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EXPERIMENTAL AND NUMERICAL INVESTIGATIONS FOR THE STRUCTURAL CHARACTERIZATION OF A HISTORIC RC ARCH BRIDGE

R. Ferrari[1*], D. Froio[1], E. Chatzi[2], C. Gentile[3], F. Pioldi[1], E. Rizzi[1]

[1] Università di Bergamo, I-24044 Dalmine (BG), Italy
[*] Corresponding Author, rosalba.ferrari@unibg.it

[2] ETH Zürich, CH-8093 Zürich, Switzerland

[3] Politecnico di Milano, I-20133 Milano, Italy

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Abstract. *A comprehensive rational methodology for the structural assessment of existing bridges is presented and specifically applied to a historic reinforced concrete arch bridge. The methodology is based on the use of non-destructive testing tools and structural model updating procedures and involves: (a) preliminary documented research and on-site geometric surveys (aimed at collecting information on the "as built'' geometry); (b) ambient vibration testing performed by using a grid of conventional high-sensitivity accelerometers, aimed specifically at investigating the vertical dynamic characteristics of the bridge and c) development of an updated Finite Element (FE) model of the structure.*
The investigated bridge, completed in May 1917, crosses the Adda river between Brivio (province of Lecco) and Cisano Bergamasco (province of Bergamo), about 50 km North-East from Milano, Northern Italy. Given the still strategic position of the bridge in the current road transportation network and within a systematic surveillance program of main infrastructures by the Province of Lecco, dynamic tests were performed under operational conditions.
Main results in terms of Operational Modal Analysis and FE modelling and updating are presented and discussed. A hierarchy of FE models with different levels of refinement is developed, with the purpose of a future selection of the model that better reproduces the current structural properties of the bridge. In this paper an automated system identification procedure has been developed and applied to the simplest of the assembled (consistent) FE models, whose results will constitute a benchmark for further studies upon the other most refined models. The aim is to perform a final baseline reference model to be used for reliability assessment within Structural Health Monitoring (SHM) purposes.

1 INTRODUCTION

Nowadays the development of methodologies for accurate and reliable condition assessment of bridges, or other typologies of civil infrastructures, is becoming increasingly important. The process of developing or improving methodologies for determining and tracking the structural integrity of infrastructures based on automated monitoring systems is a main scope of SHM [1].

FE models play a key-role in the ordinary design process of new structures and in the assessment of existing ones [2]. With the current advances in numerical modeling and computational capabilities, it is generally expected that a FE model consistently based on original technical design drawings, on-site geometric surveys, engineering judgment and assessment processes, shall reliably reproduce both static and dynamic behaviors of a structure.

However, acquired experience shows that the process of developing a FE model of a structure involves assumptions and simplifications that may induce considerable errors, which are a consequence of the underlying complexity of the structural modeling, of the uncertainty of the boundary conditions and of the real mechanical behavior of materials and structural elements [3]. Moreover, variations in these features during the lifespan of a structure may occur due to the appearance of smeared or localized damage, causing final discrepancies between the characteristics of the structure at design stage and at the current state of duty and conservation.

Structural identification via modal dynamic analysis [4] and subsequent Finite Element Model Updating [5,6] represent consistent and widespread tools towards condition assessment of existing civil constructions, like bridges or structures endowed with historical values. In fact, it is well known that changes in the physical properties of a structure correspond to changes in the modal parameters (notably frequencies, mode shapes, and modal damping ratios) [7]. In most of Model Updating techniques the stiffness, mass and damping distributions of a numerical model chosen as reference configuration, are iteratively updated, so that the differences between the measured and the analytical values of the modal parameters are minimized [2]. There appear several works in the dedicated literature in which the results obtained from modal identification have revealed useful for performing model updating of a numerical model of existing bridges [2-3,8-14]. Within such a field, this paper presents the results obtained from a research study that involved both experimental and analytical modal analysis as well as subsequent finite element model updating of a reinforced concrete bridge with parabolic arches, namely the Brivio bridge (1917), Italy, as described below.

The investigation dealt within this paper involves: (a) exploiting OMA techniques to Ambient Vibration Testing (AVT) [3]; (b) establishing three FE models of the bridge with increasing levels of detail, based on the available design drawings and on surveys performed in situ; (c) exploring the sensitivity of the natural frequencies of a 2D FE model of the bridge to changes in some uncertain structural parameters; (d) setting the parameters of such 2D FE model, that appear good candidates for the updating procedure and (e) identifying such parameters, in order to enhance the fitting between experimental and theoretical natural frequencies and mode shapes. The aim is to create an improved FE model which can be adopted as a benchmark for further scheduled analyses on more complex and detailed numerical models.

Report on this present research investigation is organized in two companion papers. Companion work [15] focuses on the analysis of the various data coming from the different adopted instrumentation, accounting also for data fusion and for reliability and uncertainty assessment of the acquired data, while the present note exposes the detailed AVT performed with conventional high-sensitivity accelerometers and the development of an updated FE models of the bridge, specifically in terms of prediction of modal properties.

The present paper is structured as follows. Section 2 describes the main characteristics of the Brivio bridge, which is the benchmark structural object taken for this study. In Section 3 the results of the output-only model identification performed on the bridge are presented. In Section 4 the three performed FE models are described in detail. Section 5 concerns the sensitivity analysis for the selection of the parameters to be considered within the model updating procedure, which is explained in Section 6. Finally, main conclusions are outlined in closing Section 7.

2 SALIENT FEATURES OF THE BRIVIO BRIDGE

The Brivio bridge (Figs. 1, 2), designed by Italian engineer Giuseppe Banfi on June 1912 and completed on May 1917, is a three-span historical reinforced concrete bridge with parabolic arches located in Lombardia, Northern Italy, about 50 km North-East away from Milano [16]. It crosses the Adda river at about 8 m from water, between the municipalities of Brivio and Cisano Bergamasco, linking the two provinces of Lecco and Bergamo. The spans of the bridge are 43.40 m, 44.00 m and 43.40 m long respectively, and consist of a deck joined on each of their sides to two lateral parabolic arches. The suspension is performed by means of sixteen hangers, per each side of each span, with rectangular cross-section that is 32 cm wide and 60 cm high. All structural elements are made of reinforced concrete.

The total width of the deck is 9.20 m, hosting a double-lane road and two pedestrian walkways, each 0.80 m wide. The deck cross section (Fig. 2) is constituted by two outer longitudinal girders framed by floor beams; girders, spaced 8.60 m center to center, display approximately rectangular cross sections with width of 45 cm and height of 100 cm, and floor beams, provided every 2.30 m, also show rectangular cross sections with width of 28 cm, but variable height along the beam axis. The floor beams are further connected to other two longitudinal ribs of width of 20 cm, placed symmetrically at a distance of 1 m with respect to the vertical longitudinal middle plane of the bridge. The resulting frame is covered by a reinforced concrete slab of 15 cm of high, which constitutes the support of the road.

Figure 1: Contemporary views of Brivio bridge seen from Brivio’s riverside (right bank).

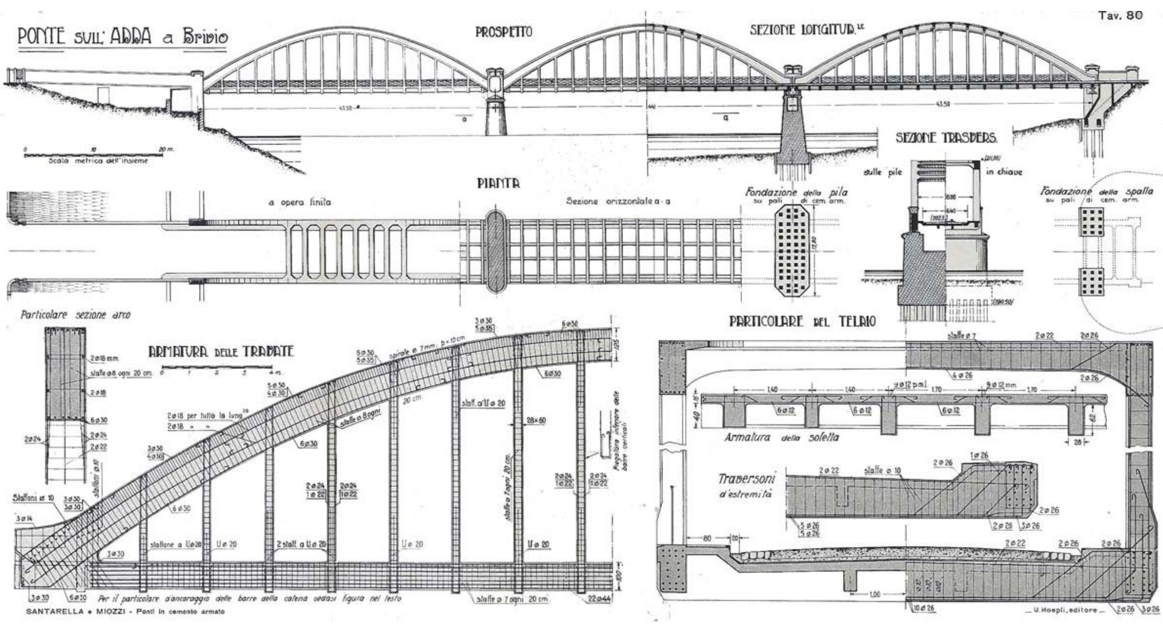


Figure 2: Historical representation of the technical drawings of the bridge [17].

The arches of the bridge show a span of about 42.80 m between the two ends and a rise of 8.00 m at the keystone. They display a rectangular cross section characterized by a constant width, equal to 60 cm, and a variable tapered height starting from around 1.50 m at the extrema to 1.25 m at the middle. To achieve higher structural stability, the arches are linked in the upper part by eight transverse girders, tapered from the end to the center.

Each span rests on either a pier or an abutment, where outer longitudinal girders end, through a mechanical system made of trusses, in order to allow little axial elongations, due e.g. to changes of temperature.

3 MODAL DYNAMIC IDENTIFICATION OF THE BRIDGE

This Section reports the modal estimates that have been obtained from output-only identification techniques based on the operational response data acquired on the bridge by using conventional high-sensitivity accelerometers.

The response of the bridge was measured at eighteen selected points, as shown in Fig. 3. Since it was decided to simultaneously use ten wired accelerometers during the tests, two set-ups were performed to measure the acceleration at opposite sides of eight cross-sections of the deck, considering two sensors as reference transducers, which were kept at the same locations in all the set-ups.

Two time windows of 3600 s were collected for each sensor layout, with a sampling rate of 200 Hz, which is higher than that required for this bridge, as the natural frequencies of the dominant modes are below 20 Hz. Hence, low pass filtering and decimation were applied to the data before the use of the identification tools, reducing the sampling rate from 200 Hz to 25 Hz.

The output-only modal identification was carried out by using both the Frequency Domain Decomposition (FDD) [18] and the data-driven Stochastic Subspace Identification (SSI-data) methods [19] available in the commercial software ARTeMIS [20].

The results of modal identification are summarized in Figs. 4-5 and in Table 1. The natural frequencies of the identified modes can be easily identified in Fig. 4 from the local maxima of the first Singular Value (SV) line resulting from the application of the FDD method; the corresponding mode shapes are shown in Fig. 5.

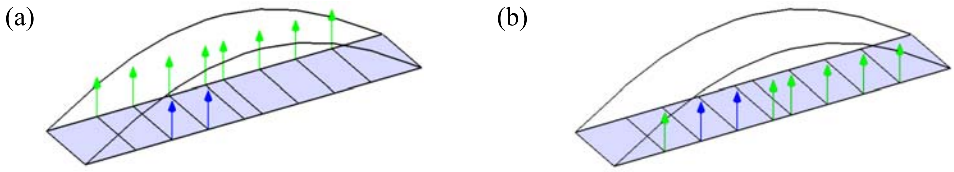


Figure 3: Points instrumented using wired accelerometers: (a) Set-up 1; (b) Set-up 2.

The inspection of Fig. 5 highlights that: (a) almost all mode shapes exhibit regular and smooth shape with dominant bending or torsion, with the exception of the 7th mode, which is characterized by coupled bending and torsion; (b) the first two modes exhibit different frequencies but practically the same mode shape. In addition, the 7th vibration mode also exhibits complex behaviour (i.e. the modal deflection phases significantly deviate from 0 or π).

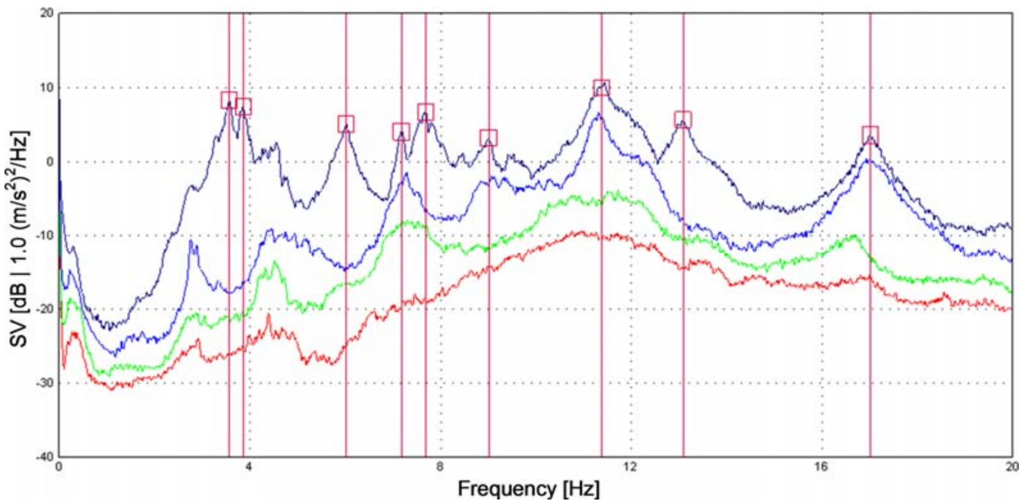


Figure 4: Singular value (SV) lines and identification of natural frequencies from the wired accelerometers data (FDD).

Mode N.	f_{FDD} *(Hz)*	f_{SSI} *(Hz)*	ζ_{SSI} *(%)*	*MAC*
1	3.564	3.449	4.60	0.997
2	3.857	3.887	4.09	0.991
3	6.018	5.968	3.17	0.998
4	7.178	7.146	1.51	0.989
5	7.690	7.592	2.82	0.991
6	9.009	8.928	1.67	0.990
7	11.377	11.390	1.28	0.938
8	13.086	13.040	2.01	0.987
9	17.017	16.990	1.44	0.935

Table 1: Identified frequencies f_i [Hz], first span, wired accelerometers.

Very close results, in terms of natural frequencies and mode shapes, are obtained by applying the SSI-data method, as it is summarized in Table 1. Furthermore, Table 1 reveals that the damping ratios of the first two modes are larger than 4%.

It should be noticed that the “splitting” of 1st mode (with quite high damping ratios) and the complex behaviour of the 7th mode deserves further investigation since both the observed phenomena might be related to the poor state of preservation and cracking of some vertical hangers. In more details, the co-existence of two close spectral peaks with similar mode shapes in place of one single mode is sometimes referred to as "dispersive phenomenon" [21] and was mainly observed in the response of cracked reinforced concrete structures. The same physical behaviour has been recently detected in ambient vibration testing of two different arch bridges [22].

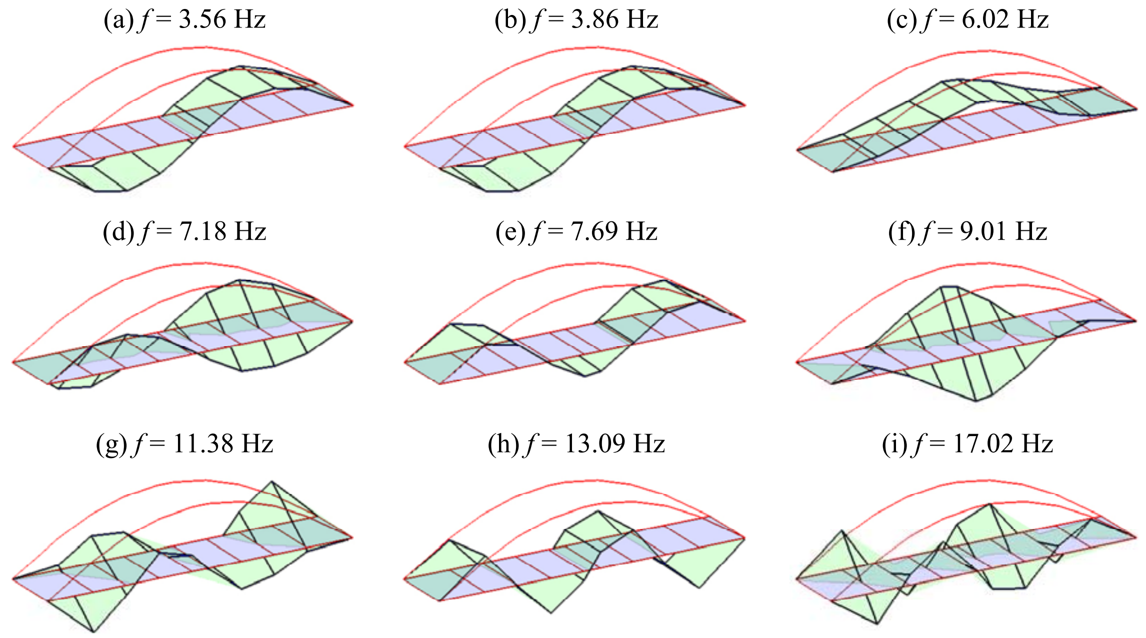


Figure 5: Vibration modes identified from the wired accelerometers data (FDD).

4 FE MODELING OF THE BRIDGE

Three Finite Element models of the Brivio Bridge with different levels of refinement have been assembled [16]. In particular, one two-dimensional model and two three-dimensional models have been implemented within the commercial FE code ABAQUS [23].

The main assumptions considered in the present FE models of the bridge are the following:

- Euler-Bernoulli beam finite elements have been used to model all the elements of the bridge, except for one of the two 3D models in which four-nodes shell elements have been employed to model the deck;
- uniform cross sections, homogeneous material properties and linear elastic mechanical behavior have been assumed; Poisson's ratio of reinforced concrete has been held constant and set equal to 0.20;
- an additional weight per unit volume of 10 kN/m^3 has been considered on the deck slab, to account for the effects of the asphalt pavement and of the walkways;
- rigid links between the concrete slab and the grid of hangers and between the latter and the arches have been applied, for taking into account the real lengths of the structural elements; each of these links provides a rigid constraint for translation and rotation of one node with respect to the degrees of freedom of the other one;
- the deck has been assumed to be able to rotate only on one side, while on the opposite side boundary conditions have been modeled according to the design characteristics of

the bearing supports (which shall allow for longitudinal displacement); hence, a hinge-roller scheme has been assumed for the boundary conditions of each span;

- a single representative span of 42.80 m has been considered and no continuous beam effects are investigated so far.

In the following Sections 4.1-4.3 a brief description of each FE model is reported.

4.1 2D FE Beam model

The assembled 2D FE model of the bridge is depicted in Fig. 6. The *x* and *y* axes represent the longitudinal axis and the vertical axis of the bridge, respectively. The model is composed of 224 elements and 178 nodes, for a total of 986 free variables in the internal code representation [23].

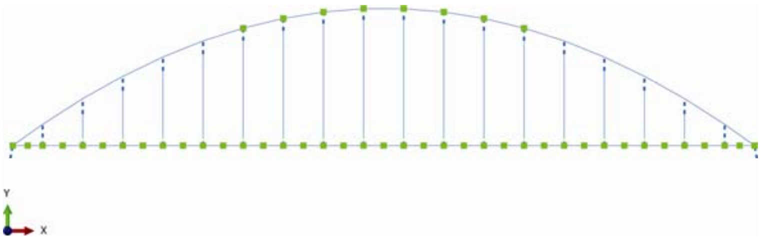


Figure 6: Assembled 2D FE Beam model (green markers specify where lumped masses have been placed).

According to the original design (see Fig. 2), the arches are composed of nineteen chunks with different heights; then, ten types of different double rectangular cross sections with height decreasing from the ends towards the top have been modeled. The hangers have been also represented by elements with a double rectangular cross section. The masses of the elements whose axes lie out of the plane of the model have been lumped at the corresponding nodal positions; the values of the lumped masses are reported in Table 2. Rotational inertia values are considered to be negligible.

Element	*Mass [kg]*
Transverse deck beam	5193
Transverse deck beam at the ends	7419
Transverse beam of the arches	3080

Table 2: Lumped masses added to the 2D FE Beam model.

The global geometrical parameters which characterize the 2D Beam model of the bridge are reported in Table 3. As a first step, the total mass has been evaluated by assuming a reinforced concrete density of 2500 kg/m^3. Further data on the geometrical characteristics are reported in [16].

Parameter	*FE model value*
Total mass	754.8 t
Component x of the center of mass	21.40 m
Component y of the center of mass	1.93 m
Moment of inertia about axis z on the center of mass	1.18 10^8 kg m^2

Table 3: Global geometric parameters of the 2D FE Beam model (concrete density = 2500 kg/m^3).

4.2 3D FE Beam model

The 3D FE beam model of the bridge is shown in Fig. 7. It has been assembled via exclusive use of only 3D beam elements. The *x*, *y* and *z* axes represent the longitudinal axis, the vertical axis and the horizontal transverse axis of the bridge, respectively. The FE model counts for 808 elements and 668 nodes, with a total number of free variables equal to 5900.

According to the original design drawings, each arch has been modeled as in the 2D FE model. Eight superior beam elements with T cross section have been added to the model, setting the two arches at a relative distance of 8.60 m. The deck has been modeled as a framework of beams. In the longitudinal direction, six beam elements have been placed, playing the role of longitudinal girders, included the reinforced concrete slab above. The cross sections of these elements have been modeled for best fitting the shape of the deck cross section, depicted in Fig. 2. In the transverse direction, beams with variable rectangular cross section have been placed.

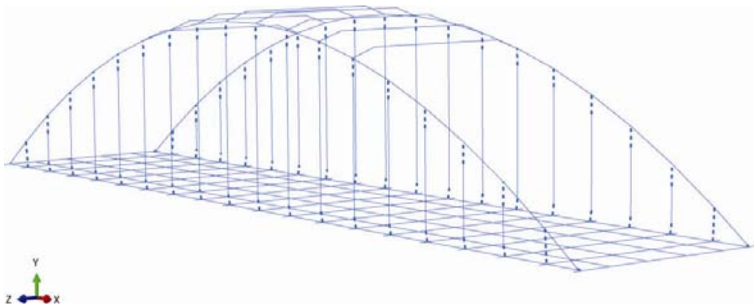


Figure 7: Assembled 3D FE Beam model.

The global geometrical characteristics of the 3D Beam model are reported in Table 4. Further data are available in [16].

Parameter	***FE model value***
Total mass	762.6 t
Component x of the center of mass	21.40 m
Component y of the center of mass	1.92 m
Component z of the center of mass	4.30 m
Moment of inertia about axis x on the center of mass	$1.53\ 10^8$ kg m^8
Moment of inertia about axis y on the center of mass	$1.23\ 10^8$ kg m^2
Moment of inertia about axis z on the center of mass	$1.20\ 10^8$ kg m^2

Table 4: Global geometric parameters of the 3D FE Beam model (concrete density = 2500 kg/m3).

4.3 3D FE Beam & Shell model

A further improvement in the FE description of the bridge has been performed by considering shell elements, instead of beam elements, in the modelization of the deck, as represented in Fig. 8. The use of shell elements allows to describe the mean line of the deck cross section, by providing a more accurate reproduction of its peculiar shape. Twelve shell elements with six different thicknesses have been employed in the model. The final assembly of the FE model counts for 3678 elements and 4188 nodes, with a total number of 22660 free variables.

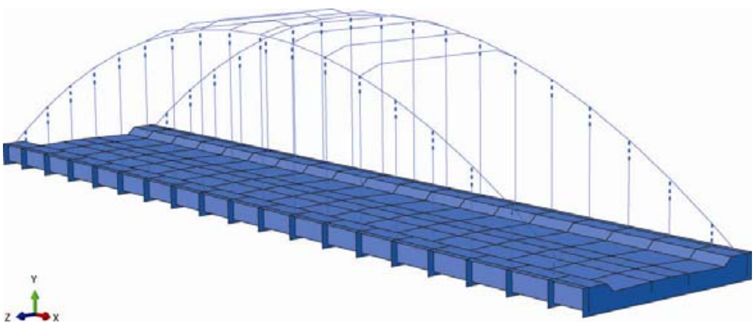


Figure 8: Assembled 3D FE Beam & Shell model.

The global geometrical parameters of the 3D Beam & Shell model of the bridge are reported in Table 5.

Parameter	*FE model value*
Total mass	767.0 t
Component x of the center of mass	21.40 m
Component y of the center of mass	1.90 m
Component z of the center of mass	4.30 m
Moment of inertia about axis x on the center of mass	$1.55\ 10^8$ kg m^2
Moment of inertia about axis y on the center of mass	$1.23\ 10^8$ kg m^2
Moment of inertia about axis z on the center of mass	$1.19\ 10^8$ kg m^2

Table 5: Global geometric parameters of the 3D FE Beam & Shell model (concrete density = 2500 kg/m^3).

As Tables 3-5 show, the FE models appear with a good level of similarity referring to the geometrical characteristics, demonstrating the consistency of the models themselves.

5 SENSITIVITY ANALYSIS

As mentioned in Introduction, the sensitivity analysis and the optimization procedure have been based on the simplest of the assembled FE models, that is, the 2D Beam model.

It is well known that the selection of the parameters to be updated is crucial, and that sensitivity analysis constitutes an efficient tool which allows for the selection of the parameters that most influence the structural responses. The sensitivity coefficients can be computed as the rate of change of a particular response of the model with respect to a change of the structural parameters [3]. Then, the sensitivity matrix **S** can be calculated as follows:

$$S_{ij} = \frac{\partial R_i}{\partial P_j} \tag{1}$$

where R_i and P_j represent a structural response index and a structural parameter, respectively, with $i=1,\dots,N$, for N response indexes and $j=1,\dots,M$, for M structural parameters. The sensitivity matrix can be computed for all physical element properties (material, geometrical, boundary, etc.), by using direct derivation or approximation techniques [2].

Eq. (1) evaluates the absolute sensitivities, which are characterized by the dimensions of responses and parameter values. If sensitivities for different types of parameters have to be compared, a normalized relative sensitivity matrix $\mathbf{S}_n$ should be better used:

$$S_{n\,ij} = \frac{\partial R_i}{\partial P_j}\frac{P_j}{R_i} \tag{2}$$

Hence, the problem may be determined, over-determined or under-determined, depending on whether matrix $\mathbf{S}_n$ is square (N=M), tall-rectangular (N>M) or wide-rectangular (N<M), respectively. If the estimation of too many parameters is attempted, then the problem may appear ill-conditioned, in particular when observations are limited, as it usually occurs in vibration testing. Therefore, to achieve a well-conditioned updating problem it is necessary to select a smaller as possible number of updating parameters, which will be the most effective ones in producing a genuine improvement in the modeling of the structure [2].

In light of this, also concerning their influence in the overall dynamic behavior of the bridge, the following structural parameters have been selected to be used in the updating procedure:

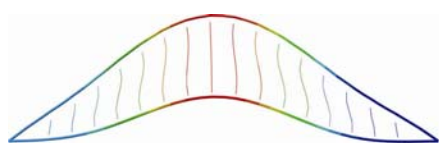
- Young's modulus of reinforced concrete deck (E_{deck});
- Young's modulus of reinforced concrete arches and hangers ($E_{arch\&hang}$);
- mass per unit volume of reinforced concrete (ρ_{conc}).

As mentioned in the work of Brownjohn et al. [2], when performing a model updating procedure, it is very important to determine a suitable initial value of a selected parameter, i.e. a reasonable starting point for the optimization process; this is because if the initial value is too far away from its real value and large discrepancies exist between the experimental and the numerical model, the iterative process may result in convergence to another (local) minimum, or even in divergence. It is usually recommended to carry out a prior manual tuning, by engineering judgment or relevant preliminarily estimations, towards obtaining a reasonable approximation of the start point and of the parameter bounds before starting the optimization procedure.

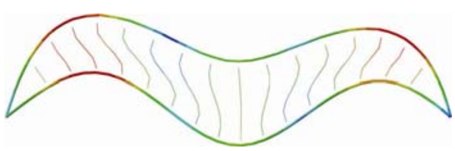
To accomplish such manual tuning, a set of preliminary modal analyses have been performed on the 2D FE model of the bridge by varying the three parameters listed above. At this first stage, the goal was that of assuring that the chosen parameters truly affected the modal response of the structure and to roughly match experimental and numerical modal results. Considering 3.564 Hz as the first modal frequency of the bridge, the manual tuning of the parameters of the FE model has provided a significant matching with respect to the experimental outcomes. In particular, very good results have been obtained using the values of 33.0 GPa, 36.5 GPa and 2400 kg/m^3, for the Young's modulus of deck (E_{deck}), the Young's modulus of arches/hangers ($E_{arch\&hang}$) and the concrete mass density (ρ_{conc}), respectively. The FE model characterized by this particular set of parameters will be referred to as "base model" in the following. Fig. 9 shows the results of the modal analysis performed through the 2D FE base model.

V1 f_{FDD} = 3.565 Hz f_{FE} = 3.601 Hz

V2 f_{FDD} = 6.018 Hz f_{FE} = 6.084 Hz



V3 f_{FDD} = 7.690 Hz f_{FE} = 7.667 Hz



V4 f_{FDD} = 13.086 Hz f_{FE} = 12.029 Hz

Figure 9: First four vibration modes of the 2D FE base model of the bridge (not updated).

The correlation between the dynamic characteristics of the FE base model and those coming from the experimental results is shown in Table 6, for the first four vibration modes, via the absolute frequency discrepancy and the modal assurance criterion (MAC) [24], to check correspondences of the mode shapes. The latter is defined as follows (MAC matrix):

$$MAC_{ij}(\boldsymbol{\varphi}_i^{exp},\boldsymbol{\varphi}_j^{num}) = \frac{\boldsymbol{\varphi}_i^{expT}\cdot\boldsymbol{\varphi}_j^{num}}{(\boldsymbol{\varphi}_i^{expT}\cdot\boldsymbol{\varphi}_i^{exp})^{1/2}\cdot(\boldsymbol{\varphi}_j^{numT}\cdot\boldsymbol{\varphi}_j^{num})^{1/2}} \tag{3}$$

where $\boldsymbol{\varphi}_i^{exp}$ and $\boldsymbol{\varphi}_j^{num}$ are the *i-th* experimental and *j-th* numerical mode shape vectors, respectively. Each value of the MAC matrix defined in Eq. (3) effectively represents a correlation coefficient ranging from 0 to 1, where a value of 1 represents a perfect correlation of the two mode shape vectors (i.e. a linear dependence), while a value close to 0 indicates uncorrelated vectors (i.e. linear independence or orthogonality condition). In general, a MAC value larger than 0.85÷0.90 is considered as a good match, while a MAC value less than 0.50 is considered to be a poor match [3].

Experimental			***2D Beam FE base model***		
Mode identifier	*Mode N*	*f_{FDD} (Hz)*	*f_{FE} (Hz)*	*Δ_f (%)*	*MAC*
V1	1	3.564	3.601	1.02	1.000
V2	3	6.018	6.084	1.10	0.991
V3	5	7.690	7.667	-0.31	0.993
V4	8	13.086	12.029	-8.08	0.889

Table 6: Correlation between experimental and 2D Beam FE base model dynamic characteristics for the first four vertical bending modes.

Some attempts have been also performed considering the value of 3.857 Hz as the frequency of the first vertical mode of the bridge, but the outcomes of the manual tuning have turned out unsatisfactory.

The normalized sensitivities (Eq. (2)) of the first six modal frequencies of the vertical bending modes with respect to the parameters above are represented in Fig. 10. The sensitivities have been evaluated by varying each time one of the parameters and keeping fixed the others to those of the base model. The plots in Fig. 10 have been obtained by linear interpolation of the point-wise values of partial derivatives of Eq. (2), which have been calculated by using a central difference evaluation.

The normalized relative sensitivities in Fig. 10 show that the chosen parameters truly affect the modal response of the structure. In particular, the plots show that: (a) the parameter that most influences the variations of the lower frequencies is the concrete mass density, with all sensitivity coefficients over 45% and almost constant for the considered frequencies; (b) concerning Young's moduli of deck and arches/hangers, the corresponding sensitivities range from 10% to 20% and from 30% to 40%, respectively; (c) the fundamental frequency f_1 of the first mode, which displays the typical antisymmetric mode shape of a vibrating arch, is indeed influenced mainly by Young's modulus of arches/hangers and, if compared to the other modes, is less influenced by the elastic modulus of deck.

Based on the obtained results, the parameters above have been set as the starting point for the optimization procedure of the 2D FE model of the bridge, as described in the following. Table 6 shows a fairly good correlation between experimental outcomes and numerical results from the base model for the first three flexural vertical modes: the higher frequency discrepancy ranges up to about 1% and the MAC index is never below 0.99. The forth vertical mode shapes displays large deviations, in particular for the frequency discrepancy which is about the 8%. Then, it has not been considered in the optimization procedure.

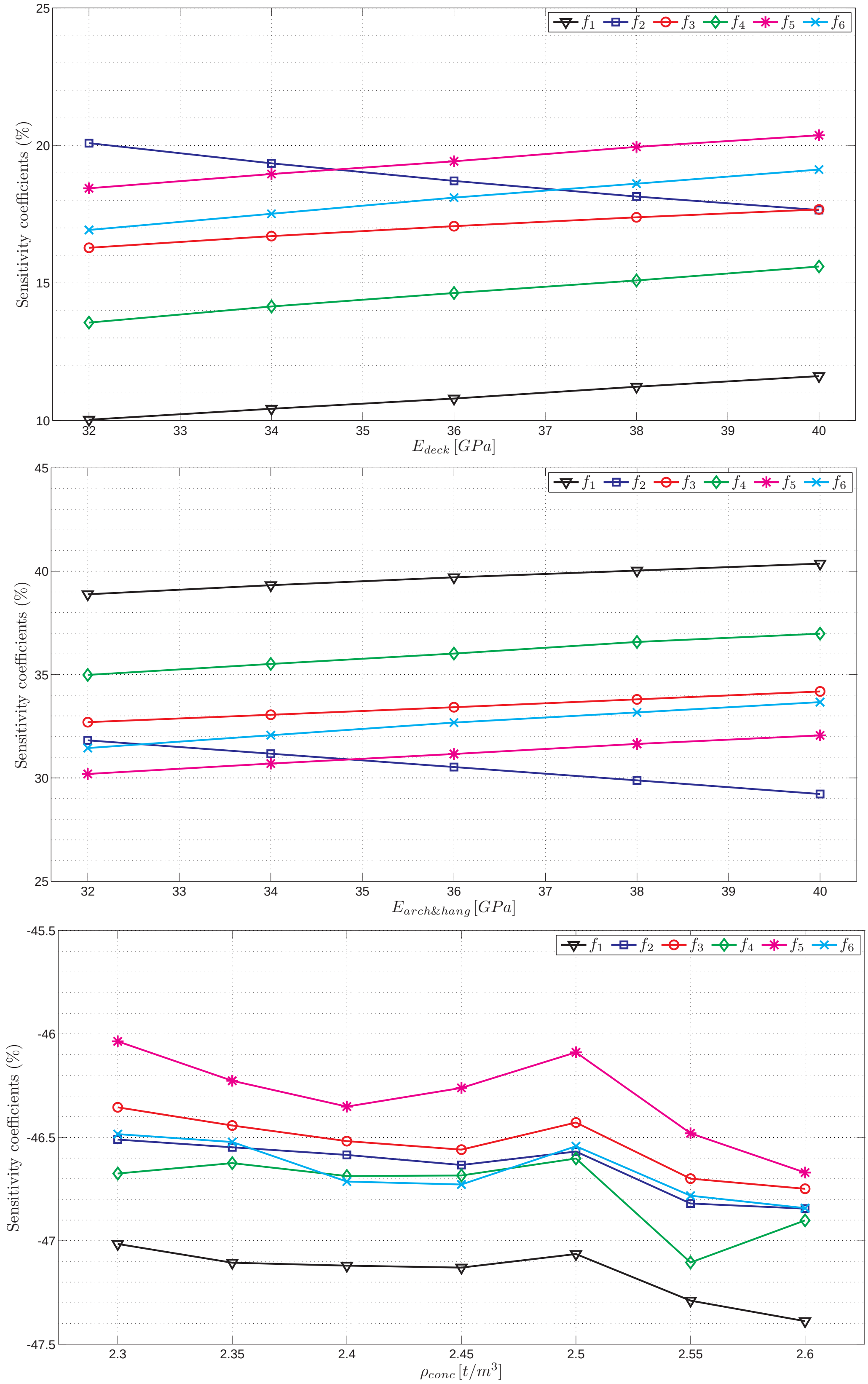


Figure 10: Sensitivity coefficients for the first six modal frequencies of the 2D Beam FE model.

6 OPTIMIZATION PROCEDURE FOR MODEL UPDATING

The optimization phase allows for obtaining the parameter values that minimize the differences between the experimental and numerical modal estimates. Then, this phase involves the definition of an appropriate objective function and the application of an optimization technique based on a non-linear least square algorithm. The algorithm of inverse analysis herein adopted is described in the following Section 6.1. It takes largely inspiration from the identification work performed in [25], in quite a different context (material indentation tests). In Section 6.2 the results obtained from the optimization procedure are reported.

6.1 Formulation

The algorithm makes use of two sources of information: experimental recorded dynamic results available before running, from which frequencies and mode shapes have been estimated through Operational Modal Analysis (FDD); numerical data that, depending on a number of modeling parameters to be identified (here three material parameters: E_{deck}, $E_{arch\&hang}$ and ρ_{conc}), arise from numerical simulations of modal analysis (Lanczos' method [23]) from the FE model.

The discrepancies among target data and simulated data are minimized, towards the identification of the material parameters allowing for most effective calibration. Such discrepancy minimization is measured in terms of an appropriate objective function, which quantifies, through a vector measure, the difference between target and predicted data. In the present case, the assumed objective function, $\boldsymbol{\omega(x)}$, corresponds to a discrete non-negative, non-dimensional, vector least-square discrepancy measure, and two types of terms, one related to the relative discrepancy of natural frequencies and another related to the MAC values [11] are considered:

$$\boldsymbol{\omega(x)} = \left[\alpha \left(\frac{f_i^{exp} - f_i^{comp}}{f_i^{exp}} \right)^2 ,\ (1-\alpha)(1-MAC\ (\boldsymbol{\varphi}_i^{exp}, \boldsymbol{\varphi}_i^{num}))^2 \right]^T , \quad i = 1, 2, 3 \tag{4}$$

where f_i^{exp} and f_i^{num} are the experimental and numerical frequencies of mode i, $\boldsymbol{\varphi}_i^{exp}$ and $\boldsymbol{\varphi}_i^{num}$ are the eigenvectors containing the experimental and numerical modal information regarding mode i and $\boldsymbol{x}$ is the (3×1) vector including the parameters to be optimized with respect to the first three flexural vertical modes. If the MAC values between measured and updated models are near to one and the frequency differences between measured and updated estimates are near to zero, the model updating is deemed to be successful.

In Eq. (4) α represents a weight coefficient [25], bounded between zero and one ($0 \leq \alpha \leq 1$), allowing to shift the importance of information from frequencies and mode shapes (possibly based also on their availability or estimated accuracy), towards the identification process. Fundamental choices are (a) α=0 (information from MAC matrix only, that is from mode shapes only); (b) α=0.5 (equal information from natural frequencies and MAC matrix); (c) α=1 (information from natural frequencies only). For other values of α ranging between 0 and 1, both test profiles could be taken into account, with variable importance, depending on the specific reliability of estimated frequencies and mode shapes.

Fig. 11 presents a synoptic flowchart that illustrates the iterative process of calibration of the numerical model. The process involves the concatenated use of two software packages: ABAQUS [23] as structural solver and MATLAB optimization toolbox [26] as optimization routine. In the ABAQUS environment the numerical algorithm for the eigenvalues and eigenvectors problem of the FE models is run based on a set of initial parameter values. In the MATLAB routine, based on the experimental modal information, the mode pairing between experimental and numerical modes is performed through the application of a least square optimization procedure. The minimization of the residuals in the objective function is achieved, by using a Trust Region method through the *"lsqnonlin"* function of the Optimization Toolbox [21].

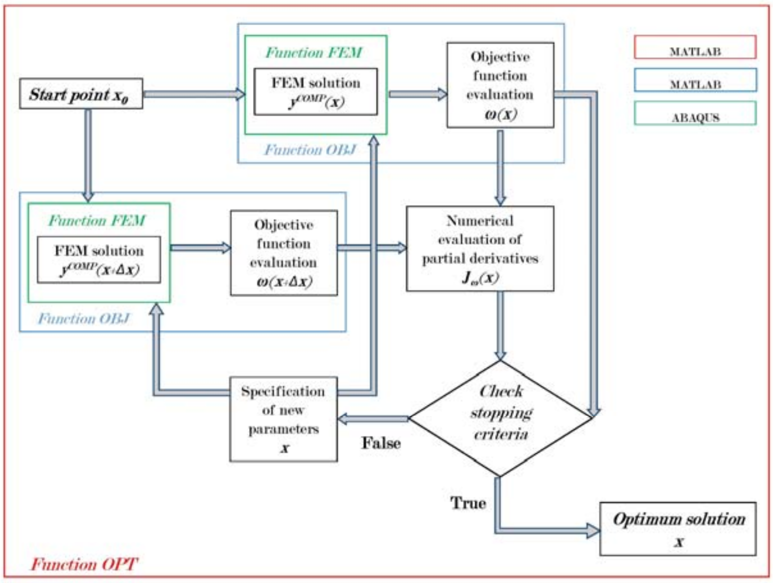


Figure 11: Flowchart of the optimization process for FE model updating (adapted from [25]).

The *"lsqnonlin"* function in MATLAB requires the following entries: the evaluation of the objective function; a start point x_0 from which the search of the absolute minimum departs; lower and upper bounds for the optimization variables, which are applied to the procedure to assure that the variations of the parameters do not lay outside some reasonable limits. Then, the function proceeds to an iterative search towards the absolute minimum, by varying the optimization variables (material parameters), evaluating through them the objective function and its jacobian at each iteration, and checking convergence/stopping criteria, as reported in [25].

6.2 Optimization results

The updated value of Young's modulus of the deck is 34.9 GPa, of Young's modulus of arches/hangers is 35.7 GPa and of concrete density is 2437 kg/m^3, with a percentage variation of 5.76%, -2.19%, and 1.54%, respectively, if compared to the initially-assumed values in the base model.

The updated frequencies f_i are listed in the fourth column of Table 7. The frequency percentage discrepancies and mode-shape correlation MAC values between the measured and updated modes are reported in the fifth and sixth column in Table 7, respectively.

Experimental			***2D updated model***		
Mode identifier	*Mode N*	*f_{FDD} (Hz)*	*f_{FE} (Hz)*	*Δ_f (%)*	*MAC*
V1	1	3.564	3.564	-0.02	1.000
V2	3	6.018	6.065	0.78	0.992
V3	5	7.690	7.627	-0.81	0.993
V4	8	13.086	11.942	-8.74	0.889

Table 7: Correlation between experimental and FE updated model dynamic characteristics of the first four vertical bending modes.

The optimization procedure has resulted quite successful. The results show maximum frequency difference of lower than 1% and very high MAC values larger than 99% for the modes within the frequency range 0-10 Hz (first three modes).

7 CONCLUSIONS

Ambient vibration testing with conventional high-sensitivity accelerometers, the assembly of three FE models with different levels of refinement and the calibration of a simplified numerical FE model (2D) of a historic reinforced concrete arch bridge have been presented in this paper.

From the results of the identification analysis based on the operational response data collected on the bridge it is possible to observe that: (a) almost all identified mode shapes exhibit regular and smooth shape with dominant bending or torsion, with the exception of the 7th mode, which is characterized by coupled bending and torsion and exhibits complex behaviour; (b) the "splitting" of first mode and the complex behaviour of the 7th mode deserves further investigation since both the observed phenomena might be related to the poor state of preservation and cracking of some vertical hangers ("dispersive phenomenon").

The calibration of the 2D FE model of the bridge has been based on the estimated dynamic characteristics of the structure determined through an operational modal analysis and it has involved a prior manual tuning of structural parameters selected by engineering judgments. Then, a sensitivity analysis and a subsequent optimization process have been performed. The sensitivity analysis has confirmed as a good choice the structural parameters selected for model updating. The application of the updating procedure has provided a 2D linear elastic model of the bridge, adequately representing the modal behavior of the structure in its present condition. In fact, good correlations with the experimental results (natural frequencies and mode shapes) have been obtained in the frequency range 0-10 Hz.

The structural parameters determined for the 2D FE model will be set as the starting point in the updating procedures of the more refined 3D FE models, in order to finally constitute a FE model as a baseline reference within a possible long-term monitoring framework of the bridge.

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