

VALIDATION OF NUMERICAL MODELS FOR RC COLUMNS SUBJECTED TO CYCLIC LOAD

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Abstract. *The paper presents results from numerical analyses of reinforced concrete (RC) elements subjected to cyclic load. Many nonlinear numerical models have been proposed in literature to capture the cyclic behaviour of reinforced concrete (RC) elements. Depending on the expected level of accuracy and computational efficiency, numerical models employ different assumptions. These assumptions ultimately affect the stiffness, strength, energy dissipation capacities and hysteretic behaviours such as strength/stiffness degradation and pinching. All of these sensitively affect the dynamic response of a structure subjected to seismic load. The objective of this work is to investigate the effects of adopted numerical models on predicted seismic performances of RC structures. Three numerical elements that are widely used in research and practice are employed; fiber element in OpenSees, frame element in VecTor5, and continuum element in VecTor2. In an element level, the behaviour responses of the different numerical models are closely compared. A few representative experimental results from PEER Column Test Database are used to investigate the accuracy of each numerical model depending on the shear demand/capacity ratio of the columns. The paper concludes with a suggestion for the applicability of different numerical models for the seismic performance assessment.*

1 INTRODUCTION

Since the philosophy of performance based seismic design (PBSD) was first introduced in mid-1990s, there has been significant efforts to implement the philosophy in structural design practice. The most salient feature in PBSD is that structures are designed to meet certain level of performance criteria at different levels of seismic hazards. For example, a structure should satisfy fully operational, damage controllable, or collapse prevention limit states depending on the level of seismic intensity. In the second and third limit states, designers allow a structure to sustain damage as long as the damage is either controllable or the structure does not collapse. To follow this design philosophy, it is essential to evaluate the seismic performance of a structure in the inelastic range, which is still quite challenging task to most practicing designers. In current state of practice, design is carried out primarily based on elastic analysis. With the development of advanced and refined numerical models, large capacity storage, and fast processing speed, however, practicing engineers tend to employ inelastic analysis. By running inelastic analyses, engineers and researchers try to model 'real' behavior of structural elements when subjected to earthquake load. The global structural response is influenced by the inelastic hysteretic behaviour of elements. Hence, it is essential to employ accurate yet computationally efficient numerical models for structural elements. With this purpose, numerous elements have been developed and proposed by researchers. For example, there exist a wide range of numerical elements for reinforced concrete (RC) elements ranging from lumped springs, fiber-based section elements, sophisticated continuum model with smeared reinforcement bars, and detailed finite element (FE) model in which concrete and reinforcement bars are modeled as FE element. Simplified models frequently employ several assumptions and they may yet require thorough and cumbersome parameter calibrations. On the other hand, sophisticated models tend to be more accurate and may often require less number of assumptions. The hysteretic behaviour used to simulate the structural response may also vary significantly with the analysis model. Thus, it is vital to shed light in the limitations and applicability of numerical models to perform seismic response analyses. The present study is aimed at understanding the limitations and/or applicability of different numerical models for RC elements. The study emphasizes the difficulties and limitations of using numerical models to predict the inelastic response of RC members and structures, especially those that are controlled by shear failure. The outcomes of the present analytical work prove that the matching between experimental and numerical simulations is acceptable for flexural-controlled failure modes. However, brittle failure modes caused by shear requires sophisticated and refined finite element models. Furthermore, existing technical guidelines and code provisions are yet far from being exhaustive for shear-dominated response of RC structures.

2 SEISMIC BEHAVIOUR OF RC STRUCTURES

Reinforced concrete structures enter into the inelastic region when subjected to moderate to high-magnitude earthquakes. Several surveys carried out in the aftermath of major earthquakes have reported widespread brittle failure to structural members and joints (i.e. Figure 1a,b), especially in existing buildings designed primarily for gravity loads. Earthquake resistant structures are nowadays designed using strength reduction, nonlinear response is generally expected for ordinary buildings which are compliant with modern seismic building codes. Such buildings are intended to experience significant structural damage before collapse; hence dissipative zones, e.g. plastic hinges, are required to dissipate and absorb seismic energy. A reliable estimate of the actual seismic capacity of RC structural systems can be obtained through accurate nonlinear analyses [3,4]. Due to the complex nonlinear behavior of structural system a in-depth knowledge at element level is required.



Figure 1: Field surveys after major devastating earthquake: (a) Shear failure of a squat column during L'Aquila earthquake 2009 [1]; (b) Soft storey mechanism with plastic hinges on columns during Emilia earthquake 2012 [2].

Numerous studies carried out in recent years have demonstrated that the cyclic behaviour of RC elements is affected by various parameters [e.g. 5,6, among many others]. The reliable assessment of the earthquake response of RC members, especially when subjected to a combination of biaxial bending moment and axial load, is still a challenging problem [7]. Based on large and comprehensive datasets of experimental tests, novel formulations which may reproduce closely the seismic behaviour of RC elements, have been developed. Many researchers [8,9] have focused on confined concrete behaviour which led to development of complex yet accurate theories. The most commonly approach to assess the cyclic behaviour of RC members is the lumped plasticity model; it consists of locating or “lumping” the system nonlinearities in ad hoc points of the members, such as the member ends. The plastic hinge length is often not an easy task to accomplish [e.g. 10]. Whenever the use of lumped models is inadequate, fiber models may be employed to account for the spreading of nonlinearities along the members (e.g. [11], among others). The classical Finite Element Method (FEM) tends to be accurate to simulate the large displacement of highly non linear systems. Nevertheless, FEM is often cumbersome for earthquake response analyses of large structures. Advanced theories have been formulated for the applications of FEM to RC structures [e.g. 12-14, among others]. The availability of high computational power has recently promoted the development of advanced and efficient algorithms and computer programs (e.g. OpenSees [15], VecTor programs [16] among others) that may simulate reliably the cyclic behaviour of RC elements or complex structural systems using refined numerical models. New cutting edges simulation capabilities have been achieved especially in interdisciplinary fields, as for instance in the assessment of soil-structure interaction [17]. The integration of such interactions with structural models is often not straightforward.

Brittle failure mechanisms of typical RC members are illustrated in Figure 2b. Brittle failure modes lead to a displacement capacity remarkably lower than the deformations experienced in ductile failure mechanisms. High ductility lateral resisting systems are required by modern earthquake-resistant design to prevent brittle failure in high seismic risk areas. However, numerous existing buildings do not conform to such design rules. Conversely, they tend to exhibit brittle shear failure of critical members, thus impairing the seismic capacity of the structural system as a whole (i.e. Figure 3). Vertical ground motion effects may also endanger the shear capacity of RC members, especially for non-conforming columns in existing framed buildings [20]. Additionally, the accurate assessment of shear critical structures is of para-

mount importance to calibrate appropriate strengthening solutions [21]. As a result, the accurate investigation of the shear response is a key point in the assessment procedure of structures. Shear response, especially when large inelastic deformations are experienced by either the members or the structure, is difficult to predict. In shear critical elements, the cracks may rapidly propagate.

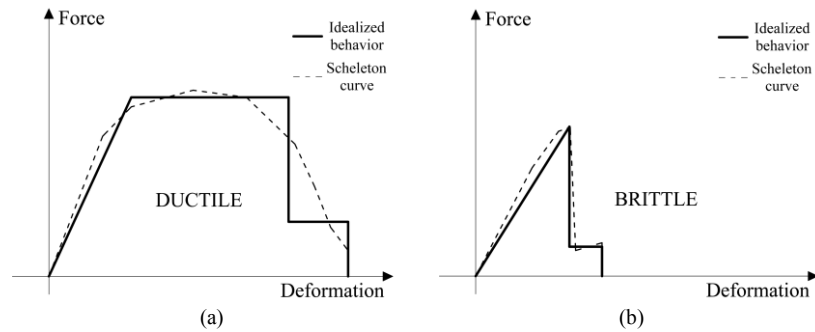


Figure 2: Typical failure modes of R.C. elements: (a) Ductile failure; (b) Brittle failure.

The most common approach to assess the shear strength of RC members uses the truss model. The shear behaviour is thus investigated by assuming that in the cracked element the resisting mechanism includes the compressive field, above the neutral axes, the diagonal concrete struts and the longitudinal. The transverse reinforcement may also be accounted for. The assumption of inclined compression diagonals, with a 45° angle [22], was removed by Mörsh [23]. The truss-based theories were largely adopted in the past and they have inspired several building codes worldwide; however, new developments have been achieved in the recent years. Extensive experimental and field investigations have shown that complex phenomena affect the shear behaviour of RC elements. From the experimental standpoints, it is worth mentioning that innovative theories have been developed; such theories are aimed at characterizing the stiffness, strength and deformability of shear critical elements (e.g. MCFT [12] and DSFM [13]). The validation of the above formulation has been achieved with the implementation of the shear response in a number of dedicated computer programs (VecTor suite [16] and tools [24]).

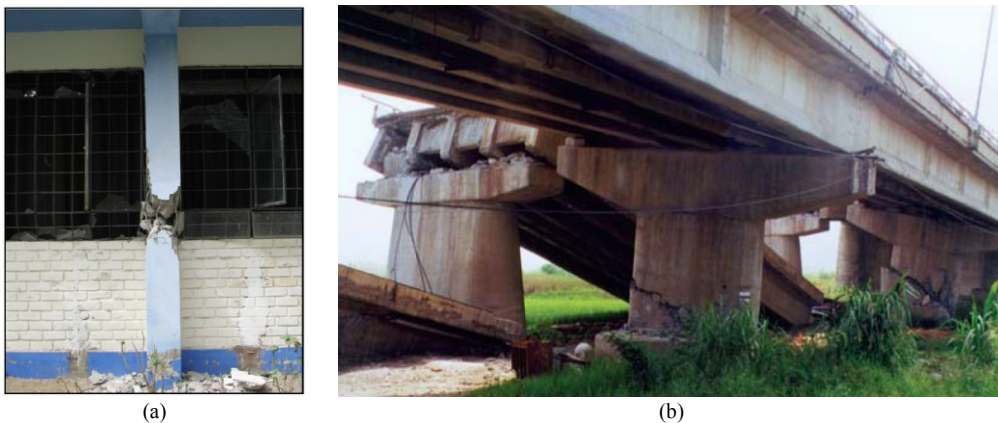


Figure 3: (a) Column shear failure during the 2007 Pisco-Chincha earthquake [18]; (b) Bridge collapse due to pier shear failure during Chi-Chi, Taiwan earthquake 1999 [19].

1 RESEARCH OBJECTIVES

Numerical models are powerful tools to simulate the nonlinear behaviour of structural elements subjected to multi-axial (shear, axial, and moment) cyclic loading. However, numerous effects that affect the seismic behaviour of RC structures are often neglected in numerical models or they not accounted for by non-expert users. This is the case, for example, of the accurate description of shear behaviour, bar buckling and reinforcement slipping. The objective of the present study is to investigate the effects of the numerical models on the seismic performance of RC members, such as beams, columns and frames. Thus, three different numerical models have been adopted: fiber element in OpenSees [15], frame element in Vec-Tor5 [16], and continuum element in VecTor2[16]. Realistic physical and mechanical parameters are utilized to set up a sound and general purpose methodology. The first step of the proposed procedure is the validation of the numerical model with respect to basic components. Then, sophisticated models with high level of accuracy are also considered. At an element level, the global hysteretic behaviour is closely assessed. A few representative experimental results from available literatures are used to investigate the accuracy of each modelling approach.

2 VALIDATION OF NUMERICAL MODELS

2.1 Specimen Selection

Reference experimental tests for structural elements and structural systems are selected such that typical failure modes in RC structures can be evaluated. The shear strength may degrade with increasing ductility demand (e.g. [20]) and shear failure after the flexural yielding may thus be attained. According to Setzler [25], five failure modes may typically occur in RC structures. Shear, flexural and mixed flexural-shear failure modes are considered the basic failure mechanisms; two additional cases are also considered when failure mode is not well detected due to the simultaneous occurrence of the two failure modes. The latter approach is also implemented in ACI 369-R11 [26].

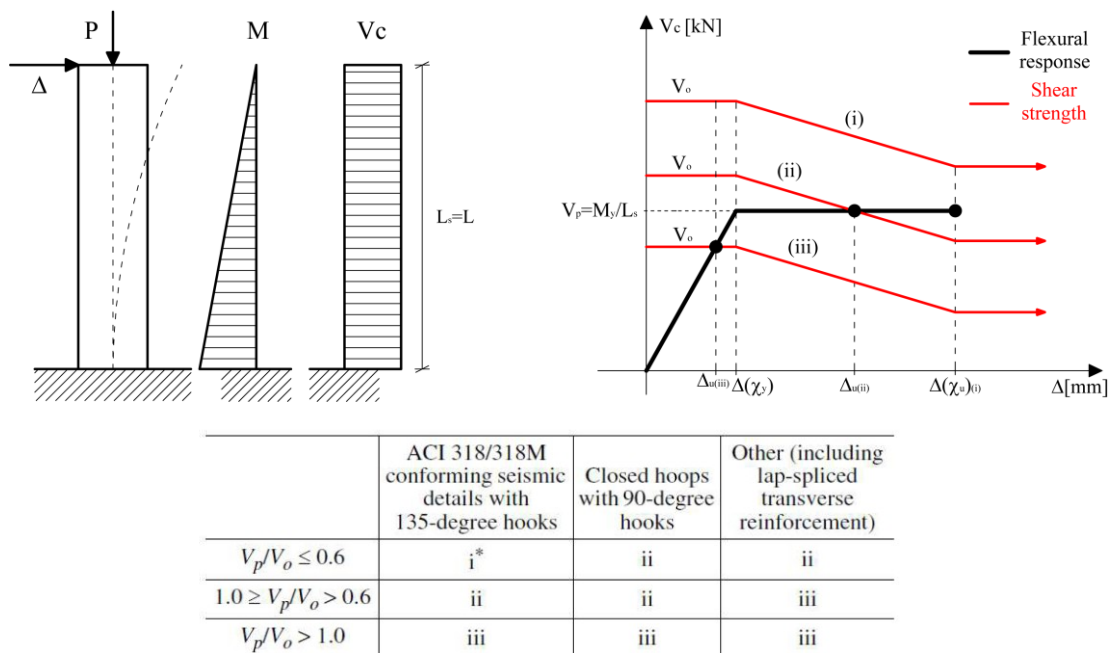


Figure 4: Failure modes: “i” Flexural; “ii” Flexural-Shear; “iii” Shear according to ACI369R-11 [26].

The ACI code is based on a large number of experimental observations; it classifies the failure mode as a function of the ratio between the shear at the flexural yielding, quoted as V_p , and the shear strength for an imposed ductility level of 1, named V_0 . As reported in Figure 4, the failure mode depends also on the transverse reinforcement details. In the present study the ACI 369-R11 [26] approach is adopted to select sample specimens corresponding to three different failure modes. Experimental results from PEER Column Test Database [27] are used to investigate the accuracy of selected numerical models. At the element level, three columns are selected to represent the three failure modes.

The selected specimens are summarized in Table 1 which includes details such as cross section shape, i.e. rectangular (R) or circular (C); concrete cylindrical compressive strength (f_c); span-to-depth ratio (L_s/D); axial load ratio (v); transverse reinforcement ratio (ρ_s) and predicted and experimental failure mode.

Reference	Spec.	Section	f_c [MPa]	L_s/D	v	ρ_s [%]	Failure Mode		Numerical Model		
							ACI 369 [26]	Exp.	Open Sees	VecTor 5	VecTor 2
Tanaka [28]	6	R	32	3	0.10	0.8	i	i	•	•	•
Lynn [29]	3CMD12	R	27.6	3.22	0.26	0.2	ii	ii	•	•	•
Umehara [30]	CUS	R	34.9	1.11	0.16	0.3	iii	iii	•	•	•

Table 1: Summary of specimen properties and numerical models adopted for the validation.

Additional details on specimen geometry, reinforcements, test setup and main experimental results can be found in the PEER Database [27] and in the related references.

2.2 Adopted numerical models

The sample numerical models were validated by investigating the cyclic behaviour of four RC columns. The experimental behaviour of a simple cantilever column with a flexural failure mode, Tanaka et al. [28] (Figure 5), is used to validate the proposed numerical models.

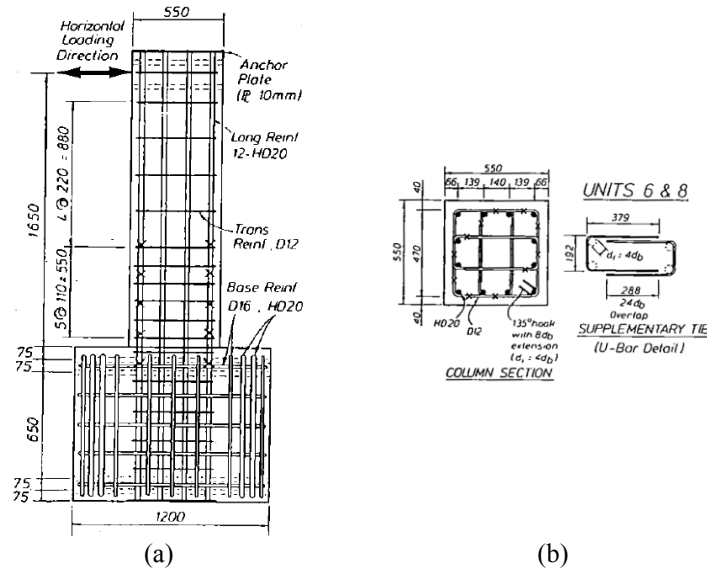


Figure 5: Tanaka [28] specimen 6 details (unit *cm*).

- *OpenSees models*

Both OpenSees [15] fiber (Figure 6a) and lumped plasticity models (Figure 6b) are adopted herein. In the fiber model, three different materials are used to represent the cyclic behaviour of column cross sections: the confined and unconfined concrete is simulated with the nonlinear *Concrete02* material; the longitudinal reinforcement is modelled with the *Steel02* uniaxial material with isotropic strain hardening. The confining effects of transverse reinforcement on the mechanical properties of the concrete core are calculated according to Mander et al. [8]. The novel *ConfinedConcrete01* (CC01) material is also adopted to account for the confining effects of the stirrups; it is computed by defining transverse reinforcement geometry and mechanical properties. The ultimate compressive strain of confined concrete is calculated according to Biskinis et al. [31] as suggested by Fardis [6].

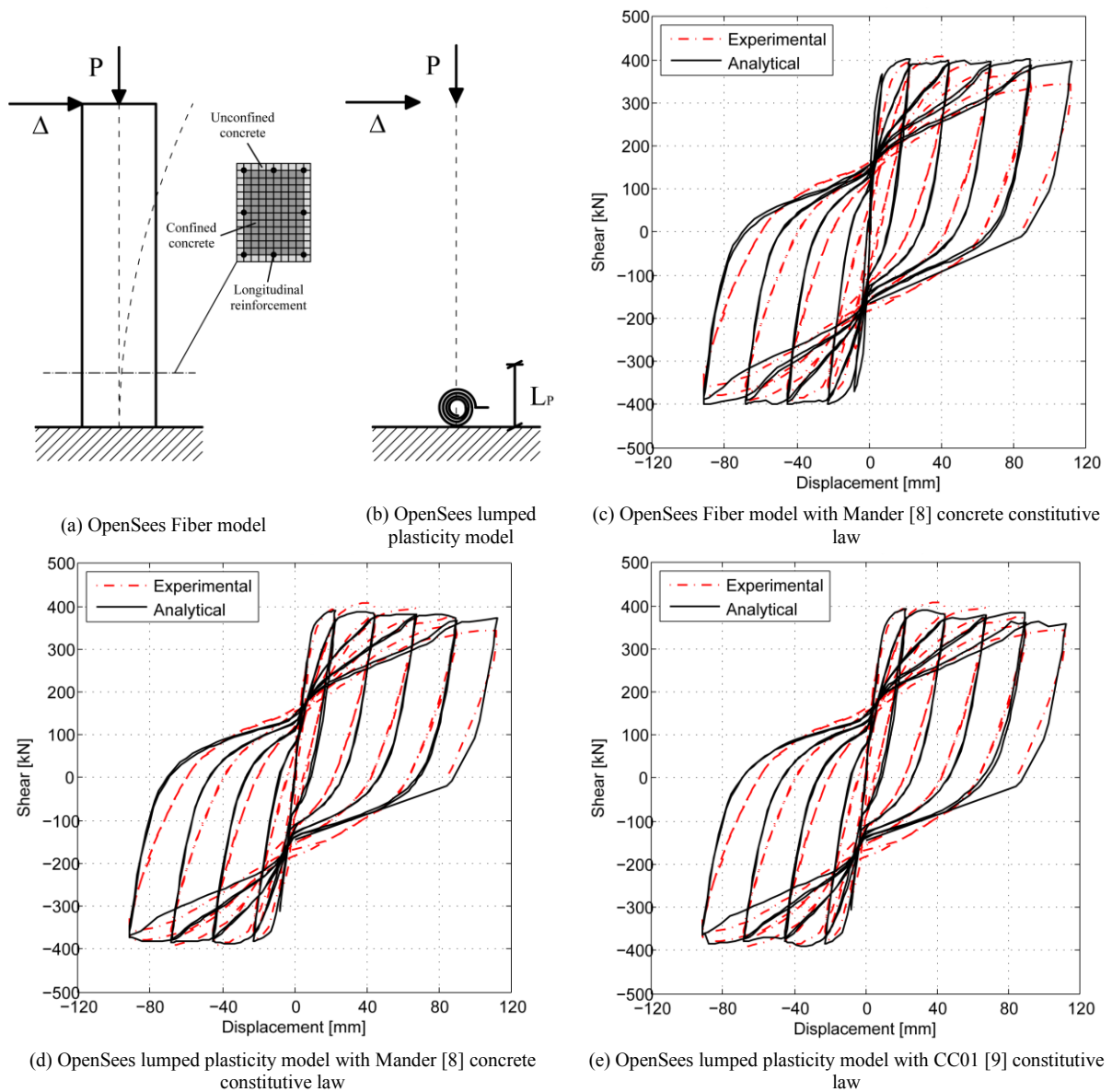


Figure 6: OpenSees model validation with Tanaka [28] spec. 6 experimental result.

Five integration points along the finite elements have been adopted, thus allowing the spreading of inelasticity to be accurately described. The cross section is uniformly divided into sixteen elements to represent closely small stress-strain variations. For the lumped plasticity model, the *2D Beam with hinges* command line is set to concentrate all system nonlinearities at the base of the column [10]. The plastic hinge length is set according to Bae et al. [32] and hinges properties are calculated assigning a fiber section. Plastic hinge stiffness is defined according to the experimental-based formulation of Elwood et al. [33]. Applied axial load (P) and cyclic displacement (Δ) are determined in accordance to the actions registered during the experimental test taking into account of P- Δ effects by means of the geometric transformation tool *PDelta Transformation*. The Newton-Raphson solution algorithm is adopted to solve model nonlinear equations. Comparisons between numerical and experimental results for the three OpenSees models in term of shear vs. imposed displacement are provided in Figure 6. Figure 6c shows the close match of experimental hysteresis loop with the OpenSees fiber model numerical results. The match is acceptable with respect to strength, deformability, and pinching. The analytical model provides an initial stiffness higher than the experimental one. This response may be related to model which do not account for the effects of bar slipping, column base cracking, support concrete block confining effects. Both the OpenSees lumped plasticity models (Figures 6d,e) provides accurate predictions of the experimental results also in terms of initial stiffness. This is due to the adoption of a realistic stiffness from a large number of experimental tests on column specimen with negligible shear deformation [33].

- *VecTor5 models*

VecTor5 is a nonlinear sectional analysis program, with a distributed nonlinearity fiber model approach, for two dimensional frame-related structures consisting of beams, columns and shear walls, subjected to static and dynamic loading. VecTor5 is based on MCFT [12] and DSFM [13] theories and is capable of considering geometric and material nonlinearities. In the model development, attention should be paid to define geometric and mechanical properties. Starting from element cross-section properties, as suggested in the manual [34] and in related literature studies [35], different thickness and mechanical properties should be assigned to each fiber. The effect of transverse and out-of-plane reinforcements is considered by assigning a specific percentage to each fiber element. Geometric and mechanical material properties of concrete, longitudinal and transverse reinforcements are defined based on experimental specimen. All analyses were performed with the use of *basic* default material behaviour models and analysis options. The Popovic [36] and Mander [8] constitutive law is adopted to reproduce concrete compressive behaviour. The nonlinear sectional analysis is performed using the Parabolic Shear Strain Distribution (Single-Layer Analysis) as shear analysis option. Optimum segment lengths, 50% of the cross section depth [35], is adopted in column modelling. Comparative analyses are carried out in order to assess the influence of the footing block. The cyclic behaviour of three different analytical models are compared in the following: a simple cantilever scheme with the length measured by column-foundation intersection (Figure 7a), an accurate model of footing schematized with beam elements according to [35] (Figure 7b), and a simplified model of the footing, modeling only the portion that goes to column base to foundation block centerline (Figure 7c). Figure 7 shows the close match with experimental results for the VecTor5 proposed models. In details the proposed VecTor5 numerical models give a good fit of experimental results in terms of peak strength, energy dissipation, pinching without neglect strength degradation effects related to large inelastic deformations. The comparisons also pointed out the importance of an appropriate modeling of the footing in order to achieve a more accurate match of element cyclic behavior.

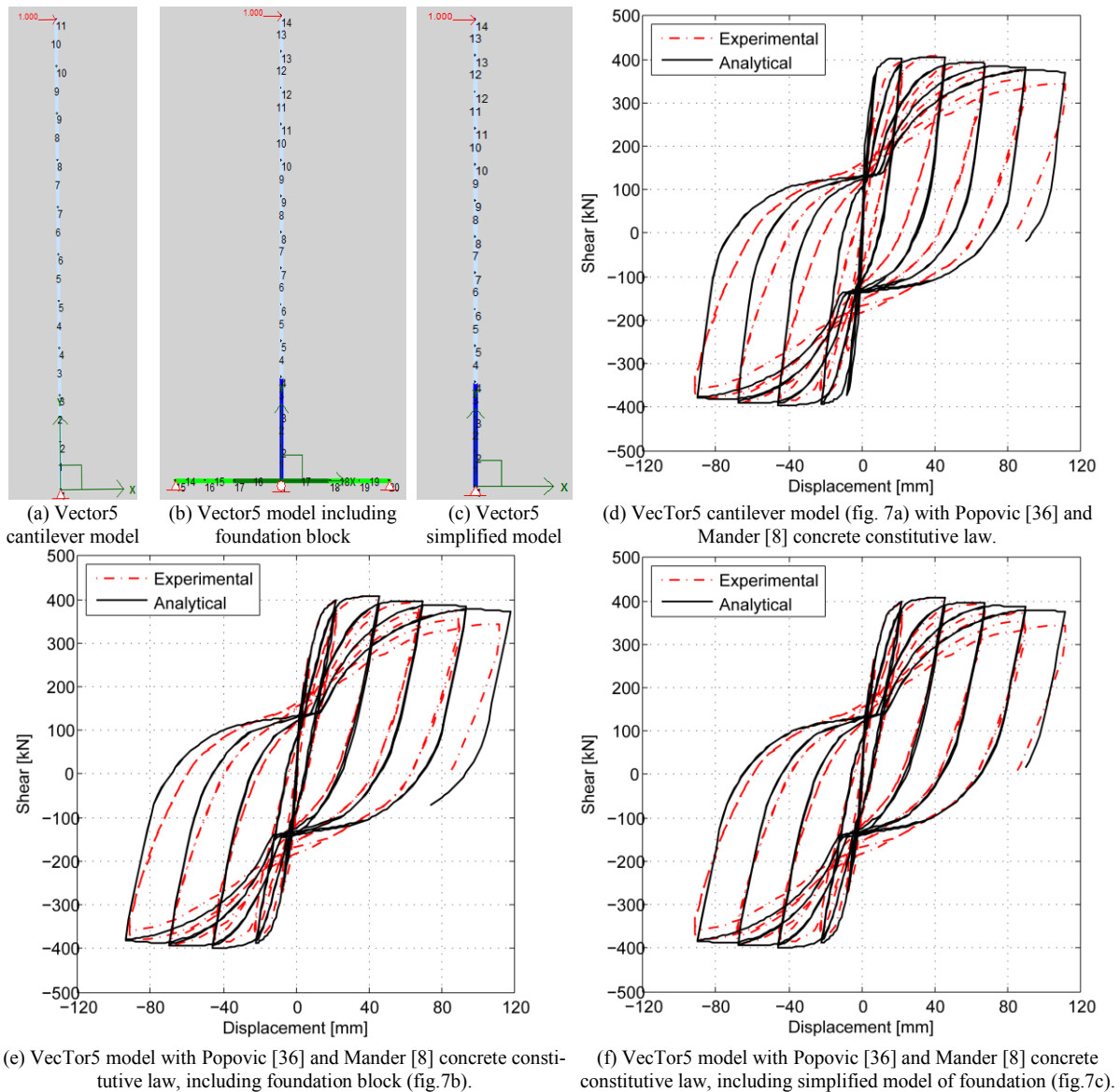


Figure 7: VecTor5 model validation with Tanaka [28] spec. 6 experimental result.

The numerical modeling of the foundation block led to more realistic estimations of the initial stiffness (Figure 7e,f). The use of a simplified scheme of the footing, i.e. adding the column portion from column base to block centerline, leads to results close to the accurate model (Figure 7f).

- *VecTor2 models*

VecTor2 is a program based on the MCFT/DSFM for nonlinear finite element (FEM) analysis of reinforced concrete membrane 2D structures that permits accurate assessments of structural performance (strength, post-peak behaviour, failure mode, deflections and cracking). The Vector2 bundle [24] includes: FormWorks, a graphics-based preprocessor program that simplifies the model building; Augustus, a complete VecTor2 post-processor that may provide all the global and local results in useful numeric or graphic formats. It is also able to display the specimen crack pattern at each stage of imposed displacement and this represents a very useful tool to detect numerical model failure mode. The FEM elements require a differ-

ent and more complex modelling approach. As showed in Figure 8a column model is composed by two different materials, confined and unconfined concrete. Confinement effects are taken into account by means of the geometric percentage of in-plane and out-of-plane reinforcements [37]. Longitudinal reinforcements are modelled with truss elements assuming perfect bond with the surrounding concrete.

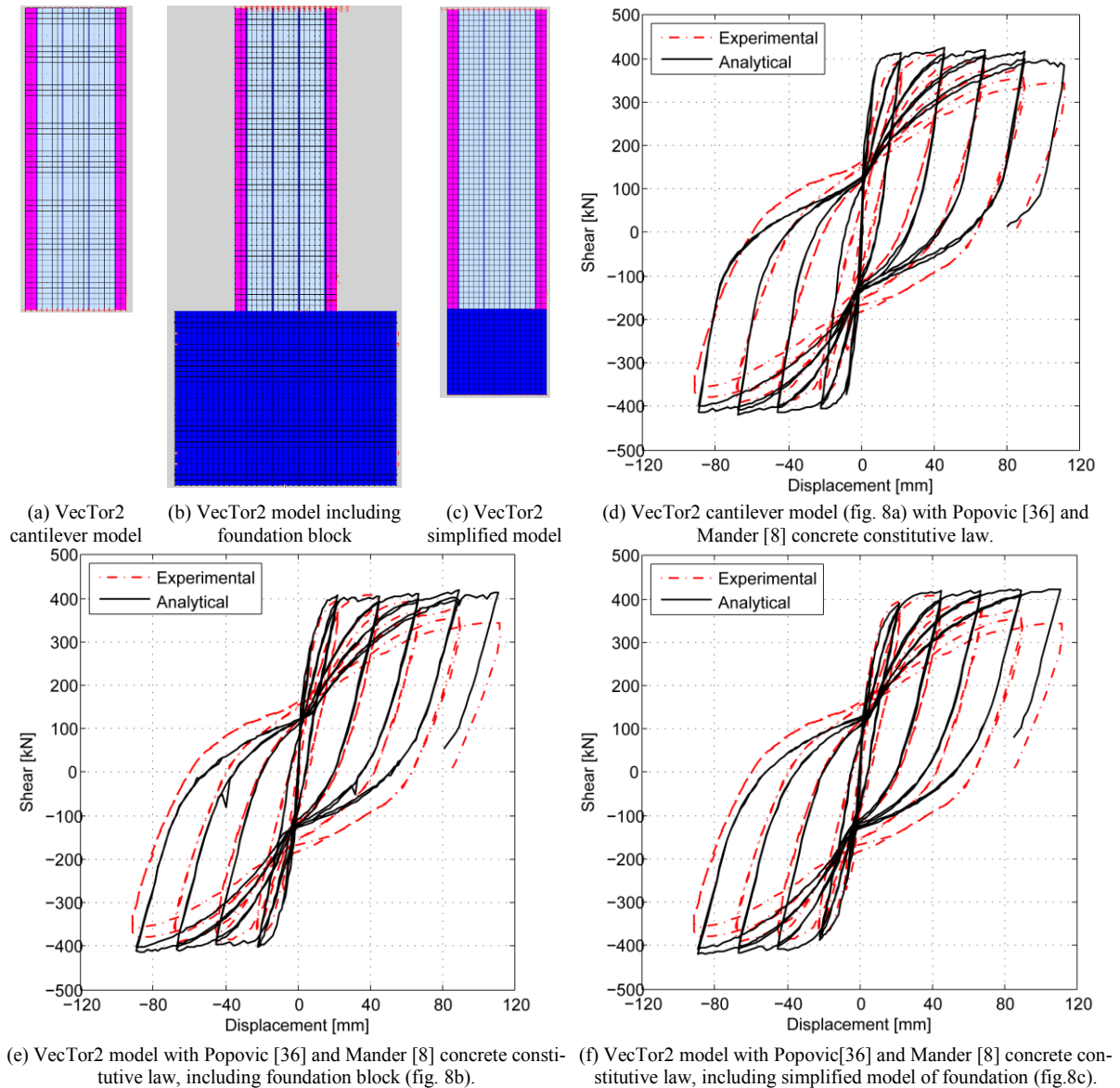


Figure 8: VecTor2 model validation with Tanaka [28] spec. 6 experimental result.

The FormWorks automatic mesh generator is used to create a model with appropriate mesh size (variable in the optimal range 30-35 mm [38]). As in the experimental test, column models are subjected to cyclic displacement and axial load is applied at the top column as a distributed load. Restrain conditions are imposed at column basement. As in the previous modeling approach, three different schemes are proposed to simulate basement effects: a simple cantilever scheme preventing the translation of column base joints (Figure 8a); foundation block accurate modeling with realistic material and reinforcement properties (Figure 8b); a simplified model of foundation block (Figure 8c). Default *basic* options are selected for material constitutive laws and analysis mode [38], except for concrete compressive behaviour

schematized with Popovic [36] and Mander [8] relations. Comparative analysis between experimental and analytical test results are depicted in Figure 8.

Also for the proposed FEM models a good match with experimental results is obtained. It is also confirmed that the foundation modelling appears to be important to correctly estimate the initial stiffness (Figure 8e); however, it can be easily simplified as in Figure 8f with a substantial reduction in computation time.

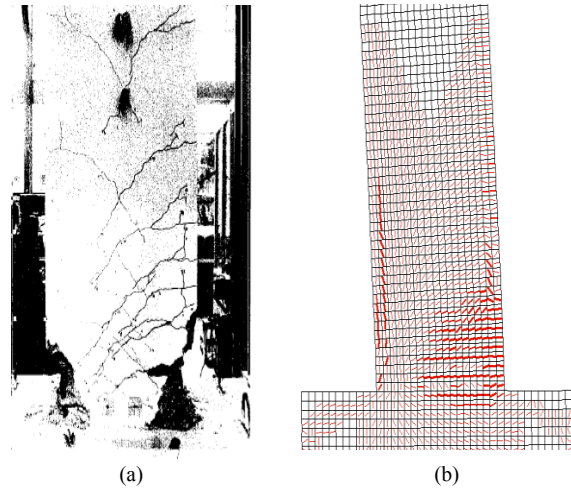


Figure 9: Tanaka [28] spec. 6 crack pattern: (a) experimental; (b) VecTor2 analytical model (Augustus view[24]).

A close match between experimental and analytical crack pattern (Figure 9) with flexural cracks concentrations at column base and the top part of the footing was also found.

2.3 Validation procedure

The validation of the numerical models presented earlier is generalized hereafter to specimens with different failure modes (see Table 1). The aim is to provide a unified modelling procedure that may lead to reasonable matches with experimental results regardless of the failure mode. Thus, in the following analyses, the models employed in the previous sections are adopted without changes or numerical calibrations. The adopted numerical models include: OpenSees lumped plasticity model with Mander [8] concrete constitutive law; VecTor5 model with Popovic [36] and Mander [8] concrete constitutive law, including simplified model of foundation; VecTor2 model with Popovic [36] and Mander [8] concrete constitutive law, including simplified model of foundation.

- Lynn spec. 3CMD12 [29]

A column with a combined flexural-shear failure mode [26] with non-negligible inelastic shear deformation is used to validate the numerical models. Specimen details are illustrated in Figure 10a. For this specimen a double curvature scheme is adopted in the numerical models. The comparisons between analytical and experimental results show that the OpenSees model (Figure 10b) led to significant overestimations of the lateral stiffness. The elastic shear deformation, neglected in the numerical model, may affect significantly the initial stiffness. Although the numerical model gives rise to a reasonable prediction of the peak strength (flexural dominated), it is not able to accurately describe the strength degradation related to shear strength degradation after column yielding. This led to significant overestimations of the dissipated energy, especially for the last cycles.

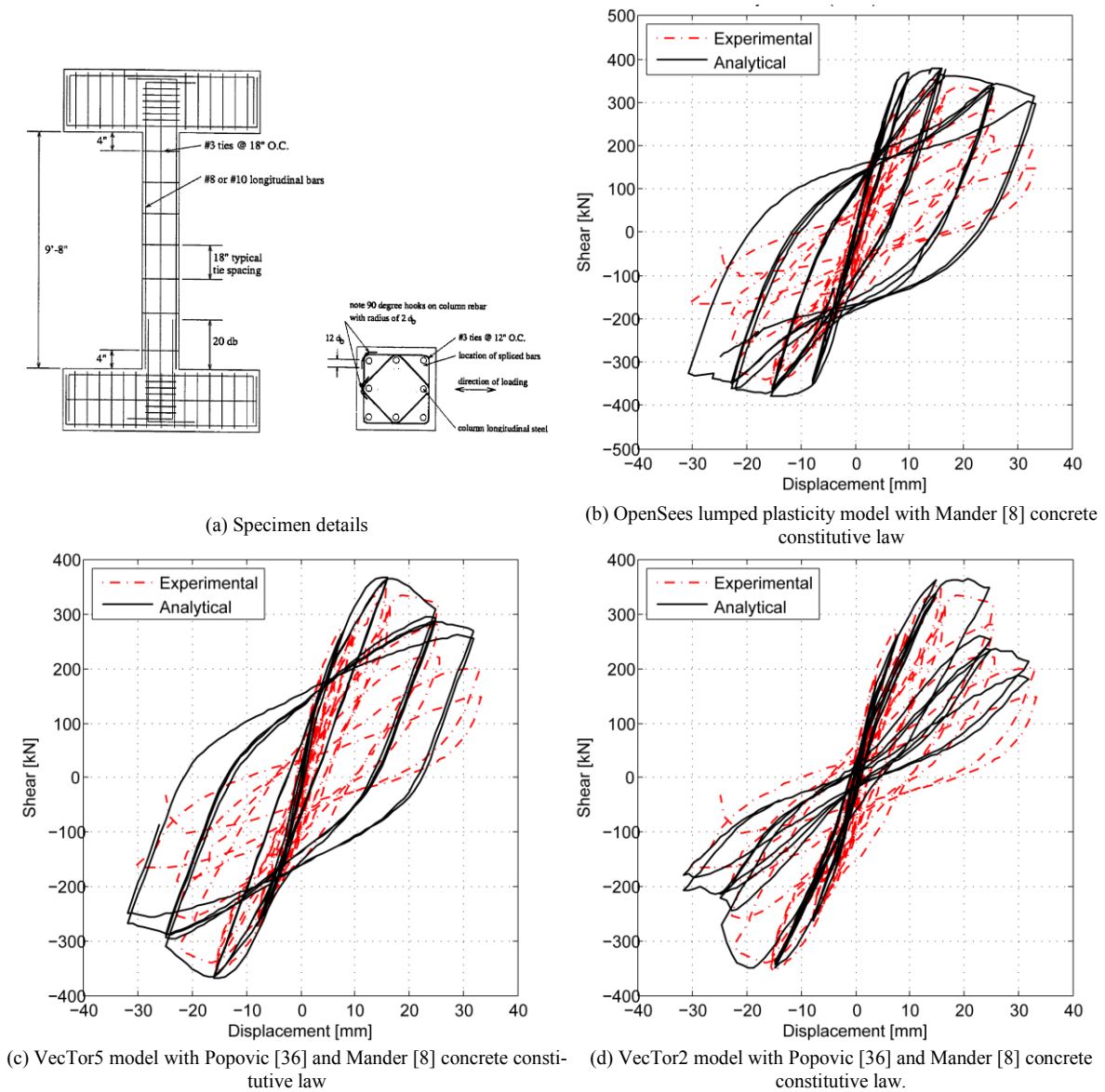


Figure 10: Validation of proposed numerical models with Lynn [29] spec. 3CMD12 experimental result.

The VecTor5 model (Figure 10c) provides a matching of the strength at the peak stage and in the degradation phase. It also describes closely the column stiffness especially at the initial stages. In the last cycle, when the effects of the shear failure became dominant, a sensitive overestimation of the dissipated energy is shown. For this kind of failure the best fit of experimental results is achieved with the VecTor2 FEM model (Figure 10d), that is able to predict with reasonable accuracy experimental test behaviour at all the stages.

- Umehara *et al.* spec. CUS [30]

The following step of the validation procedure is to compare the analytical results of the proposed numerical models with an experimental test characterized by shear failure. Geometries and reinforcement details of the selected squat column are presented in Figure 11a.

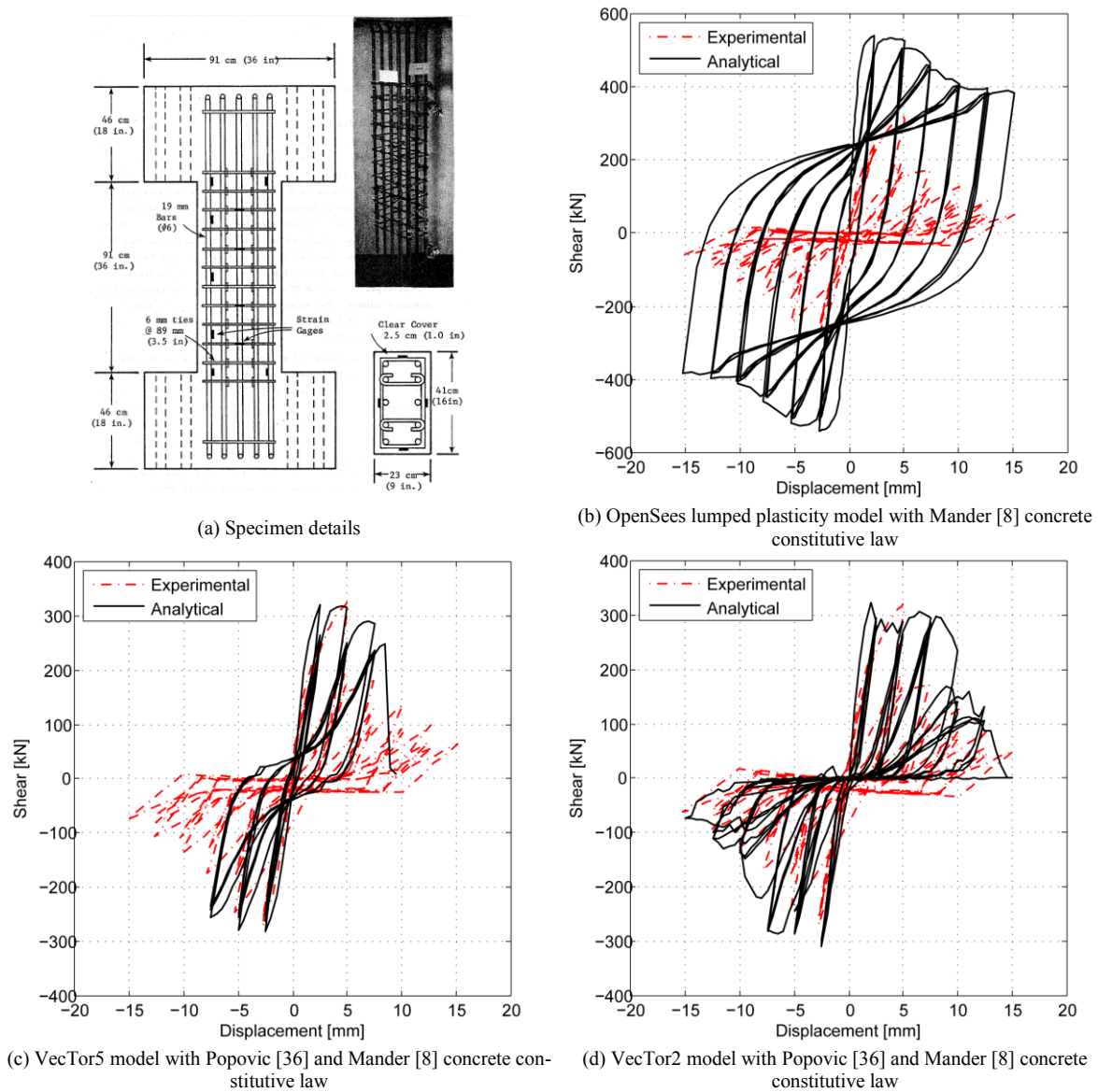


Figure 11: Validation of proposed numerical models with Umehara [30] spec. CUS experimental result.

According to the experimental test setup, a double cantilever scheme is adopted. For the VecTor5 model the single change is the segment length in the range of 10% of the cross section depth as suggested for shear walls [35]. Graphic comparisons depicted in Figure 11b show as the analytical models that do not account for shear effects in terms of strength and deformability may lead to capacity estimation sensitively different from experimental results. Both the VecTor (Figure 11c,d) models match very well with experimental results, in terms of initial stiffness, peak strength and in the degradation branch for small displacement imposed. VecTor5 capacity curve (Figure 11c) drops to zero when shear crack check detects values not compatible with specimen stability [34]. On the other hand Vector2 is able to achieve a reasonable accordance with experimental results also for high imposed displacement and important strength reductions.

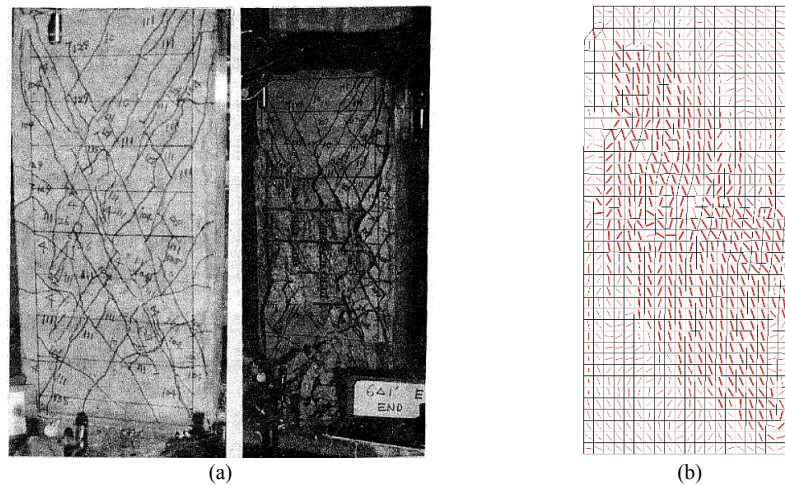


Figure 12: Umehara [30] spec. CUS crack pattern: (a) experimental; (b) VecTor2 analytical model.

Comparisons between analytical and experimental crack pattern (Figure 12) confirm the ability of Vector2 to predict crack evolution during the cyclic test.

5. DISCUSSION AND CONCLUSIONS

The structural response of RC shear critical elements may lead to simplified models based on the regression approach on experimental data. Such models are limited to an estimation of the strength capacity and are not useful to conduct advanced nonlinear analysis aimed to reproduce both the strength and deformation behaviour of shear critical elements. To achieve accurate estimation of RC buildings seismic capacity, the structural response of structural subassembly or simple elements should be more accurate. Thus, the estimation of the plastic behaviour requires a thorough assessment when the effects of strength and stiffness degradation can be significant. Especially for redundant structures, as RC frames, the failure of one or more structural elements usually does not result in a structural collapse.

In the present paper different advanced numerical model are presented and their capacity to predict the cyclic behaviour of different RC elements is closely investigated. Columns exhibiting flexural failure mode were initially selected to get the best fit between experimental and numerical results. Thus, a validation procedure was proposed to validate the three presented numerical models: OpenSees lumped plasticity model with sectional behaviour; VecTor5 nonlinear fiber model; VecTor2 FEM model. The numerical comparisons showed the accuracy of proposed models in predicting the cyclic behaviour of a flexural critical column. Relevance should be given in the modeling of the footing used to simulate the realistic boundary conditions in the experimental test. This work also pointed out the importance of an accurate modeling of shear effect in the case of shear critical elements. Neglecting elastic and inelastic effects related to shear action (as in the proposed OpenSees models) may give rise to unrealistic estimations of member capacity in terms of strength and deformability. The VecTor software may be an accurate tool to reproduce the behaviour of shear critical elements. The VecTor2 FEM models provide accurate estimations of the element capacity regardless of the failure mode. The selected software also accurately predicts the crack pattern.

Finally, it can be argued that having accurate and complete information of the cyclic behaviour of RC components might be very useful to create a reliable numerical model that will be able to take into account the full nonlinear behaviour including strength and stiffness degradation. The use of the presented models could lead to realistic estimations of the seismic

capacity of the structural system without limitations imposed by traditional capacity models and with appropriate estimation of shear deformability effects.

6. ACKNOWLEDGMENTS

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