

NONLINEAR STATIC ANALYSIS OF A PLAN IRREGULAR BUILDING ACCORDING TO EC8 PROVISIONS

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Abstract. *The paper deals with the topic of nonlinear static and nonlinear dynamic analyses of multi-storey r/c frame buildings designed according to Eurocode provisions. In particular, the open problem of the extension of N2 method to plan irregular buildings, which makes up for the underestimation of the seismic demand on stiff side of the buildings, is focused. Three methods, which take into account the accidental eccentricity of the mass (due to its uncertainties in the location) are proposed. The main differences between the three nonlinear static analysis methods are: method n.1 combines the displacements of modal response spectrum analysis obtained by the 4 models (associate to 4 positions of the centre mass) by SRSS rule and, for each frame of the multi-storey r/c frame building, the maximum normalized displacement is assumed as reference; method n.2, works on 4 models and only at the end of the procedure combines the results by SRSS rule; method n.3 considers till the end of the procedure the double sign (+ and -) of pushovers. Nonlinear static analyses are carried out using both the “modal” and “uniform” force pattern. The “orthogonal effects”, evaluated by SRSS rule, result to be negligible. The results, in terms of pushover curves, frame top displacements and interstory drifts, are compared with the ones obtained by nonlinear dynamic time-history analyses. The input adopted for nonlinear dynamic analysis (7 natural earthquakes) is selected in order to rigorously satisfy the EC8 provisions.*

1 INTRODUCTION

N2 method is one of the most spread methods for nonlinear static analysis: it is provided by EC8 [1] and other modern seismic codes. Since Fajfar [2] published it, its research group is working on its extension to plan irregular buildings, in order to make up for the underestimation of the seismic demand on the stiff side. They [3] proposed to combine the results obtained by pushover analysis of a 3D structural model with the results of a linear dynamic spectral analysis: the former results control the target displacements and the distribution of deformations along the height of the building, whereas the latter results define the torsional amplifications.

Such procedure, as proposed, does not consider that different models are to be computed due to different positions of centre of mass corresponding to different accidental eccentricities.

In the paper three different methods are proposed in order to apply the N2 method to a plan irregular building as formulated by Fajfar, taking into account the accidental eccentricity; such methods, characterised by different levels of accuracy, are described, applied to a three-storey r/c frame building and compared. It represents an enhanced version of the one proposed in Magliulo et al. [4]. The results are shown in terms of pushover curves, frame top displacements and interstorey drifts.

Dynamic analyses are performed using a set of natural records fully satisfying EC8 provisions: average, standard deviation of results are considered.

The results reported herein, for sake of brevity, concern a single structure; however they are confirmed by analyses performed at the same seismic level on different buildings.

2 GEOMETRY OF THE BUILDING AND ELASTIC ANALYSIS

The geometry of the analysed three-storey r/c frame building is reported in Figure 1. The interstorey height is equal to 3.20 m at all levels; at the first storey the columns section dimensions are 40 cm \times 65 cm, while all the beams are 40 cm \times 60 cm; at the second storey such dimensions are respectively 40 cm \times 60 cm and 40 cm \times 55 cm, at the third 40 cm \times 55 cm and 40 cm \times 50 cm. Column dimensions are kept larger than beam ones in order to take into account the capacity design. 2 m wide balconies are shown in Figure 1 as hatched areas.

Elastic analyses are performed by the computer program SAP2000 [5], according to Eurocode rules and considering a design spectrum soil B type 1 with a design ground acceleration $a_g = 0.35g$. The design is performed according to the High Ductility Class rules, a behaviour factor equal to 2.4 is computed; such value takes into account that according to EC8 the building is “torsionally flexible” and irregular in elevation (the reduction of lateral stiffness from 1st to 2nd storey is larger than 40%).

Concrete characteristic cylinder strength equal to $f_{ck} = 30 \text{ N/mm}^2$ and steel characteristic yielding strength equal to $f_{yk} = 430 \text{ N/mm}^2$ are adopted.

3 NONLINEAR MODEL

Nonlinear analyses are performed by means of the computer program CANNY99 [6, 7]. Nonlinearity regards flexural rotations, while all the other deformations are assumed linear. Both beams and columns are characterised by lumped plasticity models; in the latter case for each section two independent nonlinear springs are assigned, one for each orthogonal direction. No axial force-bending moment interaction is considered at plastic hinge.

Bending moment springs are characterised by a tri-linear skeleton curve, defined by cracking and yielding moment and corresponding rotations; the post-yielding stiffness is assumed equal to zero. An elastic-perfectly plastic steel stress-strain diagram is considered, characterised by an yielding strength equal to 530 N/mm^2 , computed as mean of tests on more than 200

steel bars performed at the laboratory of Department of Structures for Engineering and Architecture of the University of Naples Federico II.

The yielding and the ultimate rotations are evaluated as provided by EC8 [8] equations (A.10b) and (A.1) respectively, where the already cited average values are assigned to concrete maximum ($f_c=38 \text{ N/mm}^2$) and steel yielding ($f_y=530 \text{ N/mm}^2$) strength.

The hysteretic model is Takeda type; the pinching effect is also taken into account.

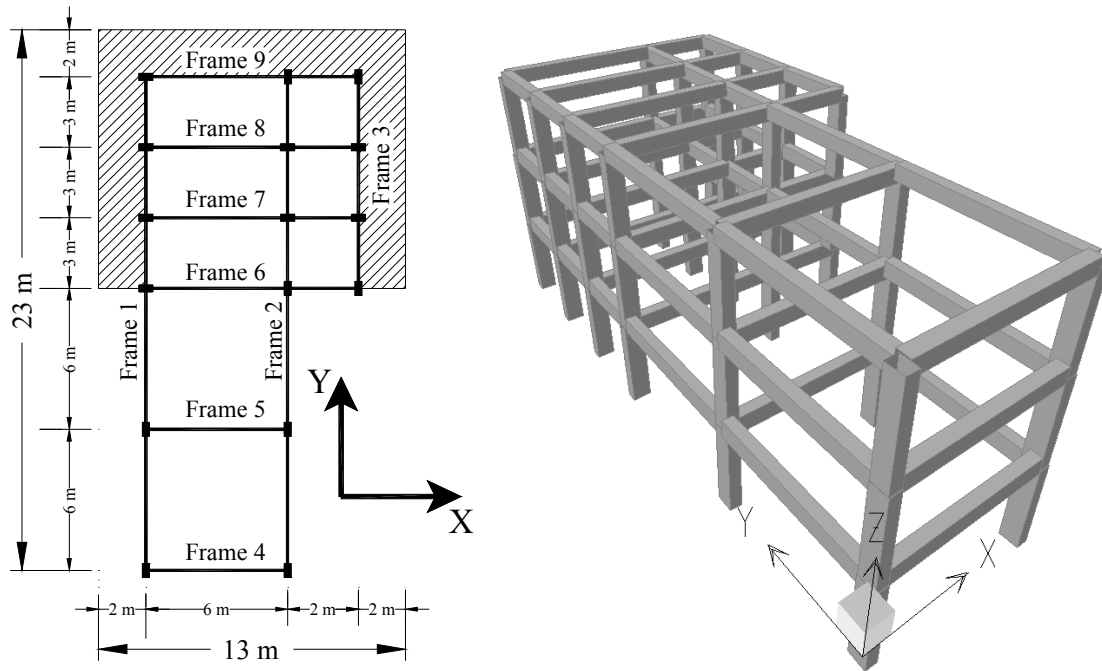


Figure 1: Geometry of the analysed building.

4 NONLINEAR STATIC ANALYSIS

Nonlinear static analyses (NLSA) are performed according to EC8, considering a force distribution proportional for each of the 2 orthogonal directions to the first modal shape in the relative direction by mass distribution (“modal” pattern). The first and the third modes of the building are translational and the square root of the ratio of torsional stiffness to lateral stiffness in each of the 2 orthogonal directions is smaller than the radius of gyration of floor mass in plan [1]; consequently the structure is torsionally flexible and the application of a specific procedure for the estimation of torsional effects is necessary.

Eight analyses, 2 opposite signs for 4 different positions of the centre of mass, are performed. In Figure 2, 2 of the 8 pushover curves in terms of adimensionalised top displacement vs base shear are shown. The bi-linear capacity curve is also presented along with capacity and demand points: “mech” indicates the mechanism of the structure, “ULS” (Ultimate Limit State) and “NC” (Near Collapse Limit State) the attainment in at least one hinge of the rotation value $3/4 \theta_u$ and θ_u respectively and “t.ULS” the demand corresponding to the EC8 elastic design acceleration spectrum built considering soil type B and $a_g = 0.35g$; θ_u is the total chord rotation capacity computed according to EC8 empirical formula (A.1) [8].

Figure 2 shows that in the examined case differences between NLSA performed on the same model and along the same direction but with opposite sign are negligible; this conclu-

sion justifies the possibility to consider only one NLSA between the two ones with opposite sign and, in particular, the one with the largest target displacement.

In order to evaluate the torsional effects three variants of the Fajfar's method are proposed and presented in the following; they consider different models characterised by different accidental eccentricities, indicating how to associate modal response spectrum analyses (MRSA) with nonlinear static ones.

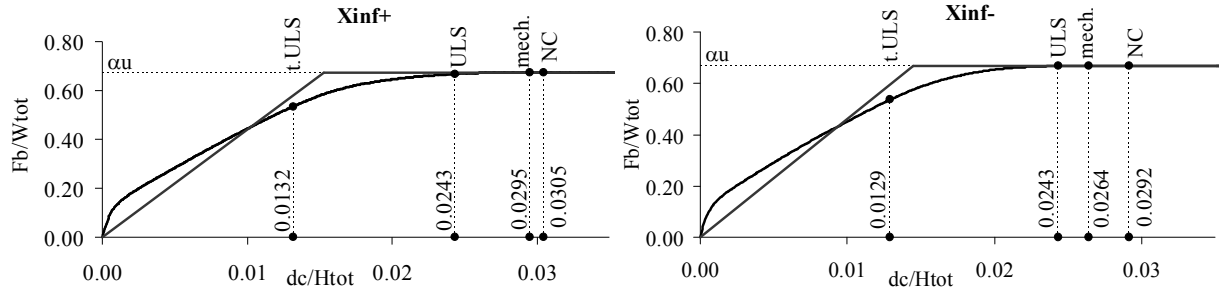


Figure 2: Capacity curves of 2 considered models given by a vertical force “modal” pattern.

4.1 Method n.1

Four modal response spectrum analyses are performed (called X_{inf} , X_{sup} , Y_{dx} , Y_{sx} , clearly depending on the direction of forces and on the position of centre of mass), one for each position of CM; obviously the load is applied along Y direction when CM is moved along X axis and, conversely, it is applied along X direction when CM is moved along Y axis;

The results (in particular top displacements of each frame and of centre of mass) obtained by each MRSA are combined by the SRSS rule:

$$\sqrt{u_{Xsup}^2 + u_{Ydx}^2}, \sqrt{u_{Xsup}^2 + u_{Ysx}^2}, \sqrt{u_{Xinf}^2 + u_{Ydx}^2}, \sqrt{u_{Xinf}^2 + u_{Ysx}^2} \quad (1)$$

For each combination and for each frame the following normalized displacement is computed:

$$\eta_{MRSA,i} = \frac{u_{MRSA,i}}{u_{MRSA,CM}} \quad (2)$$

where $u_{MRSA,i}$ is the “i” frame top displacement and $u_{MRSA,CM}$ is the CM top displacement; consequently, 4 $\eta_{MRSA,i}$ are computed for each frame and their maximum is assumed as the reference $\eta_{MRSA,i}$;

Eight nonlinear static analyses are performed, two signs (+ and -) for 4 models and for each model the NLSA with the maximum target displacement (t.ULS) is considered; consequently, 4 normalised displacements are obtained for each frame:

$$\eta_{NLSA,i} = \frac{u_{NLSA,i}}{u_{NLSA,CM}} \quad (3)$$

where $u_{NLSA,i}$ is the “i” frame top displacement and $u_{NLSA,CM}$ is the CM top displacement; For each frame 4 correction factors are computed:

$$\beta_i = \frac{\eta_{MRSA,i}}{\eta_{NLSA,i}} \quad (4)$$

where $\eta_{\text{MRSA},i}$ is the reference one and 4 $\eta_{\text{NLSA},i}$ (eq. 3) are considered;

The seismic demand related to frames on the stiff side of the structure is amplified multiplying all relevant quantities obtained by the 4 nonlinear static analyses by the corresponding correction factors;

The maximum value of demand parameters among the 4 ones given by nonlinear static analyses is considered and compared with the capacity one computed according to EC8 [8].

4.2 Method n.2

Four modal response spectrum analyses (called X_{inf} , X_{sup} , Y_{dx} , Y_{sx}) are performed, as at the 1st step of method n.1;

For each CM position and for each frame, the normalized displacement (eq. 2) is computed. Consequently, four $\eta_{\text{MRSA},i}$ for each frame are obtained;

Eight nonlinear static analyses are performed, two signs (+ and -) for 4 models and for each model the NLSA with the maximum target displacement (t.ULS) is considered; consequently, 4 normalised displacements (eq. 3) are obtained for each frame, as at step 4 method n.1;

Four correction factors (eq. 4), corresponding to the 4 positions of centre of mass, are computed for each frame;

As at method n.1, the seismic demand related to frames on stiff side of the structure is amplified multiplying all relevant quantities obtained by the 4 nonlinear static analyses by the corresponding correction factors;

The results of the four amplified NLSA are combined according to the SRSS rule:

$$\sqrt{E_{X_{\text{sup}}}^2 + E_{Y_{\text{dx}}}^2}, \sqrt{E_{X_{\text{sup}}}^2 + E_{Y_{\text{sx}}}^2}, \sqrt{E_{X_{\text{inf}}}^2 + E_{Y_{\text{dx}}}^2}, \sqrt{E_{X_{\text{inf}}}^2 + E_{Y_{\text{sx}}}^2} \quad (5)$$

where, as already said, “X” and “Y” indicate analysis direction, while “sup”, “inf”, “dx” and “sx” the centre of mass position;

The maximum value of demand parameters among the 4 ones given by SRSS combinations is considered and compared with the capacity one computed according to EC8 [8].

4.3 Method n.3

The first 2 steps are coincident with method n.2;

Eight nonlinear static analyses are performed, two signs (+ and -) for 4 models and, consequently, 8 normalised displacements (eq. 3) are obtained for each frame (as for methods n.1 and n.2 displacements at t.ULS are considered);

Eight correction factors (eq. 4), corresponding to 4 positions of centre of mass and both the signs (+ and -), for each frame are computed: to each $\eta_{\text{MRSA},i}$ correspond 2 $\eta_{\text{NLSA},i}$;

The seismic demand related to frames on stiff side of the structure is amplified multiplying all relevant quantities obtained by the 8 nonlinear static analyses by the corresponding correction factors;

The results of the 8 amplified NLSA are combined according to the SRSS rule:

$$\begin{aligned} &\sqrt{E_{+X_{\text{sup}}}^2 + E_{+Y_{\text{dx}}}^2}, \sqrt{E_{+X_{\text{sup}}}^2 + E_{-Y_{\text{dx}}}^2}, \sqrt{E_{-X_{\text{sup}}}^2 + E_{+Y_{\text{dx}}}^2}, \sqrt{E_{-X_{\text{sup}}}^2 + E_{-Y_{\text{dx}}}^2}, \sqrt{E_{+X_{\text{sup}}}^2 + E_{+Y_{\text{sx}}}^2}, \\ &\sqrt{E_{+X_{\text{sup}}}^2 + E_{-Y_{\text{sx}}}^2}, \sqrt{E_{-X_{\text{sup}}}^2 + E_{+Y_{\text{sx}}}^2}, \sqrt{E_{-X_{\text{sup}}}^2 + E_{-Y_{\text{sx}}}^2}, \sqrt{E_{+X_{\text{inf}}}^2 + E_{+Y_{\text{dx}}}^2}, \sqrt{E_{+X_{\text{inf}}}^2 + E_{-Y_{\text{dx}}}^2}, \end{aligned}$$

$$\sqrt{E_{-Xinf}^2 + E_{+Ydx}^2}, \sqrt{E_{-Xinf}^2 + E_{-Ydx}^2}, \sqrt{E_{+Xinf}^2 + E_{+Ysx}^2}, \sqrt{E_{+Xinf}^2 + E_{-Ysx}^2}, \sqrt{E_{-Xinf}^2 + E_{+Ysx}^2}, \sqrt{E_{-Xinf}^2 + E_{-Ysx}^2} \quad (6)$$

where, as already said, “X” and “Y” indicate analysis direction, “+” and “-” its sign and “sup”, “inf”, “dx” and “sx” the centre of mass position;

The maximum value of demand parameters among the 16 ones given by SRSS combinations is considered and compared with the capacity one computed according to EC8 [8].

Method n.3 provides results, which for sake of brevity are not shown, almost coincident with the ones provided by method n.2. This shows that, even when torsional effects are considered, the sign of pushover does not affect the results and, consequently, it is sufficiently accurate to consider the pushover with the maximum target displacement between the two pushovers with opposite sign.

4.4 Comments on SRSS combination rule

Table 1 shows the increment of frame top displacements due to the evaluation of orthogonal effects by the SRSS rule (eq. 5) of method n.2. In the 1st column the frames are listed, frames 1, 2 and 3 are parallel to Y axis as reported in Figure 1, while frames from 4 to 9 are parallel to X axis; the 2nd and the 3rd column show for the frames 4 to 9 and 1 to 3 respectively the top displacements resulting from (eq. 5), while the 4th and the 5th column present the corresponding displacements obtained by method n.2 before applying the combination (eq. 5); the 6th and the 7th column present the variation in percentage of the results.

It is evident that such variations are negligible; consequently the 6th step of method n.2, i.e. the application of combination (eq. 5), could be suppressed for computing such parameters.

Frame	SRSS applied		SRSS NOT applied		Variation [%]	
	u_x [m]	u_y [m]	u_x [m]	u_y [m]	u_x [-]	u_y [-]
1	-	0.126	-	0.126	-	0.24
2	-	0.124	-	0.124	-	0.01
3	-	0.133	-	0.133	-	0.14
4	0.148	-	0.147	-	0.19	-
5	0.138	-	0.137	-	0.07	-
6	0.127	-	0.127	-	0.01	-
7	0.127	-	0.127	-	0	-
8	0.129	-	0.129	-	0.02	-
9	0.132	-	0.132	-	0.05	-

Table 1: Estimation of orthogonal effects evaluated by SRSS combination rule: frame top displacements computed according to method n.2.

5 STEP BY STEP NONLINEAR DYNAMIC ANALYSES

Both the horizontal components of a set of 7 earthquakes, i.e. 14 natural records, fully satisfying the EC8 provisions [9] are used for nonlinear dynamic analyses. According to the code, if the response is obtained from at least 7 nonlinear time histories analyses, as for the performed analyses, the average of response quantities should be used as the design value of the action effect E_d in relevant verifications.

Figure 3 shows the elastic spectrum of the 14 records used, along with the average spectrum (smooth thick line), the EC8 elastic design spectrum (black thick line) and the curve whose ordinates are equal to 90% of such spectrum ones (black thin line); SF indicates the value of factors used to scale the records for the purpose to matching code spectrum: only one record is scaled by a low SF equal to 1.08.

The matched code spectrum is the one used for the design of the building analysed, assigning $q=1$.

As for nonlinear static analyses, top centre of mass and frame displacements are considered. For each of the 4 models obtained moving the centre of mass, the average of the 7 maximum results obtained applying the 7 earthquakes is considered along with the average plus the standard deviation and the maximum of the 7 maximum results; the maximum effect on the 4 models is always considered.

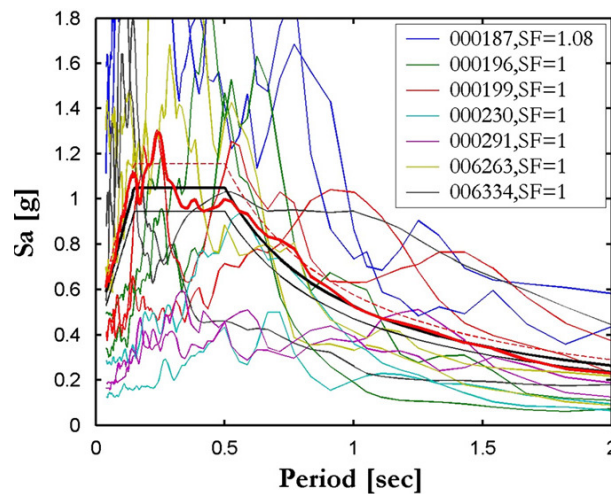


Figure 3: Spectra of records used for nonlinear analyses, their average and EC8 spectrum.

6 COMPARISONS BETWEEN THE ANALYSES RESULTS

In this chapter results of methods n.1 and n.2 are compared to the ones of nonlinear dynamic analyses (NLDA) in terms of displacements; as already written, such capacity at Ultimate Limite State is equal to $3/4 \Theta_u$.

In Figures 4 and 5 the following symbols are used: NLSA indicates results of nonlinear static analyses at “t.ULS” without considering torsional effects, i.e. the maximum of the 4 SRSS combinations (eq. 5), where the effects are not amplified by correction factors; NLSA(meth. n.1) and NLSA(meth. n.2) indicate the results of method n.1 and 2 respectively; NLDA(average) means that the average among the maximum results of the 7 nonlinear dynamic analyses is considered and the maximum among the 4 models is reported; as NLDA(av+SD) the average plus the standard deviation instead of the only average of the 7 results for each model is presented, while NLDA(maximum) means that the maximum among the maximum results of the 7 nonlinear dynamic analyses is taken and, as in the other 2 cases, the maximum of the 4 models is shown.

Figure 4 shows frame top displacements, absolute and normalized with respect to CM displacement, along X and Y direction.

It can be observed that NLSA(meth. n.1) gives displacements larger than NLSA and NLSA(meth. n.2), in particular for frames 4 and 5; this is due to the assumption of the maximum $\eta_{MRS A, i}$ in order to compute the four correction factors for each frame. This allows the NLSA(meth. n.1) displacements to better approximate the displacements of nonlinear dynam-

ic analysis when the standard deviation is also considered and, in particular, to estimate on safe side the displacements due to torsion. On the contrary, the positive aspects of NLSA(meth. n.2) are that the correction factor computed for a model coming from $\eta_{\text{MRSA},i}$ and $\eta_{\text{NLSA},i}$ computed for the same model and that the increment of displacements due to torsion effects is low, which is positive if the approximation of NLDA(average) absolute displacements is considered. Figure 4 also shows that NLSA methods provide displacements quite larger than NLDA(average) displacements and that, the standard deviation is not negligible with respect average results, as it can be observed by the difference between NLDA(average) and NLDA(av.+SD) displacements.

The comments reported above are confirmed by Figure 5, where interstorey drifts are reported at corner 1 and 2 along X and Y direction respectively. The increment of the 2nd storey drifts, shown by all the analyses, is mainly due to the reduction of the 2nd storey stiffness with respect to the 1st storey one.

This demonstrates that the safety level associated to different analyses performed according to EC8 is correlated to their accuracy, i.e. the safety verification made by nonlinear static analysis is more conservative with respect to non-linear dynamic analysis, which is the most complicated [10].

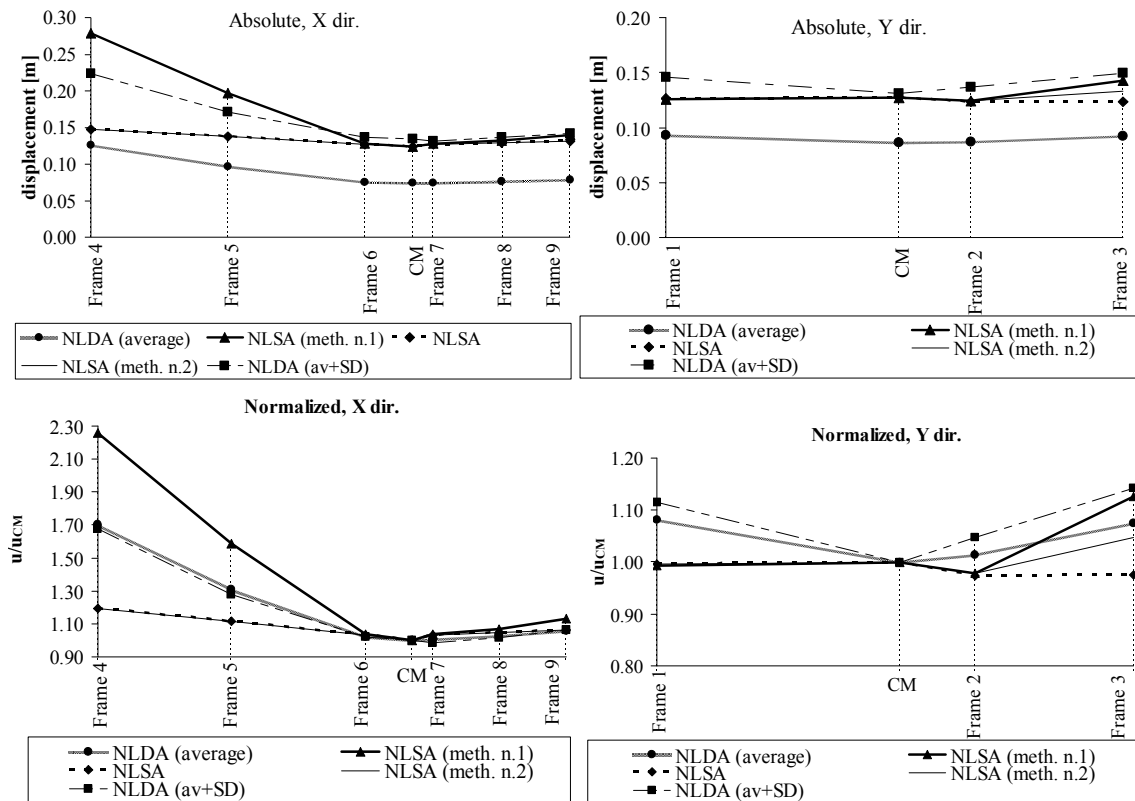


Figure 4: Absolute and normalized frame top displacements along X and Y direction.

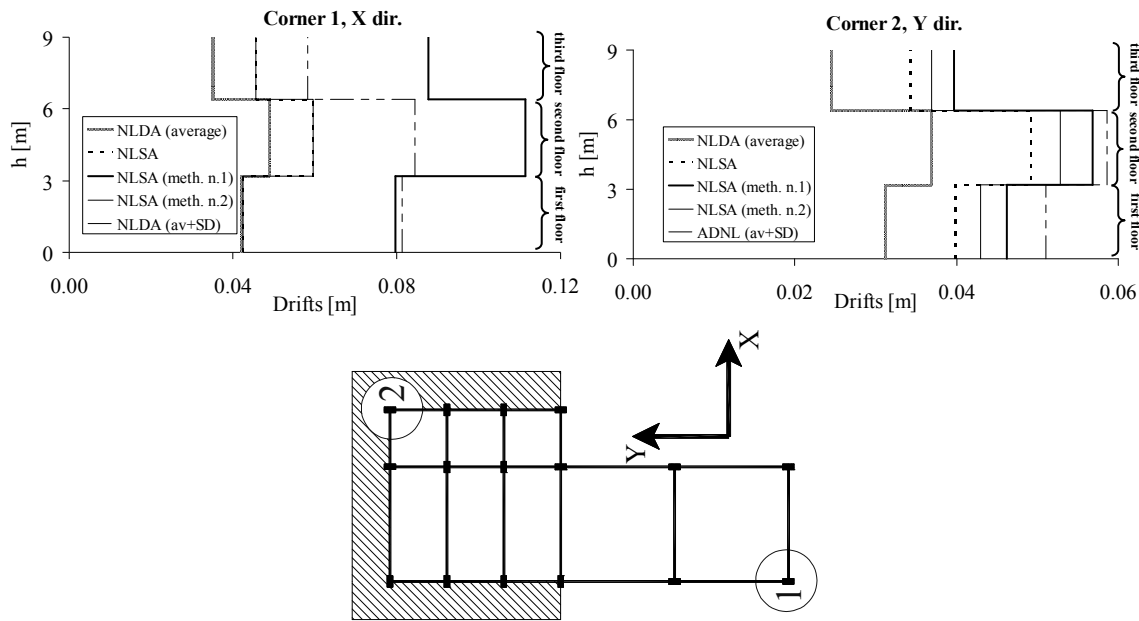


Figure 5: Drifts at corners 1 and 2 along X and Y direction.

7 CONCLUSIONS

In the paper the problem of extension of N2 method to plan irregular buildings, which makes up for the underestimate of seismic demand on stiff side, is discussed. Three methods, which take into account the accidental eccentricity (4 positions of centre of mass correspond to 4 models), are proposed, applied to a 3-storey, r/c frame plan irregular building and compared.

The main differences between the 3 nonlinear static analysis (NLSA) methods are: NLSA(meth. n.1) combines the displacements of modal response spectrum analysis obtained by the 4 models (associate to 4 eccentricities) by SRSS rule and, for each frame, the maximum normalized displacement is assumed as reference; NLSA(meth. n.2), works on 4 models and only at the end of the procedure combines the results by SRSS rule; NLSA(meth. n.3) considers till the end of the procedure the double sign (+ and -) of pushovers.

The results of NLSA(meth. n.3) are coincident with results of NLSA(meth. n.2), confirming that it is correct to consider the maximum target displacement given by the 2 pushovers performed on the same model in the same direction with different sign.

The influence of the orthogonal effects evaluated by the SRSS rule on the results of NLSA(meth. n.2) in terms of frame displacements is estimated and found negligible; this conclusion is confirmed by analyses performed on other buildings.

The input adopted for nonlinear dynamic analysis (7 natural earthquakes) is selected [9] in order to rigorously satisfy the EC8 provisions. 28 nonlinear dynamic analysis (7×4) are performed and the maximum effect of the 4 models is considered; for each model the average of 7 maximum results obtained from 7 earthquakes (NLDA(average)), the average plus standard deviation (NLDA(av+SD)) and the maximum of the 7 maximum results (NDLA(maximum)) are computed.

For some frames, NLSA(meth. n.1) gives displacements larger than NLSA(meth. n.2), approximating better the displacements of nonlinear dynamic analysis when the standard deviation is also considered and estimating on safe side the displacements due to torsion. On the contrary, the positive aspects of NLSA(meth. n.2) are that the correction factor computed for a model comes from normalised displacements computed for the same model and that the in-

crement of displacements due to torsion effects is low, which is positive considering that NLDA(average) absolute displacements are smaller.

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