use the usual notation in the theory of structures in which displacements and rotations of the ends of the beam are expressed by

$$\begin{pmatrix} d^{e1x} \\ d^{e1y} \\ \vartheta^{e1} \\ d^{e2x} \\ d^{e2y} \\ \vartheta^{e2} \end{pmatrix} = \begin{pmatrix} u_1 \\ v_1 \\ \vartheta_1 \\ u_2 \\ v_2 \\ \vartheta_2 \end{pmatrix} \tag{12}$$

the equation of the deformed beam is:

$$v = -\frac{A}{\omega^2}\cdot \cos(\omega\cdot x) - \frac{B}{\omega^2}\cdot \operatorname{sen}(\omega\cdot x) + C\cdot x + D = \begin{pmatrix} \frac{-\cos(\omega\cdot x)}{\omega^2} & \frac{-\operatorname{sen}(\omega\cdot x)}{\omega^2} & x & 1 \end{pmatrix} \cdot \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{-\cos(\omega\cdot x)}{\omega^2} & \frac{-sen(\omega\cdot x)}{\omega^2} & x & 1 \end{pmatrix} \cdot \begin{pmatrix} \Omega & \Im^{-1}{}_{11} & \Omega & \Im^{-1}{}_{12} \\ \Omega & \Im^{-1}{}_{21} & \Omega & \Im^{-1}{}_{22} \end{pmatrix} \cdot \begin{pmatrix} d^{e1x} \\ d^{e1y} \\ \vartheta^{e1} \\ d^{e2x} \\ d^{e2y} \\ \vartheta^{e2} \end{pmatrix} \tag{13}$$

The matrix $\Im^{-1}$ is:

$$\Im^{-1} = \varsigma \cdot \begin{pmatrix} -\omega^2\cdot(1-\cos(\omega\cdot L)) & \omega\cdot(-sen(\omega\cdot L)+\omega\cdot L\cdot\cos(\omega\cdot L)) & \omega^2\cdot(1-\cos(\omega\cdot L)) & \omega\cdot(sen(\omega\cdot L)-\omega\cdot L) \\ \omega^2\cdot sen(\omega\cdot L) & \omega\cdot(-1+\cos(\omega\cdot L)+\omega\cdot L\cdot sen(\omega\cdot L)) & -\omega^3\cdot sen(\omega\cdot L) & \omega^2\cdot(1-\cos(\omega\cdot L)) \\ \omega\cdot sen(\omega\cdot L) & 1-\cos(\omega\cdot L) & -\omega\cdot sen(\omega\cdot L) & (1-\cos(\omega\cdot L) \\ -1+\cos(\omega\cdot L)+\omega\cdot L\cdot sen(\omega\cdot L) & -sen(\omega\cdot L)+\omega\cdot L\cdot\cos(\omega\cdot L) & 1-\cos(\omega\cdot L) & sen(\omega\cdot L)-\omega\cdot L \end{pmatrix}. \tag{14}$$

where ς is $\varsigma = \dfrac{1}{2-2\cdot\cos(\omega L)-\omega L\cdot sen(\omega L)}$

If the loads at the ends of the beam along the x-axis are designated by p^x, the matrix k_{viga} can be deduced from the bending moment expressions (M_φ), shear force (Q) and axial force (N) by

$$p^{e1x} = -p^{e2x} = \frac{E\cdot A}{L}\cdot(d^{e1x}-d^{e2x})$$

$$(Q)_{x=0} = -p^{e1y} = [-N\cdot\frac{dv}{dx} - E\cdot I\cdot\frac{d^3v}{dx^3}]_{x=0} = -N\cdot\vartheta_1 - E\cdot I\cdot(\frac{d^3v}{dx^3})_{x=0} = E\cdot I\cdot[-\omega^2\cdot\vartheta_1 - (\frac{d^3v}{dx^3})_{x=0}] \tag{15}$$

$$(M_\varphi)_{x=0} = -M_1 = E\cdot I\cdot(v'')_{x=0}$$

$$(Q)_{x=L} = p^{e2y} = [-N\cdot\frac{dv}{dx} - E\cdot I\cdot\frac{d^3v}{dx^3}]_{x=L} = -N\cdot\vartheta_2 - E\cdot I\cdot(\frac{d^3v}{dx^3})_{x=L} = E\cdot I\cdot[-\omega^2\cdot\vartheta_2 - (\frac{d^3v}{dx^3})_{x=L}]$$

$$(M_\varphi)_{x=0} = M_2 = E\cdot I\cdot(v'')_{x=L}$$

Substituting in the above equations the derivatives of the deflection v, calculated from equation (13), and particularizing at x equals zero or L, as appropriate, we obtain:

$$\begin{pmatrix} p^{e1x} \\ p^{e1y} \\ m^{e1} \\ p^{e2x} \\ p^{e2y} \\ m^{e2} \end{pmatrix} = \left\{ E\cdot I\cdot \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \omega & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \omega\cdot sen(\omega\cdot L) & -\omega\cdot\cos(\omega\cdot L) & 0 & 0 \\ \cos(\omega\cdot L) & sen(\omega\cdot L) & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} \Omega & \Im^{-1}{}_{11} & \Omega & \Im^{-1}{}_{12} \\ \Omega & \Im^{-1}{}_{21} & \Omega & \Im^{-1}{}_{22} \end{pmatrix} + \right.$$

$$\left. + \begin{pmatrix} \frac{E\cdot A}{L} & 0 & 0 & -\frac{E\cdot A}{L} & 0 & 0 \\ 0 & 0 & \omega^2\cdot E\cdot I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{E\cdot A}{L} & 0 & 0 & \frac{E\cdot A}{L} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\omega^2\cdot E\cdot I \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \right\} \cdot \begin{pmatrix} d^{e1x} \\ d^{e1y} \\ \vartheta^{e1} \\ d^{e2x} \\ d^{e2y} \\ \vartheta^{e2} \end{pmatrix} \tag{16}$$

Replacing the four submatrices $\Im_{ij}^{-1}$ of order 2×2, in the above equation (13) and operating, we obtain the equation of beam loads movements, $p_{viga} = \bar{k}_{viga} d_{viga}$ in local coordinates:

$$\begin{pmatrix} p^{e1x} \\ p^{e1y} \\ m^{e1} \\ p^{e2x} \\ p^{e2y} \\ m^{e2} \end{pmatrix} = \bar{k}_{viga} \cdot \begin{pmatrix} d^{e1x} \\ d^{e1y} \\ \vartheta^{e1} \\ d^{e2x} \\ d^{e2y} \\ \vartheta^{e2} \end{pmatrix} \tag{17}$$

with $\bar{k}_{viga}$ equal to

$$\bar{k}_V = \begin{pmatrix} \frac{E\cdot A}{L} & 0 & 0 & -\frac{E\cdot A}{L} & 0 & 0 \\ 0 & \frac{\omega^3 \cdot sen(\omega\cdot L)}{f(\omega)} & \frac{\omega^2\cdot(1-\cos(\omega\cdot L))}{f(\omega)} & 0 & -\frac{\omega^3 \cdot sen(\omega\cdot L)}{f(\omega)} & \frac{\omega^2\cdot(1-\cos(\omega\cdot L))}{f(\omega)} \\ 0 & \frac{\omega^2\cdot(1-\cos(\omega\cdot L))}{f(\omega)} & \frac{\omega\cdot(sen(\omega\cdot L)-\omega\cdot L\cdot\cos(\omega\cdot L))}{f(\omega)} & 0 & -\frac{\omega^2\cdot(1-\cos(\omega\cdot L))}{f(\omega)} & \frac{\omega\cdot(\omega\cdot L - sen(\omega\cdot L))}{f(\omega)} \\ -\frac{E\cdot A}{L} & 0 & 0 & \frac{E\cdot A}{L} & 0 & 0 \\ 0 & -\frac{\omega^3 \cdot sen(\omega\cdot L)}{f(\omega)} & -\frac{\omega^2\cdot(1-\cos(\omega\cdot L))}{f(\omega)} & 0 & \frac{\omega^3 \cdot sen(\omega\cdot L)}{f(\omega)} & -\frac{\omega^2\cdot(1-\cos(\omega\cdot L))}{f(\omega)} \\ 0 & \frac{\omega^2\cdot(1-\cos(\omega\cdot L))}{f(\omega)} & \frac{\omega\cdot(\omega\cdot L - sen(\omega\cdot L))}{f(\omega)} & 0 & -\frac{\omega^2\cdot(1-\cos(\omega\cdot L))}{f(\omega)} & \frac{\omega\cdot(sen(\omega\cdot L)-\omega\cdot L\cdot\cos(\omega\cdot L))}{f(\omega)} \end{pmatrix}. \tag{18}$$

or in a more compact form

$$p_{viga} = \bar{k}_{viga} \cdot d_{viga} \tag{19}$$

where $f(\omega) = \frac{2 - 2\cos(\omega\cdot L) - \omega\cdot L\cdot sen(\omega\cdot L)}{E\cdot I}$ and $\bar{k}_{viga}$ is the beam stiffness matrix in local coordinates, including the influence of the axial force N.Let us note that we have identified, for the differentiation of linear stiffness matrix k_{viga}, by the stroke located on k, and which depends, viaω,on the value of the axial force on the beams (N).

Apparently, the expression of the stiffness matrix of the beam expressed in equation (18) $\bar{k}_{viga}$, is quite different from the known k_{viga}, without the inclusion of the axial force. However, we can find a similar expression when we consider that

$$tg\frac{\omega\cdot L}{2} = \frac{1-\cos(\omega\cdot L)}{sen(\omega\cdot L)} \tag{20}$$

and if we call to

$$\Psi_1 = \frac{s\cdot(1+c)}{6} - \frac{N\cdot L^2}{12\cdot E\cdot I}; \quad \Psi_2 = \frac{s\cdot(1+c)}{6}; \quad \Psi_3 = \frac{s}{4}; \quad \Psi_4 = \frac{s\cdot c}{2} \tag{21}$$

where c and s are adimensional functions, called *stability functions*

$$c = \frac{\omega\cdot L - sen(\omega\cdot L)}{sen(\omega\cdot L) - \omega\cdot L\cdot\cos(\omega\cdot L)}; \qquad s = \frac{[1-\omega\cdot L\cdot cotg(\omega\cdot L)]\cdot\omega\cdot L/2}{tg\frac{\omega\cdot L}{2} - \frac{\omega\cdot L}{2}} \tag{22}$$

which were originally derived by Lundquist and Kroll [5], and later developed by Merchant [6] under different methodology and in another context (Figure 4). We prefer the new original approach presented here[7], because we want to show the identity of procedures and goals achieved over time, even though at times the results appear to be different and have been obtained by using very different methodologies.

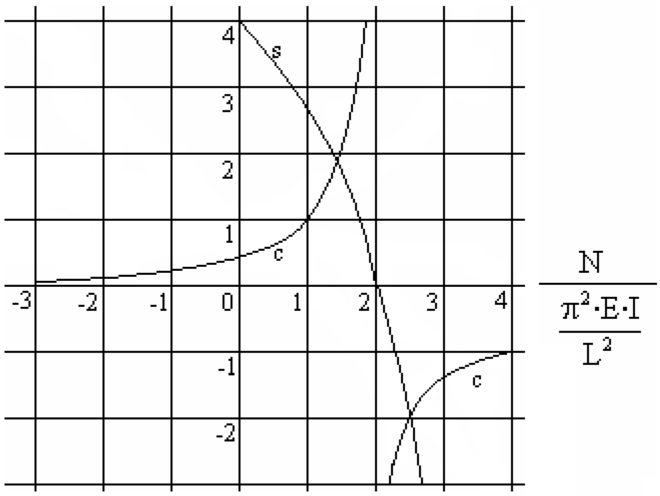

Figure 4: Stability Functions.

Operating conveniently we found that the matrix equation (16), for a beam rigidly connected at both ends and belonging to a 2D frame, is written as

$$\begin{aligned} p^{e1} &= \bar{k}^{11}\cdot d^{e1} + \bar{k}^{12}\cdot d^{e2} \\ p^{e2} &= \bar{k}^{21}\cdot d^{e1} + \bar{k}^{22}\cdot d^{e2} \end{aligned} \qquad (23)$$

or in matrix form

$$\begin{pmatrix} p^{e1} \\ p^{e2} \end{pmatrix} = \begin{pmatrix} \bar{k}^{11} & \bar{k}^{12} \\ \bar{k}^{21} & \bar{k}^{22} \end{pmatrix} \cdot \begin{pmatrix} d^{e1} \\ d^{e2} \end{pmatrix} \qquad (24)$$

being

$$\bar{k}^{11} = \begin{pmatrix} \frac{E\cdot A}{L} & 0 & 0 \\ 0 & \frac{12\cdot E\cdot I\cdot \Psi_1}{L^3} & \frac{6\cdot E\cdot I\cdot \Psi_2}{L^2} \\ 0 & \frac{6\cdot E\cdot I\cdot \Psi_2}{L^2} & \frac{4\cdot E\cdot I\cdot \Psi_3}{L} \end{pmatrix};\ \bar{k}^{12} = (\bar{k}^{21})^T = \begin{pmatrix} \frac{-E\cdot A}{L} & 0 & 0 \\ 0 & \frac{-12\cdot E\cdot I\cdot \Psi_1}{L^3} & \frac{6\cdot E\cdot I\cdot \Psi_2}{L^2} \\ 0 & \frac{-6\cdot E\cdot I\cdot \Psi_2}{L^2} & \frac{2\cdot E\cdot I\cdot \Psi_4}{L} \end{pmatrix}$$

$$\bar{k}^{22} = \begin{pmatrix} \frac{E\cdot A}{L} & 0 & 0 \\ 0 & \frac{12\cdot E\cdot I\cdot \Psi_1}{L^3} & \frac{-6\cdot E\cdot I\cdot \Psi_2}{L^2} \\ 0 & \frac{-6\cdot E\cdot I\cdot \Psi_2}{L^2} & \frac{4\cdot E\cdot I\cdot \Psi_3}{L} \end{pmatrix} \qquad (25)$$

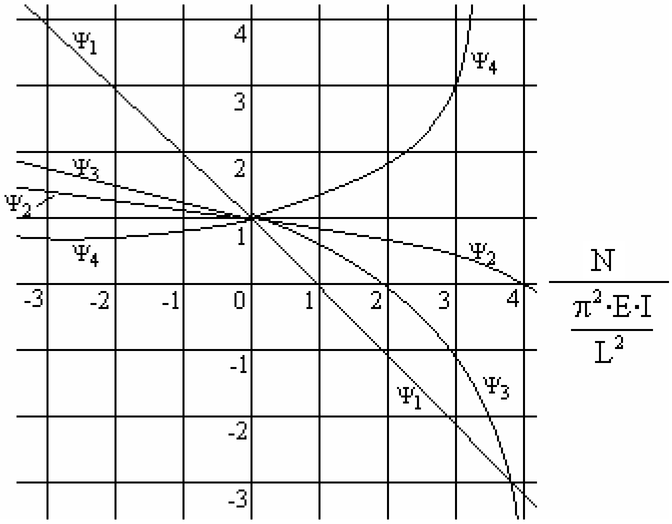

Figure 5.Stability Functions. Relation between axial force and Euler critical load.

Comparing these expressions with the known k_{ij}, it is clear that the functions Ψ are simply multiplier factors of the coefficients of the stiffness matrix of a beam without axial forces and can conveniently be expressed as functions of the relationship between axial force N and the Euler critical load $N_{cr} = \frac{\pi^2 \cdot E \cdot I}{L^2}$, as shown in Figure 5. Note that all Ψ are 1 for N = 0, so that $\bar{k}_{viga} = k_{viga}$ for N = 0.

3 GEOMETRIC STIFFNESS OF A COLUMN BELONGING TO SIMPLE SHEAR MODEL

If the beam is one of the groups of carriers of a frame modeled as a simple shear model, Figure 6, and we take the global axes of the figure

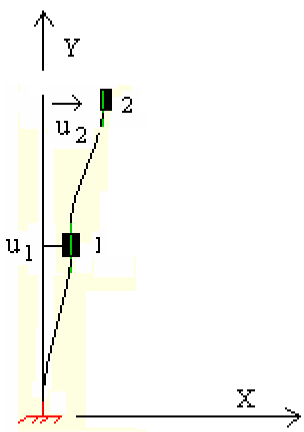

Figure 6: Simple Shear Model for a frame of 2 storys.

then equation (18) can be written as

$$\begin{pmatrix} p^{e1x} \\ p^{e1y} \\ m^{e1} \\ p^{e2x} \\ p^{e2y} \\ m^{e2} \end{pmatrix} = \begin{pmatrix} \frac{E\cdot A}{L} & 0 & 0 & -\frac{E\cdot A}{L} & 0 & 0 \\ 0 & \frac{\omega^3 \cdot sen(\omega\cdot L)}{f(\omega)} & \frac{\omega^2\cdot(1-\cos(\omega\cdot L))}{f(\omega)} & 0 & -\frac{\omega^3 \cdot sen(\omega\cdot L)}{f(\omega)} & \frac{\omega^2\cdot(1-\cos(\omega\cdot L))}{f(\omega)} \\ 0 & \frac{\omega^2\cdot(1-\cos(\omega\cdot L))}{f(\omega)} & \frac{\omega\cdot(sen(\omega\cdot L)-\omega\cdot L\cdot\cos(\omega\cdot L))}{f(\omega)} & 0 & -\frac{\omega^2\cdot(1-\cos(\omega\cdot L))}{f(\omega)} & \frac{\omega\cdot(\omega\cdot L - sen(\omega\cdot L))}{f(\omega)} \\ -\frac{E\cdot A}{L} & 0 & 0 & \frac{E\cdot A}{L} & 0 & 0 \\ 0 & -\frac{\omega^3 \cdot sen(\omega\cdot L)}{f(\omega)} & -\frac{\omega^2\cdot(1-\cos(\omega\cdot L))}{f(\omega)} & 0 & \frac{\omega^3 \cdot sen(\omega\cdot L)}{f(\omega)} & -\frac{\omega^2\cdot(1-\cos(\omega\cdot L))}{f(\omega)} \\ 0 & \frac{\omega^2\cdot(1-\cos(\omega\cdot L))}{f(\omega)} & \frac{\omega\cdot(\omega\cdot L - sen(\omega\cdot L))}{f(\omega)} & 0 & -\frac{\omega^2\cdot(1-\cos(\omega\cdot L))}{f(\omega)} & \frac{\omega\cdot(sen(\omega\cdot L)-\omega\cdot L\cdot\cos(\omega\cdot L))}{f(\omega)} \end{pmatrix} \cdot \begin{Bmatrix} d^{e1x} = 0 \\ d^{e1y} = U^1 \\ \vartheta^1 = 0 \\ d^{e2x} = 0 \\ d^{e2y} = U^2 \\ \vartheta^2 = 0 \end{Bmatrix} \quad (26)$$

Therefore, we obtain

$$p^{e1x} = p^{e2x} = 0 \; ; \; p^{e1y} = -p^{e2y} = -\frac{\omega^3 \cdot sen(\omega\cdot L)}{f(\omega)}\cdot\left(U^2 - U^1\right) \; ; \; m^{e1y} = m^{e2y} = -\frac{\omega^2\cdot\left(1-\cos(\omega\cdot L)\right)}{f(\omega)}\cdot\left(U^2 - U^1\right) \quad (27)$$

As for N=0, the functions $\frac{\omega^3 \cdot sen(\omega\cdot L)}{f(\omega)}$ y $\frac{\omega^2\cdot\left(1-\cos(\omega\cdot L)\right)}{f(\omega)}$ are preferred indeterminate, and we prefer to develop them using Taylor series and limiting the developments to the first two terms, we obtain

$$\begin{aligned} &\frac{1}{f(\omega)} = \frac{E\cdot I}{2-2\cdot\cos(\omega\cdot L)-\omega\cdot L\cdot sen(\omega\cdot L)} = \frac{12\cdot E\cdot I}{\omega^4\cdot L^4}\cdot\left(1+\frac{\omega^2\cdot L^2}{15}+\frac{67\omega^4\cdot L^4}{25200}+....\right) \\ &\frac{\omega^3\cdot sen(\omega\cdot L)}{f(\omega)} = \omega^3\cdot\left(\omega\cdot L-\frac{\omega^3\cdot L^3}{3!}+\frac{\omega^5\cdot L^5}{5!}+....\right)\cdot\frac{12\cdot E\cdot I}{\omega^4\cdot L^4}\left(1+\frac{\omega^2\cdot L^2}{15}+....\right) = \frac{12\cdot E\cdot I}{L^3}-\frac{36\cdot(1+\alpha)\cdot N}{30\cdot L} \\ &\frac{\omega^2\cdot(1-\cos(\omega\cdot L))}{f(\omega)} = \omega^2\cdot\left(1-1+\frac{\omega^2\cdot L^2}{2!}-\frac{\omega^4\cdot L^4}{4!}+....\right)\cdot\frac{12\cdot E\cdot I}{\omega^4\cdot L^4}\left(1+\frac{\omega^2\cdot L^2}{15}+....\right) = \frac{6\cdot E\cdot I}{L^2}-\frac{(1+\alpha)\cdot N}{10} \end{aligned} \quad (28)$$

If we proceed similarly for all matrix elements, the second addends constitute the geometric stiffness matrix of the first order. If we increase the terms of the development, the third summands constitute the geometric stiffness matrix of second order and so on.

For the shear efforts at beam ends we have

$$p^{e1y} = -p^{e2y} = -\left(\frac{12\cdot E\cdot I}{L^3}-\frac{36\cdot(1+\alpha)\cdot N}{30\cdot L}\right)\cdot\left(U^2-U^1\right) \quad (29)$$

Thus, the stiffness of the column of the media set of a frame modeled as a simple shear model is

$$\bar{k} = \left(\sum\frac{12\cdot E\cdot I}{L^3}\right)-\frac{36\cdot(1+\alpha)\cdot N}{30\cdot L} \quad (30)$$

Where $\frac{12\cdot E\cdot I}{L^3}$ is the stiffness of each of the columns forming the group and N is the area weight condition of all floors above the considered column. As already discussed, α marks the level of initial imperfections, in percentage terms, relative to the deformed column.

To circumvent the approach that involves limiting the Taylor series expansion of ψ_1 to the first two terms, and also based on the representation of Figure 5, we can choose to use a very approximate analytical expression for ψ_1. If we call $\beta = \frac{N}{\pi^2 \cdot E \cdot I / L^2}$ the function ψ_1 is given by $\psi_1 = -\beta + 1$, in which case the stiffness of the media set of columns of a frame modeled as a simple shear model is

$$\bar{k} = \sum_{i=1a\ n^o pilares} \frac{12 \cdot E \cdot I_i}{L_i^3} \cdot (1 - \beta_i) \qquad (31)$$

Although the differences obtained by taking (30) or (31) are not significant, we have opted for the latter as being more accurate, even slightly, and is implemented in the program for use with equal ease.

4 RESULTS OF CALCULATING THE BUILDING TAKEN AS AN EXAMPLE

Case A) Two Story Type Building.

For the ratio of stiffnesses supposed in paragraph 1, the bending moments at the top of the column in the ground floor and at the end of the lintel for static loads are

$$M_\varphi^{pilar} \simeq \frac{q \cdot L^2}{40} = \frac{41 \cdot 5 \cdot 5^2}{40} = 31.2\ kN \cdot m\ ;\ M_\varphi^{dintel} \simeq \frac{q \cdot L^2}{25} = \frac{41 \cdot 5 \cdot 5^2}{25} = 50\ kN \cdot m \qquad (32)$$

The Interstory Drift Sensitivity Coefficients and maximum deflections are

Interstory Drift Sensitivity Coefficient	
$\xi 1$ = 0,0735	$\xi 2$ = 0,0193
uMAX	
Without consideration to axial force	Coupling of bending moment and axial forces
U_1	$\overline{U}_1$
0,0335	0,0353
0,0405	0,0421

Table 1:Interstory Drift Sensitivity Coefficient for 2 Story Frame.

The stiffness k_p of the ground floor columns group is k_p = 7.0182 KN/m and the increases of shear force and bending moment, as a result of considering the coupling of axial bending, are

$$\Delta Q = \frac{12 \cdot E \cdot I}{L^3} \cdot \left(U_1 - \overline{U}_1 \right) = \frac{7018.2 \cdot 10^3}{5} \cdot \left(0.0353 - 0.0335 \right) = 2.53\ kN \qquad (33)$$

$$\Delta M_\varphi = \frac{\Delta Q \cdot L}{2} = 5.05\ kN \cdot m \qquad (34)$$

The latter increase in the bending moment on the column represents an increase of 16.2% from the value of the moment due to gravity loads, and an increase of 5% compared with the sum of the value of the moments due to static loads more dynamic loading. However, increases incurred as a result of considering a typical level of initial imperfections are not significant.

Case B) Four Story Type Building.

For the ratio of stiffnesses supposed in paragraph 2, the bending moments at the top of the column in the ground floor and at the end of the lintel for gravity loads are

$$M_{\varphi}^{pilar} \simeq \frac{q \cdot L^2}{40} = \frac{41 \cdot 5 \cdot 5^2}{40} = 31.2 \text{ kN} \cdot \text{m} \; ; \; M_{\varphi}^{dintel} \simeq \frac{q \cdot L^2}{20} = \frac{41 \cdot 5 \cdot 5^2}{20} = 62 \text{ kN} \cdot \text{m} \tag{35}$$

The Interstory Drift Sensitivity Coefficients and maximum deflections are:

Interstory Drift Sensitivity Coefficient	
ξ1 = 0,0709	ξ2 = 0,0279
ξ3 = 0,0386	ξ4 = 0,0193

uMAX	
Without consideration to axial force	Coupling of bending moment and axial forces
U_1	$\overline{U}_1$
0,0264	0,0282
0,0346	0,0364
0,0474	0,0493
0,0544	0,0562

Table2: Interstory Drift Sensitivity Coefficient for 4 Story Frame.

The stiffness k_p of the ground floor columns group is k_p = 14555 kN/m and the increases of shear force and bending moment, as a result of considering the coupling of axial bending, are

$$\Delta Q = \frac{12 \cdot E \cdot I}{L^3} \cdot \left(U_1 - \overline{U}_1 \right) = \frac{14555 \cdot 1}{5} \left(0.0282 - 0.0264 \right) = 5.24 \text{ kN} \tag{36}$$

$$\Delta M_{\varphi} = \frac{\Delta Q \cdot L}{2} = 10.48 \text{ kN} \cdot \text{m} \tag{37}$$

This latest increase of the bending moment on the column represents an increase of 33.59% from the value of the moment due to gravity loads, and an increase of 5.7% from the sum of the value of the moments due to gravity loads more dynamic loading. However, increases incurred as a result of considering a typical level of initial imperfections are not significant.

Case C) Seven Story Type Building.

For the ratio of stiffnesses supposed in paragraph 2, the bending moments at the top of the column in the ground floor and at the end of the lintel for gravity loads are

$$M_{\varphi}^{pilar} \simeq 1.15 \cdot \frac{q \cdot L^2}{35} = 1.15 \cdot \frac{41 \cdot 5 \cdot 5^2}{35} = 40.7 \text{ kN} \cdot \text{m} \; ; \; M_{\varphi}^{dintel} \simeq \frac{q \cdot L^2}{17.5} = \frac{41 \cdot 5 \cdot 5^2}{17.5} = 70.8 \text{ kN} \cdot \text{m} \tag{38}$$

The Interstory Drift Sensitivity Coefficients and maximum deflections are:

Interstory Drift Sensitivity Coefficient

ξ1 = 0,0392	ξ2 = 0,0302	ξ3 = 0,0251
ξ4 = 0,0193	ξ5 = 0.0279	ξ6 = 0.0386
	ξ7 = 0.0193	

uMAX

Without consideration to axial force	Coupling of bending moment and axial forces
U_1	$\overline{U}_1$
0,0126	0,0130
0.0201	0,0206
0,0266	0,0272
0,0370	0,0378
0.0454	0.0463
0.0588	0.0601
0.0668	0.0681

Table3: Interstory Drift Sensitivity Coefficient for 7 Story Frame.

The stiffness $k_p$of the ground floor columns group is k_p= 45999 kN/m and the increases in bending moment and shear forces, as a result of considering the coupling of axial deflection, are

$$\Delta Q = \frac{12 \cdot E \cdot I}{L^3} \cdot \left(U_1 - \overline{U}_1 \right) = \frac{45999}{5} \left(0.0130 - 0.0126 \right) = 3.68 \text{ kN} \tag{39}$$

$$\Delta M_\varphi = \frac{\Delta Q \cdot L}{2} = 7.36 \text{ kN} \cdot \text{m} \tag{40}$$

This latest increase of the bending moment on the column represents an increase of 18.08% over the value of the moment due to gravity loads, and an increase of 2.7% from the sum of the value of the moments due to static loads more dynamic loading. However, increases incurred as a result of considering a typical level of initial imperfections are not significant.

Case D) Ten Story Type building.

For the ratio of stiffnesses supposed in paragraph 2, the bending moments at the top of the column in ground floor and at the end of the lintel for gravity loads are

$$M_\varphi^{pilar} \simeq \frac{q \cdot L^2}{30} = \frac{41 \cdot 5 \cdot 5^2}{30} = 41.3 \text{ kN} \cdot \text{m} \; ; \; M_\varphi^{dintel} \simeq \frac{q \cdot L^2}{15} = \frac{41 \cdot 5 \cdot 5^2}{15} = 82.6 \text{ kN} \cdot \text{m} \tag{41}$$

The Interstory Drift Sensitivity Coefficients and maximum deflections are:

Interstory Drift Sensitivity Coefficient

ξ1 = 0,0230	ξ2 = 0,0109	ξ3 = 0,0236
ξ4 = 0,0206	ξ5 = 0,0302	ξ6 = 0,0251
ξ7 = 0,0372	ξ8 = 0,0279	ξ9 = 0,0386
	ξ10 = 0,0193	

uMAX

Without consideration to axial force	Coupling of bending moment and axial forces

U_1	$\overline{U_1}$
0,0066	0,0067
0,009	0,0091
0,0145	0,0146
0,0195	0,0196
0,027	0,0273
0,0335	0,0339
0,0438	0,0444
0,0522	0,053
0,0661	0,0673
0,0746	0,0759

Table 4.Interstory Drift Sensitivity Coefficient for 10 Story Frame.

The stiffness k_p of the ground floor columns group is k_p = 112150 KN/m and the increases in bending moment and shear forces, as a result of considering the coupling of axial deflection, are

$$\Delta Q = \frac{12 \cdot E \cdot I}{L^3} \cdot \left(U_1 - \overline{U}_1 \right) = \frac{112150}{5} \left(0.0067 - 0.0066 \right) = 2.24 \text{ kN} \tag{42}$$

$$\Delta M_\varphi = \frac{\Delta Q \cdot L}{2} = 4.49 \text{ kN} \cdot \text{m} \tag{43}$$

This latest increase of the bending moment on the column represents an increase of 10.87% from the value of the time due to gravity loads, and an increase in 1.35% from the sum of the value of the moments due to gravity loads more dynamic loading. However, increases incurred as a result of considering a typical level of initial imperfections are not significant.

5 DIMENSIONING INTERSTORY DRIFT SENSITIVITY COEFFICIENT

In the seismic calculation of a building, Spanish Instructions NCSE-02 and therefore, the CTE, establish in paragraph 3.7.7 that: “As long as the collapse of the head of the building does not exceed two per thousand of the height, it is not necessary to consider the 2nd order effects”.

On the other hand, it will not be necessary to consider the 2nd order effects, in line with that set by Eurocode Nº 8 (Paragraph1-2 Part 1-1), if for all storys the Interstory Drift Sensitivity Coefficient is less than 0.1.

As we can see, what is actually limiting the destabilizing moment is that the column is less than 10% of the stabilizing moment. Because the criterion of the 10% limit seems to be arbitrary, we propose to replace it with another limit that is more in line with the structural reality.

If we assume a square section column of side "b", for frequently used materials and for the dimensions set out in paragraph 1, the relationship between the critical load and the axial calculation is $\frac{N_{crit}}{N_{cálc}} \simeq 150 \cdot b^2$, so that we obtain $N_{calc} = 0.075 \cdot N_{crit}$ for b = 0.30m, and for b = 0.35m (corresponding to buildings of lesser heights) we have $N_{calc} = 0.055 \cdot N_{crit}$. Accordingly, we adopt

$$N_{calc} < 0.075 \cdot N_{crit} \tag{44}$$

Under this assumption,coefficient β, defined in paragraph 3, $\beta = \frac{N}{\pi^2 \cdot E \cdot I / L^2} = \frac{N_{calc}}{N_{crit}}$, gives

$$\beta \cdot N_{crit} = N_{calc} < 0.075 \cdot N_{crit} \tag{45}$$

Keeping the definition for the interstory drift sensitivity coefficient and by taking into account formula (27), we can write

$$\xi = \frac{N \cdot d}{F \cdot h} = \frac{N \cdot d}{\frac{12 \cdot E \cdot I}{L^3} \cdot \psi_1 \cdot d \cdot L} = \frac{\pi^2 \cdot \beta}{12 \cdot \psi_1} \tag{46}$$

From which we deduce $\beta = \frac{12 \cdot \psi_1 \xi}{\pi^2}$ and as $\psi_1 = -\beta + 1$,we obtain $1 - \psi_1 = \frac{12 \cdot \psi_1 \xi}{\pi^2}$. So,

$$\psi_1 = \frac{1}{1.216 \cdot \xi + 1} \tag{47}$$

By virtue of (45), it must be satisfiedthat $\beta < 0.075$, i.e., $1 - \frac{1}{1.216 \cdot \xi + 1} < 0.075$. From which we deduce

$$\xi < 0.0667 \tag{48}$$

which is consistent with the results obtained in the practical cases A) and B) of paragraph 4.

6 CONCLUSIONS

In the seismic calculation of a building, Spanish Instructions NCSE-02 and thus, the CTE, establish in paragraph 3.7.7 that: “As long as the collapse of the head of the building does not exceed two per thousand of the height, it is not necessary to consider the effects of 2nd order”.

On the other hand, it will not be necessary to consider the 2nd order effects ($P-\Delta$ effects), in line with that set by Eurocode 8 (Paragraph1-2 Part 1-1), if for all storys the interstory drift sensitivity coefficient fulfills:

$$\xi = \frac{P \cdot d}{F \cdot h} \leq 0.1 \tag{49}$$

We show in this discussion, based on the current state of knowledge, it is not justified to ignore the 2nd order effects ($P-\Delta$ effects):

1)Because it is not justified that the spectral modal analysis be simplified, because a high-level program such as Matlab can very easily address a multimodal spectral analysis study for frames, the simple model of shear or associating the nodes of the structure of the inertial properties of the frame beams and performing a calculation of stiffness.

2)Because the inclusion of 2nd order effects (effects), does not add any conceptual effort nor complicate the calculation. Additionally, if we follow the line of thought of Bazant [8] and truly think that the equilibrium, static or dynamic loading, occurs in the deformed geometry, it is logical that the calculation includes this.

3) Because we show that although the Interstory Drift Sensitivity Coefficient does not reach the value of 0.1, increases of the bending moment at the ends of the columns resulting from the refinement of the calculation including the second order effects, can account for between 15% and 34% of its value for static service loads.

4) Because most adverse effects are shown in lower height buildings (up to 5 floors), which is precisely the range in which most of the housing stock of Spain is located, to be in the highest limit allowed by the various planning regulations (PGOU).

5) As in section 5, we limited the Interstory Drift Sensitivity Coefficient to 0.0667 for buildings of lower height (up to 5 floors), so we propose that it is generally lowered to $\xi =\leq 0.06$

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