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SEISMIC ANALYSIS OF MASONRY ARCH STRUCTURES THROUGH THE FINITE ELEMENT MODEL "BLOCK-JOINT"

Francesco Pugi[1]

[1] AEDES Software
San Miniato Basso 56028, Pisa, Italy
e-mail: info@aedes.it

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Abstract. *In monumental buildings vaulted structures and arches are structures of great importance. The traditional approach with the analysis of a rigid-brittle model constitutes a reference method but it is affected by some operational limits, such as, the use of a planar model which cannot represent effectively spatial systems which include adjacent structures (walls, pillars). Moreover, disregarding elasticity, modal analysis cannot be performed. An alternative methodology is proposed. Using simple one-dimensional finite element (blocks - joints) the collapse mechanism is obtained through an incremental elastic-brittle analysis. The method easily finds a balanced solution compatible with the mechanical characteristics of the material. Also, considering the tensile stress of the mortar joints, it is possible to evaluate more accurately the effective seismic resistance of the structure. The proposed method is applied to study an important religious monument.*

1 INTRODUCTION

Under seismic loading, the existing masonry buildings and historic structures are frequently affected by partial collapses due to loss of equilibrium of portions of the structure. Local mechanisms occur in masonry walls because of out-of-plane actions, and in case of arches also under in-plane actions. In order to assess the seismic safety of the structure, the verifications in terms of local collapse mechanisms can be carried out by means of limit analysis following the kinematic approach proposed by the current standards [1-3].

The most common method of analysis, also described in the standards [3], is based on the evaluation of the horizontal action that activates the collapse mechanism selected a priori. The relevant collapse mechanisms can be assumed on the base of the seismic behaviour of similar structures or defined according to the state of damage of the structure.

In this approach, however, the selection a priori of a proper collapse mechanism is not an easy task given the wide variety of the cases that can be examined. For example, not all the arches are symmetric or have symmetrical loads; some structures may differ significantly from the reference models as in the case of complex vaults set on walls or pillars or in the case of walls with an irregular layout of the openings.

Therefore, an accurate modelling of the monumental structures requires the adoption of methodologies capable to determine the collapse mechanism avoiding the uncertainties of the selection a priori. For this reason specific methods have been developed for some important structural typologies.

Algorithms able to define the collapse mechanism of arches and vaults under seismic action [4] were developed following the Heyman theory and an approach based on rigid-brittle elements. This methodology is described in section 2.

However, the definition of the collapse mechanism, or more properly the seismic action that activates the mechanism, can also be pursued with traditional finite element methods, according to nonlinear procedures that take into account the progressive damage of the structures. At first, the structure, generally strongly hyperstatic, is analysed under vertical loads in the elastic range. The application of increasing horizontal forces leads to a gradual loss of hyperstaticity, until the structure becomes isostatic and at last unstable: at that point the collapse mechanism occurs. This type of incremental analysis, proposed as an alternative to limit analysis in recent standards [3], is described in section 3.

The result of the nonlinear static analysis is a force-displacement diagram (the capacity curve) that shows the relation between the increasing base shear and the control displacement that can be assumed at the centroid of the active masses.

In both approaches the maximum sustainable value of the horizontal action is pursued. In the kinematic approach (limit analysis) the action is obtained as the minimum force able to activate the mechanism, while in the static approach (static nonlinear analysis) the action pursued is the maximum force for which the structure remains in equilibrium. The horizontal force is due to the seismic action, therefore it is an inertial force and it corresponds to a mass multiplier. The collapse multiplier λ is defined dividing the force by the corresponding inertia.

Both the above mentioned methods aim to evaluate the collapse multiplier: its value can be assumed as a characteristic of the examined structures for given geometry, restraints, materials and applied loads.

The following paragraphs describe the two alternative methodologies for the definition of the collapse multiplier of arches and vaults.

2 LIMIT ANALYSIS

For masonry structures subjected to horizontal actions, the research of the collapse multiplier can be conducted through limit analysis using equilibrium conditions.

Limit analysis evaluates the condition of equilibrium of an unstable structure, such as an assembly of rigid wall parts in order to asses if the structure is statically determined under the applied loads. The deformation, which cannot be determined through limit analysis, is neglected assuming that the structure is still in equilibrium even in the deformed shape. However it should be considered that, dealing with existing structures, the geometrical configuration measured through geometrical survey is already the deformed one, therefore the limit analysis is capable to evaluate the actual safety level of the structure.

As said before, limit analysis methodologies are of two types: the ones based on a priori selected mechanisms, and the ones which try to determine the collapse mechanisms through calculations.

As far as the a priori selected mechanism, the recent Italian standards on monumental buildings [3] propose an abacus of different collapse mechanisms derived from the observation of the damages caused by mayor earthquakes.

In this paragraph, a limit analysis method that does not require the a priori selection of the collapse mechanism is presented. The method refers to the kinematic analysis of arches according to a rigid-brittle model [4], elaborated from Heyman's studies [5-6]. This approach, largely used in engineering practice [7], constitutes a solid reference for the calibration of alternative procedures.

Assuming that the arch is made of rigid blocks, each one able to transfer to the next one only shear and compression, the method searches – if exists – a thrust line which, under the applied load and with the structure in equilibrium, is defined within the geometric shape of the arch.

Where tensile forces occur, a hinge develops at the correspondent block interface and the thrust line at that point is tangent to the arch profile (at extrados or intrados). If necessary, a maximum of three hinges may be considered, after that a mechanism occurs. If it was possible to find an equilibrate solution under the given boundary conditions, the arch is stable.

Therefore, starting from a configuration in equilibrium under vertical loads, a system of horizontal forces is applied to the structure. Such forces are proportional to the vertical loads, that is, derived from the latters by applying a multiplier. Assessing the arch stability under increasing values of the horizontal forces, it is possible to determine the collapse multiplier, that is the maximum value of the multiplier for which the structure is still in equilibrium.

The rigid-brittle model presented in Figure 1 ignores the elasticity and the resistance of the materials. The procedure represents an important step forward with respect to traditional methods (such as the Méry method) where the position of the hinges is selected a priori. With this method instead, the hinges – if they develop – are considered in the actual position resulting from the analysis under the applied loads.

The original formulation of the procedure is for plane structures and can be used for the analysis of single arches and barrel vaults. Extensions of this procedure make possible its application even for cross vaults [7]. In general, these methodologies consider an infinite compressive strength and zero tensile strength.

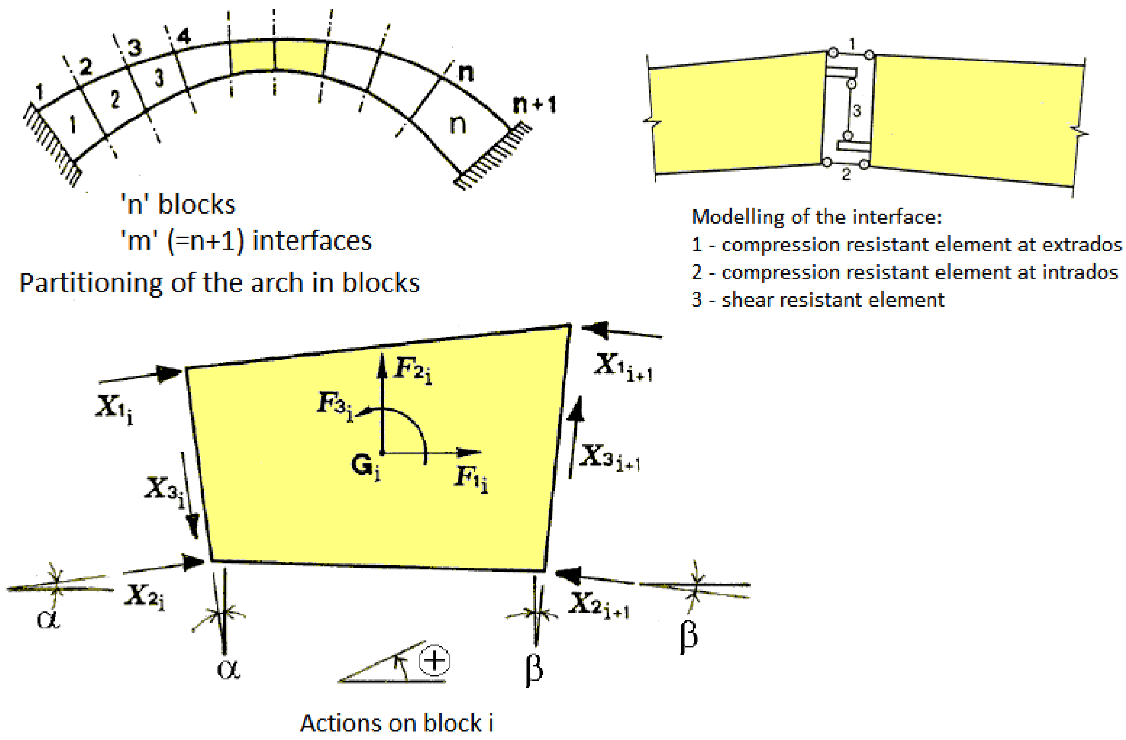

Figure 1. Rigid-brittle modelling of arch structures

3 NONLINEAR STATIC ANALYSIS: THE “BLOCK-JOINT” MODEL

As described above, according to the limit analysis approach the arch can be evaluated as a rigid-brittle system. However, the application of this method is subjected to important limits:
a) the tensile strength is considered to be null; thus, neglecting the real capacity of the mortar joints. In this way the collapse multiplier can be significantly underestimated and, although it would appear in favour of safety, this could lead to excessive strengthening interventions. The safety evaluation, in fact, should assess as close as possible the real behaviour of the structure in order to adequately design the interventions and avoid oversizing;
b) the formulation for plane structures does not allow evaluating the out-of-plane behaviour of the arches: for this purpose it is necessary to extend the method adopting more elements (at least 8) in order to correctly account for the axial force and the shear at the interface;
c) the structural scheme can be hardly generalized in case of systems of arches or structures that besides the arches include the adjacent structure or the supports (such as walls or piers);
d) the original formulation, based on a rigid-brittle behaviour, ignores elasticity; therefore, modal analysis cannot be performed.

Some of these issues can be overcome through a series of generalizations; however, finite element models and static nonlinear analyses already provide similar techniques, simple in their approach but just as effective and easily reproducible. The model proposed in this paper uses two types of finite elements, “blocks” and “joints”, under the assumption of an initial elastic behaviour of the structure.

Both blocks and joints are frame elements: the blocks have the mechanical properties of the stone and their cross section is the effective cross section of the arch, the joints represent the mortar between the blocks. Four joints for each block are used; therefore, the joint cross section is ¼ of the arch cross section. The joints have a pin-end and a fixed-end, the fixed-end provides continuity with the previous block, the pin-end transfers shear and axial force to the next block, while the joints are connected to the block ends through rigid links; in this manner the system transfers moment, shear and axial force from one block to another.

Performing a pushover analysis the system is subjected to an increasing horizontal force. At each step an axial force verification is applied to the joints: if tensile stresses occur, the internal axial force is released and the fixed-end is turned to pin so that the element loses any stiffness and the internal action remain constant at the value reached so far. As the analysis continues, the progressive deterioration of the joints lead to an unstable configuration that define the end of the pushover curve.

Assuming that the internal actions remain constant after deterioration of the joints, this procedure is capable to find easily a balanced solution compatible with the mechanical characteristics of the materials. The procedure can be applied considering or not a limited tensile strength of the material: if the tensile strength is assumed to be null a comparison with the rigid-brittle limit analysis is feasible. Passing from the rigid-brittle model to the elastic one, the collapse multiplier can either decrease or increase. However, in the elastic model, the adoption of a limited tensile strength (f_{tk}) surely leads to an increase of the collapse multiplier in comparison to the case in which it is considered to be null; thus, avoiding underestimation of the seismic capacity of the structure.

The block-joint system has the great advantage of adopting only one-dimensional elements; thus, it is easily implemented with any finite element solver.

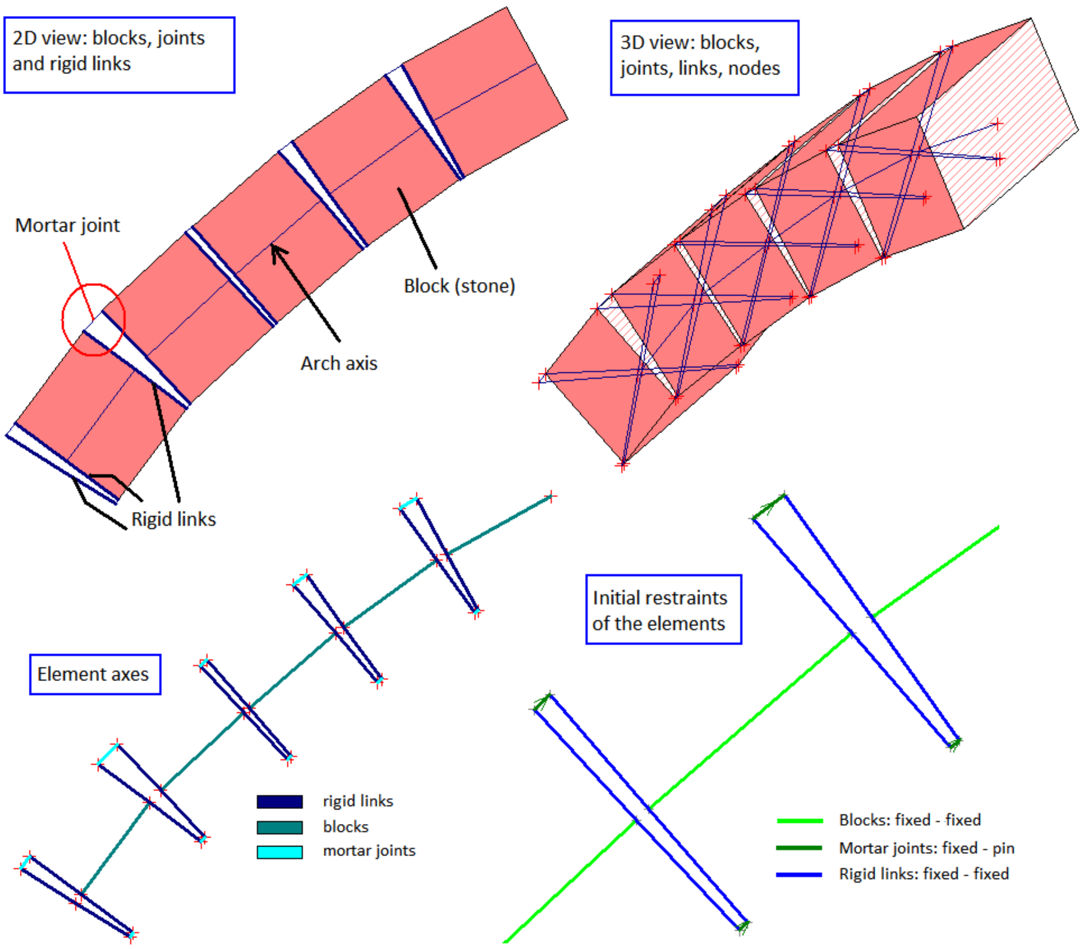

Figure 2. The finite element model block-joint

In the following several comments and remarks regarding the method are presented.

I. If excessive tensile stress occur at the joints, it is possible to intervene on the initial model modifying the internal restraints of the elements in traction. In fact, for a correct application of the nonlinear procedure, an equilibrate initial configuration is considered essential. For certain geometries and under certain loads an equilibrate static solution may not exist, this situation represents the maximum vulnerability since the system cannot withstand any horizontal seismic action. In this cases, immediate strengthening measures should be taken beyond the seismic issue.

II. As for the limit analysis procedure (cfr. Paragraph 2), the subdivision into blocks can be physical or mathematical; applications of the method show that the solution can be achieved even without a very dense subdivision. Therefore, an effective matching is not necessary between the frame elements and the real dimension of the blocks.

III. Being the modulus of elasticity of the stone much larger than the one of the mortar, the deformation is concentrated in the joints, while the blocks behave practically as rigid elements. For this reason the compression force verification is not applied to the blocks, while the tensile force verification is still applied. The linear dilatation of the block is

$$\Delta l = l_s - l_i$$

where l_s is the length of the deformed block, l_i is the original length. The linear deformation is given by

$$\varepsilon = \Delta l / l_i$$

If $\Delta l > 0$, the block is in elongation and the tensile stress is

$$\sigma_t = E \cdot \varepsilon$$

where E is the modulus of elasticity of the material. If the tensile stress exceeds the resistance of the stone, the internal restraints of the block are released as it happens for the joint elements. The deterioration of the block contributes to the mechanism affecting the value of the maximum horizontal force sustainable by the system.

IV. Since the blocks are modelled as one-dimensional prismatic elements, the joints assume a trapezoidal shape which at the extrados of the arches may be larger than the real one, especially if the arch is modelled with a limited number of blocks. This configuration, given the high deformability of the mortar, can lead to thrust lines not always coincident with the ones obtained with a rigid-brittle model; a series of comparative examples has proved that the difference is due to the internal distribution of the stresses. There are some adjustments that can be made in order to improve the block-joint model; for example, the blocks may be modelled with non-prismatic elements or the modulus of elasticity may be increased for the joints that are longer than the real ones. However, in order to preserve the ease of understanding and the repeatability of the modelling, a version of the block-joint model which disregards such adjustments has been adopted in this work.

4 CASE STUDY

The structure analysed is a great arch located in the Basilica of the Holy Sepulchre in Jerusalem. The church has been recently studied in order to evaluate its seismic vulnerability [8]. Among the several transformations undergone by the monumental building, in the period 1099-1167, the Crusaders after conquering Jerusalem rebuilt the church that was largely destroyed and they also built a new Choir (Figure 3) that remained essentially the same until today. The Crusaders created in this way a structure that still connect the Rotunda to the area of the Calvary.

Figure 3. Floor plan of the monument: Constantinian period (4th century) in black, restoration of Constantine Monomachos (11th century) in blue, structures built by the Crusaders (12th century) in red.

The floor plan of the Choir has a rectangular shape of about 12.5x24m, the apse is semi-circular with an external ray of 6.5m. The main arch that separates the Choir from the apse has an imposing size and outline the half dome on top of the apse. In recent studies [8] the apse was analysed together with the great arch; in this paper, however, we focus on the application of the finite element block-joint model only to the great arch, in order to provide a direct comparison with the rigid-brittle approach that would not be feasible dealing with a more complex structure.

Figure 4 shows the vertical section of the arch and the apse developed in 1955 by the Architects Rolando, Coupel, Antonian [10].

Figure 4. Vertical section of the apse

Figure 5 is a photo of the actual state of the structure. From left to right: (i) the half dome above the apse set on ten arches radially arranged; (ii) the great arches between the Choir and the apse analysed in this paper; (iii) the cross vault of the Choir. The great pointed arch between the Choir and the apse presents a large depth (166cm). Its thickness that at first sight appear quite large, due to the ventilation holes located above the keystone, cannot be assumed larger than 30cm. Above the arch sets a masonry wall having the same depth (166 cm) and a height over the keystone of about 160cm.

Figure 5. Internal view of the top of the apse

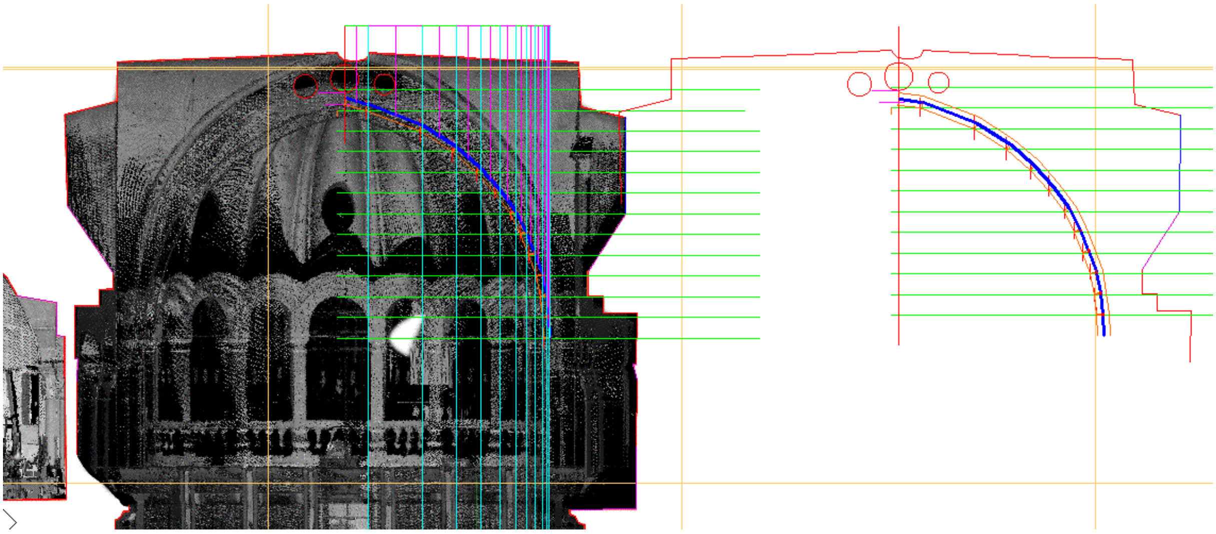

Figure 6. Study of the great arch between Choir and apse

Two structural models of the arch were analysed, the first according to the rigid-brittle approach (§4.1) and the second with the elastic nonlinear approach (§4.2) proposed in this paper.

4.1 Rigid-brittle model. Limit Analysis

The rigid-brittle model featuring zero tensile strength is presented in Figure 7. The arch was divided in blocks with a length of about 15cm.

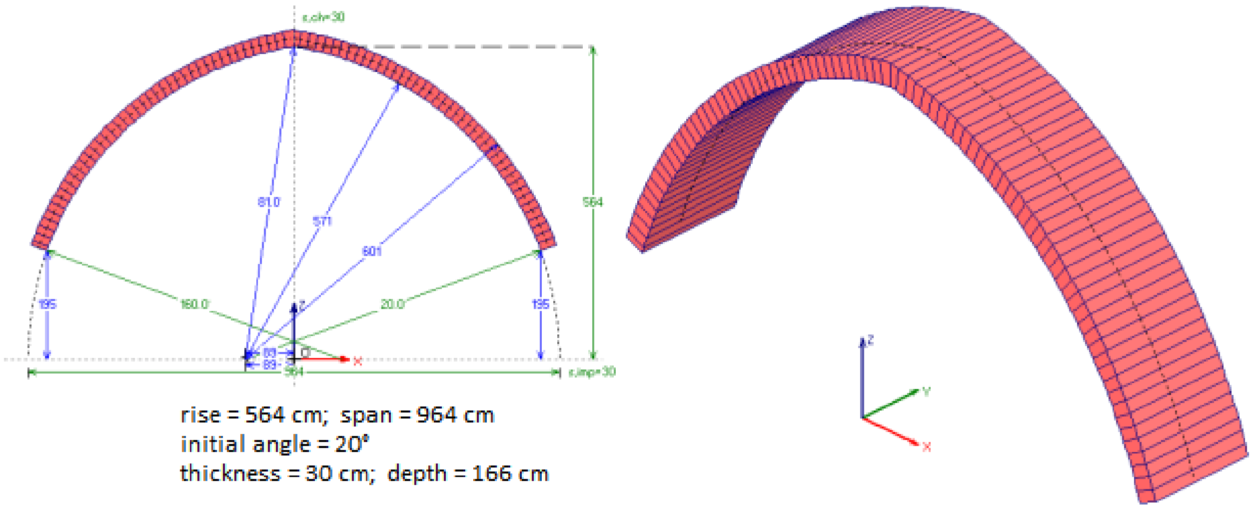

Figure 7. Rigid-brittle model of the arch

The characteristic aspects of the analysis are the following:

a) Mechanical characteristics
 - *Behaviour:* Rigid-brittle
 - *Tensile strength:* null
 - *Modulus of elasticity:* irrelevant

b) Analysis type and verifications
 - Limit analysis with analysis-determined collapse mechanism.
 - Equilibrium verification under static load and increasing seismic loads.
 - Calculation of the collapse multiplier for horizontal actions.
 - Definition of the reactions at the impost of the arch.

c) Model characteristics
 - *Geometry*: pointed arch with constant thickness and initial angle equal to 20°.
 - *Load patterns*: the self-weight and the other vertical loads are considered within the same load pattern.
 - *Restraints*: the arch is fixed at the imposts. The blocks are rigid and each block transfers to the next one only compressive stress. The joints are modelled with three frame element located at the block interfaces.
 - *Load combinations*: (i) static, with self-weight and vertical loads; (ii) seismic, with static loads and horizontal forces defined applying a multiplier to the each weight. The seismic analysis is performed for increasing values of the multiplier until the collapse mechanism occurs.

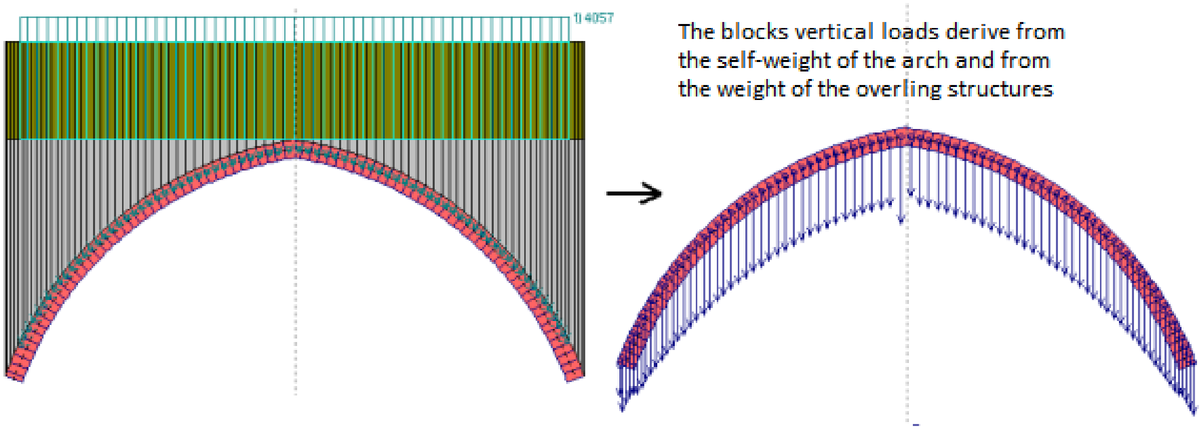

Figure 8. Loads acting on the arch

The structural analysis was performed with the aid of the software Aedes©SAV [7] according to the methods described in §2. Figure 9 presents the results of the static analysis. The arch is stable and under static load retains 3 degrees of hyperstaticity. The reaction at the impost are:

$$F_V = 839.29\ kN$$

$$F_H = 441.29\ kN$$

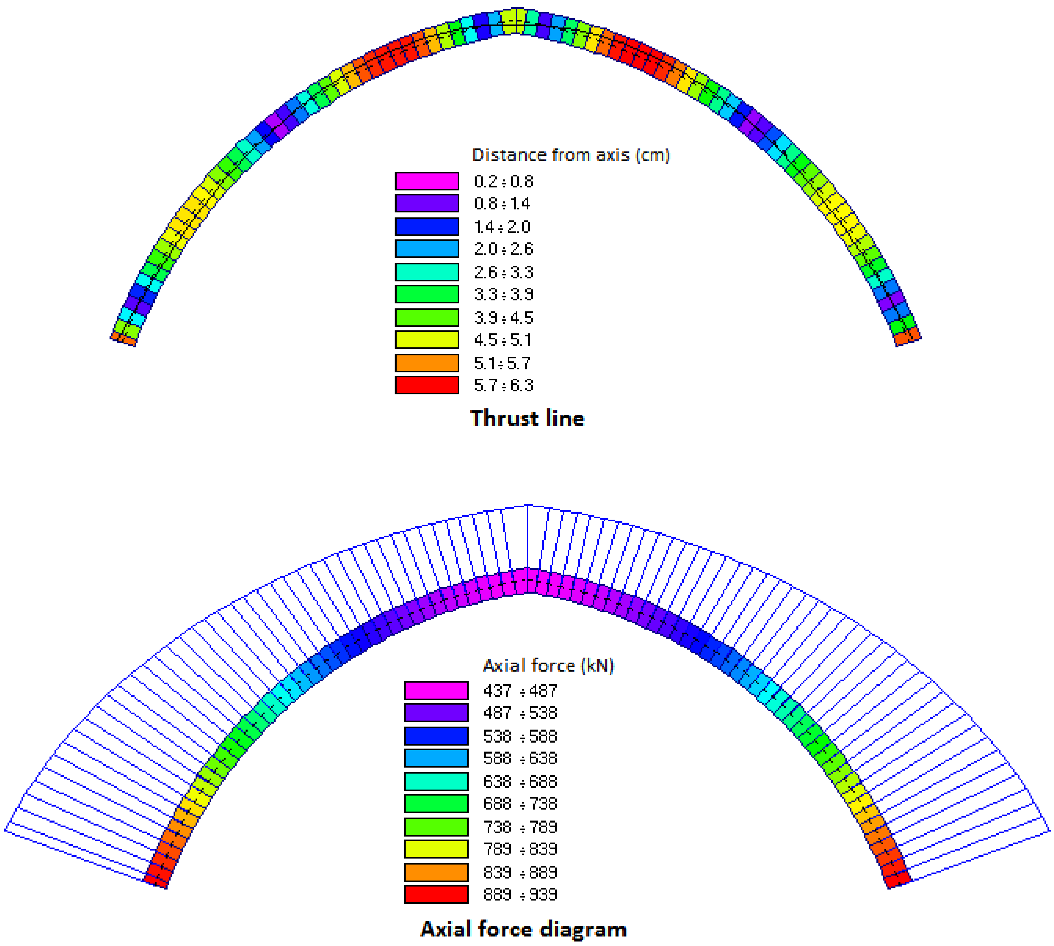

Figure 9. Static analysis results

The results of seismic analysis are provided in Figure 10.

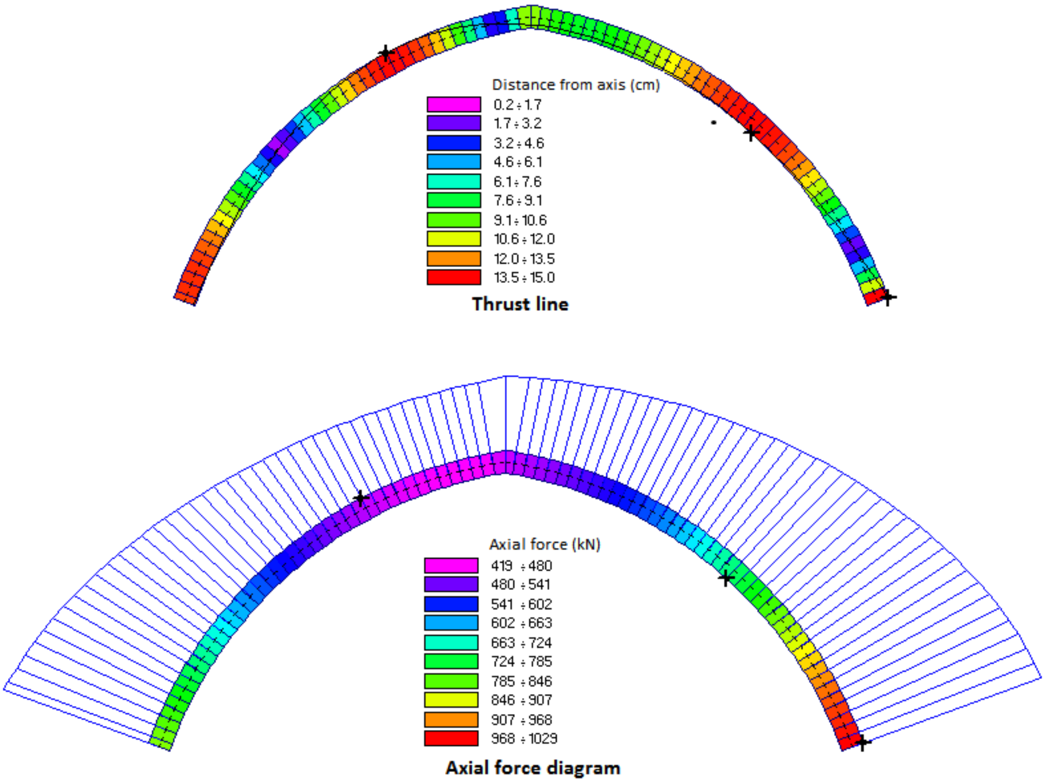

Figure 10. Seismic analysis results

The collapse multiplier is equal to 0.173; that is, the collapse mechanism shown in Figure 11 was activated applying a system of horizontal forces each one equal to the 17.3% of the corresponding weight. In the last configuration in equilibrium the arch was isostatic and presented three active hinges. A further increment of the horizontal loads led to the development of the fourth hinge at the upwind impost, then the collapse mechanism occurred. In the last configuration the reactions at the upwind (L) and at the downwind (R) imposts were

$$F_{V,L} = 794.81\ kN, \quad F_{H,L} = 293.30\ kN$$

$$F_{V,R} = 883.58\ kN, \quad F_{H,R} = 583.66\ kN$$

Therefore, for a multiplier $\lambda = 0.173$ the vertical reaction at the downwind impost increased by 6% while the horizontal reactions increased by 32%. Obviously the seismic event will provoke the thrust increment alternately at each impost.

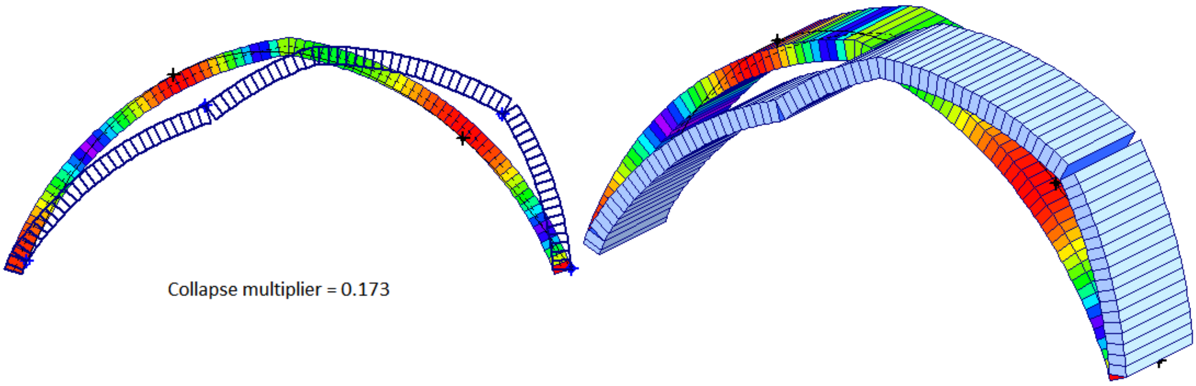

Figure 11. Collapse mechanism for seismic action

4.2 Block-joint model. Nonlinear static analysis

The analysis with the nonlinear elastic model, according to the method proposed in this paper, is able to assess the collapse multiplier taking into account the tensile strength of the mortar joints. In the structural model shown in Figure 12, first the mortar joints were considered to have zero tensile strength for a direct comparison with the rigid-brittle model results, then a tensile strength of 0.25 MPa was considered.

The characteristic aspects of the analysis are the following:

a) Mechanical characteristics
   - *Behaviour:* nonlinear elastic
   - *Tensile strength:* 0.00/0.25 MPa for the mortar joints, 3.50 MPa for the blocks
   - *Modulus of elasticity:* 660 MPa for the mortar joints, 50000 MPa for the blocks

b) Analysis type and verifications
   - Modal analysis. Static analysis.
   - Seismic static nonlinear analysis. Collapse mechanism due to the achievement of an unstable configuration after development of hinges.
   - Equilibrium verification under static load and increasing seismic loads.
   - Calculation of the collapse multiplier for horizontal actions.

c) Model characteristics
   - *Geometry:* arch modelled with block-joint system
   - *Load patterns:* (i) self-weight of the arch, (ii) weight of the overlying wall, (iii) weight of the fence on top.
   - *Load combinations:* (i) static, with self-weight and vertical loads; (ii) seismic, with static loads and increasing horizontal forces defined by predefined distributions of the total base shear.

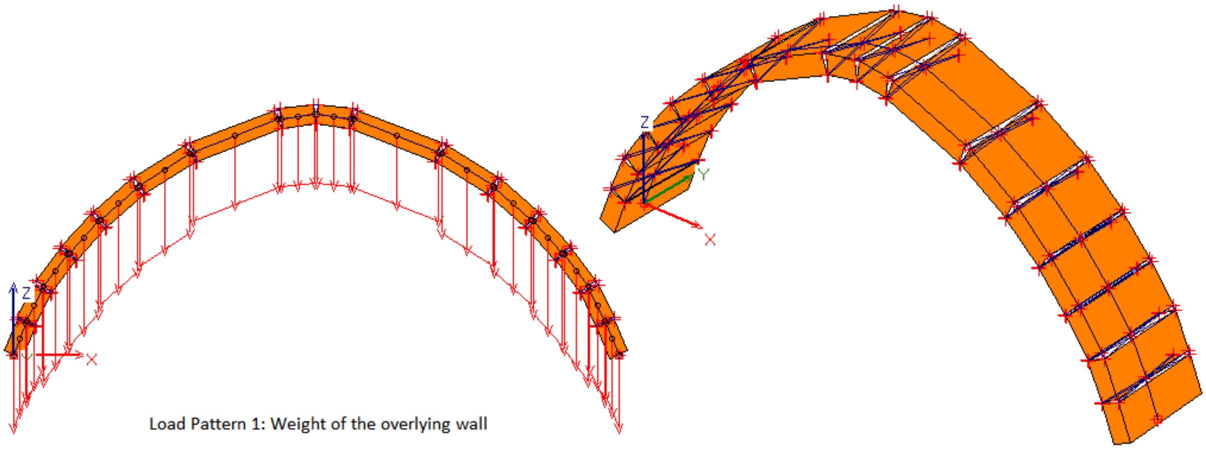

Figure 12. Nonlinear elastic model

The software Aedes©PCM [7] was used to perform the analysis.

Modal analysis is particularly significant in monumental buildings and the application to the arch proves the potential of a static approach. The knowledge of the modal characteristics allows to validate the structural model on the base of ambient vibration testing measurements and to evaluate the interaction with the adjacent macroelements.

The block-joint method allows to consider a dense distribution of the masses concentrated in the nodes which represents the real object well.

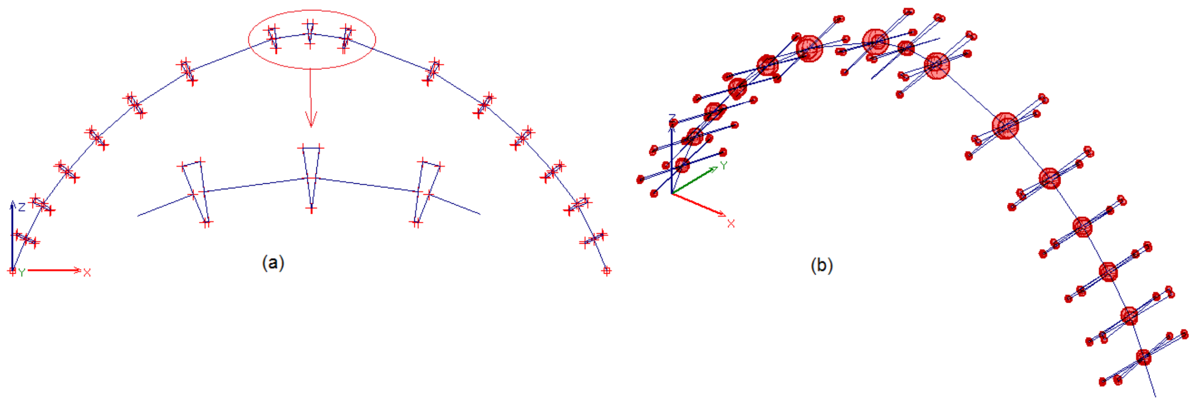

Figure 13. a) Structural model (frame elements and nodes). b) Active masses obtained from the weights applied to each node. The size of the sphere is proportional to the mass.

Given the modelling of the joints, the structure is not plane; thus, the model features 300 degrees-of-freedom and the masses are free to move in-plane and out-of-plane. Considering the in-plane behaviour (XZ plane), the first mode of vibration with a period of 0.197s (5.07 Hz) features a participating mass larger than the 85% of the total mass. Figure 14 shows the modal deformed shape.

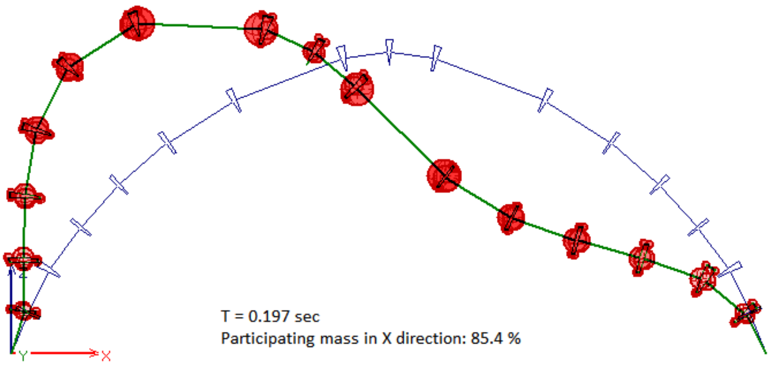

Figure 14. Modal analysis

According to the results of the non-seismic static analysis, the arch is stable and under the static loads it retains three degrees of hyperstaticity.

The nonlinear static analysis, performed applying to the structure increasing horizontal forces, allows to determine the collapse multiplier and to evaluate the capacity of the structure in terms of ground acceleration. The seismic verification that, beyond the collapse multiplier, takes into account the participating mass and the seismic action at the exact location of the structure, is independent from the model used to perform the analysis and is not presented in this paper. Attention will be paid, instead, to the calculation of the collapse multiplier with the two alternative approaches.

The nonlinear static analysis was performed applying two distributions of horizontal forces: (A) a linear distribution proportional to mass and height, that well represents the initial elastic behaviour of the structure; (E) a distribution proportional to mass and independent from height, that well represents the post-elastic phase.

The maximum sustainable force is assumed as the smallest of the ones obtained with the two distributions. The nonlinearity, that is, the non-proportionality between forces and displacements, depends on the variation of the structural model during the incremental analysis. At each step, a verification is applied to the structural elements and their mechanical characteristic and restraints are updated. In the block-joint model particularly significant is the tensile stress verification applied to the joints.

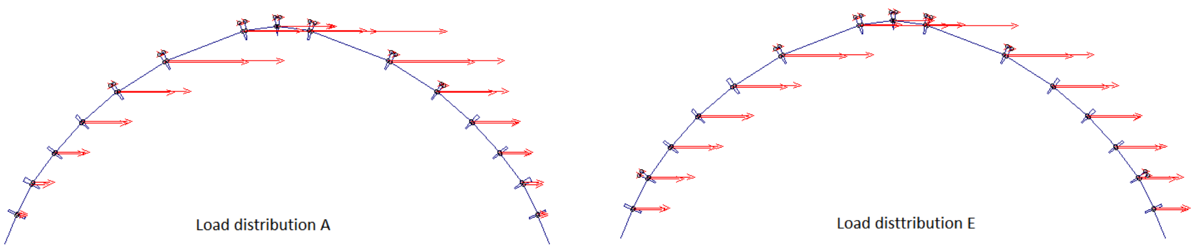

Figure 15. Load distributions in nonlinear static analysis

The total weight of the structure is 1670 kN, while the seismic active weight is 1611 kN. The nonlinear static analysis was performed with a base shear increment of 5 kN, considering first zero tensile strength of the joints and then a tensile strength equal to 0.25 MPa.

Table 1 shows the results obtained with zero tensile strength of the joints, for the two load distributions.

Distribution	Maximum force	Collapse multiplier
A	340 kN	0.211
E	415 kN	0.258

Table 1. Analysis results with zero tensile strength

The results obtained with distribution E (forces proportional to the masses), may be directly compared with the ones of the rigid-brittle model examined in §4.1; in fact, the reference software (SAV) uses the same load distribution. Compared to the rigid-brittle model, the collapse multiplier increased due to the effects of elasticity; the nonlinear curve (Figure 16) shows the end of the elastic range for a force of 360 kN and a multiplier of 0.223, that is, with an increment of 29% compared to the one obtained with the rigid-brittle model.

Figure 16 shows the deformation of the structure at the last step of the incremental analysis, when the collapse mechanism occurs: with the aid of the software [8] it is possible to visualize step-by-step the deformation process and the failures of the joints. It is worth noting that the collapse mechanism is equivalent to the one obtained with limit analysis (Figure 11) although they correspond to different values of the collapse multiplier.

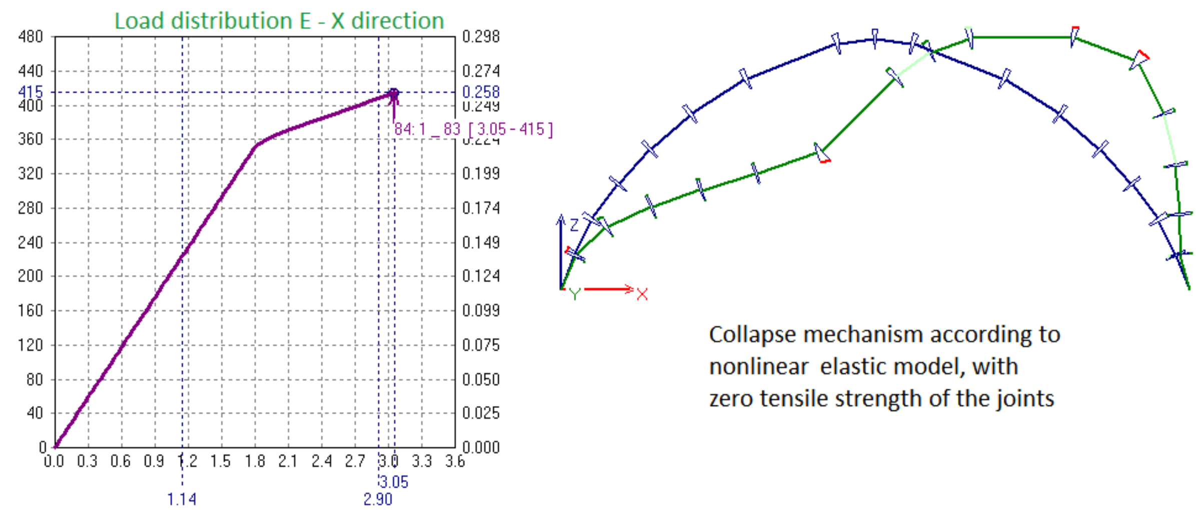

Figure 16. Nonlinear static analysis with zero tensile strength

The results of the analysis with a tensile strength of the joints equal to 0.25 MPa are shown in Table 2.

Distribution	Maximum force	Collapse multiplier
A	400 kN	0.249
E	495 kN	0.308

Table 2. Analysis results with tensile strength equal to 0.25 MPa

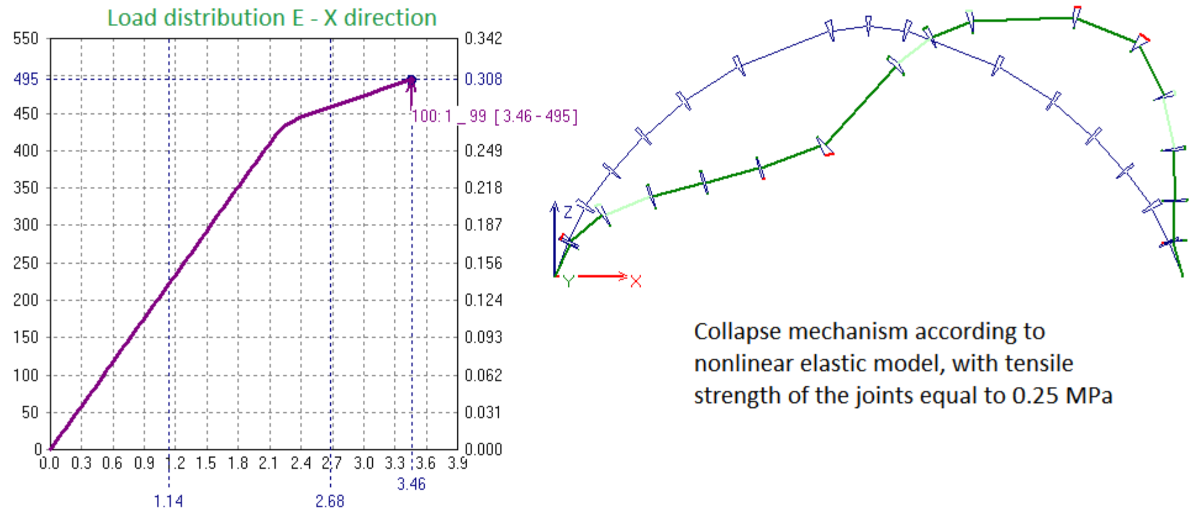

Figure 17. Nonlinear static analysis with tensile strength equal to 0.25 MPa

Due to the tensile strength of the joints, the collapse multiplier increased from 0.211 to 0.249. Finally, the application of the block-joint method led to:

- the fundamental period of the structure, equal to 0.197s,
- an increment of the collapse multiplier from 0.173 (rigid-brittle model) to 0.249 (nonlinear elastic model).

5 CONCLUSION

A new finite element model for the analysis of arch systems is proposed in this paper. The method assesses the maximum sustainable horizontal force, and thus the collapse multiplier, through an incremental procedure typical of nonlinear analysis. The ease of the approach was pursued, so that it could be performed with any finite element program that deals with one-dimensional elements. Compared to the rigid-brittle model, the new approach provides higher collapse multipliers, basically due to the possibility of taking into account the mortar tensile strength. This is favourable in order to avoid oversizing of the strengthening interventions.

The case of an arch structure located in an important monumental complex was examined; the new method was applied considering several distribution of the horizontal forces.

It is the intent of the author to further investigate the block-joint method especially regarding the following aspects: (i) curved axis blocks, in order to ensure the correct modelling of the joints avoiding excessive thickness at the extrados; (ii) calibration of the tensile strength of the joints according to their length; (iii) comparison between the nonlinear elastic and the rigid-brittle model in order to qualitatively understand the variation of the collapse multiplier according to the geometry of the structure and the applied loads.

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