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THEORETICAL STUDY ON THE REDUCTION AND RECOVERING METHOD

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Abstract. *A method for reducing Secondary DOF (Degree of Freedom) from a dynamic equilibrium system into was intended to reduce considerably the storage capacity and computation time.*

The most popular Guyan [1] method which is based on superposition of lower modes from the modal analysis is widely adopted as a basis and for further improvements in dynamic reduction procedures. As a result of neglecting the higher modes, after the dynamic responses of structures evaluated by using only the lower modes frequencies, it is not possible to recover back precisely the responses of the other reduced Secondary DOF which are obviously more dominant in higher modes.

In the present works, a new method for reducing and recovering the Secondary DOF in the dynamic equilibrium system which is based on the concept of dynamic sub-structuring method is introduced herein. To improve the computational accuracy after the reduction process, a new so called Differential DOF Replacement method is also introduced.

1 INTRODUCTION

In Japan, the safety and performance of building from damages during the occurrences of earthquake have been given the highest of importance. Therefore, it is necessary to have an understanding of the dynamic response behavior of the buildings.

To achieve the accuracy for design analysis, a designer is often been given an alternative between using a simple design procedure as a design tool combined with seismic design codes and using more complex structural model with ground accelerations applied at the base of the structure. The later tool is often called as the time history analysis with nonlinearities to evaluate the dynamic responses of the building structures. Even though the present rapid development in computing technology and hardware, it seems like for design calculation and practical analysis purposes there is still a need in seeking for a simple and easy yet reliable design procedure.

Most of seismic design codes including charts, tables and graphs are developed based on an equivalent single DOF model for evaluating dynamic responses of building structures against the ground motion excitations. The response spectral of accelerations, velocities and displacements are still a valuable and practical tool for design practice due to its simplicity.

Although in the current state of the art in the dynamic vibration analysis techniques and computing progressions, computers are capable of solving systems with million hundred degrees of freedoms, they still are not adequate for treating directly the mathematical idealizations used in the analysis of complex structures. Moreover, it is seldom of interest to determine more than dozen vibration mode shapes even in the most complex structural system because the mode-superposition method generally is applied to structures in which the loading excites significantly only at the lowest modes. For these reasons, various eigensystem enhancements procedures have been proposed to predict the lower modes precisely for the reduction purposes. Therefore, simplification against the Secondary DOF is made forward-wise and not reversible. Once the responses of reduced structures evaluated by using modes lower frequency modes, it is not possible to get the responses of the reduced higher frequencies of the Secondary DOF precisely.

In present study, a new theory for reduction and recovering degree of freedoms which is based on the Rayleigh-Ritz method and expressed in a series form is formulated. Figure 1 shows a schematic of the proposed reduction and recovering method between the Secondary DOF and Main DOF systems. Present study is a continuation work of several research works [2-4] in an attempt for eliminating the role of modal analysis in the reduction and recovering methods for dynamic analysis problems.

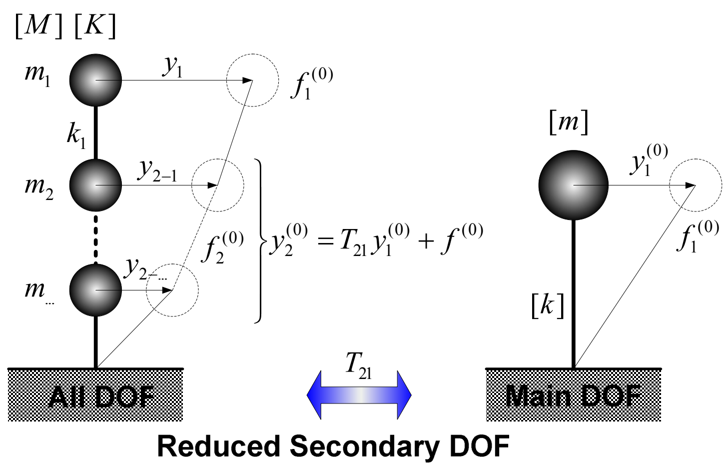

Figure 1: The schematic concept of reduction and recovering methods.

2 RAYLEIGH-RITZ METHOD

The differential equation of motion for an undamped Multi-DOF system can be written in the following

$$[M]\{y_i^{(2)}\}+[K]\{y_i^{(0)}\}-\{f_i^{(0)}\}=\{0\} \tag{1}$$

where, $[M]$, $[K]$, $\{f_i^{(0)}\}$, $\{y_i^{(0)}\}$ and $\{y_i^{(2)}\}$ are the mass matrix, stiffness matrix, external loading vector, displacement vector and acceleration vector, respectively. The time dependent displacement vector and acceleration vector are generally can be expressed in differential form as $y_i^{(n)}=\dfrac{d^n y_i}{dt^n}$; $n=1,2,...,N$ in which t is the time and n is the order of derivative with respect to time. The displacement $\{y_i^{(0)}\}$ and acceleration $\{y_i^{(2)}\}$ vector are then separated into two parts of vectors, a Main DOF vector $\{y_1^{(0)}\}$ and the remaining Secondary DOF vector $\{y_2^{(0)}\}$. The Secondary DOF vector and its characteristic properties, such as mass and stiffness matrices, are going to be reduced into the Main DOF vector and its characteristic properties.

Rewriting the Eq. (1), the following expressions are obtained as

$$\begin{aligned}
\{y_i^{(0)}\}&=\begin{Bmatrix} y_1^{(0)} \\ y_2^{(0)} \end{Bmatrix}=\begin{bmatrix} I_{11} \\ T_{21} \end{bmatrix}\{y_1^{(0)}\}+\begin{Bmatrix} 0 \\ f^{(0)} \end{Bmatrix}=[T]\{y_1^{(0)}\}+\{\overline{F}_i^{(0)}\} \\
\{y_i^{(2)}\}&=\begin{Bmatrix} y_1^{(2)} \\ y_2^{(2)} \end{Bmatrix}=\begin{bmatrix} I_{11} \\ T_{21} \end{bmatrix}\{y_1^{(2)}\}+\begin{Bmatrix} 0 \\ f^{(2)} \end{Bmatrix}=[T]\{y_1^{(2)}\}+\{\overline{F}_i^{(2)}\}
\end{aligned} \tag{2}$$

in which the following relationship can be obtained as

$$y_2^{(0)}=T_{21}y_1^{(0)}+f^{(0)} \tag{3}$$

where $[T]$, I_{11}, T_{21} being the transformation matrix, unit matrix and main to secondary transformation matrix, respectively. By applying the principle of virtual displacement to the Eq. (1) as given below

$$\delta\{y_i^{(0)}\}^T\left([M]\{y_i^{(2)}\}+[K]\{y_i^{(0)}\}-\{f_i^{(0)}\}\right)=0 .$$

The dynamic equation of motion in Eq. (1) can be rewritten by the following equation

$$[m]\{y_1^{(2)}\}+[k]\{y_1^{(0)}\}-\{f_1^{(0)}\}=\{0\} \tag{4}$$

where the mass matrix, stiffness matrix and external loading vector are given as

$$[m]=[T]^T[M][T]=\begin{bmatrix} I_{11} & T_{21}^T \end{bmatrix}\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}\begin{bmatrix} I_{11} \\ T_{21} \end{bmatrix}=m_{11}+T_{21}^T m_{21}+m_{12}T_{21}+T_{21}^T m_{22}T_{21}$$

$$[k]=[T]^T[K][T]=\begin{bmatrix} I_{11} & T_{21}^T \end{bmatrix}\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}\begin{bmatrix} I_{11} \\ T_{21} \end{bmatrix}=k_{11}+T_{21}^T k_{21}+k_{12}T_{21}+T_{21}^T k_{22}T_{21}$$

$$
\begin{aligned}
\{f_1\} &= [T]^T\left([M]\{\overline{F}_i^{(2)}\} + [K]\{\overline{F}_i^{(0)}\} - \{f_i^{(0)}\}\right) \\
&= \begin{bmatrix} I_{11} & T_{21}^T \end{bmatrix}\left(\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}\begin{bmatrix} 0 \\ f^{(2)} \end{bmatrix} + \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}\begin{bmatrix} 0 \\ f^{(0)} \end{bmatrix} - \begin{Bmatrix} f_1^{(0)} \\ f_2^{(0)} \end{Bmatrix}\right) \\
&= \left(m_{21} + T_{21}^T m_{22}\right) f^{(2)} + \left(k_{21} + T_{21}^T k_{22}\right) f^{(0)} - \left(f_1^{(0)} + T_{21}^T f_2^{(0)}\right)
\end{aligned}
$$

3 REDUCTION THEORY

By using matrix notation, the Main and Secondary DOF vectors and matrices of the undamped Multi-DOF dynamic equilibrium equation can be separated into the Main and Secondary DOF equations of rows as given in the following equations,

$$
\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}\begin{Bmatrix} y_1^{(2)} \\ y_2^{(2)} \end{Bmatrix} + \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}\begin{Bmatrix} y_1^{(0)} \\ y_2^{(0)} \end{Bmatrix} - \begin{Bmatrix} f_1^{(0)} \\ f_2^{(0)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{5}
$$

$$
m_{12} y_2^{(2)} + k_{12} y_2^{(0)} = f_1^{(0)} - m_{11} y_1^{(2)} - k_{11} y_1^{(0)} \tag{5a}
$$

$$
m_{22} y_2^{(2)} + k_{22} y_2^{(0)} = f_2^{(0)} - m_{21} y_1^{(2)} - k_{21} y_1^{(0)} \tag{5b}
$$

with the subscript notations are showing the index of vectors or matrices, and the superscript number in brackets are showing the number of derivative with respect to time.

3.1 Guyan's Reduction

The oldest theory in the dynamic reduction theory was firstly introduced by Guyan [1], where the Eq. (5b) was solved for the Secondary DOF vector $y_2^{(0)}$ as,

$$
y_2^{(0)} = -k_{22}^{-1}\left(k_{21} y_1^{(0)} - f_2^{(0)}\right) - k_{22}^{-1}\left(m_{21} y_1^{(2)} + m_{22} y_2^{(2)}\right)
$$

and by neglecting the second derivative terms, results in

$$
y_2^{(0)} = T_{21} y_1^{(0)} + f^{(0)} \tag{6}
$$

where, the transformation matrix which relates the Main DOF and Secondary DOF vectors is given by $T_{21} = -k_{22}^{-1} k_{21}$ and the loading vector is given by $f^{(0)} = k_{22}^{-1} f_2^{(0)}$.

It should be noted that the reduction method introduced by Guyan used this approximation transformation method to recover the Secondary DOF vectors from the Main DOF vectors which is solved by the Eq. (4). Because the reduction method proposed by Guyan is only make use of the Eq. (5b) to obtain the transformation matrix T_{21}, which contains only parts of the stiffness matrix of the dynamic equation, the computed displacement modes are approximated values.

3.2 0th-Order Transformation Matrix

In order to include the influence of Eq. (5a) into the transformation process, the term $y_1^{(2)}$ in the Eq. (5a) is obtained as,

$$
y_1^{(2)} = m_{11}^{-1}\left(f_1^{(0)} - k_{11} y_1^{(0)} - m_{12} y_2^{(2)} - k_{12} y_2^{(0)}\right) \tag{7}
$$

and substitution into the Eq. (5b) to get the revised $y_2^{(0)}$ term for constructing the new transformation matrix T_{21} can be expressed as,

$$y_2^{(0)} = \bar{k}_{21}^{(0)} y_1^{(0)} + \bar{F}_2^{(0)} + \bar{m}_{22}^{(0)} y_2^{(2)} = \overset{0}{T}_{21} y_1^{(0)} + \overset{0}{f}^{(0)} + \overset{0}{m}_{22} y_2^{(2)} \tag{8}$$

where,

$\overset{0}{T}_{21} = \bar{k}_{21}^{(0)} = -k_{22}^{(0)-1} k_{21}^{(0)}$, $\overset{0}{f}^{(0)} = \bar{F}_2^{(0)} = k_{22}^{(0)-1} F_2^{(0)}$, $\overset{0}{m}_{22} = \bar{m}_{22}^{(0)} = -k_{22}^{(0)-1} m_{22}^{(0)}$,

$k_{22}^{(0)} = k_{22} - m_{21} m_{11}^{-1} k_{12}$, $k_{21}^{(0)} = k_{21} - m_{21} m_{11}^{-1} k_{11}$

$m_{22}^{(0)} = m_{22} - m_{21} m_{11}^{-1} m_{12}$, $F_2^{(0)} = f_2^{(0)} - m_{21} m_{11}^{-1} f_1^{(0)}$.

The number above the variables, $\overset{*}{(\ )}$, indicates a series of numbering to be summarized the end of the formulations.

By neglecting the second derivative with respect to time of the last third term in the Eq. (8), the equation becomes

$$y_2^{(0)} = \overset{0}{T}_{21} y_1^{(0)} + \overset{0}{f}^{(0)} . \tag{9}$$

Because the Eq. (5a) is used in solving the Secondary DOF vector $y_2^{(0)}$ term, the parts of mass matrix from the dynamic system of equation are considered, therefore it will give better approximation and higher accuracy compared to the Guyan's approach. By substituting the Eq. (8) into Eq. (5a), the vector $y_1^{(2)}$ can be obtained as,

$$y_1^{(2)} = \bar{k}_{11}^{(0)} y_1^{(0)} + \bar{F}_1^{(0)} + \bar{m}_{12}^{(0)} y_2^{(2)} \tag{10}$$

where,

$\bar{k}_{11}^{(0)} = -m_{11}^{-1}\left(k_{11} + k_{12}\bar{k}_{21}^{(0)}\right)$, $\bar{m}_{12}^{(0)} = -m_{11}^{-1}\left(m_{12} + k_{12}\bar{m}_{22}^{(0)}\right)$, $\bar{F}_1^{(0)} = m_{11}^{-1}\left(f_1^{(0)} - k_{12}\bar{F}_2^{(0)}\right)$.

3.3 1st-Order Transformation Matrix

In this study, a new concept is introduced by having the dynamic equation of motion in Eqs. (5a) and (5b) are further derived twice with respect to time which results in the following equations,

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{Bmatrix} y_1^{(4)} \\ y_2^{(4)} \end{Bmatrix} + \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{Bmatrix} y_1^{(2)} \\ y_2^{(2)} \end{Bmatrix} - \begin{Bmatrix} f_1^{(2)} \\ f_2^{(2)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{11}$$

$$m_{12} y_2^{(4)} + k_{12} y_2^{(2)} = f_1^{(2)} - m_{11} y_1^{(4)} - k_{11} y_1^{(2)} \tag{11a}$$

$$m_{22} y_2^{(4)} + k_{22} y_2^{(2)} = f_2^{(2)} - m_{21} y_1^{(4)} - k_{21} y_1^{(2)} \tag{11b}$$

Under the assumption at $t = 0$, the initial conditions of the Main DOF vector $y_1^{(n)} = 0$, and the Secondary DOF vector $y_2^{(n)} = 0 \ldots (n = 0,1,2,3,4)$ are detained, the solutions for the homogeneous equations in Eq. (5) are exist and hence, all of its derivatives equations can be used to obtain the approximation of the transformation matrix.

By substitution of Eq. (10) into the Eq. (11), the term $y_1^{(4)}$ in the Eq. (11a) can be solved as follow

$$y_1^{(4)} = m_{11}^{-1}\left(-k_{11}\bar{k}_{11}^{(0)} y_1^{(0)} + \left(f_1^{(2)} - k_{11}\bar{F}_1^{(0)}\right) - \left(k_{12} + k_{11}\bar{m}_{12}^{(0)}\right) y_2^{(2)} - m_{12} y_2^{(4)}\right), \tag{12}$$

then, by further substituting the above equation into Eq. (11b), the $y_2^{(2)}$ term can obtained as

$$y_2^{(2)} = \bar{k}_{21}^{(2)} y_1^{(0)} + \bar{F}_2^{(2)} + \bar{m}_{22}^{(2)} y_2^{(4)} \tag{13}$$

where,
$\bar{k}_{21}^{(2)} = -k_{22}^{(2)-1} k_{21}^{(0)} \bar{k}_{11}^{(0)}$, $\bar{m}_{22}^{(2)} = -k_{22}^{(2)-1} m_{22}^{(0)}$, $\bar{F}_2^{(2)} = k_{22}^{(2)-1}\left(F_2^{(2)} - k_{21}^{(0)} \bar{F}_1^{(0)}\right)$.

Substitution of Eq. (13) into the Eq. (8) and rearranging the similar terms, result in

$$y_2^{(0)} = \overset{1}{T}_{21} y_1^{(0)} + \overset{1}{f}{}^{(0)} + \overset{1}{m}_{22} y_2^{(4)} \tag{14}$$

where,
$\overset{1}{T}_{21} = \bar{k}_{21}^{(0)} + \bar{m}_{22}^{(0)} \bar{k}_{21}^{(2)}$, $\overset{1}{f}{}^{(0)} = \bar{F}_2^{(0)} + \bar{m}_{22}^{(0)} \bar{F}_2^{(2)}$, $\overset{1}{m}_{22} = \bar{m}_{22}^{(0)} \bar{m}_{22}^{(2)}$.

Neglecting the fourth derivative with respect to time of the last third term in the Eq. (14), results in the following equation,

$$y_2^{(0)} = \overset{1}{T}_{21} y_1^{(0)} + \overset{1}{f}{}^{(0)} . \tag{15}$$

Repeated herein, the number above the variables, $\overset{*}{(\ )}$, indicates a series for summation purpose.

3.4 M^th-Order Transformation Matrix

By repeating the similar derivation procedures as given in previous sub-sections, for deriving the M^th-Order transformation matrix can be obtained by further make a $(N-1)$ derivation with respect to time to the dynamic equation which results in,

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{Bmatrix} y_1^{(2n)} \\ y_2^{(2n)} \end{Bmatrix} + \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{Bmatrix} y_1^{(2n-2)} \\ y_2^{(2n-2)} \end{Bmatrix} - \begin{Bmatrix} f_1^{(2n-2)} \\ f_2^{(2n-2)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{16}$$

$$m_{12} y_2^{(2n)} + k_{12} y_2^{(2n-2)} = f_1^{(2n-2)} - m_{11} y_1^{(2n)} - k_{11} y_1^{(2n-2)} \tag{16a}$$

$$m_{22} y_2^{(2n)} + k_{22} y_2^{(2n-2)} = f_2^{(2n-2)} - m_{21} y_1^{(2n)} - k_{21} y_1^{(2n-2)} . \tag{16b}$$

Under the same assumption at $t = 0$, the initial conditions of the Main DOF vector $y_1^{(2n-2)} = 0$, and Secondary DOF vector $y_2^{(2n-2)} = 0 \ldots (n = 0,1,2,3,...,N)$ are detained, the solutions for the homogeneous equations in Eq. (5) are exist and hence, all of its derivatives equations can be used to obtain the approximation of the transformation matrix.

If the procedure is repeated following the same procedure as given in the previous section, the $y_2^{(*)}$ terms can be obtained as follow,

$$\begin{gathered} y_2^{(0)} = \bar{k}_{21}^{(0)} y_1^{(0)} + \bar{F}_2^{(0)} + \bar{m}_{22}^{(0)} y_2^{(2)} \\ y_2^{(2)} = \bar{k}_{21}^{(2)} y_1^{(0)} + \bar{F}_2^{(2)} + \bar{m}_{22}^{(2)} y_2^{(4)} \\ \ldots \\ y_2^{(2n)} = \bar{k}_{21}^{(2n)} y_1^{(0)} + \bar{F}_2^{(2n)} + \bar{m}_{22}^{(2n)} y_2^{(2n+2)} \end{gathered}$$

And in a form of series, all the derivatives of $y_2^{(*)}$ terms can be expressed as

$$y_2^{(0)} = \overset{M}{T}_{21} y_1^{(0)} + \overset{M}{f}{}^{(0)} + \overset{M}{m}_{22} y_2^{(2M+2)} \tag{17}$$

with

$$\overset{M}{T}_{21}=\sum_{m=0}^{M}\Pi_{n=0}^{m}\overline{\overline{m}}_{22}^{(2n)}\overline{k}_{21}^{(2m)}$$

$$\overset{M}{f}^{(0)}=\sum_{m=0}^{M}\Pi_{n=0}^{m}\overline{\overline{m}}_{22}^{(2n)}\overline{F}_{2}^{(2m)}$$

.

$$\overset{M}{m}_{22}=\Pi_{n=0}^{M+1}\overline{\overline{m}}_{22}^{(2n)}\ ,\ \overline{\overline{m}}_{22}^{(2n)}=\overline{m}_{22}^{2(n-1)}\ ,\ \overline{\overline{m}}_{22}^{(0)}=\overline{m}_{22}^{(-2)}=I_{22}$$

4 RECOVERING OF THE SECONDARY DOF

It can be noted that from the Eq. (17), the tolerance of the present reduction theory is due to the negligence of the twice higher order derivatives terms which are considered very small.

$$\left|\overset{M}{T}_{21}\,y_1^{(0)}+\overset{M}{f}^{(0)}\right|\rangle\rangle\left|\overset{M}{m}_{22}\,y_2^{(2M+2)}\right|$$

Hence, the Main DOF and Secondary DOF vectors can be written by using the a series form as follow,

$$\begin{Bmatrix} y_1^{(0)} \\ y_2^{(0)} \end{Bmatrix}=\begin{Bmatrix} I_{11} \\ \overset{M}{T}_{21} \end{Bmatrix}y_1^{(0)}+\begin{Bmatrix} 0 \\ \overset{M}{f}^{(0)} \end{Bmatrix} \tag{18}$$

.

The second row of the above equation can be used to recover the reduced Secondary DOF vector from the Main DOF vector as a result from solving the reduced dynamic equation in Eq. (4).

5 DIFFERENTIAL DOF REPLACEMENT METHOD

In a complex dynamic system, there are many occasions where the difficulties to determine the Main DOF and Secondary DOF appeared, because different boundary of conditions and loadings will effect the behavior of solutions. If there is a case, still the above formulations can be applied by using relative differences between DOFs instead of each independent DOF. The relative differences between DOFs will make the Secondary DOF vector consists of small values. Components of the Secondary DOF vector $y_2^{(0)}$ are then replaced by $y_{ij}^{(0)}$, where the Secondary DOF components vector now consist of relative difference values to an arbitrarily selected component DOF $y_i^{(0)}$ from the Main DOF which is given as,

$$y_{ij}^{(0)}=y_i^{(0)}-y_j^{(0)} \tag{19}$$

.

Rewriting the above equation, each component $y_j^{(0)}$ in the Secondary DOF vector can be expressed by the following relative difference relationship,

$$y_j^{(0)}=y_i^{(0)}-y_{ij}^{(0)} \tag{20}$$

As a result, all the components vector of the Multi-DOF dynamic system consist of only either $y_i^{(0)}$ or $y_{ij}^{(0)}$ terms, where the values of $y_{ij}^{(0)}$ are very small compared to the components $y_i^{(0)}$ in the Main DOF vector.

By successively replacing each other components, the DOF vectors can be reconstructed as $\begin{Bmatrix} y_1^{(0)} & y_2^{(0)} \end{Bmatrix}^T$, even with the low order approximation of transformation matrix, high accuracy will be obtained.

6 CONCLUSIONS

- A theory for reduction and recovering methods of large degree-of-freedom in structural dynamic analysis is proposed. The theory is based on the Rayleigh-Ritz method, and then further developed to create a transformation matrix which is finally summarized in a series form; the series form of transformation matrix implies the inclusions of higher order modes inside the formulations. This newly developed theory is expected to give a new kind of reduction theory without using the modal analysis.
- By using the DOF vectors transformation matrix, calculation results from the Main DOF equations can be used to recover back the reduced Secondary DOF while retaining high accuracy results.
- The present proposed method is suitable and recommended for practical design method.

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