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FINITE ELEMENT FORMULATION FOR AN ARCH STRUCTURE FORMED BY USING BUILT-UP BEAMS ELEMENT

Eiji Nouchi[1], Mitsuharu Kurata[2] and Buntara S. Gan[3]

[1,2,3] Nihon University, College of Engineering
Department of Architecture

1-Nakagawara, Tokusada, Koriyama City, Fukushima, Japan 963-8642
[1]nouchi@arch.ce.nihon-u.ac.jp
[2]kurata@arch.ce.nihon-u.ac.jp
[3]buntara@arch.ce.nihon-u.ac.jp

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Abstract. *In architecture of building history, the arch type of structures, especially made from timber elements, are aesthetically very attracting creation. Since the primitive era, human being has learned how to build the arch type of structures that have been used as roofs or beams for houses or bridges structures. They have learned from failures and experiences how to stack and connect pieces of timbers in order to assemble a large span arch type of structure.*

Present study reports the theoretical derivation and finite element formulation for an arch structure formed by using built-up beams elements.

Static experimental works were conducted to an arch structure formed by using built-up beams elements. The behavior of the arch structure formed by using built-up beams elements are evaluated numerically by means of the finite element method.

An analytical formulation is proposed to transform beam elements which can be commonly connected at both beam ends or can be intermittently built-up together as an element member. The formulation was derived based on the assumed stress distribution, shear deformation and expansion-contraction of the beam cross section within the context of linear elasticity theory.

1 INTRODUCTION

In architecture of building history, the arch type of structures, especially made from timber elements, are aesthetically very attracting creation. Since the primitive era, human being has learned how to build the arch type of structures that have been used as roofs or beams for houses or bridges structures. They have learned from failures and experiences how to stack and connect pieces of timbers in order to assemble a large span arch type of structure.

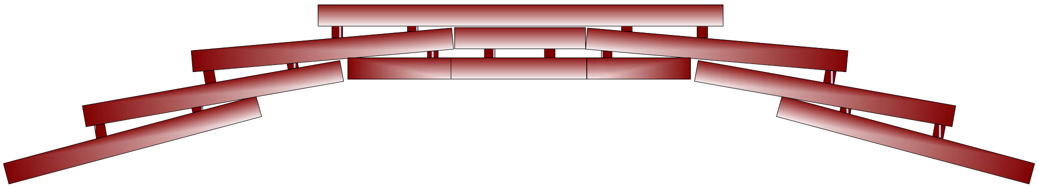

Figure 1: Illustration of a timber arch type structure.

Kawaguchi [1] reported a study on Japanese Traditional buildings which revealed that the builders had an excellent understanding of static mechanisms and structural behavior of the roofs; however their understanding were based on their long period experiences in building the structures. A hanging roof is considered rational and economical, because the pieces of timbers are only in tension, thus the strength requirement of timber is not need to be reduced to consider the buckling effects.

In structural design works, to analyze the deformations and strengths of each member of the roofs, the standard beam element combined with modified joint element for connection is usually adopted [2-4].

This paper is intended to derive a formulation in which each element of the timber is modeled by using four nodes with two displacement degree of freedoms at each node. The fundamental theory is based on the beam element formulation considering shear deformation [5].

2 BASIS FOR THEORETICAL FORMULATIONS

In this study, the standard four component of degree of freedoms which are used in the traditional two nodes beam element are converted into four arbitrary nodes lying within the beam element with two degree of freedoms attached at each node.

Based on the principle of virtual work displacement, the displacements of those four arbitrary nodes within the beam element are transformable to the beam end displacements is formulated. The present theory is applicable for analyzing an arch type structure built-up by four arbitrary nodes along the boundary of a beam element which is transformed from the standard beams element model.

3 PRINCIPLE OF VIRTUAL WORK

By applying virtual displacements, the virtual works experienced by a one dimensional elastic body lying on an *x*-*y* plane under the assumption $\sigma_y = \sigma_z = 0$ can be defined as

$$\int_V (\delta U \cdot X + \delta V \cdot Y) dV + \int_{S_\sigma} (\delta U \cdot X_v + \delta V \cdot Y_v) ds = \int_V (\delta \varepsilon_x \sigma_x + \delta \gamma_{xy} \tau_{xy}) dV \tag{1}$$

where, δU and δV being the virtual displacements in the x and y directions, X and Y being the body forces applied in the x and y directions, X_v and Y_v being the surface tractions applied in the x and y directions, $\sigma_x = E\varepsilon_x$ and $\tau_{xy} = G\gamma_{xy}$ are the elastic properties of the beam following the linear Hooke's law.

By changing the integration boundaries to the length of the beam and both ends conditions, Eq. (1) can be expressed as

$$\begin{aligned} &\int_0^\ell \int_A (\delta U \cdot X + \delta V \cdot Y) dA dx + \int_{A_i} (\delta U \cdot X_\nu + \delta V \cdot Y_\nu) dA_i + \int_{A_j} (\delta U \cdot X_\nu + \delta V \cdot Y_\nu) dA_j \\ &= \int_0^\ell \int_A (\delta \{\boldsymbol{\varepsilon}\}[\mathbf{E}]\{\boldsymbol{\varepsilon}\}) dA dx \end{aligned} \tag{2}$$

where, $dV = dA\, dx$, dA_i is a small integration area at node $i(x=0)$, dA_j is a small integration area at node $j(x=\ell)$, ℓ is the length of the beam, with the strain and modulus properties defined as follow, $\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \gamma_{xy} \end{Bmatrix}$, $[E] = \begin{bmatrix} E & 0 \\ 0 & G \end{bmatrix}$.

4 STRESS AND STRAIN RELATIONSHIP WITHIN A BEAM

The stress assumption of a beam can be given as

$$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x) = 0 \, , \tag{3}$$

with the strains in both directions can be obtained from

$$\varepsilon_y = \frac{\partial V}{\partial y} \text{ and } \varepsilon_x = -\frac{\partial U}{\partial x} \tag{4}$$

where, ν being the Poisson's ratio of the beam material.

The horizontal displacement of an arbitrary point can be expressed as

$$U = u + y\beta + \bar{f}_1(x,y) \tag{5}$$

where, $\varepsilon_y = \dfrac{\partial V}{\partial y} = -\nu\varepsilon_x = -\nu\dfrac{\partial U}{\partial x} = -\nu\left(\varepsilon + y\kappa + \dfrac{\partial \bar{f}_1(x,y)}{\partial x}\right)$, being $\varepsilon = \dfrac{du}{dx}$, $\kappa = \dfrac{d\beta}{dx}$.

It can be noted that the last term $\bar{f}_1(x,y)$ in the above equation is a function to incorporate the effect of shear deformation of the beam cross section.

The vertical displacement of an arbitrary point can be defined as

$$V = v - \nu\left(y\varepsilon + \frac{y^2}{2}\kappa + \int_0^y \frac{\partial \bar{f}_1(x,y)}{\partial x} dy\right) \tag{6}$$

where, $\dfrac{\tau_{xy}}{\tau_0} = \dfrac{QS}{b(y)I} \bigg/ \dfrac{QS_0}{b(0)I} = \dfrac{b(0)S}{b(y)S_0} = \dfrac{G\gamma_{xy}}{G\gamma}$ being $\gamma_{xy} = \dfrac{\partial V}{\partial x} + \dfrac{\partial U}{\partial y} = \dfrac{b(0)S}{b(y)S_0}\gamma$. For a uniform width of beam cross section in which $b(y) = b(0) = b$, the term $b(0)/b(y)$ will vanish from the shear stress and strain equations.

By substituting Eq. (6), into the ε_y term, and integrating along the depth of the beam cross section, results in the following explicit equation considering shear deformation effect.

$$\frac{\partial \bar{f}_1(x,y)}{\partial y} = -\left(1 - \frac{b(0)S}{b(y)S_0}\right)\gamma + \nu\left(y\frac{d\varepsilon}{dx} + \frac{y^2}{2}\frac{d\kappa}{dx} + \int_0^y \frac{d^2 \bar{f}_1(y)}{dx^2} dy\right) \tag{7}$$

with,

$$\bar{f}_1(y) \approx -\left(\int_0^y \left(1-\frac{S}{S_0}\right)dy\right)\gamma + \nu\left(\frac{y^2}{2}\frac{d\varepsilon}{dx}+\frac{y^3}{6}\frac{d\kappa}{dx}\right),$$

where,

$$\int_0^y \int_0^y \frac{d^2 \bar{f}_1(y)}{dx^2} dydy \approx 0 .$$

5 FORMULATION OF BEAM REPRESENTED BY FOUR ARBITRARY NODES

Figure 2 shows a standard two-node beam element with four arbitrary nodes (*k, l, m and n*) lying along the boundary of the beam element.

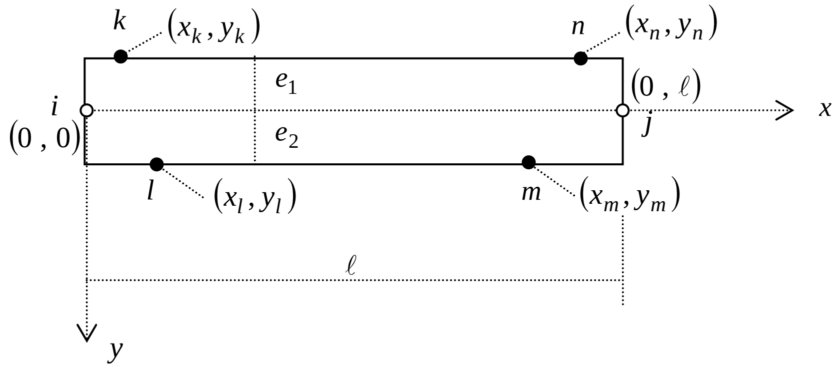

Figure 2: The schematic beam element with four arbitrary nodes.

5.1 General displacements functions

Displacements and rotations of an arbitrary node inside a beam element interpolated from both beam ends values can be expressed by using polynomial as follow

$$\begin{aligned}
u(x) &= \left(1-\frac{x}{\ell}\right)u_i + \frac{x}{\ell}u_j \\
v(x) &= v_i + \theta_i \cdot x + a \cdot x^2 + bx^3 \\
\theta(x) &= \frac{dv}{dx} = \theta_i + 2ax + 3bx^2 \\
\gamma(x) &= \beta + \theta = c + dx \\
\beta(x) &= -\theta + c + dx = -\theta_i - 2ax - 3bx^2 + c + dx
\end{aligned} \tag{8}$$

where, $u(x)$, $v(x)$, $\theta(x)$, $\gamma(x)$ and $\beta(x)$ are the horizontal displacement, vertical displacement, slope of the beam axis, shear rotation and beam cross section rotation, respectively, at a distance *x* along the beam axis.

By defining the boundary conditions of both ends values of the beam as

$$\begin{aligned}
&u_i = u(x=0),\ v_i = v(x=0),\ \theta_i = \theta(x=0),\ \beta_i = \beta(x=0) \\
&u_j = u(x=\ell),\ v_j = v(x=\ell),\ \theta_j = \theta(x=\ell),\ \beta_j = \beta(x=\ell)
\end{aligned} \tag{9}$$

and solving for all the parameters in Eq. (8), result in

$$
\begin{aligned}
a &= -\frac{1}{\ell}\left(2\theta_i + \theta_j + 3\frac{v_j - v_i}{\ell}\right) \\
b &= \frac{1}{\ell^2}\left(\theta_i + \theta_j + 2\frac{v_j - v_i}{\ell}\right) \\
c &= \beta_i + \theta_i \\
d &= \frac{1}{\ell}\left(\beta_j + \theta_j\right) - \frac{1}{\ell}\left(\beta_i + \theta_i\right)
\end{aligned}
\tag{10}
$$

Thus, an interpolation matrix between the arbitrary point displacement values and both ends values of the beam can be described as follow

$$
\begin{Bmatrix} u \\ v \\ \theta \\ \beta \end{Bmatrix} =
\begin{bmatrix}
1-\frac{x}{\ell} & 0 & 0 & 0 & \frac{x}{\ell} & 0 & 0 & 0 \\
0 & 1-\frac{3x^2}{\ell^2}+\frac{2x^3}{\ell^3} & x-\frac{2x^2}{\ell}+\frac{x^3}{\ell^2} & 0 & 0 & \frac{3x^2}{\ell^2}-\frac{2x^3}{\ell^3} & -\frac{x^2}{\ell}+\frac{x^3}{\ell^2} & 0 \\
0 & -\frac{6x}{\ell^2}+\frac{6x^2}{\ell^3} & 1-\frac{4x}{\ell}+\frac{3x^2}{\ell^2} & 0 & 0 & \frac{6x}{\ell^2}-\frac{6x^2}{\ell^3} & -\frac{2x}{\ell}+\frac{3x^2}{\ell^2} & 0 \\
0 & \frac{6x}{\ell^2}-\frac{6x^2}{\ell^3} & \frac{3x}{\ell}-\frac{3x^2}{\ell^2} & 1-\frac{x}{\ell} & 0 & -\frac{6x}{\ell^2}+\frac{6x^2}{\ell^3} & \frac{3x}{\ell}-\frac{3x^2}{\ell^2} & \frac{x}{\ell}
\end{bmatrix}
\begin{Bmatrix} u_i \\ v_i \\ \theta_i \\ \beta_i \\ u_j \\ v_j \\ \theta_j \\ \beta_j \end{Bmatrix}
\tag{11}
$$

or in a vector and matrix forms, Eq. (11) can be written as $\{\mathbf{u}\} = [\mathbf{x}]\begin{Bmatrix} \mathbf{u}_i \\ \mathbf{u}_j \end{Bmatrix}$.

5.2 Strains based on general displacement functions

Based on the general displacement functions derived in the previous section, the strains at any arbitrary location can be obtained from the following relationships.

$$
\frac{d\varepsilon}{dx} = \frac{d^2u}{dx^2} = 0 \ , \ \frac{d^2\kappa}{dx^2} = \frac{d^3\beta}{dx^3} = 0 \ , \ \frac{d^2\gamma}{dx^2} = 0
\tag{12}
$$

Where, κ being the curvature rate of the beam axis. By integrating Eq. (7) with respect to *y*, results in the following

$$
\bar{f}_1(x,y) = f_1(y)\gamma + \nu\frac{y^3}{6}\frac{d\kappa}{dx}
\tag{13}
$$

where, $f_1(y) = -\left(\int_0^y \left(1 - \frac{b(0)S}{b(y)S_0}\right)dy\right)$.

Substituting Eq. (13) into Eqs. (5) and (6), will give the following displacements,

$$
\begin{aligned}
U &= u + f_1(y)\theta + \left(y + f_1(y) + \nu\frac{y^3}{6}\frac{d^2}{dx^2}\right)\beta \\
V &= -\nu y\frac{du}{dx} + v - \nu\int_0^y f_1(y)dy\frac{d\theta}{dx} - \nu\left(\frac{y^2}{2} + \int_0^y f_1(y)dy\right)\frac{d\beta}{dx}
\end{aligned}
\tag{14}
$$

thus, the axial and shear strains can be obtained from the following

$$
\begin{aligned}
\varepsilon_x &= \frac{du}{dx} + f_1(y)\frac{d\theta}{dx} + \left(y + f_1(y)\right)\frac{d\beta}{dx} \\
\gamma_{xy} &= \left(1 + \frac{df_1}{dy}\right)\gamma = f_2(y)\left(\theta + \beta\right)
\end{aligned}
\tag{15}
$$

where, $f_2(y) = 1 + \dfrac{df_1}{dy}$.

5.3 Virtual Work Equations

Applying the virtual nodal displacements $\delta\{\mathbf{u}_q\}$ to Eq. (2), results in

$$\delta\{\mathbf{u}_q\}^T\{\mathbf{P}\} = \delta\{\mathbf{u}_q\}^T[\mathbf{K}]\{\mathbf{u}_q\} \tag{16}$$

and the following equilibrium equation can be obtained

$$\{\mathbf{P}\} = [\mathbf{K}]\{\mathbf{u}_q\} \tag{17}$$

where,

$$\{\mathbf{u}_q\}^T = \{u \quad v \quad \theta \quad \beta\}^T$$

$$\{\mathbf{P}\} = \{\mathbf{q}\} + \{\mathbf{p}\}$$

$$\{\mathbf{q}\} = \int_0^\ell \int_A \left([\mathbf{y}'][\mathbf{x}]\right)^T \{\mathbf{X}\} dA dx$$

$$\{\mathbf{p}\} = \int_{A_i} \left([\mathbf{y}'][\mathbf{x}(0)]\right)^T \{\mathbf{X}_v\} dA_i + \int_{A_j} \left([\mathbf{y}'][\mathbf{x}(\ell)]\right)^T \{\mathbf{X}_v\} dA_j$$

$$[\mathbf{K}] = \int_0^\ell \int_A \left([\mathbf{x}'][\mathbf{x}]\right)^T \left([\mathbf{E}]\left([\mathbf{x}'][\mathbf{x}]\right)\right) dA dx$$

$$\{\mathbf{X}\} = \begin{Bmatrix} X \\ Y \end{Bmatrix}$$

$$\{\mathbf{X}_v\} = \begin{Bmatrix} X_v \\ Y_v \end{Bmatrix}$$

$$[\mathbf{x}'] = \begin{bmatrix} \dfrac{d}{dx} & 0 & f_1(y)\dfrac{d}{dx} & \left(y + f_1(y)\right)\dfrac{d}{dx} \\ 0 & 0 & f_2(y) & f_2(y) \end{bmatrix}$$

$$[\mathbf{y}'] = \begin{bmatrix} 1 & 0 & f_1(y) & y + f_1(y) + \nu\dfrac{y^3}{6}\dfrac{d^2}{dx^2} \\ -\nu y\dfrac{d}{dx} & 1 & -\nu\displaystyle\int_0^y f_1(y)dy\dfrac{d}{dx} & -\nu\left(\dfrac{y^2}{2} + \displaystyle\int_0^y f_1(y)dy\right)\dfrac{d}{dx} \end{bmatrix}$$

6 TRANSFORMATION MATRIX FOR ARBITRARY 4 NODAL POINTS

The transformation matrix for arbitrary 4 nodal points in the beam can be obtained by substituting each point's coordinates $(x_k, y_k),(x_l, y_l),(x_m, y_m),(x_n, y_n)$ into the Eq. (14) by using Eqs. (8) and (10). The transformation matrix defined for the arbitrary 4 nodal points related with the beam ends nodal displacements can be given as follow,

$$\begin{Bmatrix} U(x_k,y_k) \\ V(x_k,y_k) \\ U(x_\ell,y_\ell) \\ V(x_\ell,y_\ell) \\ U(x_m,y_m) \\ V(x_m,y_m) \\ U(x_n,y_n) \\ V(x_n,y_n) \end{Bmatrix} = \begin{bmatrix} {}_kS_{11} & {}_kS_{12} & {}_kS_{13} & {}_kS_{14} & {}_kS_{15} & {}_kS_{16} & {}_kS_{17} & {}_kS_{18} \\ {}_kS_{21} & {}_kS_{22} & {}_kS_{23} & {}_kS_{24} & {}_kS_{25} & {}_kS_{26} & {}_kS_{27} & {}_kS_{28} \\ {}_\ell S_{11} & {}_\ell S_{12} & & & & & & \\ {}_\ell S_{21} & {}_\ell S_{22} & & & & & & \\ {}_mS_{11} & {}_mS_{12} & & & & & & \\ {}_mS_{21} & {}_mS_{22} & & & & & & \\ {}_nS_{11} & {}_nS_{12} & & & & & {}_nS_{17} & {}_nS_{18} \\ {}_nS_{21} & {}_nS_{22} & {}_nS_{23} & {}_nS_{24} & {}_nS_{25} & {}_nS_{26} & {}_nS_{17} & {}_nS_{28} \end{bmatrix} \cdot \begin{Bmatrix} u_i \\ v_i \\ \theta_i \\ \beta_i \\ u_j \\ v_j \\ \theta_j \\ \beta_j \end{Bmatrix} \tag{18}$$

rewritten into a matrix notation,

$$\{\mathbf{U}_q\} = [\mathbf{S}]\{\mathbf{u}_q\} \tag{19}$$

where,

$${}_kS_{11} = 1 - \frac{x_k}{\ell}$$

$${}_kS_{12} = y_k\left(\frac{6x_k}{\ell^2} - \frac{6x_k^{\ 2}}{\ell^3}\right) - \nu\frac{2y_k^{\ 3}}{\ell^3}$$

$${}_kS_{13} = f_1(y_k)\left(1 - \frac{x_k}{\ell}\right) + y_k\left(\frac{3x_k}{\ell} - \frac{3x_k^{\ 2}}{\ell^2}\right) - \nu\frac{y_k^{\ 3}}{\ell^2}$$

$${}_kS_{14} = (y_k + f_1(y_k))\left(1 - \frac{x_k}{\ell}\right)$$

$${}_kS_{15} = \frac{x_k}{\ell}$$

$${}_kS_{16} = -y_k\left(\frac{6x_k}{\ell^2} - \frac{6x_k^{\ 2}}{\ell^3}\right) + \nu\frac{2y_k^{\ 3}}{\ell^3}$$

$${}_kS_{17} = f_1(y_k)\left(\frac{x_k}{\ell}\right) + y_k\left(\frac{3x_k}{\ell} - \frac{3x_k^{\ 2}}{\ell^2}\right) - \nu\frac{y_k^{\ 3}}{\ell^2}$$

$${}_kS_{18} = (y_k + f_1(y_k))\frac{x_k}{\ell}$$

$$ {}_kS_{21} = \nu \frac{y_k}{\ell} $$

$$ {}_kS_{22} = \left(1 - \frac{3{x_k}^2}{\ell^2} + \frac{2{x_k}^3}{\ell^3}\right) - \nu \frac{{y_k}^2}{2}\left(\frac{6}{\ell^2} - \frac{12x_k}{\ell^3}\right) $$

$$ {}_kS_{23} = \left(x_k - 2\frac{{x_k}^2}{\ell} + \frac{{x_k}^3}{\ell^2}\right) - \nu \frac{{y_k}^2}{2}\left(\frac{3}{\ell} - \frac{6x_k}{\ell^2}\right) + \nu \frac{1}{\ell}\int_0^{y_k} f_1(y)dy $$

$$ {}_kS_{24} = \nu \frac{1}{\ell}\left(\frac{{y_k}^2}{2} + \int_0^{y_k} f_1(y)dy\right) $$

$$ {}_kS_{25} = -\nu \frac{y_k}{\ell} $$

$$ {}_kS_{26} = \left(\frac{3{x_k}^2}{\ell^2} - \frac{2{x_k}^3}{\ell^3}\right) + \nu \frac{{y_k}^2}{2}\left(\frac{6}{\ell^2} - \frac{12x_k}{\ell^3}\right) $$

$$ {}_kS_{27} = \left(-\frac{{x_k}^2}{\ell} + \frac{{x_k}^3}{\ell^2}\right) - \nu \frac{{y_k}^2}{2}\left(\frac{3}{\ell} - \frac{6x_k}{\ell^2}\right) - \nu \frac{1}{\ell}\int_0^{y_k} f_1(y)dy $$

$$ {}_kS_{28} = -\nu \frac{1}{\ell}\left(\frac{{y_k}^2}{2} + \int_0^{y_k} f_1(y)dy\right) $$

being the matrix coefficients for node *k* which fill the above two rows of the matrix $[\mathbf{S}]$. The other three nodes (*l*,*m*,*n*) construct similar matrix coefficients by replacing the coordinates (x_k, y_k) of node *k* with the other three nodes' coordinates $(x_l, y_l), (x_m, y_m), (x_n, y_n)$.

6.1 Equilibrium Equation

By substituting Eq. (19) into the virtual work equation of the beam in Eq. (16), the virtual work equation in terms of the arbitrary four node displacements can be given as

$$ \delta\{\mathbf{U}_q\}^T\left([\mathbf{S}]^{-1}\right)^T\{\mathbf{P}\} = \delta\{\mathbf{U}_q\}^T\left([\mathbf{S}]^{-1}\right)^T[\mathbf{K}][\mathbf{S}]^{-1}\{\mathbf{U}_q\}, \tag{20} $$

and the static equilibrium equation for the arbitrary four nodal points in the beam element can be given as

$$ \{\overline{\mathbf{P}}\} = [\overline{\mathbf{K}}]\{\mathbf{U}_q\} \tag{21} $$

where,

$$ \{\overline{\mathbf{P}}\} = \left([\mathbf{S}]^{-1}\right)^T\{\mathbf{P}\} $$

$$ [\overline{\mathbf{K}}] = \left([\mathbf{S}]^{-1}\right)^T[\mathbf{K}]\left([\mathbf{S}]^{-1}\right). $$

7 CONCLUSIONS

- A finite element formulation for an arch structure formed by using built-up beams elements is derived.
- A transformation matrix was analytically derived which can be used to transform the degree of freedoms from the standard two nodes beam element to the arbitrary four nodes lying along the beam boundary.

- Based on the virtual works principle, the equilibrium equation for the developed element model can be obtained by using the transformation matrix with the standard two nodes beam element matrices.

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