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DISCRETE IMPEDANCE MATRIX FOR FLEXIBLE SURFACE FOUNDATIONS

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Abstract : *Knowledge on the subject of dynamic soil-structure interaction (SSI) has been derived mainly from studies on rigid foundations. The seismic response of structures resting on flexible foundations has received considerably less attention and the results of these efforts have not yet lead to established computational procedures of general acceptance, such as those developed for structures resting on rigid foundations (ATC-3, NEHRP-03, IBC, EC-8). In this paper, a rigorous computational procedure for the analysis of soil-flexible foundation–structure interaction is developed based on the finite element method. Thus, a full model of the soil-flexible foundation system is constructed and the discrete (in space), frequency dependent, impedance matrix of the system is obtained. To this end the special purpose finite element program ACS SASSI is utilized taking into consideration the infinite extent of the soil medium. A simplified, under certain conditions, frequency independent impedance matrix is also presented. The above impedance matrices are used in a straightforward manner for the dynamic/seismic analysis of structures resting on flexible mat foundations in frequency domain.*

1 INTRODUCTION

During the last decades the problem of dynamic soil-structure interaction (SSI), has received considerable attention. The main research effort has been focused on the dynamic behavior of rigid foundations and some of the produced publications have become standard references in the field, e.g. Richart *et al.* [1], Veletsos and his co-workers [2-4], Luco and his co-workers [5-10], Vaish and Chopra [11] and Wolf [12]. In comparison to this effort, the study of flexible foundations, particularly extensive mat foundations, has received limited attention, e.g. Refs [13-17]. In contrast to rigid foundations, the study of which is based on classic fundamental solutions for the half space, e.g. Refs [18,19], or discretization of the half space by either the FEM, e.g. Refs [20-22], or the BEM, e.g. Refs [23-25], the dynamic analysis of flexible foundations requires the discretization of both the half space and the foundation itself, a task usually accomplished with the help of the FEM for both domains, e.g. Ref. [26], or the BEM, e.g. Ref. [24], or a combination of FEM and BEM, e.g. Refs [14, 27, 28]. Comprehensive reviews of these matters can be found in various sources, e.g. Gazetas [29], Beskos [30], Karabalis [31], etc.

In this work, the main goal is the study of the dynamic response of extensive mat foundations resting on a homogeneous, isotropic, linear elastic half space. The FEM, in frequency and time domain, is used throughout this work. For the purposes of these analyses (a) the half space impedances are computed with the help of the computer code ACS SASSI [32] and subsequently are transformed to a non-dimensional form, and (b) the half space impedances are matched with the discretization of the mat foundation, within the platforms of the computer codes ACS SASSI [32] or ANSYS [33], and the dynamic analysis of the half space-foundation system is computed in frequency or time domain. Numerical results pertaining to half space impedances and dynamic analyses of flexible foundations, as well as comparisons with results from other published works are also presented in order to verify the accuracy and practical efficiency of the proposed methodologies.

2 PROBLEM STATEMENT AND ANALYSIS PROCEDURES

The problem under consideration consists of determining the dynamic response of linear flexible mat foundations resting on a homogeneous soil medium idealized as a linear elastic, isotropic half space. The FEM is used throughout this work and requires the discretization of both the half space and the finite foundation slab. The half-space is simulated with the help of the computer code ACS SASSI which, via the usage of transmitting boundaries, provides the means for efficient modeling of a semi-infinite medium. The superstructure, in this case the foundation slab, can also be modeled with ACS SASSI, thus providing for a total solution of the problem. However, the analysis of the entire soil-structure system can also be accomplished in two steps: (a) the impedances of a dense network of nodal points on the surface of the half space are computed by ACS SASSI, and stored in a non-dimensional form, and (b) the above impedances are imported to any standard general purpose FEM code, in this case ANSYS, where the solution of the soil-structure system can be solved for a variety of structural configurations and loading scenarios. Both approaches are used in this work and the obtained results are compared with results obtained in the literature.

2.1 Impedance of a half space

The soil simulation in the computer code ACS SASSI consists of two parts: (a) the original site model (“free field”) which is assumed to consist of horizontal soil layers overlying either a rigid base or an elastic half space boundary condition and (b) the boundary condition at the base of the “free field”. The material properties for the soil layer system are assumed to be viscoelastic, i.e. complex modulus representation of the stiffness and damping properties of each soil layer. In the case of a half space simulation, the case of this work, the same material properties are used for both parts of the model and the elastic half space boundary condition is modeled using the variable depth method, where n extra layers are added to the base of the top soil layers.

The thickness of each soil layer depends on the minimum wavelength λ_{min} of the analysis and it should not exceed the value $\lambda_{min}/5$. The total depth of soil layers depends on the discretized surface dimensions, and for the purposes of this work is extended to at least ten times the dimension of the discretized free surface. The total depth H of the added layers (simulating the half space boundary condition under the "free field"), varies with frequency and is set to be equal to 1.5λ. The surface of the half space is discretized using three-dimensional quadrilateral thin shell elements. The nodes of the elements are the interaction points where the impedance values are calculated. The maximum distance between these nodes is set to be lower than $\lambda_{min}/8$.

The steps of the analysis are: (i) every surface interaction point is consecutively loaded with a unit harmonic load in the direction of a specific degree of freedom, (ii) the half-space flexibility complex matrix is calculated for all interaction points and (iii) the impedance matrix is computed by inverting the flexibility matrix.

Results of the above analysis are presented for a range of frequencies and half-space material properties. These results are normalized using the shear modulus of the half-space G_s. Impedance values are plotted as a function of dimensionless distance d_o, defined as

$$d_0 = \frac{d^2 G_s}{\sum F_{ext}} \tag{1}$$

where d is the distance from the loaded point and F_{ext} the external load. A characteristic set of impedance values for vertical and rocking oscillations, as a function of d_o and frequency f, are presented in Figure 1.

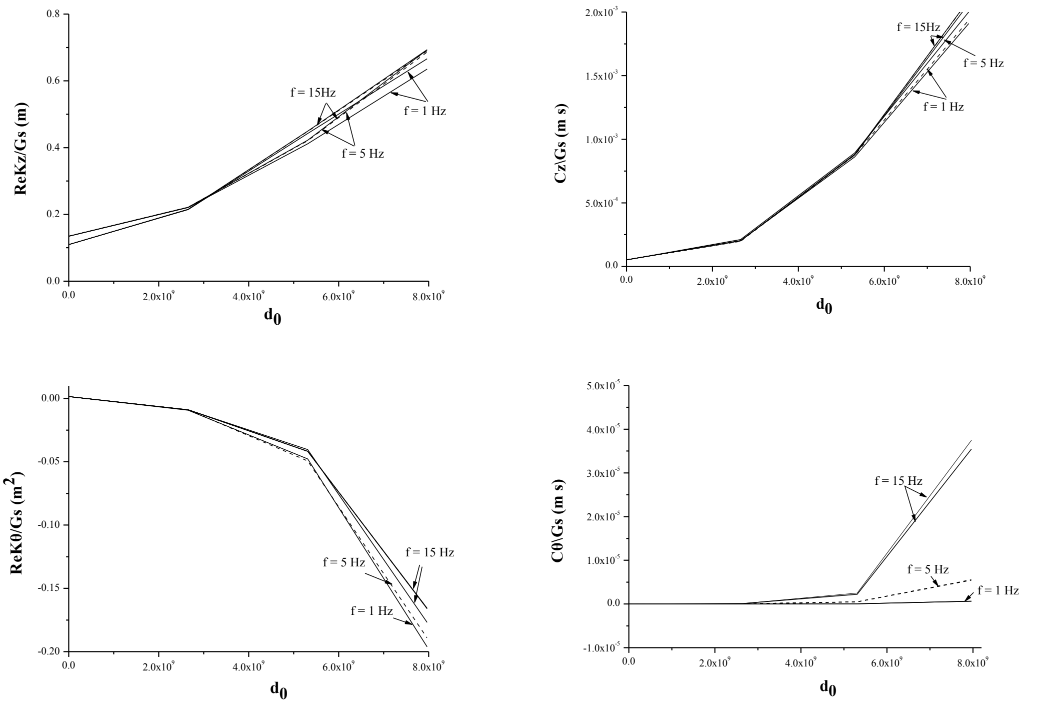

Figure 1. Half-space impedances for vertical and rocking oscillations as a function of dimensionless distance d_0 (ν_s= 0.30)

It is evident that impedance values depend on the spatial distance from the loading point, and they are practically independent of the frequency for a certain distance from the loading point. Moreover, the imaginary values (damping effects) increase substantially with increasing distance and, thus, the effect of the dynamic loading fades with distance. In general, imaginary values tend to be more sensitive to frequency variation, although this is observed at significant distances from the loading point.

2.2 Soil-Flexible Foundation Interaction

In this work, the dynamic response of a flexible foundation is computed using two distinct approaches. The first approach, used for comparison purposes, is based on a full soil-foundation model using the computer code ACS SASSI. To this end, the discretization of the half space described earlier, is supplemented with the finite model of a flexible slab resting on the surface of the half space, as shown in Figure 2. It is assumed that each nodal point of the discretized foundation medium coincides with an interaction point of the half space disretization. For the purposes of this work, a thin plate element is used for the foundation medium and separation (uplift) between the soil and the foundation is not allowed. The soil-foundation system can be loaded with a harmonic load at any nodal point and the frequency domain response (impedance) at any other nodal point can be computed following standard FEM procedures.

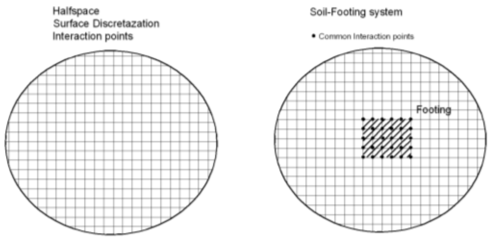

Figure 2. Soil-Foundation System. Surface Interaction Points

The second approach, of considerable practical importance, is based on the non-dimensional impedances of the previous section which can be used, in the context of any commercial FE code, in this case ANSYS, to simulate the half space medium. Thus, on top of these frequency dependent "springs" any structural model can be "built" and analyzed following, again, standard FEM procedures.

2.3 Presentation of Numerical Results

Solutions are presented for a series of harmonic excitations of the soil-foundation system. The main system parameters are the dimensionless stiffness ratio K and frequency α_o. The dimensionless stiffness ratio, expresses the relative stiffness between the soil and the foundation and is defined as [13]

$$K = \frac{E_f t_f^{\,3}(1-\nu_s)}{12(1-\nu_f^2)G_s B^3} \tag{2}$$

while the dimensionless frequency as

$$a_0 = \frac{\omega B}{Vs} \tag{3}$$

where E_f, ν_f, t_f and B are the elastic modulus, Poisson ratio, thickness and length of the footing, respectively, G_s, V_s and ν_s are the shear modulus, the shear wave velocity and Poisson ratio of the half-space and ω is the excitation frequency.

In addition, the dimensionless displacement Δ is defined as [13]

$$\Delta = \frac{uG_sB}{(1-\nu_s)\sum F_{ext}} \tag{4}$$

where u is the displacement due to dynamic loading F_{ext}.

In Figure 3 the values of computed impedances using ACS SASSI for various points of a flexible footing are plotted as a function of dimensionless frequency α_o, for vertical and rocking excitation. In this case only the center of the foundation is loaded. As expected impedance values differ from point to point due to foundation flexibility. It is also evident that the impedance values are more sensitive to spatial distance from the loading point, than to the excitation frequency.

In Figures 4 and 5 the dynamic response of a square, flexible foundation slab, is shown. The foundation is loaded with a harmonic concentrated vertical force at its center and the dimensionless vertical displacements, Δ, at 3 characteristic nodal points, i.e. center (loaded point), middle of side, and corner, are plotted versus the dimensionless frequency α_o. The foundation response shown in Figure 4 corresponds to a rather "soft" foundation (K=0.004) while the response shown in Figure 5 to a rather "stiff" foundation (K=0.06). For comparison purposes, the related results found in Refs [13,14] are also plotted in Figures 4 and 5, showing a good agreement between the various methods.

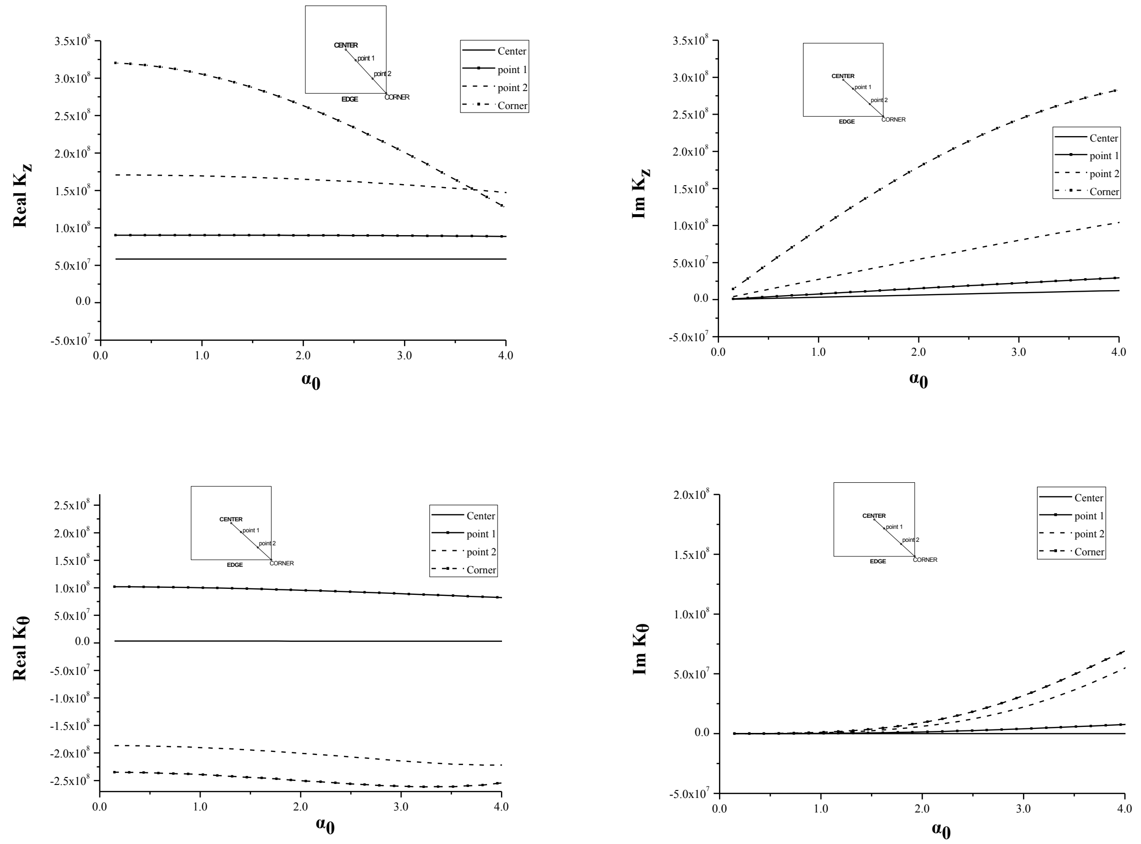

Figure 3: Real and Imaginary impedance values for vertical (K_z) and rocking (K_θ) oscillations as a function of frequency. (K = 0.004, ν_s = 0.45)

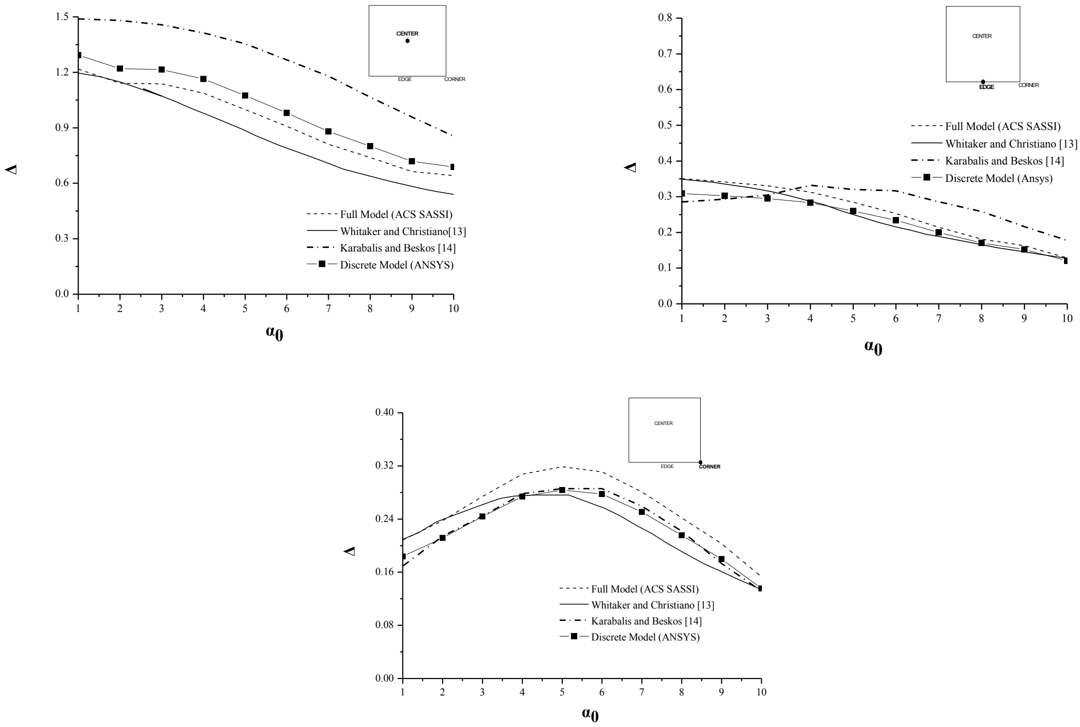

Figure 4: Flexible footing dynamic response versus frequency. (K = 0.004, ν_s = 0.45)

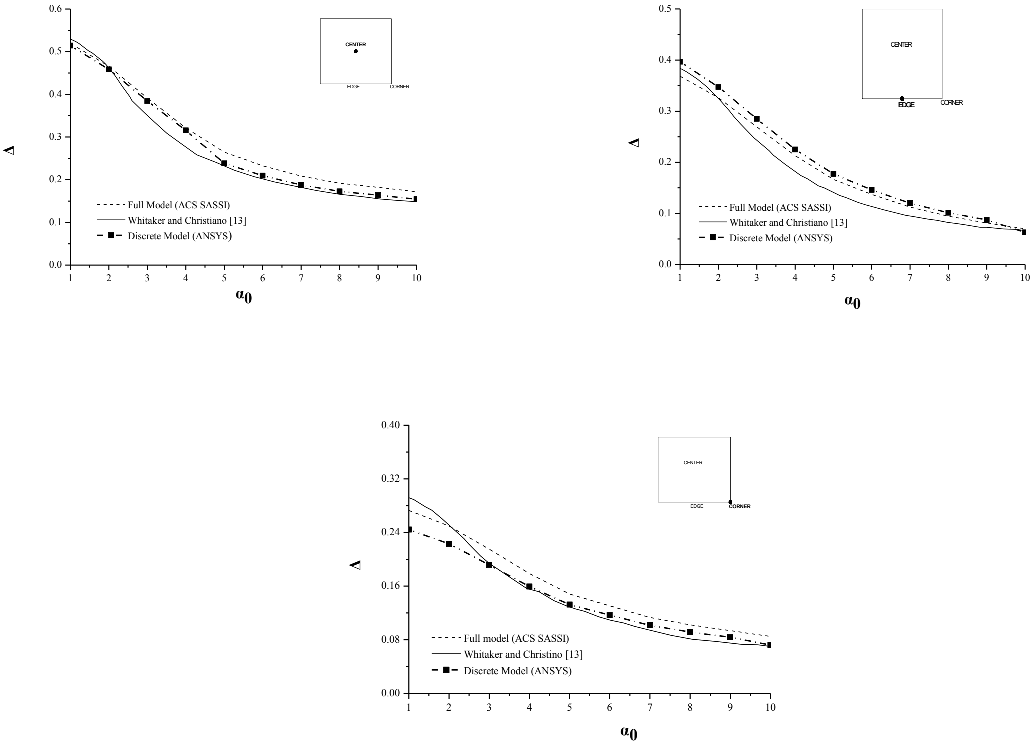

Figure 5: Flexible footing dynamic response versus frequency. (K = 0.06, ν_s = 0.45)

3 CONCLUSIONS

In this paper, a computational procedure for the analysis of soil-flexible foundation interaction is developed based on the finite element method. Impedance values for vertical and rocking oscillations of a half space are computed and used as the basis for the foundation-soil interaction problem. The discrete (in space), frequency dependent, impedance matrix of the soil-foundation system is also obtained. The main conclusions of this article are:

1) Impedance values of the half-space are practically frequency independent for relatively low excitation frequencies, at the neighborhood of the loaded point.
2) The proposed approaches are readily applicable to the analysis of flexible foundations and superstructures.
3) The proposed approaches are in good agreement with previous methodologies and easy to use.

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