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ON NUMERICAL MODELING OF DYNAMICS OF IRREVERSIBLE DEFORMING AND FRACTURE OF OIL-BEARING LAYER IN THE VICINITY OF A BOREHOLE

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Abstract. A paper dealing with numerical simulation in two-dimensional plane deformed state of dynamics for oil-gas saturated medium near borehole under sudden removal pressure on internal wall of the borehole. The layer is represented by the model of damageable thermoelastoplastic material with two parameters of damaging. The criterion of the beginning of new free surfaces within the material uses the principle of the critical value of specific dissipated energy. For explicitly construction of coasts of macroscopic infraction of material continuity we apply algorithm of decomposition Lagrangian mesh.

1 INTRODUCTION

In engineering practice widely used originating drilling artificial mining cavity circular cross-section having different diameters (holes, mine trunks, etc). Character of fracture of mining rock near hole depends from presence in rock structural nonhomogeneity different scales (pores, cracks). Present paper dealing with numerical simulation of dynamical deforming, micro- and macrofracture near borehole under sudden removal pressure on internal wall of the borehole.

Main used assumptions are: mass forces neglected; parameters of problem not depend of space coordinate which normal to plane of layer; assume that take place plane deformation of layer, problem solution in two-dimensional statement; layer modeling as damageable thermoelastoplastic medium; process of deforming is adiabatic.

Models of damageable deformable solids which describing so called continuum or dispersed fracture beginning from classical papers of L.M. Kachanov , Yu.N. Rabotnov and A.A. Il'yshin [1-3]. In [1, 2] for the first time was introduced one scalar damage parameter which describe accumulation of damages in material under creeping, and in [3] – tensor damaging degree for describing wide class of processes of irreversible and microfracture for different models of media. Use in following works of thermodynamical principals make possible construct many correct models of solids in which mechanical, thermal and processes of microfracture are correlated. This direction in mechanics takes very wide development and number of papers counts may hundreds. Therefore indicate only some new monographs [4-7] and review [8] which contained extensive bibliography.

Model of deformable damageable solid which used in present paper for describing dynamics of oil-gas layer based on works [9-13] which constructed with use thermomechanical approach and model of elastoplastic flow Prandtl-Rice type.

2 STATEMENT OF PROBLEM

Describing of deforming process produce in cylindrical coordinate system $Ozr\theta$, axis Oz coincide with axis of borehole (figure 1). Then any parameter depend on space coordinates r, θ and time t.

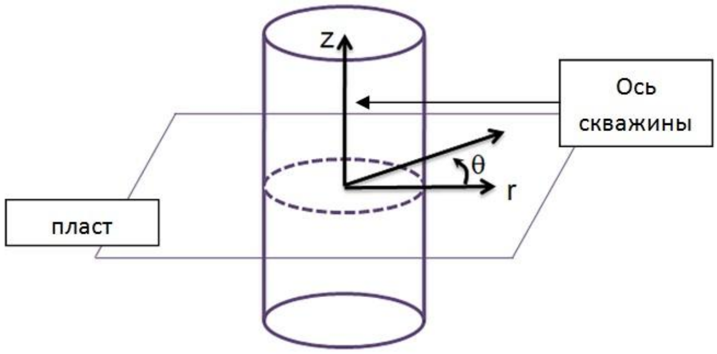

Figure 1: Cylindrical coordinate system connecting with borehole and layer.

Write equation of mass, momentum and internal energy:

$$\frac{\dot{\rho}}{\rho} = -\dot{\varepsilon}_r - \dot{\varepsilon}_\theta; \quad \rho\dot{v}_r = \frac{\partial\sigma_r}{\partial r} + \frac{1}{r}\frac{\partial\sigma_{r\theta}}{\partial\theta} + \frac{\sigma_r - \sigma_\theta}{r}, \quad \rho\dot{v}_\theta = \frac{\partial\sigma_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial\sigma_\theta}{\partial\theta} + 2\frac{\sigma_{r\theta}}{r}; \qquad (1)$$

$$\rho c_\sigma \dot{T} + \alpha_v \dot{\sigma} T = S_r \dot{\varepsilon}_r^p + S_\theta \dot{\varepsilon}_\theta^p + S_z \dot{\varepsilon}_z^p + 2S_{r\theta}\dot{\varepsilon}_{r\theta}^p + A\dot{\alpha}^2 + \Lambda\dot{\omega}^2$$

A last equation writing in form obtained in [9-13]. In (1) and further dote under symbol designate material derivation on time and introduce next notations: ρ - density; v_r, v_θ - components of velocity vector; $\sigma_r, \sigma_\theta, \sigma_z, \sigma_{r\theta}$ - components of stress tensor, which decompose on spherical and deviator parts: $\sigma_r = \sigma + S_r, \sigma_z = \sigma + S_z$, $\sigma_\theta = \sigma + S_\theta, \sigma_{r\theta} = S_{r\theta}$, $\sigma = (\sigma_r + \sigma_z + \sigma_\theta)/3$; $\dot{\varepsilon}_r$, $\dot{\varepsilon}_\theta$, $\dot{\varepsilon}_{r\theta}$ - components of velocity strain tensor; $\dot{\varepsilon}_r^p$, $\dot{\varepsilon}_\theta^p$, $\dot{\varepsilon}_z^p$, $\dot{\varepsilon}_{r\theta}^p$ - plastic components of velocity strain tensor; T - temperature; ω, α - scalar damage parameters of media ($0 \le \omega < 1$ – first invariant of symmetric damage tensor ω_{ij} $(\omega = \omega_{kk}/3)$ which interpreted as volume matter of micropores in material which filling liquid and/or gas; $0 \le \alpha < 1$– second invariant of deviator damage tensor ω_{ij} describing shear microfracture of material ($\alpha = \sqrt{(\omega_{ij} - \omega\delta_{ij})(\omega_{ij} - \omega\delta_{ij})}$); in no damage material $\omega = \alpha = 0$; c_σ - heat capacity under constant stresses; α_V - module of volume extension; Λ, A - media parameters which connecting thermal processes with processes of damage accumulation.

Velocity strain expressed from components of velocity vector:

$$\dot{\varepsilon}_r = \frac{\partial v_r}{\partial r}, \quad \dot{\varepsilon}_\theta = \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}, \quad \dot{\varepsilon}_{r\theta} = \frac{1}{2}\left(\frac{\partial v_\theta}{\partial r} + \frac{1}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r}\right) \tag{2}$$

and decomposed on elastic and plastic components:

$$\dot{\varepsilon}_r = \dot{\varepsilon}_r^e + \dot{\varepsilon}_r^p, \quad \dot{\varepsilon}_\theta = \dot{\varepsilon}_\theta^e + \dot{\varepsilon}_\theta^p, \quad \dot{\varepsilon}_{r\theta} = \dot{\varepsilon}_{r\theta}^e + \dot{\varepsilon}_{r\theta}^p, \quad \dot{\varepsilon}_z = \dot{\varepsilon}_z^e + \dot{\varepsilon}_z^p \equiv 0 \tag{3}$$

Plastic flow is incompressible: $\dot{\varepsilon}_r^p + \dot{\varepsilon}_\theta^p + \dot{\varepsilon}_z^p \equiv 0$.

System of constitutive equations coupled model of damageable thermoelastoplastic media have next form [9-13]:

$$\begin{cases} \dot{\sigma}' = K_0\left(\dot{\varepsilon}_r + \dot{\varepsilon}_\theta - \alpha_v \dot{T} - \dfrac{\Lambda}{3}\dot{\omega}\dfrac{\partial \dot{\omega}}{\partial \sigma}\right) \\ \left(S'_{ij}\right)^\nabla + \lambda S'_{ij} = 2\mu_0 \dot{e}_{ij} - 2A\dot{\alpha}\dfrac{\partial \dot{\alpha}}{\partial S_{ij}} \\ S'_{ij}S'_{ij} \le \dfrac{2}{3}Y_0^2(\sigma) \\ Y_0 = c_1\sigma + c_2 \end{cases} \tag{4}$$

Here symbol ∇ designate Jaumann derivative from components of deviator stress tensor on time: $\left(S'_{ij}\right)^\nabla = \dot{S}'_{ij} - S'_{ij}\Omega_{jk} - S'_{jk}\Omega_{ik}$; $\dot{e}_{ij}$ - deviator of velocity strain tensor; $\sigma' = \sigma/(1-\omega)$, $S'_{ij} = S_{ij}/(1-\omega)(1-\alpha)$; K_0 и μ_0 - volume module and shear module of no damageable media respectively; Ω_{ij} - rotation tensorтензор which components in considerate case are:

$$\Omega_r = \Omega_\theta = 0, \qquad \Omega_{r\theta} = -\Omega_{\theta r} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r} - \frac{\partial v_\theta}{\partial r}\right) \tag{5}$$

A last formula in (4) – low Mizes-Shleixer which connect yield limit under simple tension Y_0 and pressure in layer $(-\sigma)$; c_1, c_2 – material constants.

System of equation (1) - (5) close kinetic equations for damage parameters ω and α :

$$\frac{\dot{\omega}}{\omega} = B\left(\frac{\sigma}{1-\omega} - \sigma_*\right)H\left(\frac{\sigma}{1-\omega} - \sigma_*\right) + \frac{\sigma - \sigma^+}{4\eta_0}H(\sigma - \sigma^+) + \frac{\sigma - \sigma^-}{4\eta_0}H(\sigma^+ - \sigma),$$

$$\sigma^+ = -\frac{2}{3}Y_0 \ln\omega - p_0\left(\frac{\omega_0}{\omega}\right)^\gamma, \quad \sigma^- = +\frac{2}{3}Y_0 \ln\omega - p_0\left(\frac{\omega_0}{\omega}\right)^\gamma, \tag{6}$$

$$\dot{\alpha} = C\left(\frac{S_u}{(1-\omega)(1-\alpha)} - S_u^*\right)H\left(\frac{S_u}{(1-\omega)(1-\alpha)} - S_u^*\right)$$

Here η_0 - dynamic viscosity of no damage material; p_0 - initial pressure in pore ("mountainous" pressure); γ - index of media adiabatic which filling pore; ω_0 - initial porosity; $S_u = \sqrt{S_{ij}S_{ij}}$ - intensity of deviator stress tensor; S_u^*, σ_*, B, C - constants of material; $H(x)$ - Heviside function.

From equations (6) see that damage parameter ω depend with ball part of stress tensor σ and depending on which component in right part of first equation from (6) enclose parameter ω may increase or decrease. This parameter describes damage micropore type (so called viscous fracture). Parameter α depend from intensity of deviator stress tensor and describe shear fracture.

As criteria of beginning macrofracture of material (appearance new surfaces free from loading) used entropy criteria limit specific dissipation [9-15]:

$$D = \int_o^{t^*} \frac{1}{\rho}\left(S_{ij}\dot{\varepsilon}_{ij}^p + \Lambda\dot{\omega}^2 + A\dot{\alpha}^2\right)dt = D^* \tag{7}$$

Here t^* – time of beginning of fracture; D^* - limit specific dissipation (material constant); $d_M = S_{ij}\dot{\varepsilon}_{ij}^p$ - mechanical dissipation; $d_F = \mathrm{A}\dot{\alpha}^2 + \Lambda\dot{\omega}^2$ - dissipation of continuum fracture. Thermal dissipation $d_T = -\boldsymbol{q}\cdot grad\, T / T$ in (7) is absent because process consider as adiabatic ($\boldsymbol{q}$ – vector of heat flow).

Such model, but with one parameter of volume damage ω, used early for calculation problem of hydraulic fracture of oil layer ([14, 15]).

3 INITIONAL AND BOUNDARY CONDITIONS

In initial moment $t = 0$ layer is in rest condition: $v_r = v_\theta = 0$. In calculation for initial condition was take that $\rho = \rho(r,\theta) = \rho_0 = const$. It is necessary to set initial distribution of stresses $\sigma_r, \sigma_\theta, \sigma_z, \sigma_{r\theta}$ in layer, damage parameters ω and α as function of space coordinates r and θ.

As initial distribution of stresses in layer we used solution a next static linier-elastic problem. Consider infinite cylindrical body with circular cup; on infinite in two mutually orthogonal directions apply contractive stresses σ_1 and σ_2 corresponding so called "mountain pressure". In common case $\sigma_1 \neq \sigma_2$ that makes possible modeling nonhomogeneity stress condition of layer. On surface of circular cup applied contractive stress σ_3 which correspond pressure into borehole (figure 2). This problem have solution in linear-elastic statement and in elastoplastic statement which well-known as Galin problem [17]. Some generalizations of Galin problem reduce in monograph [17] and in bibliography in this book.

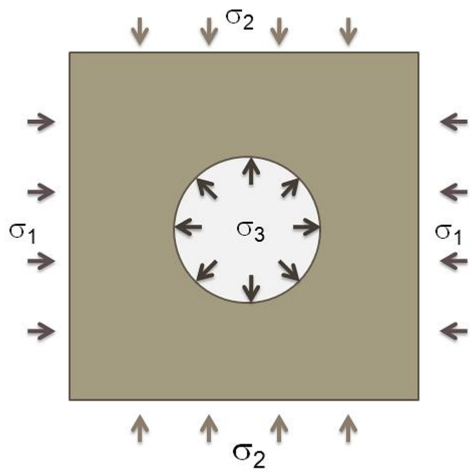

Figure 2: scheme of application of loading.

Solution in linear-elastic statement obtained as superposition of three solutions separately for stresses $\sigma_1, \sigma_2, \sigma_3$ and have next form [18]:

$$\begin{cases} \sigma_r = \dfrac{\sigma_1+\sigma_2}{2} + \dfrac{\sigma_1-\sigma_2}{2}\cos 2\theta + \dfrac{a^2}{r^2}\left(-\sigma_3 - \dfrac{\sigma_1+\sigma_2}{2} - 2(\sigma_1-\sigma_2)\cos 2\theta\right) + \\ +\dfrac{a^4}{r^4}\left(\dfrac{3}{2}(\sigma_1-\sigma_2)\cos 2\theta\right), \\ \sigma_\theta = \dfrac{\sigma_1+\sigma_2}{2} - \dfrac{\sigma_1-\sigma_2}{2}\cos 2\theta + \dfrac{a^2}{r^2}\left(\sigma_3 + \dfrac{\sigma_1+\sigma_2}{2}\right) + \dfrac{a^4}{r^4}\left(-\dfrac{3}{2}(\sigma_1-\sigma_2)\cos 2\theta\right), \\ \sigma_z = \nu(\sigma_r + \sigma_\theta), \\ \sigma_{r\theta} = -\dfrac{\sigma_1-\sigma_2}{2}\sin 2\theta - \dfrac{a^2}{r^2}\left(2(\sigma_1-\sigma_2)\sin 2\theta\right) + \dfrac{a^4}{r^4}\left(\dfrac{3}{2}(\sigma_1-\sigma_2)\sin 2\theta\right). \end{cases} \tag{8}$$

In moment $t = 0$ happen sharp fall pressure into borehole from value σ_3 to value $\Delta p > 0$. Therefore on borehole wall take place pressure $\sigma_3 + \Delta p.$ Case then $\Delta p = -\sigma_3$ corresponded with condition on free surface.

4 CONSTRUCTION OF MACROFRACTURE DOMAINS

Considered problem solve numerically by method Wilkins type on Lagrange meshes [19, 20]. Modeling of macrofracture of layer realized by using method of decomposition Lagrange mesh [21-23]. As criteria of beginning of macrofracture we used entropy criteria of fracture (7): media lose continuity when specific dissipation D reach limit value D_*. In point in which realized criteria of fracture realize explicit construction of macrofracture banks (cracks). For this doing separation of nodes of calculated net on boundary meshes (figure 3) – internal nodes and corresponding them edges of meshes become boundary on which give condition of free surface, definite pressure or contact condition depending on situation. In detail this algorithm presented in thesis [23]. Mark what earlier we used for explicit construction boundary of fracture in problem of hydraulic fracture of layer procedure of bifurcation Lagrange calculation nets [14, 15] which detailed described in [27].

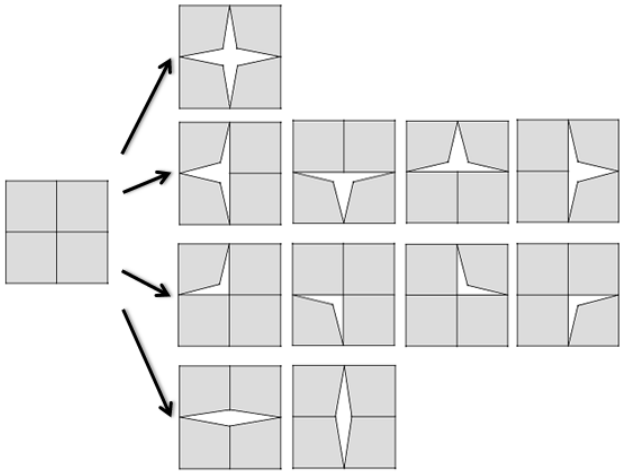

Figure 3: Alternative versions for “elemental” cracks for internal node.

Variant of decomposition defined from analysis of stress vector on adjacent for present note edge. Interaction of “elemental” cracks with already exist cracks to form fracture zones larger scale. In result of decomposition on new formed boundary edges set boundary conditions: condition of free boundary or contact conditions depending on situation.

Using in present paper algorithm for realization of boundary conditions on contact surfaces contained in correction of coordinates and velocities of boundary nodes of calculated meshes [24-26]. This algorithm is symmetric as under correction of coordinates interaction boundaries enter symmetric form.

5 RESULTS OF CALCULATIONS

Problem has two axis of symmetry therefore calculations conducted for one fourth of domain. On boundaries $\theta = 0$ and $\theta = \pi / 2$ under integration of momentum equations take into account contribution symmetric meshes so that be realized condition: $v_\theta = 0$. External boundary of calculated domain choose so remote that it is possible no take into account reflection waves from external boundary which extends from wall of borehole.

Calculations conducted in three different statements. *In first statement* under numerical calculation no realize explicit construction of fracture domains. *In second statement* fracture domains constructed explicit way, however it is not taking into account contact interaction on new free surfaces under decomposing of mesh. *In third variant* on free surfaces contact interaction take into account.

Under calculations used next values of parameters: $a = 0,5$ m, $\rho_0 = 2000$ kg/m^3; $K_0 = 14$ GPa; $\mu_0 = 8,4$ GPa; $c_1 = -0,09$; $c_2 = 0,04$ GPa; $S_u^* = 36$ GPa; $\eta_0 = 100$ Pa$\cdot$c; $\Lambda = 1500$ Pa$\cdot$c; $C = 8 \cdot 10^{-5}$ (Pa$\cdot$c)$^{-1}$; $A = 1200$ Pa$\cdot$c; $\gamma = 1,4$; $D_* = 334,4$ J/kg.

In lower represented calculations were take uniform distribution of initial volume damage (porosity) ω o material: $\omega_0 = \omega_0(r,\theta) = const = 0,05.$ Initial pressure in pores p_0 in formula (6) - $p_0 = -\sigma|_{t=0}$ (under that $\sigma|_{t=0}$ defined from formula (8)).

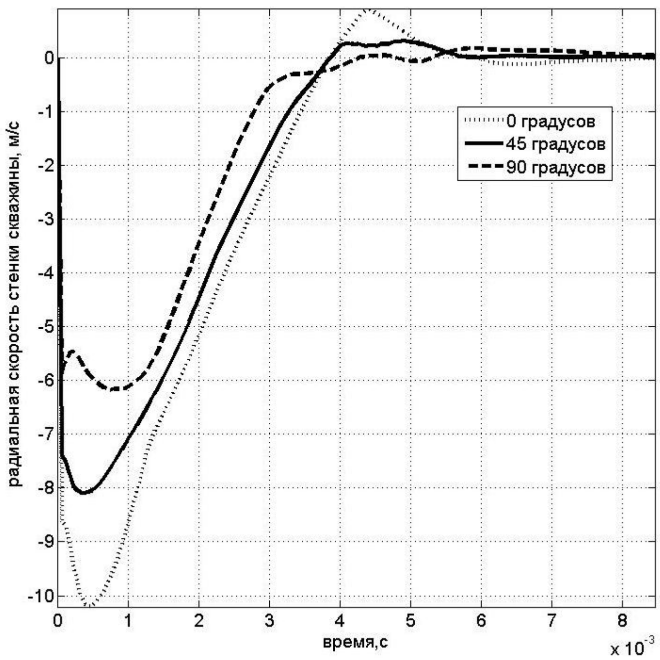

Figure 4: Dependence from time of radial velocity for different point of borehole wall (*first statement*).

Lower present results of calculations with initial distribution of stresses (8) for next case: $\sigma_1 = -65$ MPa, $\sigma_2 = -75$ MPa, $\sigma_3 = -75$ MPa; $\Delta p = |\sigma_3|$.

On figure 4 present dependence from time of radial velocity for points of borehole for different values of angle θ (*first statement*); difference caused thus that что $\sigma_1 \neq \sigma_2$.

As advancing wave unloading into layer perturbed domain in which media beginning motion to center $(v_r < 0)$ is increase. Later velocity of layer motion falls until zero.

After removal pressure into borehole wall of borehole turn to centre and media stretched in radial direction, radial stresses σ_r are growth remained negative (unloading wave propagation into layer). In ring direction media conversely contract and ring stresses σ_θ decrease (in absolute value they increase). Shear stresses for angles $\theta = 0$ and $\theta = \pi/2$ equal zero for all moments of time in view of symmetry of problem.

On figure 5 show distribution of specific dissipation in two sequential moments of time (*first statement*). Critical level D_* correspond white color. Obviously that fracture domain localized in zone which correspond to concentrate of biggest contractive stress (on vertical).

Thus from obtained results show that under modeling without explicit of fracture surfaces in layer its fracture take place directly near wall of borehole.

Fracture domain under realization of *second statement* quality changes her character. Now fracture realized as turnpike curvilinear crack (stretched comparatively restricted zones of fracture), which originated on wall of borehole and germinate into layer. On figure 6 show distributions of specific dissipation in three sequential moments of time on which it is possible to trace development of crack. Number of cracks, direction and velocity of their growth depend on relation σ_1 / σ_2. For symmetric case $(\sigma_1 = \sigma_2)$ zone of fracture is circle around borehole.

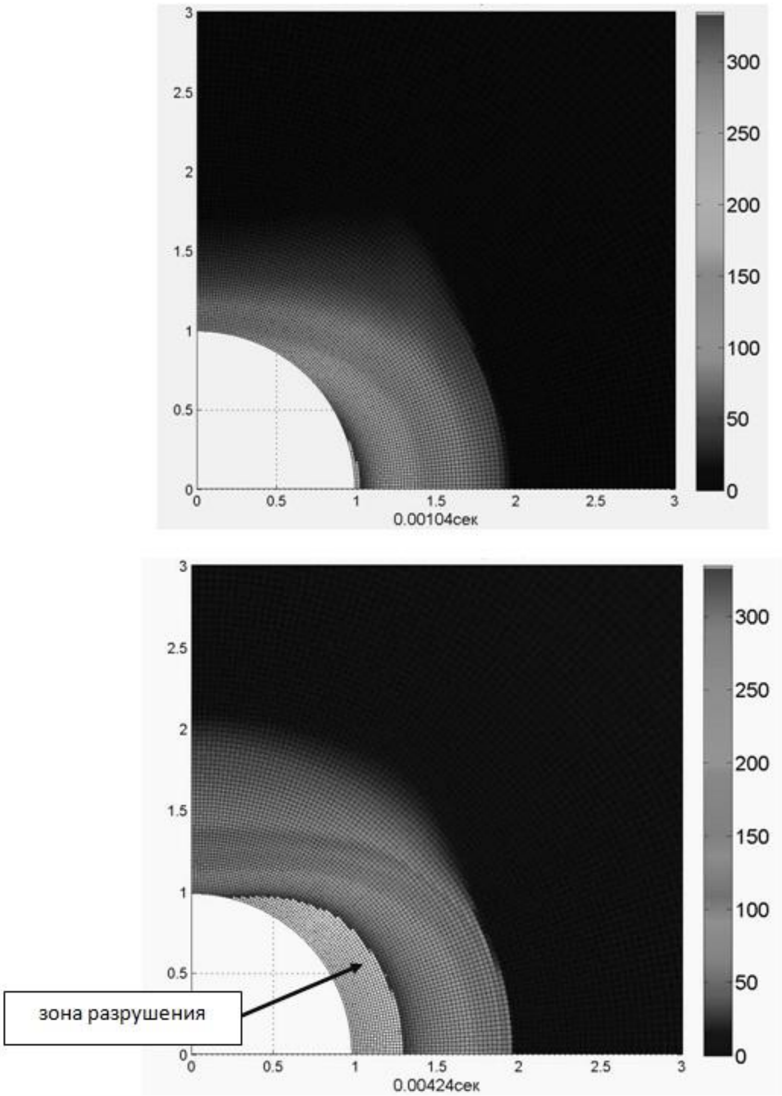

Figure 5: Distribution of specific dissipation D in sequential time moments (*first statement*).

Note that simple character of fracture with development of magisterial cracks, their rotation in processes of growth as on figure 6 observed and under modeling cracks of hydraulic fracture which formed in result of bury to layer liquid from borehole [14, 15]. In problem of hydraulic fracture cracks banks no closing and the more so no take place their constriction as presence liquid in cracks.

Graph of dependence of depth propagation of crack into layer from time for different relations σ_1 / σ_2 represent on figure 7 ($0,5 < \sigma_1 / \sigma_2 \le 1$). Obviously that ratio σ_1 / σ_2 less than 1 by that earlier crack appear and her depth of penetration more.

Modeling with taking account contact interaction of cracks banks (*third statement*) give qualitative another result then in case of separation of cracks banks without contact interaction (*second statement*). Fracture domain now not restricted lengthy zone but continuous domain which initial localized near borehole wall. Fracture domain represent on figure 8; here as above white color corresponding critical value of dissipation D_*. Character of fracture having “fragmentation” character: into borehole fly fragments of layer (clusters of calculation Lagrangian particles which separated from layer in result of realized procedure of mesh decomposition) thus take place of bring down of borehole wall.

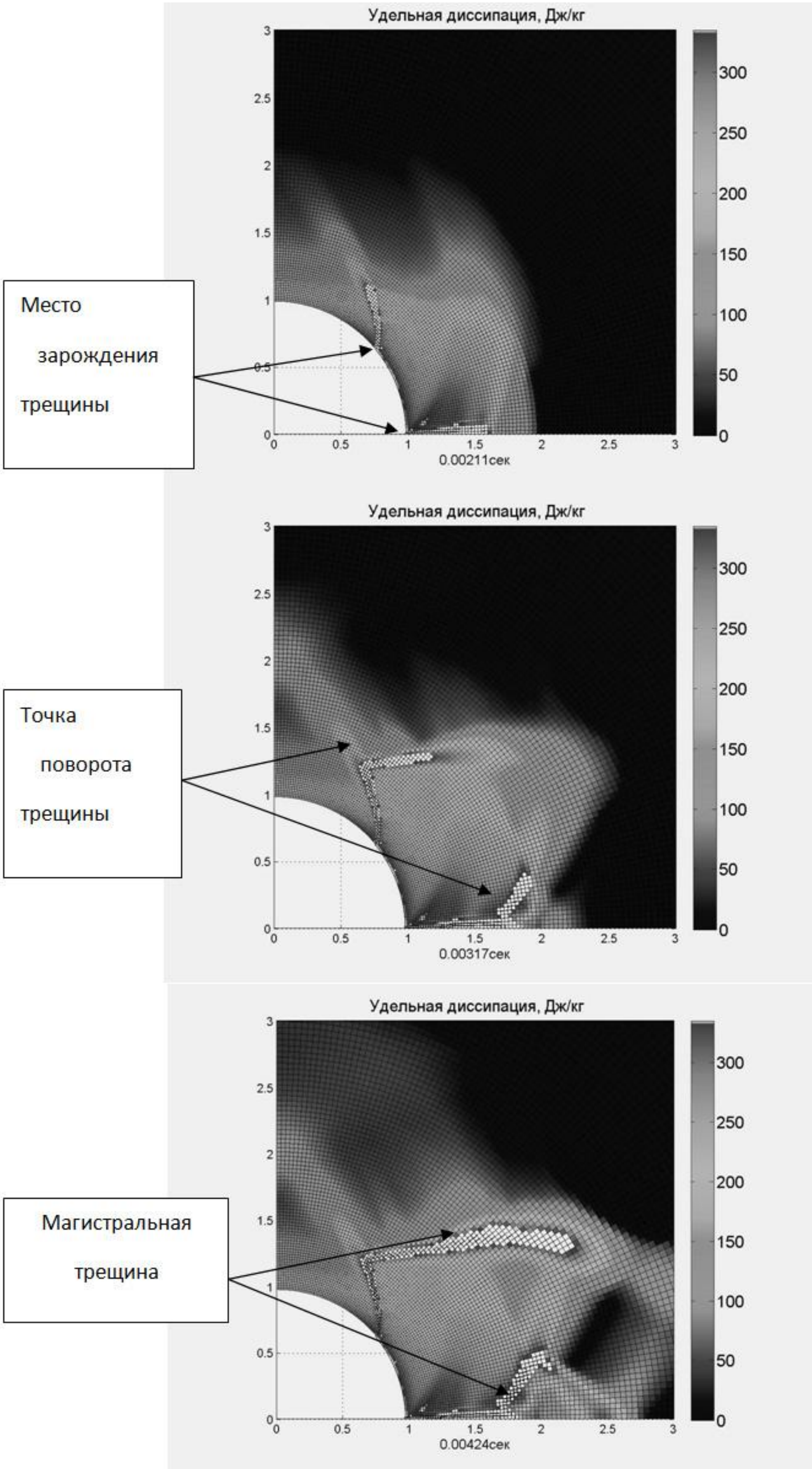

Figure 6: Forming of turnpike crack (*second statement*).

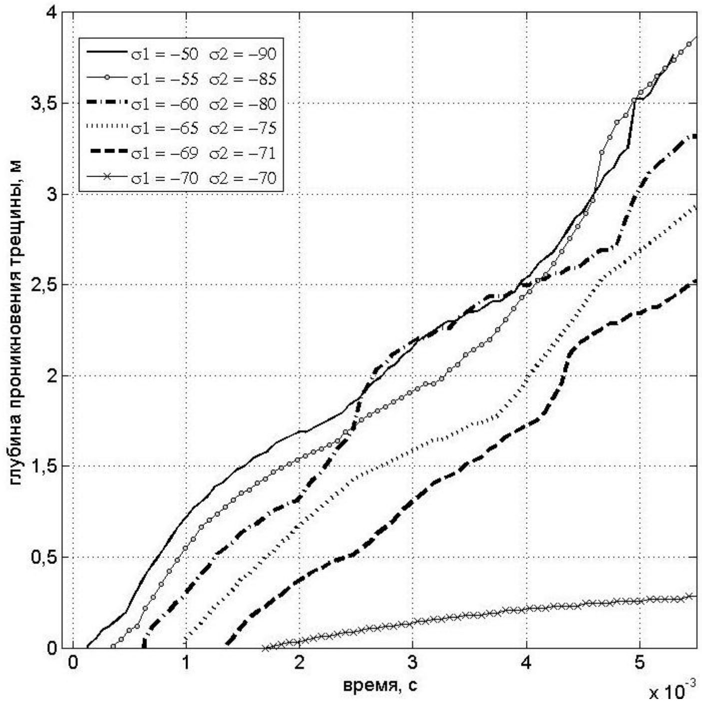

Figure 7: Dependence of depth of crack interaction in layer from time (*second statement*).

Shear damage α localize near borehole but on some distance from it. Volume damage ω too growth near borehole in domains of in intensive expansion of media originating as result its moving to center of borehole after harsh taking off pressure into borehole. Biggest contribution to dissipation D give mechanical dissipation $d_M = S_{ij}\dot{\varepsilon}_{ij}^{p}$. Dissipation of continuum volume fracture $d_F^{\omega} = \Lambda\dot{\omega}^2$ and shear fracture $d_F^{\alpha} = \mathrm{A}\dot{\alpha}^2$ types give noticeable contribution to full dissipation D only in fracture domains of layer (see formula (7)).

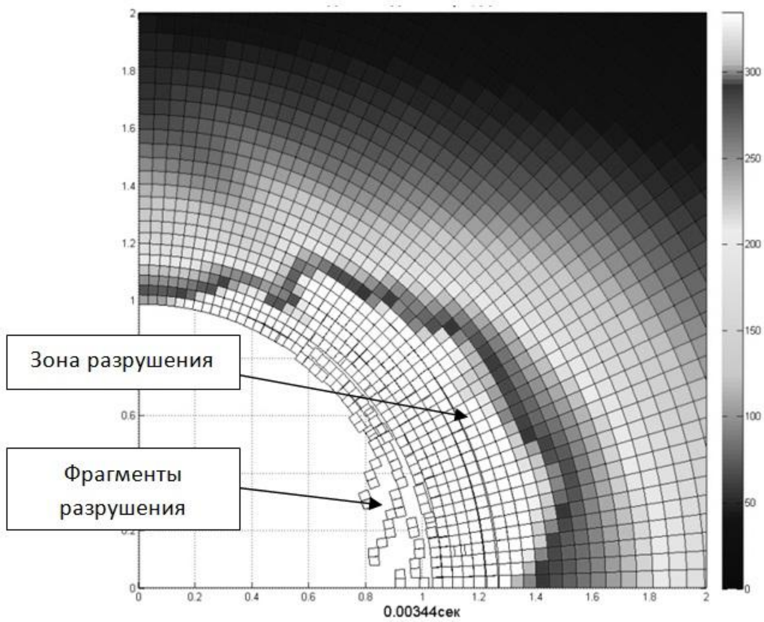

Figure 8: Dependence of specific dissipation (*third statement*).

6 CONCLUTIONS

Numerically investigated a problem of irreversible dynamic deforming and fracture of rock layer near borehole under sudden removal pressure in borehole. Solution was obtained in nonsymmetrical two-dimensional statement with taking into account both microfracture (volume and shear characters) as explicit structure of domains of macroftacture (curvilinear cracks which separated from layer).

Analyzed character and degree of fracture in layer depending on relation rock pressure far from borehole which acts on two orthogonal directions and from degree numerical realization of boundary conditions on banks of macrocracks.

Showed what simplification of algorithm calculation boundary conditions on crack banks reduced to principal different characters of layer fracture. Wherefore for obtain physically real results its need conduct numerical investigations under maximum fool realization boundary conditions on crack banks which formatted into layer.

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