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STOCHASTIC REDUCED-ORDER MODELS FOR CALCULATING RESPONSE STATISTICS OF STRUCTURAL SYSTEMS SUBJECTED TO RANDOM INPUT

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Abstract. *Probabilistic characterization of a system's response under random dynamic input is essential for performance-based engineering. Monte Carlo simulation is the only general method available for estimating response statistics. However, the method is computationally expensive for realistic structural systems since it involves repeated deterministic dynamic analyses for a large number of samples of the random input. The framework of random vibration is also inadequate for calculating response statistics for linear systems under non-Gaussian inputs and non-linear systems subjected to both Gaussian and non-Gaussian excitation.*

We propose a novel, highly efficient method for calculating response statistics. The method is based on stochastic reduced-order models (SROM), i.e., stochastic processes that have a finite number of samples selected in an optimal manner from samples of the input process. The SROM-based method can be viewed as a smart Monte-Carlo. Like Monte-Carlo simulation, the method uses random samples of the input to characterize structural response. Unlike Monte Carlo, which uses a large number of samples selected at random, SROM selects just a small number of samples in an optimal way. Numerical examples of the newly proposed method are presented for single-degree-of-freedom linear and non-linear Duffing and Bouc-Wen systems.

1 INTRODUCTION

Calculation of response statistics of structures subjected to seismic loads is an essential part of performance-based seismic design. Monte Carlo is the only general method available for calculating this type of statistics. However, the method is computationally impractical for realistic structural systems since it involves repeated deterministic dynamic analyses for randomly selected samples of seismic load processes. This procedure makes Monte Carlo method computationally expensive.

The framework of linear/non-linear random vibration is also inadequate for calculating response statistics. Classical theory of linear random vibration provides efficient tools for calculating the first two response moments for linear systems. For example, unless the input is Gaussian and the system is linear or the output can be assumed to be Gaussian, the first two moments of the response are insufficient for calculating exceedance probability ([1], Sect. 7.2). This is a significant limitation since structural systems behave non-linearly under strong seismic loads.

The main goal of this paper is to propose two novel, conceptually-simple, accurate, non-intrusive and computationally-efficient methods for calculating response statistics for structures subjected to seismic loads. The methods are based on stochastic reduced order models (SROMs) [2], i.e. stochastic processes that have a finite number of samples selected in an optimal manner from the samples of the target process. Like Monte Carlo simulation, the methods use samples of seismic load processes to characterize structural response and is not intrusive in the sense that their construction uses deterministic solutions. However, the proposed methods use a small number of load samples selected in an optimal manner. In contrast, Monte Carlo simulation uses a large number of samples selected at random.

The first method, referred to as *the first-order SROM*, constructs a SROM using directly the input samples. The second method, referred to as *the second-order SROM*, requires a parametric model, with random parameters, whose samples are evaluated from input samples. A SROM is constructed for the random parameters, which implicitly defines a SROM for the random input.

The use of optimally selected samples allows to reduce the number of simulations required by the Monte Carlo method by one or two orders of magnitude while retaining accuracy. The method has been originally developed for dynamic response [3]. Recently, it has been shown that for static problems the SROM-based method can be improved significantly [4]. Preliminary studies indicate that similar improvements will provide accurate solutions for random vibration problems [5]. Optimization algorithms have been developed for constructing SROMs for random random vectors [2, 6]. Applications of SROM in earthquake engineering, for efficient calculation of fragility surfaces, were already developed in [7].

Response statistics are calculated for linear and non-linear systems subjected to samples of random input through the two new SROM methods and results are compared with the reference Monte-Carlo ones. The SROM match satisfactory the expected results with a fraction of the computational effort.

2 PROBLEM DEFINITION

Simplified models of real systems are often used in structural engineering. Three simple examples of models of single-degree-of-freedom (SDOF) systems are represented by the fol-

lowing three ordinary differential equations:

$$\ddot{Y}(t) + 2\zeta\nu\dot{Y}(t) + \nu^2 Y(t) = X(t) \tag{1}$$

$$\ddot{Y}(t) + 2\zeta\nu\dot{Y}(t) + \nu^2(Y(t) + \rho Y^3(t)) = X(t) \tag{2}$$

$$\ddot{Y}(t) + 2\zeta\nu\dot{Y}(t) + \nu^2(\rho Y(t) + (1-\rho)W(t)) = X(t) \tag{3}$$

$$\dot{W}(t) = -\gamma\dot{Y}(t) + \alpha|W(t)|^{\eta-1}W(t) + \beta\dot{Y}(t)|W(t)|^{\eta}$$

where $X(t)$ is the random input represented by a stochastic process; $Y(t)$, $\dot{Y}(t)$ and $\ddot{Y}(t)$ are the displacement, velocity and acceleration of the SDOF system, respectively; and ν, ζ, α, β, γ, ρ and η are scalars representing system parameters. Equations (1-3) define known systems, i.e. linear, Duffing and Bouc-Wen oscillators, respectively. Parameters ν and ζ in Eq. (1) are also known as the natural frequency and the damping ratio of the linear system.

We want to find statistics of the extrema for the processes $Y(t)$ defined in Eqs. (1-3). For example, we will calculate the probability of exceedance $\mathbb{P}\left(\max_{0\le t\le\tau}|Y(t)| > y_{cr}\right)$, where τ is the duration of the process $X(t)$ and y_{cr} is some critical value of the displacement.

The input process $X(t)$ is a stochastic process and it can represent any type of random loading, such as earthquake or wind. Usually the input process is available in the form of a catalog of recorded events, i.e. samples of $X(t)$. Therefore, for generality we consider that our input is represented by a specified large number n of samples $x(t)$ of $X(t)$.

3 SOLUTION

Two SROM-based methods are used to solve the problem and results are compared with the reference Monte-Carlo results. SROM can be seen as a smart Monte-Carlo method. Like Monte-Carlo, it uses randomly generated samples of the stochastic process $X(t)$. Unlike Monte-Carlo, in which samples are equally likely, in SROM the samples are chosen in an optimal way and are weighed by probabilities.

3.1 Monte-Carlo Solution

The Monte-Carlo method relies on solving the ODEs in Eqs. (1-3) for all available samples $\{x_i(t),\ i = 1, ..., n\}$ of $X(t)$. Then the exceedance probability of interest is calculated as

$$P^{MC}(y_{cr}) = \mathbb{P}\left(\max_{0\le t\le\tau}|Y(t)| > y_{cr}\right) = \frac{1}{n}\sum_{i=1}^{n}\mathbb{1}\left\{\max_{0\le t\le\tau}|y_i(t)| > y_{cr}\right\}. \tag{4}$$

3.2 First-order SROM Solution

The first-order stochastic reduced order model is defined directly for the samples of the input process $X(t)$. Any number m_1 of samples $\{\tilde{x_k}(t),\ k = 1, ..., m_1\}$ of $X(t)$ and probabilities $\{p_k \ge 0,\ k = 1, ..., m_1\}$ such that $\sum_{k=1}^{m_1} p_k = 1$ define a stochastic reduced order model. We denote the SROM as $\tilde{X}(t) = \{(\tilde{x}_k, p_k),\ k = 1, ..., m_1\}$. It is shown [2] that it is possible to find a selection of $\{(\tilde{x_k}(t), p_k),\ k = 1, ..., m_1\}$ such that $X(t)$ and $\tilde{X}(t)$ have similar probability laws for $m_1 << n$.

We are looking for a SROM $\tilde{X}(t)$ with a range $\{\tilde{x_k}(t),\ k = 1, ..., m_1\}$ of a relatively small number m_1 of independent samples of $X(t)$. Our objective is to find an optimal vector $p^{opt} = (p_k^{opt},\ k = 1, ..., m_1)$ that minimizes the discrepancies between the probability laws of $X(t)$ and $\tilde{X}(t)$ [9]. We obtain p^{opt} by solving an optimization problem with an objective function which

measures differences between various statistics of $X(t)$ and $\tilde{X}(t)$. It is suggested in [2] to use an objective function of the form

$$h(p) = \sum_{i=1}^{3} e_i h_i(p), \tag{5}$$

where $h_1(p),\ h_2(p),\ h_3(p)$ measure the differences between the moments, marginal distributions and covariance functions of $X(t)$ and $\tilde{X}(t)$, respectively, where $p = (p_i,\ i = 1, 2, 3)$ and $(e_i \geq 0,\ i = 1, 2, 3)$ are some weights which account for the relative importance of components $h_i(p)$.

Statistics for $X(t)$ can be calculated directly from its probability law if $X(t)$ is well defined. However, in our case, in order to maintain the idea of the input being defined as a catalog of samples, statistics for both $X(t)$ and $\tilde{X}(t)$ are estimated from their samples. Thus, the moments of order q are

$$\mu(t; q) = \mathbb{E}[X^q(t)] = \frac{1}{n} \sum_{i=1}^{n} x_i^q(t), \tag{6}$$

$$\tilde{\mu}(t; q) = \sum_{k=1}^{m_1} p_k \tilde{x}_k^q(t), \tag{7}$$

the marginal distributions are

$$F(\xi_j; t) = \mathbb{P}(X(t) \leq \xi) = \frac{1}{n} \sum_{i=1}^{n} \mathbb{1}\left\{x_i(t) \leq \xi_j\right\}, \tag{8}$$

$$\tilde{F}(\xi_j; t) = \sum_{k=1}^{m_1} p_k \mathbb{1}\left\{\tilde{x}_k(t) \leq \xi_j\right\},\ j = 1, ..., n_F \tag{9}$$

and, finally, the covariance functions can be calculated as

$$\Sigma(t, s) = \mathbb{E}[X(t)X(s)] = \frac{1}{n-1} \sum_{i=1}^{n} x_i(t)x_i(s) \tag{10}$$

$$\tilde{\Sigma}(t, s) = \sum_{k=1}^{m_1} p_k \tilde{x}_k(t)\tilde{x}_k(s). \tag{11}$$

Components of the objective function in Eq.(5) are calculated using the statistics estimated above.

$$h_1(p) = \max_{0 \leq t \leq \tau} \left\{ \sum_{q=1}^{n_q} w_\mu(t, q) \left|\mu(t; q) - \tilde{\mu}(t; q)\right| \right\} \tag{12}$$

$$h_2(p) = \max_{0 \leq t \leq \tau} \left\{ \sum_{j=1}^{n_F} w_F(\xi_j, t) \left|F(\xi_j; t) - \tilde{F}(\xi_j; t)\right| \right\} \tag{13}$$

$$h_3(p) = \max_{0 \leq t,s \leq \tau} \left\{ w_\Sigma(t, s) \left|\Sigma(t, s) - \tilde{\Sigma}(t, s)\right| \right\}, \tag{14}$$

where $w_\mu, w_F, w_\Sigma \geq 0$ are weighing functions. Minimizing the objective function in Eq. (5) results in obtaining the vector p^{opt}. The optimization problem will be repeated for a specified

number of independent sets of m_1 samples of $X(t)$, $\{\tilde{x_k},\ k = 1, ..., m_1\}$. The minimum value of $h(p^{opt})$ indicates a sub-optimal solution p^{opt}.

The selected SROM pairs $\{(\tilde{x_k}(t), p_k^{opt}),\ k = 1, ..., m_1\}$ are used to solve the ordinary differential equations and the target probability is calculated as

$$P_f^{SROM1}(y_{cr}) = \sum_{k=1}^{m_1} p_k^{opt} \mathbb{1}\left\{\max_{0\le t\le\tau} |\tilde{y}_k(t)| > y_{cr}\right\}, \tag{15}$$

where $\tilde{y}_k(t)$ are the solutions of Eqs. (1-3) to $\tilde{x}_k(t),\ k = 1, ..., m_1$.

3.3 Second-order SROM Solution

The second-order SROM method involves using a parametric model for the input process $X(t)$ of the form

$$X(t) = X(t, Z) \approx \sum_{i=1}^{d} Z_i \varphi_i(t), \tag{16}$$

where $Z = (Z_i,\ i = 1, ..., d)$ is a random vector and $\{\varphi_i(t),\ , i = 1, ..., d\}$ are deterministic specified functions of time. Parametric models of this type can be calculated for any stochastic process. We use a truncated Karhunen-Loève model to calculate functions $\{\varphi_i(t),\ i = 1, ..., d\}$ by calibrating it to the first two moments of $X(t)$, that is, the mean and covariance functions [10]. Functions $\varphi_i(t)$ are the eigenvectors of the covariance matrix $\Sigma(t, s)$ defined for $0 \le t, s \le \tau$. Unlike the Karhunen-Loève expansion, for which $X(t) = \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{\lambda_i} X_i \varphi_i(t),\ t \ge 0$, where λ_i are the eigenvectors and X_i are random variables with zero mean and unit variance, we use the approximation in Eq. (16). The random vector Z_i will be characterized by its samples. Samples $z_{k,i}$ of the random vector Z_i are obtained by minimizing the difference between the actual samples $x_k(t)$ of $X(t)$ and their approximations $\sum_{i=1}^{d} z_{k,i}\varphi_i(t)$, where $z_{k,i}$ denotes the i-th component of sample z_k of Z. In other words, for each sample $x_k(t)$ available we find a sample of $z_k = \{z_{k,i},\ i = 1, ..., d\}$ by minimizing, for example, the mean squared error $\int \left(x_k(t) - \sum_{i=1}^{d} z_{k,i}\varphi_i(t)\right)^2 dt$.

In this second method we construct a SROM $\tilde{Z}$ for Z using the samples of the random vector Z. Let $\tilde{Z}$ be a random vector of the same dimension as Z and samples $\{\tilde{z}_k,\ k = 1, ..., m_2\}$ selected from samples of Z. We associate probabilities $(p_1, ..., p_{m_2})$ for each sample $\tilde{z}_k$ such that $\sum_{k=1}^{m_2} p_k = 1$ and the probability laws of Z and $\tilde{Z}$ are similar, following the same procedure presented in the first-order SROM solution. We call $\tilde{Z}$ a stochastic reduced order model of Z. Moreover, we denote by $\{\Gamma_k,\ k = 1, ..., m_2\}$ the cells of a Voronoi partition of the range of Z, with centers $\{\tilde{z}_k\}$ [4]. A cell Γ_k contains all samples z_i of Z that are closer to $\tilde{z}_k$ than any other $\tilde{z}_l$, i.e. $|z_i - \tilde{z}_k| \le |z_i - \tilde{z}_l|,\ l \ne k$. Note that the probability p_k defines also the probability that Z takes values in cell Γ_k.

The SROM $\tilde{Z}$ of Z also defines a SROM $\tilde{X}(t; Z)$ of $X(t; Z)$ with samples $\{\sum_{i=1}^{d} \tilde{z}_{k,i}\varphi_i(t)\}$ with probabilities $p_k,\ k = 1, ..., m_2$. As in the previous SROM method, $\tilde{X}(t; Z)$ has only m_2 samples $\tilde{x}(t; \tilde{z}_k)$, much smaller than the large number n of available samples of $X(t)$. We denote by $\tilde{y}(t; \tilde{z}_k)$ the solutions of Eqs. (1-3) to $\tilde{x}(t; \tilde{z}_k)$. The response process is approximated by

$$\tilde{Y}(t; Z) = \sum_{k=1}^{m_2} \mathbb{1}\{Z \in \Gamma_k\} \left[\tilde{y}(t; \tilde{z}_k) + \nabla\tilde{y}(t; \tilde{z}_k)(Z - \tilde{z}_k)\right], \tag{17}$$

where $\tilde{y}(t;\tilde{z}_k)$ are the responses as defined previously with their corresponding probabilities p_k and $\nabla\tilde{y}(t;\tilde{z}_k) = \partial Y(t;Z)/\partial Z_i,\ i = 1,...,d$ are called sensitivity factors of the response $Y(t;Z)$ calculated at $Z = \tilde{z}_k$. Function $\mathbb{1}$ denotes the indicator function.

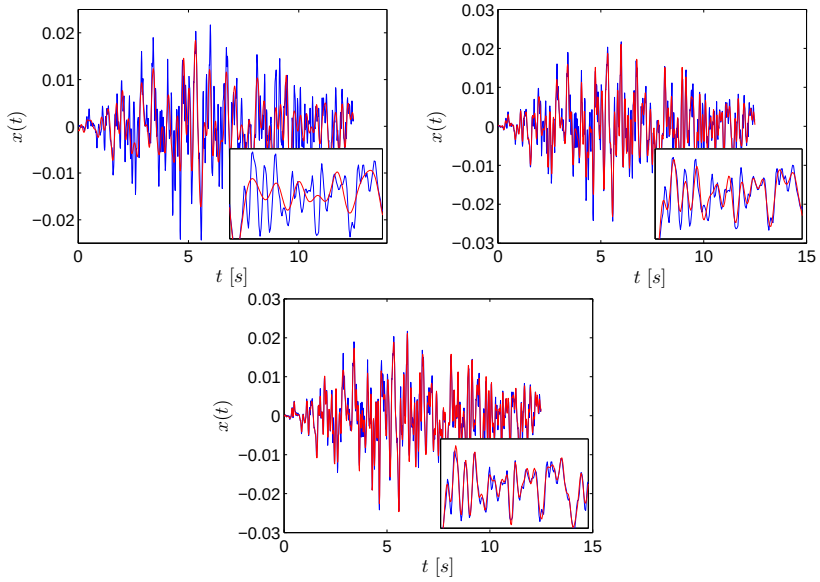

Figure 1: Samples of $X(t)$ and their approximation $\sum_{i=1}^{d} Z_i\varphi_i(t)$ for $d = 100, 200, 300$.

The sensitivity factors can be approximated numerically from the differences between the responses $Y(t;Z)$ calculated at $Z = \tilde{z}_k$ and $Z = \tilde{z}_k + \Delta\tilde{z}_k$, where $\Delta\tilde{z}_k$ denotes a perturbation of $\tilde{z}_k$. They can also be determined by solving deterministic differential equations as illustrated below. The sensitivity factors for the linear system are the solutions of the follwoing ODE with constant coefficients

$$\frac{\partial\ddot{X}(t,Z)}{\partial Z_i} + 2\zeta\nu\frac{\partial\dot{X}(t,Z)}{\partial Z_i} + \nu^2\frac{\partial X(t,Z)}{\partial Z_i} = \varphi_i(t), \tag{18}$$

where $i = 1,...,d$ and they do not depend on the values of $\tilde{z}_k$. The sensitivity factors for the Duffing oscillator in Eq.(2) are the solutions of

$$\frac{\partial\ddot{X}(t,Z)}{\partial Z_i} + 2\zeta\nu\frac{\partial\dot{X}(t,Z)}{\partial Z_i} + \nu^2\frac{\partial X(t,Z)}{\partial Z_i}\left(1 + 3\rho X^2(t;\tilde{z}_k)\right) = \varphi_i(t). \tag{19}$$

Finally, the sensitivity factors $\nabla\tilde{y}(t;\tilde{z}_k)$ for the Bouc-Wen system are the solutions of the following system of deterministic ordinary differential equations

$$\frac{\partial\ddot{X}(t,Z)}{\partial Z_i} + 2\zeta\nu\frac{\partial\dot{X}(t,Z)}{\partial Z_i} + \nu^2\left(\rho\frac{\partial X(t,Z)}{\partial Z_i} + (1-\rho)\frac{\partial W(t,Z)}{\partial Z_i}\right) = \varphi_i(t) \tag{20}$$

$$\frac{\partial W(t,Z)}{\partial Z_i} = -\gamma\frac{\partial\dot{X}(t,Z)}{\partial Z_i} + \alpha\left((n-1)\,\mathrm{sgn}(\tilde{W}_k)|\tilde{W}_k|^{n-2}\tilde{W}_k + |\tilde{W}_k|^{n-1}\right)\frac{\partial W(t,Z)}{\partial Z_i}\bigg|_{Z=\tilde{z}_k}$$
$$+ \beta\left(\frac{\partial\dot{X}(t,Z)}{\partial Z_i}|\tilde{W}_k|^n + n\,\mathrm{sgn}(\tilde{W}_k)\dot{\tilde{X}}_k|\tilde{W}_k|^{n-1}\frac{\partial W(t,Z)}{\partial Z_i}\bigg|_{Z=\tilde{z}_k}\right), \tag{21}$$

where $\tilde{W}_k = (\partial W(t;Z)/\partial Z_i)|Z = \tilde{z}_k$ and $\dot{\tilde{X}}_k = (\partial \dot{X}(t;Z)/\partial Z_i)|Z = \tilde{z}_k$ are deterministic quantities and are calculated from Eq. (3).

Note that the larger the dimension d of vector Z, the better the approximation of $X(t,Z)$ to $X(t)$ is. Figure 1 show one sample of Z using different lengths for the truncation of the KL expansion, i.e. $d = 100$, $d = 200$ and $d = 300$, respectively. Figure 2 shows the mean-squared-error probability density functions between the original samples of $X(t)$ and their approximation by $X(t,Z)$ for various values of d.

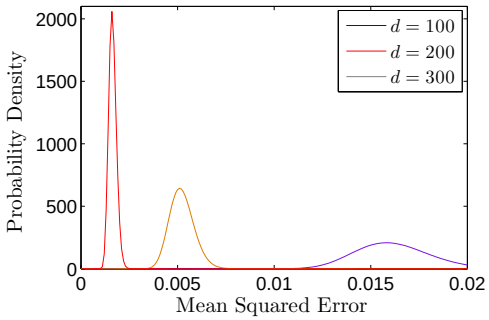

Figure 2: Error densities between samples of $X(t)$ and $\sum_{i=1}^{d} Z_i \varphi_i(t)$ for various d.

The selected SROM pairs $\{(\tilde{z}_k, p_k^{opt}),\ k = 1, ..., m_2\}$ are used to solve the ordinary differential equations and the target probability is calculated as

$$P_f^{SROM2}(y_{cr}) = \sum_{k=1}^{m_2} p_k^{opt} \mathbb{1}\left\{ \max_{0\le t\le \tau} \tilde{y}(t;z_k) > y_{cr} \right\}, \tag{22}$$

where $\tilde{y}(t;z_k)$ are the samples of $\tilde{Y}(t;Z)$ calculated in Eq.(17).

4 Numerical Examples

A numerical example is considered in order to compare the performance of the two SROM-based methods with respect to the reference Monte-Carlo results. We calculate the response statistics for all three systems described above.

For the current example we consider the input process $X(t)$ to be a non-stationary, zero-mean, Gaussian process with known second-order moment properties. We use $n = 10,000$ samples of $X(t)$ in our catalog of input samples. Note that the methodology described in the previous section works also for non-Gaussian processes. The Monte-Carlo statistics are calculated by solving Eqs. (1-3) to all n samples.

First-order SROM The first-order SROM $\tilde{X}(t)$ is calculated using just $m_1 = 20$ samples of $X(t)$. Figure 3 shows the means of processes $X(t)$ and $\tilde{X}(t)$, respectively. Figure 4 and 5 compare marginal distribution and covariance function of $X(t)$ estimated from n samples with the same statistics obtained from the SROM $\tilde{X}(t)$ of $X(t)$.

While the means and the marginal distributions of $X(t)$ and $\tilde{X}(t)$ are reasonably close to each other, it is more difficult to capture the covariance function of $X(t)$ with just the $m_1 = 20$ samples used in the construction of $\tilde{X}(t)$.

Note that the first-order SROM may be improved by either increasing the dimension its dimension m_1 or by improving the selection of the samples in order to minimize the discrepancies between the probability laws of $X(t)$ and $\tilde{X}(t)$.

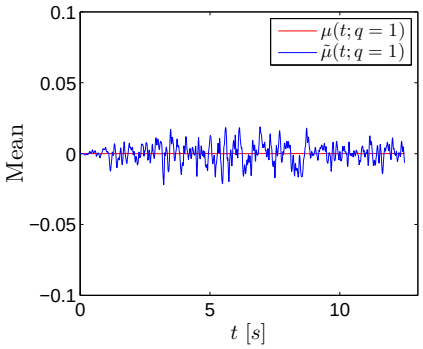

Figure 3: Estimated mean of $X(t)$ and $\tilde{X}(t)$

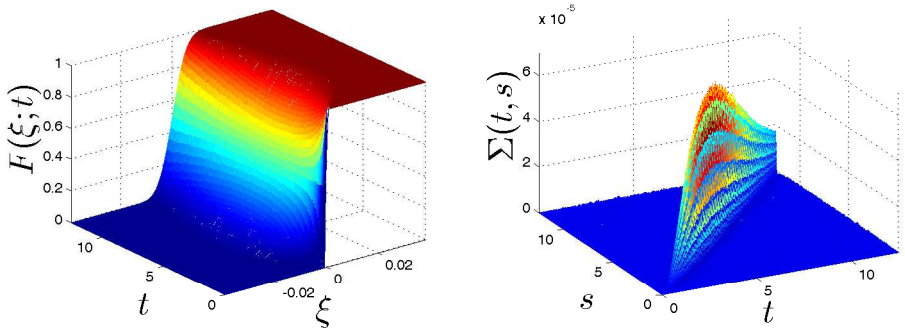

Figure 4: Estimated distribution function and covariance matrix for $X(t)$

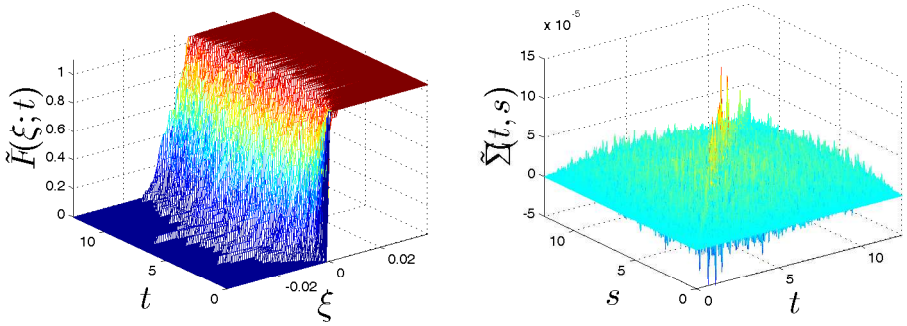

Figure 5: Estimated distribution function and covariance matrix for $\tilde{X}(t)$

Second-order SROM The second-order SROM method involves the construction of a SROM $\tilde{Z}$ for the random-vector parameter Z. We use only $m_2 = 20$ samples of Z. The dimension of Z used is $d = 200$, for which the mean squared error between samples of $X(t)$ and their approximates is around 5×10^{-3} as shown in Figure 2.

Figure 6 shows the means of random-vector parameters Z and $\tilde{Z}$, respectively. Figure 7 and 8 compare the mean, marginal distribution and covariance function of Z and $\tilde{Z}$, respectively. As previously, out of the three statistics, the covariance function is the hardest to match with just $m_2 = 10$ samples of Z.

Note that the second-order SROM built may be improved by either increasing the dimension of the SROM m_2, by increasing the dimension d of the parameter Z, or by using a more efficient methodology of selecting the range of $\tilde{Z}$ in order to optimize the discrepancies between the

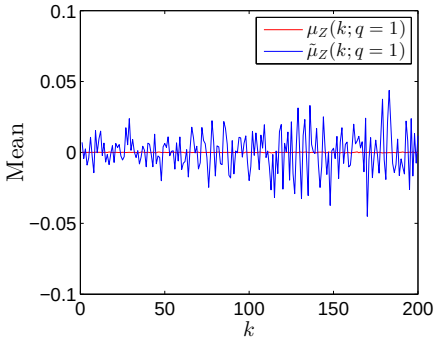

Figure 6: Estimated mean of Z and $\tilde{Z}$

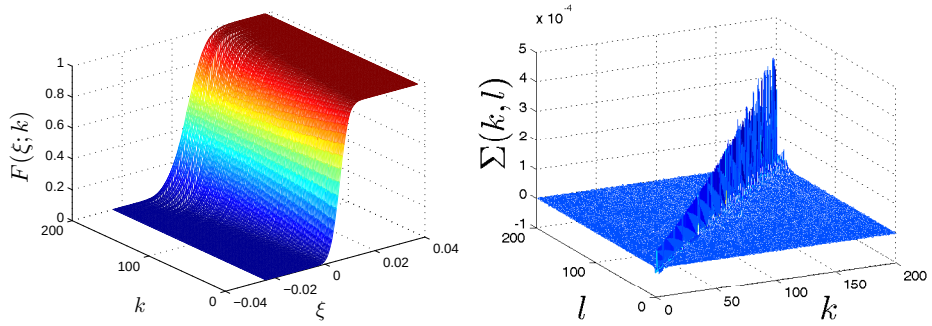

Figure 7: Estimated distribution function and covariance matrix for Z

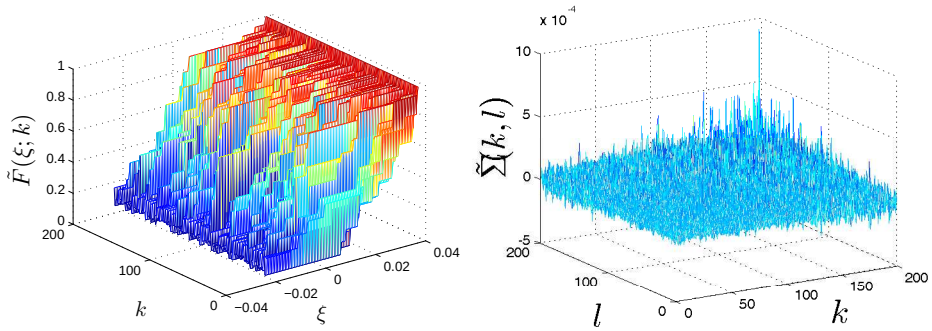

Figure 8: Estimated distribution function and covariance matrix for $\tilde{Z}$

statistics of Z and $\tilde{Z}$.

Once the SROMs are constructed, Eqs. (1-3) may be solved using just the samples selected and exceedance probability curves calculated as shown in Eqs. (15) and (22), respectively. Finally, the probability that the response $Y(t)$ exceeds a critical value y_{cr} is compared in Figure 4 calculated by using each method, i.e. first-order SROM (black dashed line), second-order SROM (red dashed line) and Monte Carlo (solid blue line).

A good approximation within a 5% mean squared error of the Monte-Carlo solution is obtained using the first-order SROM with the least computational effort. The solution obtained by the second-order SROM is almost identical with the Monte-Carlo results. Table 4 summarizes the error and the computational effort in term of the number of dynamic analyses required to calculate the statistics of interest.

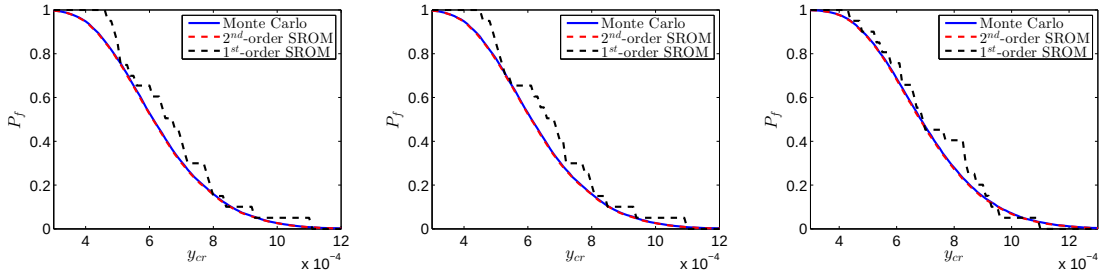

Figure 9: Exceedance probability for Linear, Duffing and Bouc-Wen oscillators

Model	1st order SROM	2nd order SROM	MC
Dynamic Analyses	m_1	$m_2 \times d$	n
Dynamic Analyses (example)	20	$2,000$	$10,000$
Error	$\sim 5\%$	~ 0	0

Table 1: Computational effort vs. relative error by comparing SROM-based methods with Monte Carlo

An important difference between the two SROM-based methods can clearly be noticed in this table. Unlike the first-order SROM, the second-order SROM produces almost zero error with respect to Monte Carlo at the expense of d-times more analayses.

5 CONCLUSIONS

Two novel, highly efficient methods have been proposed to calculate response statistics for structural systems subjected to random input. The methods are based on the stochastic reduced order models that can be viewed as smart Monte Carlo simulations. Unlike the Monte Carlo method which uses a large number of samples selected at random, the new methods use much smaller numbers of samples that are selected in optimal manner. The methods are implemented to calculate exceedance probabilities of linear and non-linear single-degree-of-freedom systems subjected to samples of stochastic processes. The SROM estimates of exceedance probabilities match satisfactorily the Monte-Carlo estimates of these probabilities although they are based on a small number of input samples. Even though both methods produce very good approximations of the reference results, one method produces almost zero error at the expense of more dynamic analyses.

REFERENCES

[1] M. Grigoriu. *Stochastic systems. Uncertainty quantification and propagation.* Springer-Verlag Ltd, London, UK, 2012.

[2] M. Grigoriu. Reduced order models for random functions. application to stochastic problems. *Applied Mathematical Modelling*, 33:161–175, 2009.

[3] M. Grigoriu. Linear random vibration by stochastic reduced order models. *International Journal for Numerical Methods in Engineering*, 82:1537–1559, 2010.

[4] M. Grigoriu. A method for solving stochastic equations by reduced order models and local approximations. *Journal of Computational Physics*, 231(19):64956513, 2012.

[5] M. Grigoriu. A new method for solving random vibration problems. *Proceedings of The Eleventh International Conference on Computational Structural Technology*, 2012.

[6] J. Warner, M. Grigoriu and W. Aquino. Stochastic reduced order models for random vectors. applications to random eigenvalue problems. *Probabilistic Engineering Mechanics*, 31:1–11, 2013.

[7] A. Radu and M. Grigoriu. Structural performance by fragility surfaces for simple systems under non-Gausssian seismic input. *Earthquake Engineering and Structural Dynamics*, (under review), 2015.

[8] A. Radu and M. Grigoriu. An efficient approach on dynamic-response analyses for structural systems. *Proceedings of the 10th National Conference on Earthquake Engineering*, Anchorage, AK, 2014.

[9] A.C. Miller and T.R. Rice. Discrete approximations of probability distributions. *Management Science*, 29(3):352–362, 1983.

[10] M. Grigoriu. Evaluation of Karhunen-Loève, spectral, and sampling representations for stochastic processes *Journal Engineering Mechanics (ASCE)*, 132(2):179–189, 2006.