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LONGITUDINAL SEISMIC VIBRATIONS OF A SEGMENTED PIPELINE CONSIDERED AS THE PERIODICALLY INHOMOGENEOUS ROD

Mukhady Sh. Israilov[1]

[1] Research Institute on Mathematical Physics and Seismodynamics. Grozny,

Chechen Republic, Russia

israiler@hotmail.com

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Abstract. *Underground segmented pipeline with flexible joints is considered as a periodically piecewise homogeneous rod. Longitudinal vibrations of the rod are investigated by utilizing E.A. Ilyushina's method which reduces this problem to vibration study of separation boundaries between homogeneous parts of the rod. In this way, the averaged differential equations of motion and the average wave velocities are obtained for inhomogeneous rod when its cell of periodicity contains two, three, and more elastic components. It is shown that the average velocities are the same as the ones calculated through effective static moduli of the composite rod.*

The iron/steel pipelines with rubber and lead rings at joints are considered as examples. In these examples the average wave velocities in segmented pipeline (calculated by the method mentioned above) are significantly less than the wave velocity in the iron/steel pipe itself. The last result justifies the necessity of supersonic case study (the case in which wave velocity in the pipe less than seismic wave velocity in the soil) when earthquake response of buried pipelines is investigated. Usually this case is ignored assuming that the wave velocity in the pipe exceeds the wave velocity in the soil. The results of this paper clearly demonstrate that for segmented pipeline this assumption may not be true.

1 LONGITUDINAL VIBRATIONS OF PIECEWISE INHOMOGENEOUS PIPELINE: STUDY METHOD

Let us consider longitudinal displacements $u(z,t)$ of an elastic pipeline $z_1 \le z \le z_{N+1}$ consisting of N different homogeneous segments with interfaces at $z = z_r$ $(r = 2, \dots, N)$. As boundary conditions stresses $\sigma_{11}(t)$ or displacements $u(t)$ can be defined at the ends of the pipeline at $z = z_1$ and $z = z_{N+1}$. We will consider stationary vibrations of the pipeline assuming that the boundary conditions are harmonic functions of time. Hence displacements of r-th segment $u_r(z,t) = U_r(z)\exp(-i\omega t)$ will obey the following equation:

$$U_r''(z) + \omega_{1r}^2 U_r(z) = 0 \tag{1}$$

where the prime denotes the derivative with respect to the argument and $\omega_{1r} = \omega/c_r$ ($c_r = \sqrt{E_r/\rho_r}$ is a velocity of wave propagation in r-th segment with Young modulus E_r and density ρ_r).

The general solution of equation (1) has the form

$$U_r(z) = A_1^r \cos\omega_{1r} z + A_2^r \sin\omega_{1r} z \tag{2}$$

Following the method of H.A Ilyushina [1] developed for oscillation study of an inhomogeneous structure, we find the relation between the displacement U and stress Σ_{11} ($\sigma_{11}(z,t) = \Sigma_{11}(z)\exp(-i\omega t)$) on the left side of the r-th segment ($z = z_r$) with that on its right side ($z = z_{r+1}$).

We introduce a two-dimensional vector $\boldsymbol{Y}^r$ in the following way:

$$Y_1^r = U_r(z), \qquad Y_2^r = \frac{1}{E_0\omega_{10}}\Sigma_{11}^r = \frac{E_r}{E_0\omega_{10}} U_r', \tag{3}$$

where E_0, c_0- normalization constants and $\omega_{10} \equiv \omega/c_0$.

From the common solution of equations (2) and (3) it follows that

$$A_1^r = Y_1^r \cos\omega_{1r} z - P_r Y_2^r \sin\omega_{1r} z,$$
$$A_2^r = Y_1^r \sin\omega_{1r} z + P_r Y_2^r \cos\omega_{1r} z,$$
$$P_r \equiv \frac{E_0\omega_{10}}{E_r\omega_{1r}}$$

or in matrix form:

$$\boldsymbol{A}^r = C^r \boldsymbol{Y}^r;\; C^r \equiv \begin{Vmatrix} \cos\omega_{1r} z & -P_r \sin\omega_{1r} z \\ \sin\omega_{1r} z & P_r \cos\omega_{1r} z \end{Vmatrix} \tag{4}$$

Writing the relation (4) for the r-th and r+1-th segments and using the condition of continuity of U and Σ_{11} on the interface between these segments, we obtain (where $h_r \equiv z_{r+1} - z_r$ is length of r-th segment):

$$Y_1^{r+1}(z_{r+1}) = Y_1^r(z_{r+1}) = A_1^r \cos\omega_{1r}(z_r + h_r) + A_2^r \sin\omega_{1r}(z_r + h_r)$$
$$= Y_1^r(z_r)\cos\omega_{1r} h_r + P_r Y_2^r(z_r)\sin\omega_{1r} h_r,$$
$$Y_2^{r+1}(z_{r+1}) = Y_2^r(z_{r+1}) = [-A_1^r \sin\omega_{1r}(z_r + h_r) + A_2^r \cos\omega_{1r}(z_r + h_r)]/P_r =$$
$$= -(Y_1^r(z_r)\sin\omega_{1r} h_r)/P_r + Y_2^r(z_r)\cos\omega_{1r} h_r$$

or in matrix form:

$$\boldsymbol{Y}^{r+1}(z_{r+1}) = D^r \boldsymbol{Y}^r(z_r)\,,\ D^r \equiv \begin{Vmatrix} \cos\omega_{1r}h_r & P_r \sin\omega_{1r}h_r \\ -\frac{1}{P_r}\sin\omega_{1r}h_r & \cos\omega_{1r}h_r \end{Vmatrix} \tag{5}$$

Recurrence equations (5) give the relation between the values of the vector $\boldsymbol{Y}^r$ of r-th segment at $z = z_r$ and values $\boldsymbol{Y}^{r+1}$ r+1-th section at $z = z_{r+1}$ and allow us to find the values of U and Σ_{11} on any internal interface of the pipeline, if the values on one of its ends are defined.

2 VIBRATIONS OF TWO-COMPONENT AND MULTICOMPONENT PERIODIC PIPELINES

Let us consider the case of an infinite two-component (two-segment) pipeline which consists of repetitive homogeneous sections of two types (Figure 1) with the constants: r-th interval - E_1, ω_{11}, h_1; r+1-th interval - E_2, ω_{12}, h_2. This case is very important for practical applications. Writing the relation (5) for two adjacent segments, we find

$$\boldsymbol{Y}^{r+2} = D\,\boldsymbol{Y}^r,\ D = D^2 D^1, \tag{6}$$

where

$$D = \begin{Vmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{Vmatrix}, \tag{7}$$

$$\begin{aligned}
d_{11} &= \cos\omega_{11}h_1\ \cos\omega_{12}h_2 - \frac{P_2}{P_1}\sin\omega_{11}h_1\ \sin\omega_{12}h_2\,,\\
d_{12} &= P_1\ \sin\omega_{11}h_1\ \cos\omega_{12}h_2 + P_2\cos\omega_{11}h_1\sin\omega_{12}h_2\,,\\
d_{21} &= -\frac{1}{P_2}\cos\omega_{11}h_1\ \sin\omega_{12}h_2 - \frac{1}{P_1}\sin\omega_{11}h_1\cos\omega_{12}h_2\,,\\
d_{22} &= -\frac{P_1}{P_2}\sin\omega_{11}h_1\ \sin\omega_{12}h_2 + \cos\omega_{11}h_1\cos\omega_{12}h_2
\end{aligned}$$

The whole set of interfaces of infinite periodic two-component pipeline is divided into two classes. The first class consists of the interfaces for which the segments of the material 1are located on the right side and the segments of the material 2 – on the left side (the interfaces of this class are marked in Fig. 1 by bold dots); and the second class consists the interfaces for which, on the contrary, the segments of the material 2 are located to the right and the segments of the material 1 - to the left (in Fig. 1 they are marked by crosses).

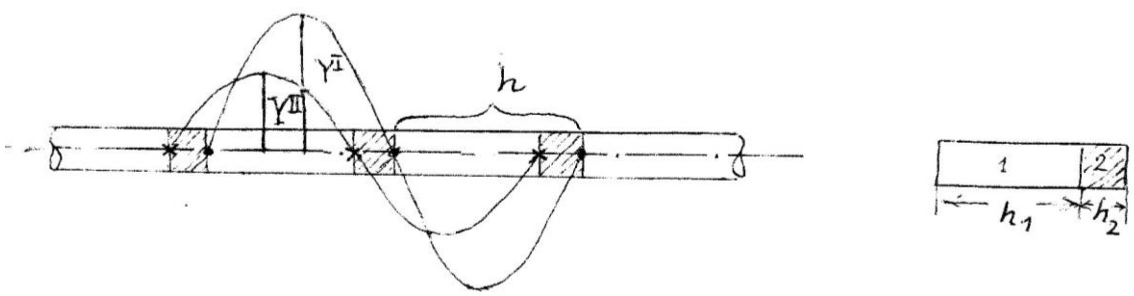

Figure 1: Two-component periodic pipeline geometry

For such periodic structure is natural to assume that the interfaces of the first and second classes vibrate in similar way, i.e. with the same frequency, but with different amplitudes, as shown in (Fig. 1). Below, in Sec. 3, it was investigated when this assumption is satisfied and have place a phenomenon of the transmission of frequency by the periodic structure, and when it fails, and there is aperiodic motion of the structure (the phenomenon of locking or filtering out frequencies). In this assumption vector $\boldsymbol{Y}^r$ can be represented as ($h \equiv h_1 + h_2$)

$$\boldsymbol{Y}^r = \begin{cases} Y^I e^{ikrh/2}, & r = \pm 2, \pm 4, \pm 6, \dots; \\ Y^{II} e^{ik[\frac{(r+1)h}{2} - h_2]}, & r = \pm 1, \pm 3, \pm 5, \dots \end{cases} \tag{8}$$

Then after substituting (8) into equation (6) we obtain the following characteristic equation

$$\begin{vmatrix} d_{11} - e^{ikh} & d_{12} \\ d_{21} & d_{22} - e^{ikh} \end{vmatrix} = 0 \tag{9}$$

Using in (9) expressions for d_{ij} from (7), we obtain the spectral equation of two-component segmented pipeline

$$\cos kh = \cos(\omega_{11} h_1 + \omega_{12} h_2) - \gamma \sin \omega_{11} h_1 \sin \omega_{12} h_2, \tag{10}$$

$$\gamma \equiv \frac{1}{2}\left(\frac{P_1}{P_2} + \frac{P_2}{P_1}\right) - 1 > 0$$

Since length of seismic wave $\Lambda = 2\pi/k$ (many tens to hundreds of meters) is much longer than length of an pipeline's cell of periodicity h (in practice up to ten meters), for the spectral equation (10) is a good approximation, the so-called, long-wave approximation, when in it for small k and ω, and hence small $\omega_{11} = \omega/c_1$ and $\omega_{12} = \omega/c_2$, in the Taylor series of incoming functions we keep members up to the second order. Then equation (10) proceeds to the next:

$$k^2 h^2 = \omega^2 \left[\frac{h_1^2}{c_1^2} + \frac{h_2^2}{c_2^2} + 2(1+\gamma)\frac{h_1}{c_1}\frac{h_2}{c_2}\right]$$

This shows that the average or "averaged" velocity of propagation of longitudinal perturbations in two-component pipeline for small k and ω is

$$c_L \equiv \langle c \rangle = \frac{h}{\sqrt{\left(\frac{h_1}{c_1}\right)^2 + \left(\frac{h_2}{c_2}\right)^2 + \left(\frac{P_1}{P_2} + \frac{P_2}{P_1}\right)\frac{h_1}{c_1}\frac{h_2}{c_2}}} \tag{11}$$

Thus, when the length of the seismic wave substantially larger than the characteristic size of inhomogeneity (the length of the periodicity cell) of the segmented pipeline, it may be considered as a homogeneous rod which has averaged, or as they say, effective mechanical properties. In this case, averaged equation of longitudinal vibrations of such homogeneous pipeline has the form

$$U''(z) + \frac{\omega^2}{\langle c \rangle^2} U(z) = 0$$

For average velocity can be easily obtained the following important theorems.

Theorem 1. The average velosity of propagation of longitudinal waves in the segmented pipeline can be calculated on the basis of its effective static characteristics, considering the pipeline as a composite [2], *namely,*

$$c_L \equiv \langle c \rangle = \sqrt{\langle E \rangle / \langle \rho \rangle}\,, \tag{12}$$

where

$$\langle \rho \rangle = \frac{h_1 \rho_1 + h_2 \rho_2}{h}\ , \quad \langle E \rangle = \frac{h}{h_1/E_1 + h_2/E_2} \tag{13}$$

are the average density and the effective Young's modulus of bi-component rod, respectively.

The expression for the average density in (13) is obvious, and for the effective Young's modulus obtained by introducing the concept of the average strain of the periodicity cell by the formula (σ - longitudinal stress)

$$\langle \varepsilon \rangle h \equiv \frac{\sigma}{\langle E \rangle} h = \varepsilon_1 h_1 + \varepsilon_2 h_2 = \frac{\sigma}{E_1} h_1 + \frac{\sigma}{E_2} h_2.$$

To prove statement of the theorem we transform the right-hand side of (12) using (13). That leads to

$$\sqrt{\frac{\langle E \rangle}{\langle \rho \rangle}} = \sqrt{\frac{h}{h_1/E_1 + h_2/E_2} \cdot \frac{h}{h_1\rho_1 + h_2\rho_2}} = \ h \Big/ \sqrt{\left(\frac{h_1}{c_1}\right)^2 + \left(\frac{h_2}{c_2}\right)^2 + h_1 h_2 \left(\frac{\rho_1}{E_2} + \frac{\rho_2}{E_1}\right)}$$

From which we obtain the desired result if we note that

$$\frac{\rho_1}{E_2} + \frac{\rho_2}{E_1} = \frac{1}{c_1 c_2}\left(\frac{P_1}{P_2} + \frac{P_2}{P_1}\right)$$

Theorem 2. Without loss of generality, assume that $c_1 > c_2$. *Then* $c_L \equiv \langle c \rangle < c_1$, *that is, the average velocity is less than the greater of the wave velocities in the internal parts of the two-component cell of the pipeline.*

To prove the theorem let us use elementary transformation:

$$\langle c \rangle = \frac{h}{\sqrt{\left(\frac{h_1}{c_1}\right)^2 + \left(\frac{h_2}{c_2}\right)^2 + \left(\frac{P_1}{P_2} + \frac{P_2}{P_1}\right)\frac{h_1}{c_1}\frac{h_2}{c_2}}} = \frac{h}{\sqrt{\left(\frac{h_1}{c_1} + \frac{h_2}{c_2}\right)^2 + \left(\frac{P_1}{P_2} + \frac{P_2}{P_1} - 2\right)\frac{h_1}{c_1}\frac{h_2}{c_2}}}$$

The second term under the radical in the last expression is positive (because of the inequality $x + \frac{1}{x} > 2$ for $x > 0$ and $x \neq 1$ (for $x = P_1/P_2$)), therefore

$$\langle c \rangle < \frac{h}{\frac{h_1}{c_1} + \frac{h_2}{c_2}} \equiv c_*$$

But

$$c_* = \frac{c_1 c_2}{\frac{h_1}{h} c_2 + \frac{h_2}{h} c_1} = c_1 \frac{1}{\frac{h_1}{h} + \frac{h_2}{h}\frac{c_1}{c_2}} = c_1 \frac{1}{1 + \frac{h_2}{h}\left(\frac{c_1}{c_2} - 1\right)} < c_1$$

since by assumption $c_1 > c_2$. Thus, it is found that $\langle c \rangle < c_* < c_1$.

It should be noted that the average velocity can be less than the smallest velocity of wave propagation in the segments of the pipeline (which is equal to c_2 by assumption).This is easily seen if we transform expression (11) or (12) to the form

$$\langle c \rangle = \frac{c_2}{\sqrt{1 + \left(\frac{h_1}{h}\right)^2 \left[\left(\frac{c_2}{c_1}\right)^2 - 1\right] + \frac{h_1 h_2}{h\, h}\left[\frac{\rho_1}{\rho_2} + \frac{\rho_2}{\rho_1}\left(\frac{c_2}{c_1}\right)^2 - 2\right]}}$$

From this formula, it is also clear that the average velocity $\langle c \rangle$ is less than c_2 in the case of $c_2 = c_1$ (but with segments of different materials).

Thus, from the prof provided above, we can conclude that the use of damping material on the joints reduces the velocity of wave propagation in the pipeline and that runs the risk of transition to supersonic regime when resonance may occur [3].

Outlined procedure can be applied to the calculation of the average wave propagation velocity in the three and $N-$ component periodic pipeline. Although this involves considerable analytical calculations but the result can be obtained. In these cases, the matrix D in the formulas, analogous to (6), is a product of three or more (generally N) matrices of type (5) and, therefore, it does not work to write the characteristic equations as for the two-component periodic rod (equations like (10)). However, one can easily write down the asymptotic analogs of these characteristic equations for small k and ω with retention of members to the second order (a situation that is just implemented during strong earthquakes), of which have been set to the average velocity. We write out the results obtained in this way. In the case of the three-component periodic pipeline the averaged velocity is

$$\langle c \rangle = \frac{h}{\sqrt{\left(\frac{h_1}{c_1}\right)^2 + \left(\frac{h_2}{c_2}\right)^2 + \left(\frac{h_3}{c_3}\right)^2 + \left(\frac{P_1}{P_2} + \frac{P_2}{P_1}\right)\frac{h_1}{c_1}\frac{h_2}{c_2} + \left(\frac{P_1}{P_3} + \frac{P_3}{P_1}\right)\frac{h_1}{c_1}\frac{h_3}{c_3} + \left(\frac{P_2}{P_3} + \frac{P_3}{P_2}\right)\frac{h_2}{c_2}\frac{h_3}{c_3}}},$$

and for pipeline with N components in the periodicity cell we obtain

$$\langle c \rangle = \frac{h}{\sqrt{\sum_{k=1}^{N}\left(\frac{h_k}{c_k}\right)^2 + \sum_{k,l=1;l>k}^{N}\left(\frac{P_k}{P_l} + \frac{P_l}{P_k}\right)\frac{h_k}{c_k}\frac{h_l}{c_l}}}$$

3 EXAMPLES: THE EXISTENCE OF PERIODIC SOLUTIONS AND THE NUMERICAL VALUES FOR AVERAGE VELOCITY IN TWO-COMPONENT PIPELINE

To quantify the effect of flexible joints consider two examples: a) pipeline consisting of iron pipes (large diameter) length of 5.97 m, connected in complex joints with rubber gaskets width of 3 cm and b) a pipeline consisting of iron pipe length 5.90 m, in complex junctions connected with lead strips of 10 cm width. In both cases, the length of the periodicity cell h = 6 m. Necessary for calculating the average velocity by the formula (11) mechanical properties of the pipeline material (iron, lead and rubber) are taken from Kaye and Laby [4] and are given in SI by values:

a) $c_1 = 4{,}3 \cdot 10^3$ m/s, $c_2 = 0{,}7 \cdot 10^2$ m/s, $E_1 = 13{,}0 \cdot 10^{10}$ N/m^2, $E_2 = 0{,}4 \cdot 10^7$ N/m^2;

b) $c_1 = 4{,}3 \cdot 10^3$ m/s, $c_2 = 1{,}2 \cdot 10^3$ m/s, $E_1 = 13{,}0 \cdot 10^{10}$ N/m^2, $E_2 = 1{,}62 \cdot 10^{10}$ N/m^2.

For our two-component periodic pipeline has been assumed the existence of periodic solutions of the form (8) with a real k, meaning that both types of interface oscillate with the same frequency. We define in the examples the regions of small ω, in which this assumption holds, or otherwise, the intervals in which the right-hand side of the characteristic equation (10) is less than one in modulus. The latter is valid in intervals where the graph of the function $y_1 = \cos X$, $X \equiv (\lambda_1 + \lambda_2)\omega$ is located between curves $y_2 = \alpha \cos \varkappa X + \beta$ and $y_3 = \alpha \cos \varkappa X - \beta$, where $\lambda_1 \equiv h_1/c_1$, $\lambda_2 \equiv h_2/c_2$, $\varkappa \equiv (\lambda_2 - \lambda_1)/(\lambda_2 + \lambda_1)$, $\alpha \equiv \gamma/(2+\gamma)$, $\beta \equiv 2/(2+\gamma)$.

Fig. 2 and 3 present graphs of these functions for small ω for examples a and b, respectively. As can be seen, there is a periodic solution for positive $X \lesssim 0.2$ (which corresponds to the frequency $\omega/2\pi \equiv f \lesssim$ 17 Hz) in Example a and for $X \lesssim 2.9$ (which corresponds $f \lesssim 325$ Hz) in Example b. Thus, in the case of rubber gaskets in complex junctions, since moderate earthquakes of intensity 2 - 3 on the Richter scale (the predominant frequency of seismic vibrations ~ 10 Hz [5]), there are periodic oscillations of the pipeline. And only in the case of weak shocks it is possible aperiodic motion. With lead seals periodic oscillations occur except in narrow regions near the points of change X in vicinity of X = -3, and X = 3 (Fig. 3), which is explained not as big a difference in velocity of wave propagation in the elements of the periodicity cell of the pipeline in this case.

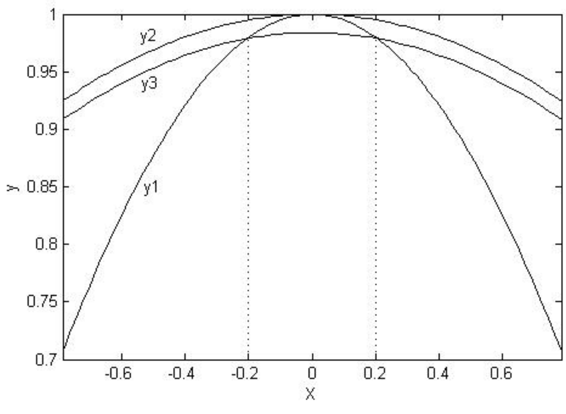

Figure 2: Region of periodic oscillations for iron pipeline with rubber gaskets

The average velocities of wave propagation, calculated by the formula (11), take in examples under consideration the values: *a)* $\langle c \rangle \approx 350$ m/s and *b)* $\langle c \rangle \approx 4.05 \cdot 10^3$ m/s. In the first example (with rubber gaskets), the average velocity of an order of magnitude smaller than the velocity of waves in the iron segment of the composite pipe and less than the velocity of propagation of longitudinal waves in typical soils (~ 500 ÷ 1500 m/s), which may have a significant impact and influence to earthquake safety of buried pipelines because of the nature of the solutions in the supersonic region [3].

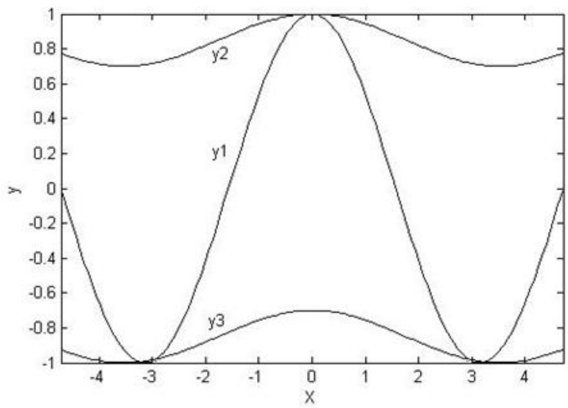

Figure 3. Region of periodic oscillations for iron pipeline with lead strips at junctions

4 CONCLUSION

The main conclusions derived from this study are summarized as follows:

1). A new method developed for study of longitudinal vibrations of multi-component periodically inhomogeneous pipeline (segmented pipeline with flexible joints) subjected to seismic waves. This method reduces the problem to vibration problem for corresponding homogeneous rod (or pipe) with average mechanical properties. It is shown that the reduction is a valid approximation when the length of seismic wave is much longer than the length of the of pipeline's periodicity cell.

2). Analytical expressions for average wave velocities are found for two, three, and N-component pipelines. It is proved that the average velocity is less than the greatest velocities of all pipeline components and can even be less than the smallest of them.

3). Theoretical results of this study are applied to the periodic steel pipeline with flexible joints consisting of rubber or lead gaskets. In examples of common practical interest numerical values for average velocities, which were calculated on the base of the developed theory, are significantly less than the wave velocity in the steel segment of the pipeline. And in the example with rubber gaskets the average velocity (~ estimates as 350 m/s) is less than the velocity of propagation of longitudinal waves in typical soils. Thus, this, so-called, supersonic regime cannot be ignored (as is usually done) in earthquake response analysis of segmented underground pipelines.

4). Here during investigation of the segmented pipeline as a periodic piecewise homogeneous structure it was assumed that all interfaces vibrate with the same frequency. The frequency intervals, in which this assumption is satisfied, are discussed and illustrated by the plots (Figures 2 and 3) in the examples.

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