

EXPERIMENTAL INVESTIGATION ON COHERENCE FACTOR OF RHYTHMIC JUMPING AND ITS APPLICATION FOR SIMULATION OF CROWD JUMPING LOAD

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Abstract. *This paper investigates the coherence factor of rhythmic jumping and its application for simulation of crowd jumping load. Experiments were conducted using three-dimensional motion capture technique on single person jumping. Seventy three test subjects participated in the experiments and each of them conducted four test cases of different frequencies. Thirty nine markers were attached to each test subject during the test. The jumping frequency was simulated by a metronome. The time lag between metronome time and landing time was proposed as the coherence factor of rhythmic jumping. The statistical analysis of experimental data shows that probability distribution characteristics of coherence factor can be described as the combination of uniform distribution and Beta distribution, which are corresponding to individual random jumping and coherent jumping in crowd respectively. The parameters of factor model functions are determined by experimental results. Simulation of crowd jumping load is carried out by combining the coherence factor with single random jumping load model. The simulating method is proved to be reasonable by application examples.*

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1 INTRODUCTION

With the application of high-strength and light-weight building materials and building owner's demands for large column-free open space, large-span floor structures have become more and more popular in assembly buildings, cantilever grandstands, waiting halls, dance halls and so on. For example, the Qingdao Training Center adopt 72m×42m prestressed concrete floor^[1] and the Chongqing University Sports Center had a 10m cantilever grandstand^[2]. Although these structures are designed for different functions, they share common structural features as low fundamental frequency and small damping. Therefore, these structures are prone to human-induced vibrations which may, when larger than certain threshold value, cause vibration serviceability issue.

There are typical two ways to address the vibration serviceability problem. One is to limit the structure's fundamental frequency to be higher than a given threshold value, the other is to limit the structure's maximum vibration amplitude (typically acceleration) to be lower than a given threshold value. The second way is more reliable since the vibration amplitude is the key parameter for judging vibration level. It is acknowledged that an accurate load model is an important premise for the engineering design. For human-induced load especially for group (crowd) -induced load, such a model is still absent. Experimental investigations have been conducted on human jumping load^[2-6]. In China, little research has been done so far on the crowd-jumping load. Due to the difference of body features like average heights, body weights and even culture background, studies in biomechanical fields already prove that different population can have different gait leading to different GRF. Therefore, it is not suitable to use overseas human-induced load directly. Among all human-induced loads such as running, walking, jumping, bobbing, marching on the spot and so on, jumping load has maximum dynamic load factor. Moreover, jumping load is frequently encountered in the design of structures of gymnasium, dancing halls, and music halls.

In this paper, experiments were conducted on human jumping load using three dimensional motion capture system in conjunction with force plates. Seventy-three volunteers participated in the experiments. Based on the experimental data, a new synchronization factor has been introduced to represent the coherence of the crowd jumping load. Group-induced jumping load can then be simulated by combining the synchronization factor with the single person's jumping load model.

2 EXPERIMENT ON INDIVIDUAL JUMPING LOAD

2.1 Experimental arrangement

A series of experiments were conducted to collect individual jumping load records. The experiments were conducted in Gait lab of Ruijin Hospital, Shanghai, China. Vicon 3D motion capture system consisting of 10 T40 infrared cameras was used in the experiments. Moreover, two AMTI OR6-7 force plates were employed to record test subject's jumping load. The forces were sampled at 1000Hz. Thirty-nine reflective markers were attached to every test subject according to a standard template for gait experiment. The trajectories of all the markers were captured by the Vicon system at a sampling frequency of 200 frames per second (i.e. 200 Hz). The whole experiments were monitored by a DV recorder. More details about the jumping experiments can be found in Zhao (2013)^[1].

2.2 Test cases

Ginty^[3] has analyzed the frequency contents of 210 pop songs and found that 96.2% frequencies of those songs range from 1.0Hz to 2.8Hz, which indicates that persons rhythmic

jumping frequencies also belong to this range when they jumped to music beat. However, it has been considered when the music frequency is lower than 1.5Hz, most people can't jump constantly and consistently. For example, The British Standard BS6399:Part 1(1996) indicates the frequency of the crowd consistent jumping ranges from 1.5Hz to 2.8Hz, when beyond this range, it's difficult for people to jump normally. On the other hand, Littler^[4] proposed the maximum of the jumping frequency should exceed 2.8Hz, and can be as high as 3.0Hz to 3.5Hz. Consequently, on the basis of those achievements above and the feedback of the volunteers in the preliminary experiment, it is decided to set the jumping test frequency as 1.5Hz, 2.0Hz, 2.67Hz, and 3.5Hz in our experiment. Therefore, each test subjected performed seven test cases: four cases at the above four frequencies which were instructed by a metronome, and three cases at self-chosen slow, normal and fast jumping frequency. 73 volunteers from Tongji University participated in this experiment.

2.3 Statistics of test subjects

After data quality check on the original experimental data, 37 groups jumping load with complete records were selected for further analysis, in which contain 27 man groups and 10 woman groups. The genders, ages, weights, and heights of volunteers are showed as table 1 as below.

Gender	Number	Age (20-29)		Weight (39-88kg)		Height (1550-1850mm)	
		Average	Standard deviation	Average	Standard deviation	Average	Standard deviation
Man	27	23.4	1.7	68.4	10.4	1759	57
Woman	10	22.8	1.5	51.6	7.9	1633	24

Table 1: The basic information statistics of the volunteers.

3 THE COHERENCE ANALYSIS OF THE CROWD JUMPING EXCITATION

The synchronization of crowd jumping is the foundation of modeling the crowd jumping load proposing a reasonable coherence factor (or coherence index) based on experimental records, and studying on its statistical property is a core work and also an essential part where the difficulty lies.

3.1 The definition of coherence factor

Coherence factor is a parameter for analyzing the synchronization of individuals in a jumping crowd. Sim and Blakeborough^[5], Fitts^[6] and Parkhouse and Ewing^[7] introduced different coherence factors based on their experiments. Sim and Blakeborough's suggestion doesn't have clear physical meanings. Ebrahimpour and Fitts's suggestion is only suitable for the condition of two persons, which isn't available for the crowd jumping cases. Parkhouse and Ewing's suggestion actually utilizes the first order phase of Fourier series to replace the time history of the jumping load, however, this method will introduce new truncation error.

This paper defines a new coherence factor that is taken as the time difference between time instant of metronome's sound and the time instant of test subject's tiptoe touching the ground. The new coherence factor is illustrated in Figure 1, where T_b is the metronome period. $t_{b,i}$ is the moment of the number i beat. $t_{d,i}$ is lag time difference of the number i jumping, and $t_{t,i}$ is the moment of the number i tiptoe touching ground.

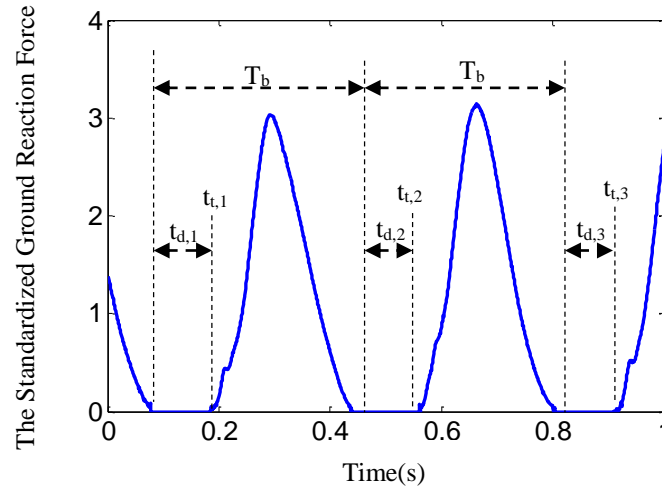


Figure 1: Definition of coherence factor.

3.2 Statistical property of the coherence factor

First of all, split every actually measured reaction force time history into several intervals due to the metronome's sound time instant (shown as the middle part between the two dotted lines in Figure 1). In other words, every interval is equal to the metronome period, and every interval can be treated as an independent jumping case. In this way, n reaction force impulses can be treated as n persons jumping one time.

Take the 2.0Hz jumping case for instance, a 30s reaction force time history usually contains 60 jumping periods, which we can regard as 60 persons jumping one time. This experiment has totally measured 37 reliable reaction force time history curves, so we can regard these 37×60 reaction force impulses as 37×60 persons jumping one time. Finally, diversity of jumping coherence can be obtained by statistically analyzing every time lag of those reaction force impulses.

Every time lag of reaction force impulse has been given as the following Equation:

$$t_{d,i} = t_{t,i} - t_{b,i} \quad (0 < t_{d,i} < T_b) \quad (1)$$

In order to compare the frequencies under different test cases, the time lag $t_{d,i}$ should be standardized by the metronome period T_b , like the following Equation:

$$\delta_{d,i} = t_{d,i}/T_b \quad (0 < \delta_{d,i} < 1) \quad (2)$$

Where, $t_{t,i}$ is the number i moment when tiptoe touching the ground, and $t_{b,i}$ is the number i metronome corresponding moment. After standardization, the range of the time lag changes from $0 < t_{d,i} < T_b$ to $0 < \delta_{d,i} < 1$.

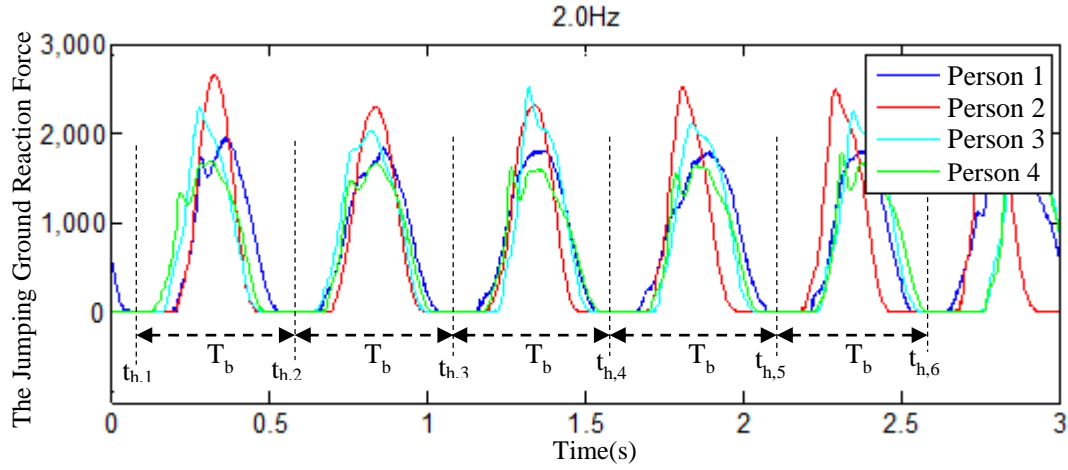


Figure 2: The crowd jumping load coherence diagram

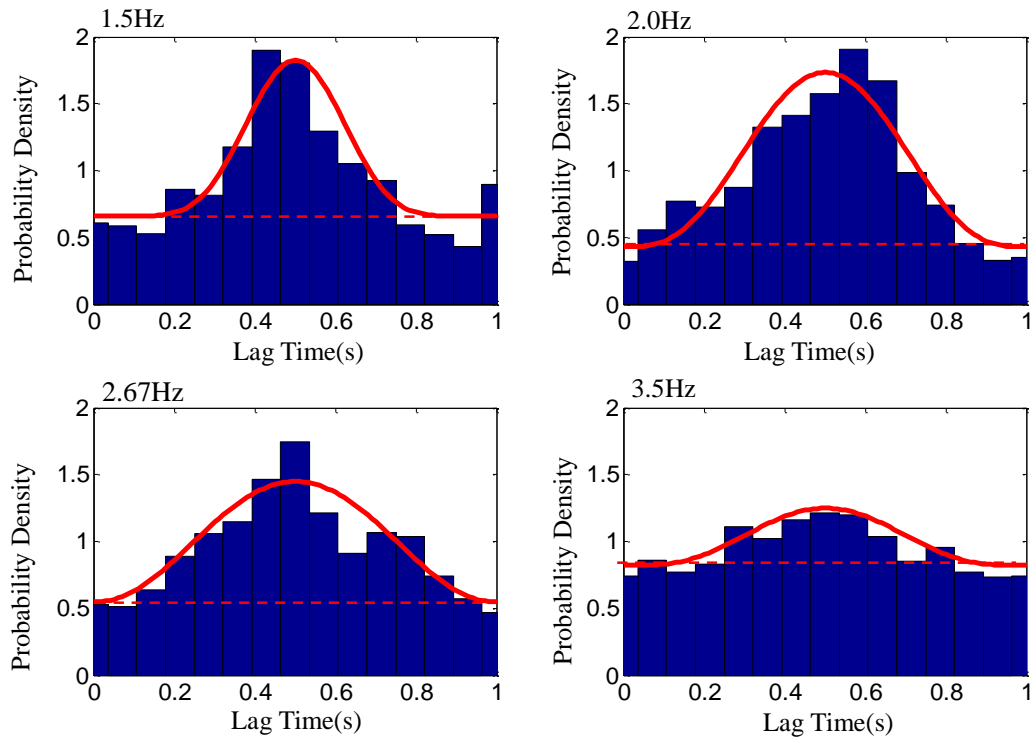


Figure 3: Standardized lag time probability density diagram.

Figure 3 presents the standardized probability density diagram of the time lag under four different frequency cases. Furthermore, every diagram can be approximately treated as a symmetric figure superposed by two parts, namely the rectangle under the red dotted line and the convex above the dotted line. In conclusion, this probability density function can be

modeled by adding up the two functions which represent each part respectively, and the Equation has been provided below.

$$f(x) = p_1 f_1(x) + p_2 \quad (3)$$

Where, $f_1(x)$ is standard Beta probability density distribution function, as shown in Equation (4), and p_1, p_2 are real numbers with the range of $[0,1]$ and $p_1 + p_2 = 1$.

$$f_1(x, \gamma, \eta) = \frac{1}{B(\gamma, \eta)} x^{\gamma-1} (1-x)^{\eta-1} \quad (4)$$

Where, $B(\gamma, \eta) = \int_0^1 x^{\gamma-1} (1-x)^{\eta-1} dx, \gamma > 0, \eta > 0, 0 \leq x \leq 1$.

In Equation (3), p_2 stands for the position of the dotted line on the vertical coordinate, namely the height of the rectangle under that red dotted line, and the height value reflects the random part of this rhythmic jumping excitation. Moreover, it also means the bigger p_2 is, the greater proportion of random will be, namely, the worse coherence will be shown.

The shape of the Beta probability density distribution function depends on two parameters γ and η , which can change from uniform distribution to approximately normal distribution and from symmetry to asymmetry with the domain of 0 to 1. When $\gamma = \eta$, the shape of this function is symmetry.

When $\gamma \neq \eta$, the shape of the function is asymmetry. It has been discovered in Figure 3 that lag time has a symmetry distribution, so it is reasonable to utilize symmetry fitting function to approach it, namely let $\gamma = \eta$ in Equation (4). Thus, the number of unknown parameters are reduced from 3 to 2. At last, utilize the least square method to evaluate unknown parameters, and the result are shown in Table 2.

Frequency (Hz)	P_2	γ
1.50	0.66	9.46
2.00	0.43	4.36
2.67	0.55	3.37
3.50	0.82	4.71

Table 2: The result of fitting parameter.

Note from Table 2 that when the jumping frequency is 2.0Hz, or 2.67Hz, p_2 is relatively smaller, which also means the jumping action has a relatively better coherence. However, when the jumping frequency is relatively lower, like 1.5Hz, a long time interval exists between the tiptoe touching the ground and the begin of the next jumping, which make the jumping become a discontinuous action, thus causing a worse coherence. In contrast, when the jumping frequency is 3.5Hz, this case has already approached the upper boundary of the crowd jumping action, as a results, it's hard to keep jumping by such high frequency, thus causing the coherence worse as well.

3.3 The relation between coherence and jumping frequency

Jumping frequencies other than the above four jumping frequencies will be considered for practical application. Therefore, it's necessary to investigate the relation between the jumping frequency and coherence factor. Based on the existing research achievements for the range of jumping frequency and the knowledge of the parameter variation tendency in Table 2, this

paper assumes that when metronome frequency is under 1.0Hz or above 4.0Hz, because of the restriction of physiological condition, it is impossible for people to follow the rhythm of metronome to complete random jumping action. In conclusion, it is assumed that when the metronome frequency changes in the range of $f \leq 1.0$ or $f \geq 4.0$, $p_2 = 1$.

As shown in Figure 4, the red circles stand for actual measurement in Table 2, and the red boxes stand for the assumed values when the jumping frequency is 1.0Hz or 4.0Hz. By utilizing polynomial fitting, it has been found that cubic polynomial fitting is able to reflect the parameter variation tendency very well, and also has a better fitting goodness. Therefore, the relation between coherence factor p_2 and the jumping frequency can be fitted by cubic polynomial well, which is given in Equation (5). Additionally, what can be presented from Figure 4 is when the jumping frequency is changing around 2Hz, the jumping action has the best coherence, or the jumping coherence will become worse.

$$p_2 = \begin{cases} -0.0821f^3 + 0.8425f^2 - 2.4920f + 2.7420 & 1.0 < f < 4.0 \\ 1 & f \leq 1.0 \text{ or } f \geq 4.0 \end{cases} \quad (5)$$

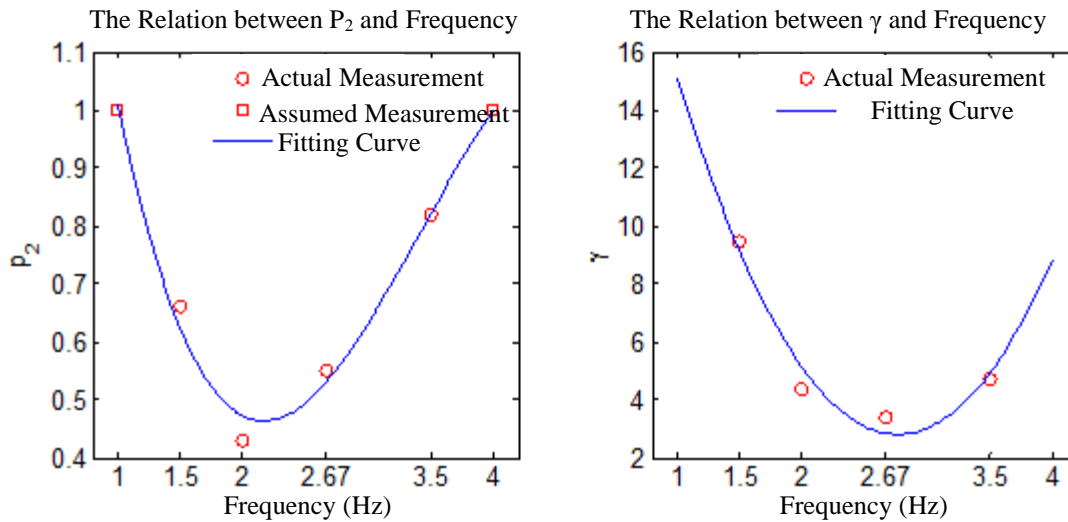


Figure 4: The relation between jumping coherence and jumping frequency.

As for the second parameter γ , utilize quadratic polynomial to fit the actually measured points, as shown in Figure 4. The red circles stand for the measured points, and the blue lines stand for quadratic polynomial fitting curve. In Equation(3), the constrain condition is $p_1 + p_2 = 1$, and when $f \leq 1.5$ or $f \geq 3.5$, parameter p_1 is relatively small, which also means the function $f_1(x)$ has less influence on $f(x)$. In other words, the value of parameter γ in $f_1(x)$ has less effect on the whole function $f(x)$, so we won't discuss the details of γ .

$$\gamma = 3.9550f^2 - 21.9200f + 33.1200 \quad 1.0 < f < 4.0 \quad (6)$$

4 MODELING OF THE CROWD JUMPING LOAD

Based on the above jumping coherence factor and single person's jumping load model, the crowd jumping load model can be simulated.

4.1 The single jumping load model

A complete jumping process can be divided into three parts, namely touching the ground, the beginning of jumping and soaring in the air. These three parts are demonstrated in Figure 5. In order to precisely describe these three actions, this paper defines three parameters, namely jumping period, contacting time and impulse factor. In Figure 5, the blue full line stands for the ground reaction force normalized by body weight, and T_p is the jumping period, namely the time difference between two adjacent moments when people touching ground, and t_p is the contacting time, namely the time between the moment of touching and leaving in one period. K_p is the peak factor that defines the maximum value of the ground reaction force.

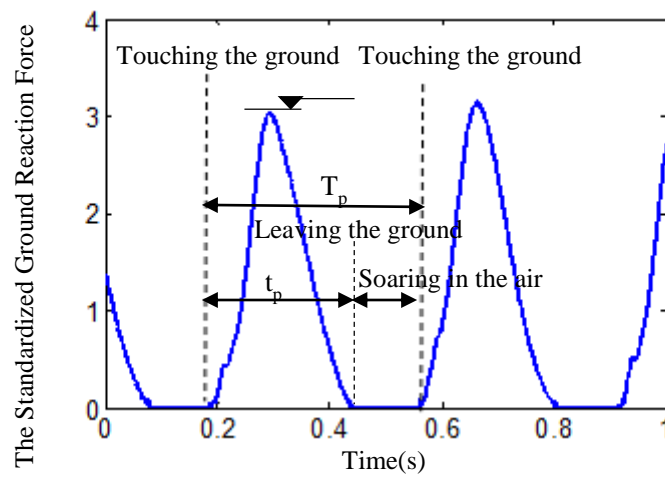


Figure 5: Parameter definitions.

Wang ^[8] has studied on the dynamic properties of the jumping load and proposed a mathematical model for single person jumping load. The model assumed that the jumping period T_p is a constant value, so only the contact time and the pulse factor are required to model the ground reaction force curve. Contact ratio, which is defined as the ratio between contact time over jumping period, follows approximately normal distribution. The mean and standard deviations of contact ratios for different jumping frequencies are summarized in Table 3.

Frequency of Test cases	1.5Hz	2.0Hz	2.67Hz	3.5Hz
Mean	0.7023	0.6660	0.6734	0.6692
Standard deviation	0.0913	0.0823	0.0685	0.0686

Table 3: The statistical property of the contact rate ^[8]

We also found that the peak factor could be expressed by the contact ratio as given in Equation (7). Finally the jumping load curve of each jumping event can be represented by piece-wise function as given in Equation (8).

$$K_p = \begin{cases} 1.554/\alpha & (1.5\text{Hz}) \\ 1.913/\alpha & (2.0\text{Hz}) \\ 2.145/\alpha & (2.5\text{Hz}) \\ 2.097/\alpha & (3.5\text{Hz}) \end{cases} \quad (7)$$

Where, contact rate α can be represented by $\alpha = t_p/T_p$.

$$F(t) = \begin{cases} K_p G \sin(\pi t/t_p), & 0 \leq t \leq t_p, f_p = 1. \\ K_p G \sin^2(\pi t/t_p), & 0 \leq t \leq t_p, f_p = 2.0 - 3.5\text{Hz}. \\ 0 & t_p \leq t \leq T_p. \end{cases} \quad (8)$$

4.2 The application steps of the crowd jumping load model

Based on proposed coherence index and single person's jumping load model, the simulation of the crowd jumping load can performed by the following steps:

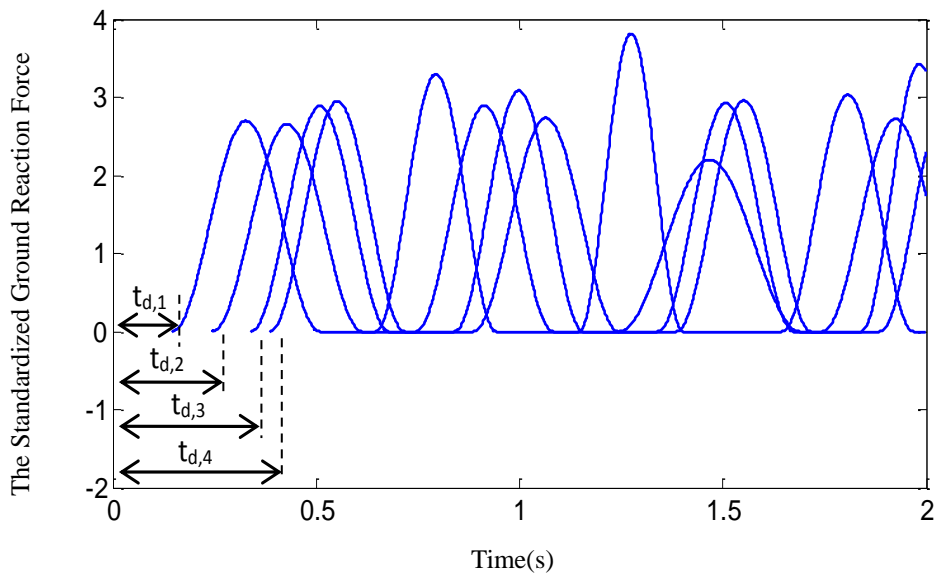


Figure 6: The start points of the crowd jumping load time history curve

(1) Determine the number of persons N in the crowd. Determine the number of impulses M in every jumping time history curve. That's to say we have a crowd of N persons each of them jumps M times.

(2) Determine the jumping frequency f_p , and accordingly determine the mean value and standard deviation of contact ratio from Table 3.

(3) Under normal distribution assumptions, randomly generate M contact ratios α_i in one jumping load time history curve. ($i=1 \sim M$)

(4) Calculate M impulse factor K_p^i , the jumping period $T_p = 1/f_p$, and the touching time $t_p^i = T_p \alpha_i$ by Equation (8).

(5) Put K_p^i, T_p and t_p^i into Equation (9) to get M single jumping load curves, and integrate M curves into one jumping load time history curve.

(6) Repeat step (2) to step (5) N times. N time history curves can be generated for N persons. Then based on the probability density distribution function (4) of the time lag to randomly generate N the start points of the time history curve, and finally ensure the time differences between each curve (Figure 6).

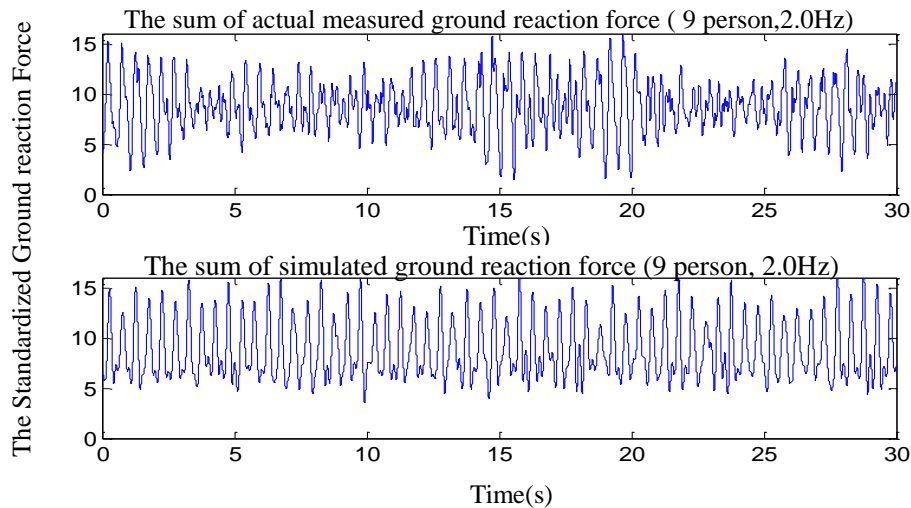


Figure 7: Actual measured load versus simulated load.

Figure 7 presents the comparison diagram between the actual measurement and simulation of the crowd jumping load, where horizontal coordinate stands for time, and vertical coordinate stands for the total reaction force of the nine-people jumping loads, in addition, metronome frequency has been set as 2.0Hz.

5 CONCLUDING REMARKS

Based on single person's jumping load experiments, this paper has suggested a new coherence factor that is defined as the difference between the time instant of the feet touching the ground and metronome beat. The new factor has clear physical meaning and easy-to-use in simulating the crowd load. Statistical properties of the factor have been investigated using measurements from jumping tests at four fixed frequencies as 1.5, 2.0, 2.67 and 3.0Hz. Crowd-induced loads can be simulated by combing coherence factor with single persons jumping load. Application examples show that the suggested simulation procedure gives reasonable results.

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