COMPDYN 2015
5th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, V. Papadopoulos, V. Plevris (eds.)
Crete Island, Greece, 25–27 May 2015

# DYNAMIC ANALYSES OF A CURVED CABLE-STAYED FOOTBRIDGE UNDER HUMAN INDUCED VIBRATIONS: NUMERICAL MODELS AND EXPERIMENTAL TESTS

**Elisa Bassoli[1], Paola Gambarelli[1], Laura Simonini[1] and Loris Vincenzi[1]**

[1] DIEF – University of Modena and Reggio Emilia
via Pietro Vivarelli, 10 – 41125 Modena
e-mail: elisa.bassoli@unimore.it; paola.gambarelli@unimore.it; laura.simonini@unimore.it; loris.vincenzi@unimore.it

**Keywords:** Footbridge, human-induced vibrations, dynamic identification, model updating, experimental tests.

**Abstract.** *Nowadays, pedestrian bridges are increasingly lively and slender structures due to the development of improved structural materials and aesthetic requirements. As a result of this trend, contemporary footbridges are more and more prone to human-induced vertical and lateral vibrations that can compromise the comfort serviceability conditions. The goal of this paper is to characterize the dynamic behaviour of a curved cable-stayed footbridge subjected to pedestrian loads starting from experimental tests and numerical dynamic analyses. The dynamic behaviour of the footbridge is investigated thanks to an experimental campaign performed by means of an advanced MEMS-based SHM system. Accelerations due to ambient vibrations are recorded and the modal parameters of the structure are identified by means of a classic identification method. Then, to investigate the dynamic response of the footbridge subjected to pedestrian actions, a wide number of experimental tests were performed with different-sized groups of pedestrians crossing the footbridge, running, free or synchronized walking with different pacing frequencies. Then, a finite element model of the footbridge is developed and calibrated so that the numerical dynamic predictions agree with the experimental modal properties. Then, to simulate dynamic loading conditions due to a single pedestrian or a crowd of people crossing the footbridge, two mathematical models are examined. In the first approach both the non-calibrated and the updated FE model are adopted to evaluate the vertical dynamic response of the footbridge when subjected to pedestrian loads. Dynamic analyses are performed by simulating the pedestrian walking through a periodic load model representing the human-induced force as a deterministic force. The second approach is based on the solution of the equation of motion via modal decomposition, considering multi-harmonic forces and experimental mode shapes and frequencies. Finally, the accelerations obtained through the mathematical approaches are compared with the experimental results.*

## 1 INTRODUCTION

Over the last decades, cable-stayed bridges and footbridges have reached great importance and popularity thanks to the ease of construction, the economical convenience and the reduction of the deck bending moments due to the benefit of cables. Nevertheless, footbridges are generally slender structures due to the structural solutions adopted even for aesthetic requirements and the using of deformable elements, and so they are increasingly sensitive to dynamic vibrations induced by pedestrian actions [1, 2] that can compromise the comfort serviceability conditions. Hence, the full assessment of footbridge dynamic behaviour with reference to pedestrian dynamic amplifications is a topical issue in the vibration serviceability analyses [3-5].

Accurate FE models are often needed to investigate the dynamic behaviour of these kinds of structures and to reproduce the pedestrian effects through dynamic analyses considering adequate human-induced load models. However, the complex shapes of these structures usually involve difficulties in accurate structural modelling and simulation. Therefore, model updating procedures are required to develop reliable models starting from the comparison between the structural dynamic behaviour investigated through dynamic tests and its numerical model response [6].

This paper is a part of a research that aims to characterize the dynamic behaviour of a curved cable-stayed footbridge subjected to pedestrian loads starting from experimental tests and numerical dynamic analyses. The case study is the Pasternak footbridge that is about 270 meters long and 3 meters wide and it crosses an important freeway in Modena (Italy). It is composed of two steel towers 18 meters high that support the steel-concrete deck by means of 6 pairs of cables.

First of all, the dynamic behaviour of the footbridge is investigated thanks to an experimental campaign performed by means of an advanced MEMS-based SHM system [7]. Through the dynamic tests the accelerations due to ambient vibrations (wind) are recorded and the modal parameters (frequencies, mode shapes and damping ratios) are identified [8]. To study the dynamic behaviour of the footbridge subjected to pedestrian loads, several experimental tests were performed with different-sized groups crossing the footbridge running, free or synchronized walking with different pacing frequencies. To simulate dynamic loading conditions due to a single pedestrian or a crowd of people crossing the structure, two mathematical models are examined. In these models, vertical dynamic actions depend on the pacing frequency, the walking (or running) speed, the step length, the number of people involved and the synchronous action modelling. To apply the first approach, a finite element model is developed and calibrated so that the numerical dynamic predictions agree with the experimental modal properties [9, 10]. Both the updated and the non-calibrated models are used to evaluate the dynamic response of the footbridge when subjected to pedestrian loads. In the second approach, the equation of motion is solved via modal decomposition, considering a multi-harmonic force model and experimental mode shapes and frequencies. The numerical accelerations obtained through dynamic analyses are finally compared with the experimental results.

## 2 THE PASTERNAK FOOTBRIDGE

The dynamic tests were performed on a steel curved cable-stayed footbridge, the Pasternak footbridge (see Figure 1*a*), by means of the SHM602 system. The structure was built in 2008 and it overpasses an important freeway crossroad in Modena (Italy). The footbridge is about 270 meters long and it is composed of a curved central span 60 meters long and two access ramps with bent layout. The main span is sustained by cables linked with two V-shaped steel towers each of them composed of inclined masts connected with each other by X-bracings

(Figure 1*b,c*). Each mast is approximately 18 meters high with an inclination of about 15 degrees with respect to the vertical. The lateral access ramps are composed of several spans supported by V-shaped precast concrete piers. The structure is about 3 meters wide and it has a constant radius of approximately 32 meters. The deck is composed of steel trusses that support a concrete slab. The main span stiffening spatial truss girder (see Figure 1*d*) is composed of circular hollow profiles, connected with the steel-concrete deck. At the deck level trusses are made up of two open-shaped concrete-filled chords connected in plane with H- and L-shaped profiles. The masts are constituted by prismatic hollow profiles with variable sections linked with each other with H-shaped bracings by means of bolted connections. Each cable is composed of two or three elements connected with each other and anchored to the towers with plates and bolts. Finally, the footbridge is provided with steel parapets along the entire length.

## 3 DYNAMIC IDENTIFICATION

Dynamic tests were performed to investigate the dynamic behavior of the footbridge subjected to ambient vibrations. Then, the modal properties of the footbridge are identified by applying identification techniques to the experimental measurements [8].

The experimental campaign was carried out by means of an advanced MEMS-based SHM system. The main components of the SHM602 system consist in a controller and storage unit and in several intelligent bus-connected sensing units. Each sensing unit can record the accelerations along two orthogonal axes and the temperature. Thanks to local digital filtering techniques and oversampling rates implemented, these units relied on MEMS sensors can exhibit

(*a*) (*b*) (*c*) (*d*)

Figure 1: (*a*, *b*) General view of the Pasternak footbridge; (*c*) detail of the V-shaped steel towers and (*d*) the spatial truss girder.

a noise level of about 0.3÷0.5 mg [7] and thus lower than those measured by other MEMS-based systems [11].

### 3.1 The dynamic tests

To estimate as many natural modes as possible, several setups were arranged. For each set-up, 12 sensors were installed on the vertical elements of the steel parapets at the deck level by means of magnets. The sensors were mainly placed in the curved central span to acquire accelerations along two orthogonal axes, in the vertical and horizontal directions. The sensors located at the towers measured accelerations in the radial (transverse) and longitudinal directions, and some sensors in the main span were installed to record vertical and longitudinal accelerations. In the first setup, sensors were arranged quite equally spread on both deck sides along the whole central span. In the second and third setups, sensors were mainly arranged on the north or the south side of the central span to cover the maximum possible length of the footbridge and to better identify the greater number of modes. For all test arrangements, some sensors were placed at the same locations to combine data obtained from different setups. Setups no. 2 and 3 are reported in Figure 2*a*,*b* respectively.

### 3.2 Results of dynamic identification

To identify the footbridge dynamic parameters, the Power Spectral Density (PSD) [12] matrix of the acquired accelerations is calculated and the Enhance Frequency Domain Decomposition method is applied [13].

Examples of PSDs achieved from vertical accelerations are reported in Figure 3. Ten modes (6 vertical modes and 4 torsional ones) involving the central span with frequencies in the range 1.462÷12.061 Hz are clearly identified. Values of identified damping ratios are in the range 0.12÷0.67%. The identified natural frequencies and damping ratios are reported in

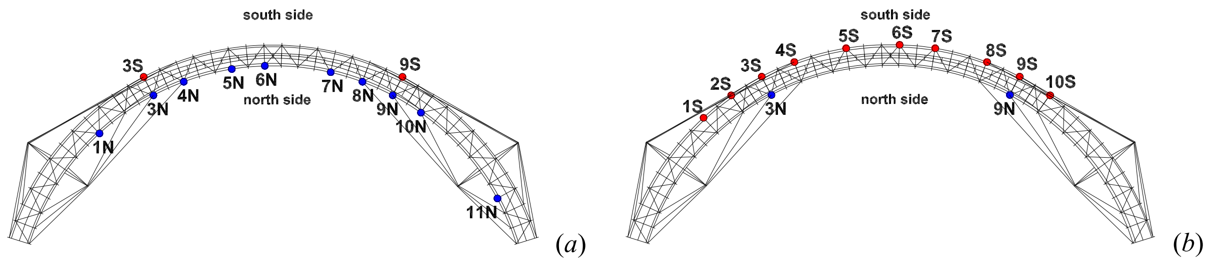



Figure 2: The second (*a*) and the third (*b*) experimental setups

| Mode Type | Experimental Frequency [Hz] | Damping Ratio [%] |
|---|---|---|
| 1st Vertical | 1.462 | 0.67 |
| 2nd Vertical | 2.274 | 0.56 |
| 3rd Vertical | 3.210 | 0.39 |
| 4th Vertical | 4.949 | 0.40 |
| 1st Torsional | 6.551 | 0.30 |
| 2nd Torsional | 7.605 | 0.15 |
| 6th Vertical | 8.772 | 0.22 |
| 3rd Torsional | 9.409 | 0.24 |
| 7th Vertical | 11.453 | 0.18 |
| 4th Torsional | 12.061 | 0.12 |

Table 1: Experimental modes

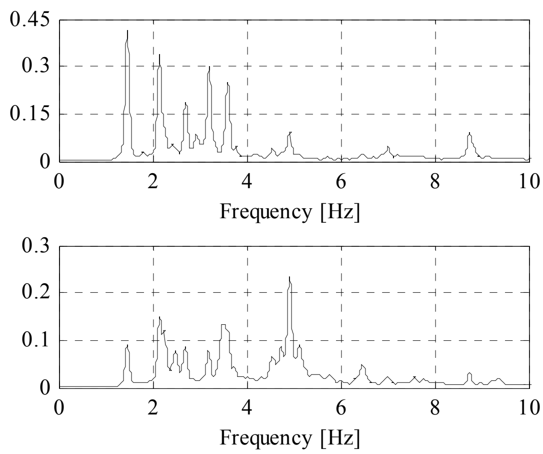



Figure 3: Examples of PSDs obtained from vertical accelerations

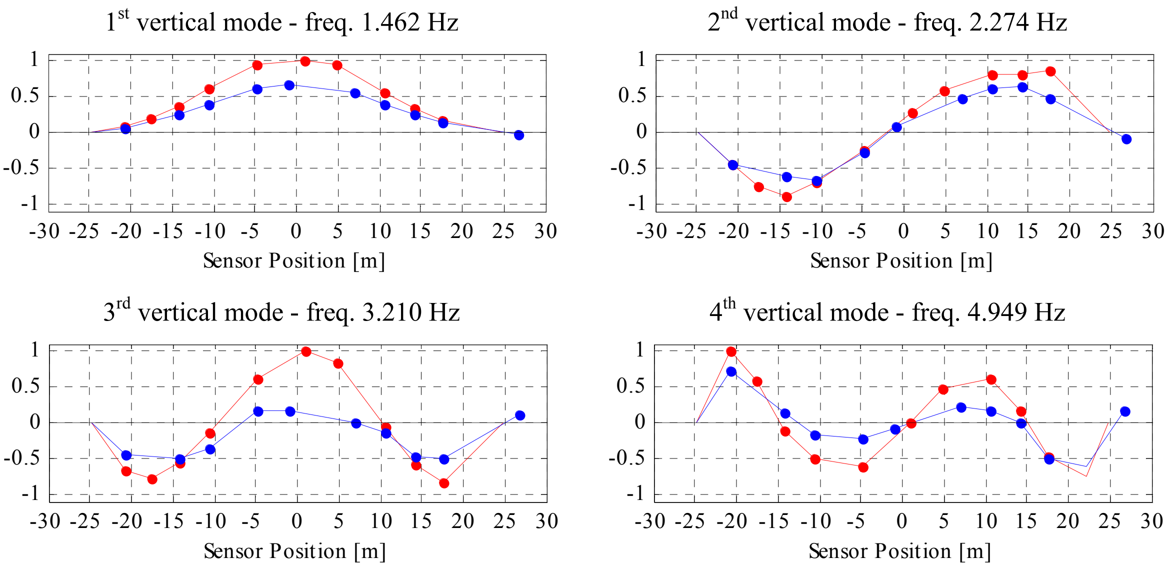



Figure 4: Vertical components of some of the more relevant experimental modes

Table 1 while Figure 4 shows the vertical components of the first 4 mode shapes. The blue and the red colour represent the north and the south side of the footbridge respectively.

## 4 FINITE ELEMENT MODEL AND MODEL UPDATING

A FE model of the footbridge is developed representing the curved main span and portions of the lateral spans. To accurately model the steel-concrete deck and the towers, beam and shell elements are used; particularly, steel members and cables are modeled adopting 1D elements while the concrete slabs and the towers with 2D elements. Constraints are applied at the base of towers and piers. The equivalent density of longitudinal chords is defined by considering the weight increment due to the parapets and the concrete fillings.

The modal analysis provides for frequencies and mode shapes. The first four mode shapes are shown in Figure 5. Referring to the developed model (called FE Model A in the following) numerical frequencies are reported and compared with the experimental ones in Table 2. Numerical results match quite well all the experimental modes, with errors in frequencies in the range 2÷4% and MAC values greater than 94%. All the identified modes are characterized by prevailing vertical displacements, thus the MAC values are calculated considering only the vertical components of the mode shapes.

Then, the FE model is adjusted by means of a model updating procedure so that the numerical frequencies and mode shapes match as better as possible the experimental ones; the updated model is identified as FE Model B in the following. The optimization process is carried out through an improved surrogate-assisted evolutionary algorithm. Surrogate-assisted evolutionary strategies use efficient computational models, as response surface or high polynomial functions, to approximate the objective function. Recently, they received considerably increasing interest in reducing the computational effort in optimization problems, mainly when the evaluation of the objective function is highly time consuming [14]. In this work, the so-called DE-Q algorithm is adopted [9, 10]; it combines the robustness of the DE algorithm with the computational efficiency due to a second-order surrogate approximation of the objective function. The optimal parameters are obtained through the minimization of an objective function defined as the relative error between numerical and experimental modal frequencies and mode shapes. To obtain a reliable comparison, the numerical and the experimental mode shapes are coupled by using the MAC. In the dynamic analyses described in Section 6, the human-induced walking forces are simulated by using harmonics with frequencies not greater than 4.5 Hz. Therefore, the first four vertical modes (modes no. 1÷4 in Table 1) with frequencies in the range 1.462÷4.949 Hz are selected for the optimization process. Moreover, only the vertical components of mode shapes are considered as the purpose of the presented work is to evaluate the vertical effects of pedestrians on the footbridge.

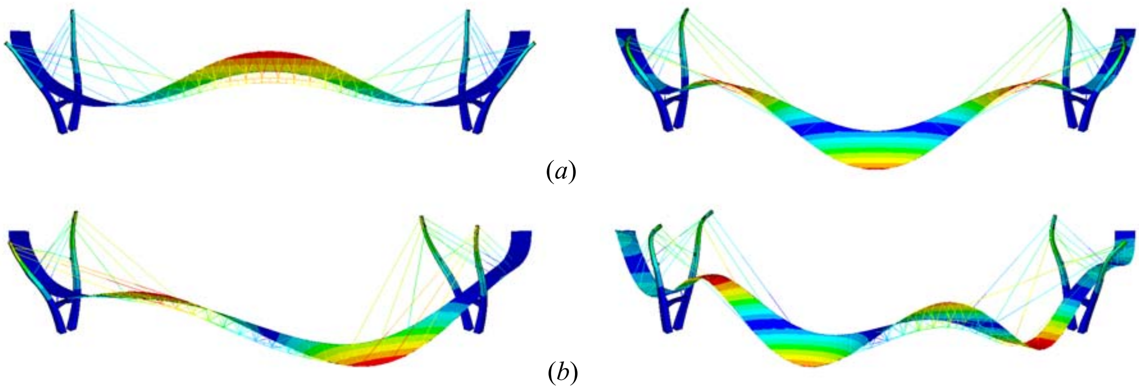
In Table 2, frequencies of the updated model (FE Model B) are reported and compared with the experimental results. The numerical frequencies match very well the experimental ones, with error values smaller than 1% for the first 2 modes and about 1% for the 3rd and the 4th modes; MAC reaches values greater than 95% for all modes.

It is worth noting that after the updating procedure the correlation between numerical and experimental mode shapes is almost perfect as regards the vertical components (high MAC values that are computed considering only the vertical components of the mode shapes). Instead, the transverse mode shape components are not accurately appreciated by the numerical model; as a matter of fact the model exhibits transverse deformations considerably higher than those obtained experimentally particularly referring to the first mode.

## 5 PEDESTRIAN-INDUCED ACCELERATIONS

To study the dynamic behaviour of the footbridge subjected to pedestrian loads, several experimental tests were performed with pedestrians crossing the footbridge running and free or synchronized walking with different pacing frequencies.

A number of crowd tests were carried out with different-sized groups of pedestrian walking on the footbridge. In some tests, the crowd was asked to cross the footbridge at their own normal walking pace. Afterwards, 12 pedestrians were asked to walk in single file starting from one end of the footbridge. Tests with pedestrians on the north (see Figure 6*a*) or the south side of the deck were performed considering different pacing frequencies. In addition, several tests were carried out arranging a stream of 44 walking pedestrians that completely filled the deck. To achieve a better synchronisation between pedestrians a megaphone and a



(*a*) (*c*) (*b*) (*d*)

Figure 5: Numerical mode shapes: 1st (*a*), 2nd (*b*), 3rd (*c*) and 4th (*d*) vertical modes

| Mode Type | Experimental Frequency [Hz] | FE Model A Freq. [Hz] | FE Model A Error [%] | FE Model A MAC [%] | FE Model B Freq. [Hz] | FE Model B Error [%] | FE Model B MAC [%] |
|---|---|---|---|---|---|---|---|
| 1st Vertical | 1.462 | 1.419 | -2.94 | 99.7 | 1.455 | -0.51 | 97.7 |
| 2nd Vertical | 2.274 | 2.365 | +4.00 | 97.4 | 2.269 | -0.23 | 97.1 |
| 3rd Vertical | 3.210 | 3.249 | +1.21 | 94.4 | 3.256 | +1.42 | 95.9 |
| 4th Vertical | 4.949 | 4.837 | -2.25 | 97.9 | 4.904 | -0.91 | 97.6 |

Table 2: Experimental and numerical modes

metronome were always used to control the step frequency that generally ranged between 1.2 Hz to 2.2 Hz. Several tests were performed with a step frequency close to the first vertical frequency of the footbridge (i.e. $f_s$ = 1.45 Hz) with an increasing number of pedestrians. Moreover, some tests were also carried out with few people (from 1 up to 4) running at about 2.2 Hz, with the purpose to excite the second vertical mode of the structure. Finally, tests with 44 pedestrians walking in a closed loop on the main span were also performed to excite the first torsional mode of the structure (see Figure 6*b*).

### 5.1 Recorded accelerations and comfort criteria

In Figure 7 vertical accelerations induced by pedestrians are reported. In each graph, the black line represents the measured accelerations while the red line denotes the root-mean-squared (RMS) accelerations. The experimental measurements show that vertical accelerations caused by pedestrians in unsynchronized tests are about 15÷18 mg (where g indicates the gravity acceleration) corresponding to 0.15÷0.18 m/s$^2$. The accelerations measured in a test with 44 pedestrians walking at a pacing frequency of 1.2 Hz are reported in Figure 7*a*. In this scenario, vertical acceleration amplitudes between 20÷25 mg were recorded without any evidence of excessive vibration amplitudes. High dynamic amplifications are obtained for a pacing frequency close to the first vertical frequency of the structure. Figure 7*b* shows accelerations induced by 12 pedestrians crossing the footbridge in single file on the north side of the deck with a step frequency of 1.45 Hz (close to the frequency of the first vertical mode). The RMS accelerations reach maximum values of about 80 mg.

Then, accelerations induced by 44 pedestrians walking in a closed loop on the main span with a step frequency of 1.45 Hz are reported in Figure 7*c*. It can be noticed that acceleration values greater than 120 mg are reached only about 25 seconds after the test start; these high accelerations produced feelings of discomfort for pedestrians and then, the test was immediately suspended. After that, accelerations gradually decreased up to 20 mg in about 30 seconds. The long period spent to reach low accelerations confirms the small values of damping ratio obtained by the identification procedure. Even larger vibration amplitudes may have been measured if people had been allowed to continue walking.

Finally, RMS accelerations reached maximum values of about 4 mg when a single person run with a step frequency of 2.2 Hz (Figure 7*d*).

Peaks and maximum RMS values of the vertical accelerations recorded by the monitoring system during the dynamic tests are compared with the comfort criteria given by Eurocode, ISO 10137, Sétra and HiVoSS guidelines, for each different pacing frequency.

Eurocode [15] recommends maximum acceptable acceleration values for the deck of 71 mg (0.7 m/s$^2$) for vertical vibrations and 20 and 41 mg for horizontal vibrations in normal use and for exceptional crowd conditions, respectively. These recommendations mainly neglect


(*a*)


(*b*)

Figure 6: Dynamic tests: 12 pedestrians in single file on the north side of the deck (*a*) and 44 pedestrians walking in a closed loop on the main span (*b*)

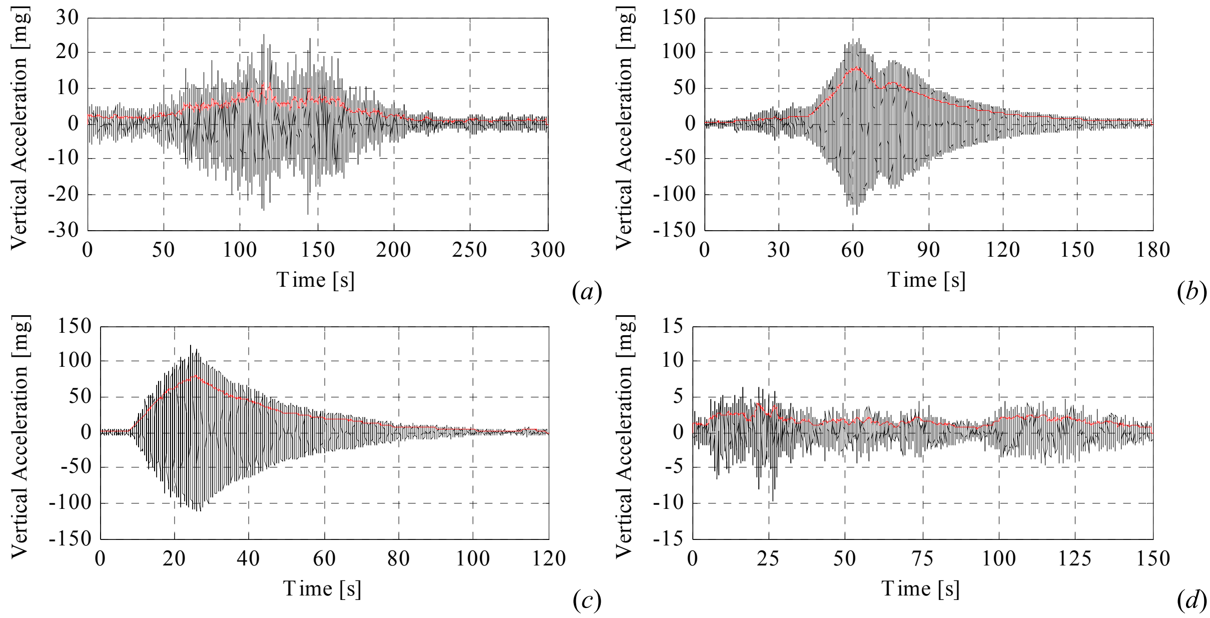



Figure 7: Measured vertical accelerations and RMS acceleration traces due to different loading conditions: a stream of 44 pedestrians walking at a pacing frequency of 1.2 Hz (*a*), 12 pedestrians at 1.45 Hz in single file (*b*), 44 pedestrians at 1.45 Hz in a closed loop (*c*) and a single individual running at 2.2 Hz (*d*)

the vibration frequencies of the structure, the kind and the duration of the human activity. ISO 10137:2007 [16] gives recommendations for the evaluation of serviceability against vibrations of buildings and walkways. Different scenarios are defined considering different crowd concentrations. Threshold acceleration is provided, depending on the natural frequency of the structure and on the traffic condition. Sétra [17] and HiVoSS [18] guidelines present four intervals of acceleration levels with corresponding comfort levels, defined for vertical and horizontal vibrations. Levels are maximum, mean, minimum comfort and uncomfortable.

Results obtained for vertical accelerations are shown in Figure 8. Peak acceleration values measured in the case of 12 pedestrians crossing the footbridge are reported. Eurocode considers acceptable all induced vertical accelerations except for the peak acceleration of 129 mg caused by 12 pedestrians crossing the footbridge at a pacing frequency of 1.45 Hz. This peak exceeds also comfort standards prescribed in ISO 10137. For Sétra and HiVoSS, the peak acceleration of 129 mg satisfies a minimum comfort level; however, for these guidelines, the reached accelerations are only allowed in very dense traffic situation. Accelerations reached for other step frequencies are included in the so-called “mean comfort level”.

## 6 PEDESTRIAN DYNAMIC LOADS

Dynamic analyses are performed to reproduce pedestrian dynamic effects on the footbridge and, consequently, to compare experimental and numerical maximum accelerations. To carry

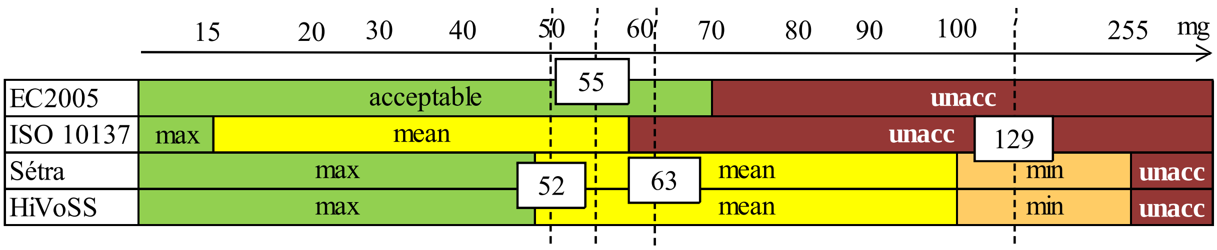



Figure 8: Comparison between comfort criteria and recorded accelerations

out dynamic analyses, mathematical models representing the dynamic forces due to a single pedestrian or a crowd of people crossing the structure are needed.

Pedestrian walking on a structure produces a dynamic force that has components in all three directions. Most of the studies performed to characterize pedestrian-induced force [19-23] show that it depends on the pacing frequency, the walking (or running) speed and the step length. The present work is focused on the simulation of the human-induced force in the vertical direction, adopting the numerical models described in the following. The dynamic walking excitation is commonly modelled as a series of moving single footsteps, for which the time-history of the vertical component is described by a Fourier series. In this study two models to simulate the pedestrian action are adopted. The first model (Model I) is proposed by Li [24] and it represents the human-induced force as a deterministic periodic force, while the second one (Model II), developed by Živanović [25], allows to take into account the inter- and intra-subject variability during walking.

The first model is adopted to perform numerical dynamic analyses with the FE models of the structure. It provides the single step force that is applied to consecutive nodes of the footbridge to simulate the pedestrian crossing. Besides the numerical dynamic analyses performed with the FE models, the dynamic behaviour of the footbridge under pedestrian loads is also investigated solving the differential equations of motion for the decoupled vibration modes. The modal force acting on each SDOF system is characterized through the pedestrian load model developed by [25].

To take into account the effect of pedestrian groups, the load induced by a single pedestrian has to be increased considering the pedestrian number $N$. In [19, 21] an amplification factor equal to $N$ in the case of pedestrian synchronization or $\sqrt{N}$ otherwise is suggested.

### 6.1 Simulation Model I

According to the model proposed by [24], the walking force induced by a single pedestrian is simulated using the step-by-step load model described in the following. The step frequency $f_s$, along with the static weight of the pedestrian $G$, determines the vertical single foot force $F_v(t)$ as

$$F_v(t) = G\sum_{n=1}^{5} A_n \sin\left(\frac{\pi n}{T_C} t\right), \qquad 0 \le t \le T_C \tag{1}$$

where $A_n$ are the Fourier coefficients, normalized to the weight of the pedestrian, and $T_C$ is the duration of the contact between the foot and the ground, both defined as functions of the step frequency. Values of the Fourier coefficients for the first five harmonics, depending on the pacing frequency, are shown in Table 3. According to the statistical results reported in [26], the ratio of the one foot contact duration ($T_C$) to the period during which both feet have contact with ground ($\Delta t$) is relatively insensitive to the different walking speeds [24] and it is equal to

$$\frac{T_C}{\Delta t} = 4.165 \tag{2}$$

With reference to the Figure 9 the one foot contact duration $T_C$ can be expressed as the sum between $\Delta t$ and the cycle of the continuous walking force $T_S$ (i.e. the inverse of the step frequency $f_s$), leading to a constant relation between the step frequency and the contact duration

$$T_C = \frac{1}{0.76 f_S} \tag{3}$$

An example of the single foot force obtained for a step frequency of 1.8 Hz is presented in Figure 9, while Figure 10 shows the continuous foot force. Both forces are normalized to the average weight of the pedestrian *G*.

The described pedestrian load model is adopted to perform dynamic analyses considering the vertical forces due to a single step applied to consecutive nodes of the footbridge model, to simulate the pedestrian walking. For each node, forces are defined by the Eq. 1 for values of time $t$ between $t_i$ and $t_i+T_C$ and equal to zero otherwise.

### 6.2 Simulation Model II

Besides the finite element solution, the dynamic behaviour of the footbridge under pedestrian loads can be investigated solving the decoupled equations of motion. At first, the well-known equation of motion of the MDOF system is defined, where the external force represents the pedestrian moving across the structure. The moving load $P$ is expressed as

$$P(t,x) = F(t)\,\delta(x - v_p t) \tag{4}$$

where $v_p$ is the pedestrian velocity, $\delta(x-v_p\,t)$ is the Dirac delta function and $F(t)$ is the pedestrian-induced force, defined through the analytical model described in the following.

Based on the principle of modal superposition, the equation of motion is transformed into modal coordinates. Then, integrating over the structure length $x$ and using the modal orthogonality conditions, the equation of motion for the $n$-th decoupled mode is obtained:

$$\ddot{y}(t) + 2\xi_n(2\pi f_n)\dot{y}(t) + (2\pi f_n)^2 y(t) = \frac{F(t)}{M_n}\phi_n(t) \tag{5}$$

where $\ddot{y}(t)$, $\dot{y}(t)$ and $y(t)$ are the modal acceleration, velocity and displacement respectively, while $\xi_n$, $M_n$, $f_n$ are the modal damping, mass and frequency, respectively. The right hand side of the equation represents the modal force acting on the SDOF system, expressed as the product of the walking force $F(t)$ and the mode shape $\phi_n(t)$ transformed from the space- to the time-domain. As such, the human-induced force is weighted by the mode shape $\phi_n$ in order to take into account that the force moves across the footbridge and it has a limited duration. The

| Fourier coefficients | 1.6 Hz $\le f_s \le$ 2.32 Hz | 2.32 Hz $< f_s \le$ 2.4 Hz |
|---|---|---|
| $A_1$ | -0.0698 $f_s$ + 1.211 | -0.1784 $f_s$ + 1.463 |
| $A_2$ | 0.1052 $f_s$ - 0.1284 | -0.4716 $f_s$ + 1.210 |
| $A_3$ | 0.3002 $f_s$ - 0.1534 | -0.0118 $f_s$ + 0.5703 |
| $A_4$ | 0.0416 $f_s$ - 0.0288 | -0.2600 $f_s$ + 0.6711 |
| $A_5$ | -0.0275 $f_s$ + 0.0608 | 0.0906 $f_s$ - 0.2132 |

Table 3: Fourier coefficients for the first five harmonics – Model I

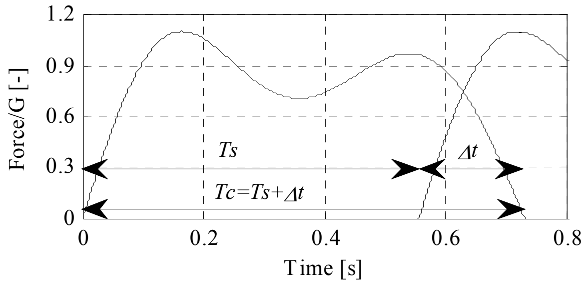



Figure 9: Normalized single foot force obtained for $f_s$=1.8 Hz – Model I

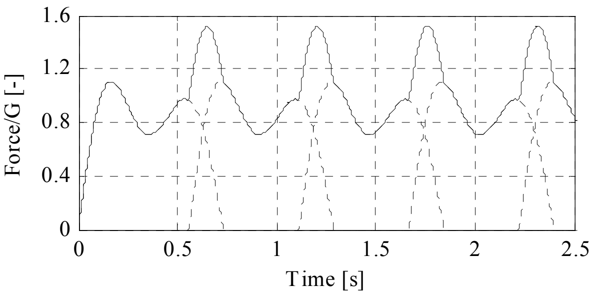



Figure 10: Normalized continuous foot force obtained for $f_s$=1.8 Hz – Model I

weighted modal force is used to calculate the footbridge modal response based on the time integration of the equation formulated for the modal coordinates. Then, the modal responses obtained for individual vibration modes have to be summed according to the mode superposition principle.

The walking force $F(t)$ is represented through the pedestrian load model proposed by [25] and based on the results obtained from [27]. By analyzing walking force time-histories measured on a treadmill, [27] found that human-induced walking force is not periodic, but rather it is a narrow band random process, meaning that there is a leaking of energy around the main harmonics in the force spectrum. Based on the results provided from Brownjohn [27], Živanović [25] developed a multi-harmonic force model for the calculation of the multi-mode structural response to a single person walking across a footbridge. The proposed model takes into account the inter- and intra-subject variability in the induced walking force. The former term implies that different pedestrians generate different dynamic forces, and the latter means that even a single pedestrian induces a walking force that differs with each step. Variability in the walking force is considered via probability-based modeling, defining the variables that describe the human-induced force through their probability density function. Parameters that describe the variability in walking forces induced by different pedestrians (inter-subject variability) are the walking frequency, the step length and the magnitude of the walking force. As regards the intra-subject variability, imperfections in the human walking can be described through slight changes in the walking frequency and, consequently, in the spectrum of the walking force, resulting in a narrow band spectrum. Using walking force time-histories measured on the treadmill, [27] investigated the frequency content of the force and observed that main harmonics appear at frequencies equal to the pacing frequency and its integer multiples but some sub-harmonics appear at frequencies between the main harmonics. Starting from the frequency content of the walking force, a model representation of the spectrum amplitudes and phases was achieved and the multi-harmonic force model was formulated. For all details about the model see [25].

For the $i$-th harmonic, occurring at frequency $i\,f_s$, the force in the time domain has the following expression:

$$F_i(t) = G \cdot DLF_i \cdot \sum_{\bar{f}_j = i-0.25}^{i+0.25} \overline{DLF}_i\left(\hat{f}_j\right)\cos\left(2\pi \hat{f}_j f_s t + \theta\left(\hat{f}_j\right)\right) \tag{6}$$

while for the $i$-th sub-harmonics it is

$$F_i^s(t) = G \cdot DLF_i^s \cdot \sum_{\bar{f}_j{}^s = i-0.75}^{i-0.25} \overline{DLF}_i{}^s\left(\hat{f}_j^s\right)\cos\left(2\pi \hat{f}_j^s f_s t + \theta\left(\hat{f}_j^s\right)\right) \tag{7}$$

where $f_s$ is the step frequency, the product $\hat{f}_j\, f_s$ is a frequency line within the energy range of the (sub-) harmonic analysed and $\theta$ is the phase assigned to the current line in the spectrum. This assignment is based on a uniform distribution of phases in the interval [-π,+π]. The dynamic load factors for the harmonics $DLF_i$ and sub-harmonics $DLF_i^s$ analysed are reported in Table 4, while the normalized amplitudes for the same harmonics $\overline{DLF}_i$ and sub-harmonics $\overline{DLF}_i{}^s$ for each spectrum line can be found in [25]. Finally, $G$ is the average weight of a pedestrian. The total force induced by a single pedestrian can be obtained as

$$F(t) = \sum_{i=1}^{2} F_i\left(t\right) + \sum_{i=1}^{2} F_i^s\left(t\right) \tag{8}$$

The force should be reconstructing for time equal to the crossing time $T_T$ that is obtained as

$$T_T = \frac{L}{l_s f_s} \tag{9}$$

where $L$ is the length of the footbridge and $l_s$ is the step length. Note that the model proposed by [25] covers the frequency range of the walking force related to the first five harmonics and sub-harmonics while in the present study only the first two harmonics and sub-harmonics are considered.

In the present study the simulations are performed considering pedestrians crossing the footbridge at defined pacing frequencies, to reproduce the in-situ test conditions. Thus, the variability in the walking frequency is neglected and the step length $l_s$ is deterministically imposed proportional to the step frequency and the pedestrian height $h$:

$$l_s = 0.24 h f_s \tag{10}$$

As regards the magnitude of the walking force, the mean values of the *DLFs* are considered instead of their probability distributions. Thus, in this study the inter-subject variability is neglected.

An example of the walking force normalized to the weight of the pedestrian $G$ is reported in Figure 11. Note that, differently from the Model I, the continuous foot force is not perfectly periodic, as the intra-subject variability during walking is taken into account.

## 7 COMPARISON BETWEEN EXPERIMENTAL AND SIMULATED ACCELERATIONS INDUCED BY PEDESTRIANS

Numerical simulations are performed to predict the dynamic behaviour of the Pasternak footbridge under pedestrian load actions. The essential components of the simulation model are the dynamic behaviour of the footbridge, the characterization of the dynamic footfall load and the response calculation.

Two types of simulation analyses are performed: the first type is based on a finite element resolution with the numerical model of the footbridge described in Section 4 and with the step-by-step load model (Model I), where the walking forces are simulated as a series of moving single foot forces as proposed by [24]. The numerical dynamic analyses are performed considering both the un-calibrated FE model (Model A) and the updated FE model (Model B). In the following, results obtained from the un-calibrated model are referred as “Model I-A”, while “Model I-B” is used to indicate the results obtained from the updated FE model. The second type of simulation is based on the resolution of the equation of motion in the modal coordinates. In this case, the dynamic properties of the footbridge are those determined experimentally and the walking forces are represented through the model proposed by [25] (Model II). For the presented simulation, the first three natural modes are taken into account.

| | |
|---|---|
| $DLF_1$ | $-0.2649 f_s^3 + 1.3206 f_s^2 - 1.7597 f_s +$ $+0.7613$ |
| $DLF_2$ | 0.07 |
| $DLF_1^s$ | $0.026\ DLF_1 + 0.0031$ |
| $DLF_1^s$ | $0.074\ DLF_1 + 0.01$ |

Table 4: Dynamic load factors for the first 2 harmonics and sub-harmonics – Model II

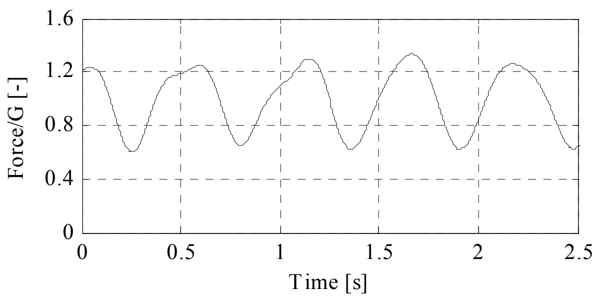



Figure 11: Normalized walking force obtained for $f_s$=1.8 Hz – Model II

Accelerations obtained from the Model I and II are first compared with each other by means of the amplification factors of the vertical accelerations, considering a single pedestrian crossing the footbridge with different pacing frequencies. Particularly, the pacing frequency is varied from 1 Hz up to 2.5 Hz for the Model I and from 1 Hz to 3.5 Hz for the Model II. Note that the Fourier coefficients for Model I are defined for a pacing frequency from 1.6 Hz up to 2.4 Hz while in the numerical simulations the frequency is varied in the range 1÷2.5 Hz. In the opinion of the authors, this assumption is necessary to investigate also the dynamic behaviour of the footbridge close to the first natural frequency but it may cause inaccuracies in the amplitude response at upper and lower frequencies.

In Figure 12 the response amplification factors obtained from the Model I-A and the Model II are compared with reference to a position at midspan (Figure 12*a*) and at one quarter (Figure 12*b*). Simulation results illustrate the strong dependency of the footbridge acceleration levels on the step frequency of the pedestrian. Regarding results obtained from Model II (blue line), three peaks are clearly present, corresponding to the first three natural modes of the structure. Moreover, two other peaks are obtained for a pacing frequency of 1.14 Hz and 1.60 Hz due to the contribution of higher order harmonics of the walking forces. Particularly, at a step frequency of about 1.14 Hz the fourth harmonic component generates near resonance conditions with the second natural mode, while at a step frequency of about 1.60 Hz the fourth harmonic component generates near resonance conditions with the third natural mode.

Regarding the Model I-A, dynamic amplifications are obtained for pacing frequencies rather different from the experimental ones, as the FE Model A is not calibrated with respect to the experimental modal quantities. It is worth noting the difference in the response amplitude of the two models, especially for frequencies corresponding to the first and the second natural modes (i.e. at about 1.4 and 2.2 Hz).

Then, the acceleration levels recorded at the different measuring positions and for the dif-

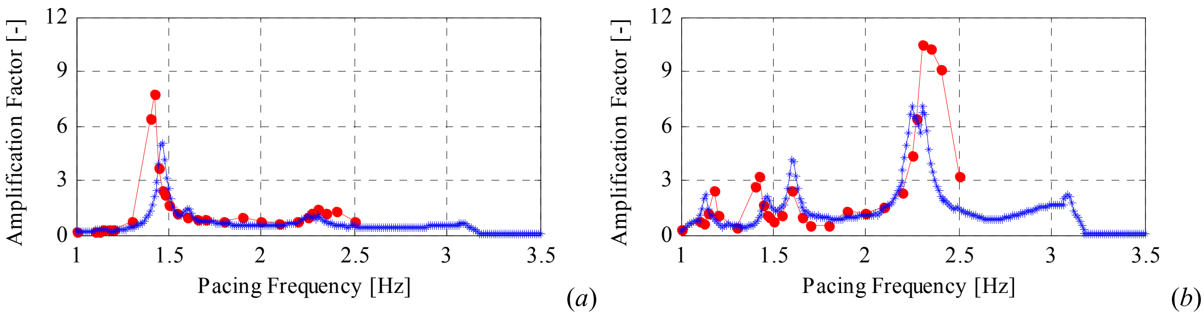



Figure 12: Response amplification factors vs. pacing frequency obtained from the Model I-A (red line) and the Model II (blue line) at midspan (*a*) and at one quarter (*b*)

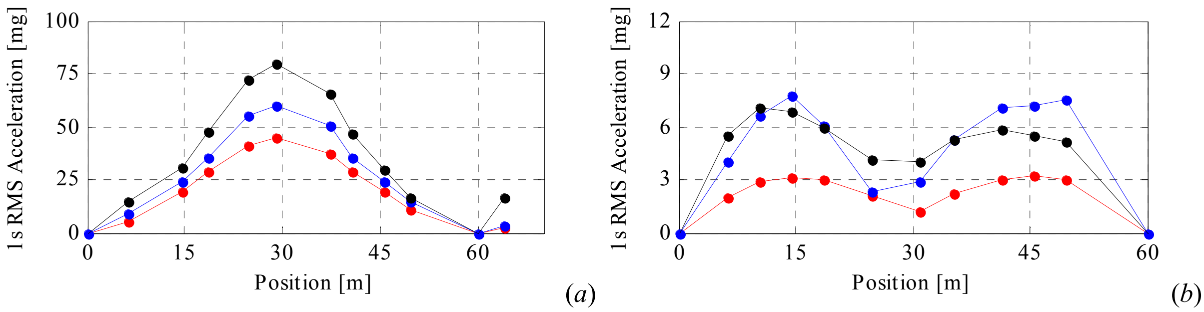



Figure 13: Acceleration levels recorded at the different measuring positions and for different test conditions: 12 pedestrians walking at a step frequency of 1.45 Hz (*a*) and a single pedestrian running at a step frequency of 2.2 Hz (*b*). Black line: experimental RMS accelerations; red line: RMS accelerations from Model I-A; blue line: RMS accelerations from Model II

ferent test conditions (described in Section 5) are compared with those obtained from the Model I-A and II. Results are compared in terms of maximum root-mean-square values of the vertical acceleration, with an averaging period of 1 s. Maximum RMS values of the vertical accelerations caused by 12 pedestrians walking at a step frequency of 1.45 Hz are presented in Figure 13*a*. Blue line represents accelerations simulated through the Model II, the red line indicates values obtained from Model I-A and the recorded response is the black line. It can be clearly seen how the pedestrian action mainly excites the first natural mode of the structure. In the same way, the maximum RMS accelerations caused by one pedestrian walking at a step frequency of 2.2 Hz, close to the second natural mode of the structure, is shown in Figure 13*b*. As regards the Model II, good agreement between the simulated and measured response can be noted, while smaller response amplitudes are obtained from the Model I-A. These inaccuracies are mainly related to the fact that the FE Model A is not calibrated, and so its natural frequencies do not match properly the experimental ones.

This is further demonstrated by the fact that results obtained from the updated FE model match very well the experimental ones, as show in Figure 14,15. At first, the response amplification factors obtained with the updated FE model (Model I-B) and with the Model II are

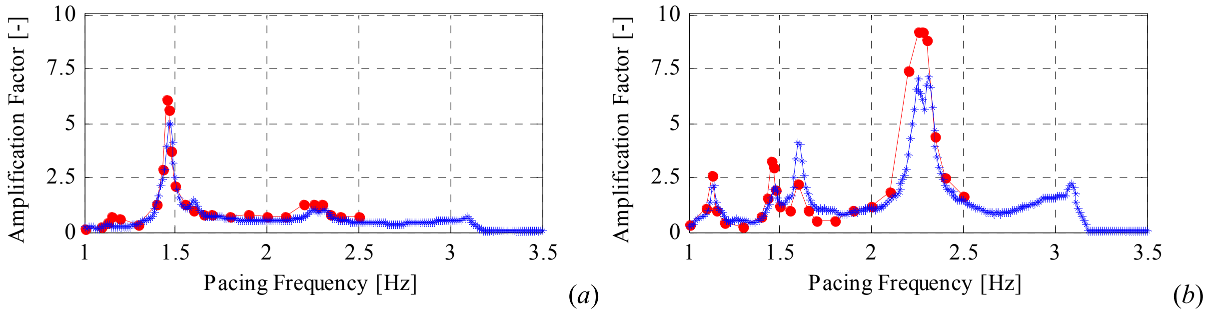



Figure 14: Response amplification factors vs. pacing frequency obtained from the Model I-B (red line) and the Model II (blue line) at midspan (*a*) and at one quarter (*b*)

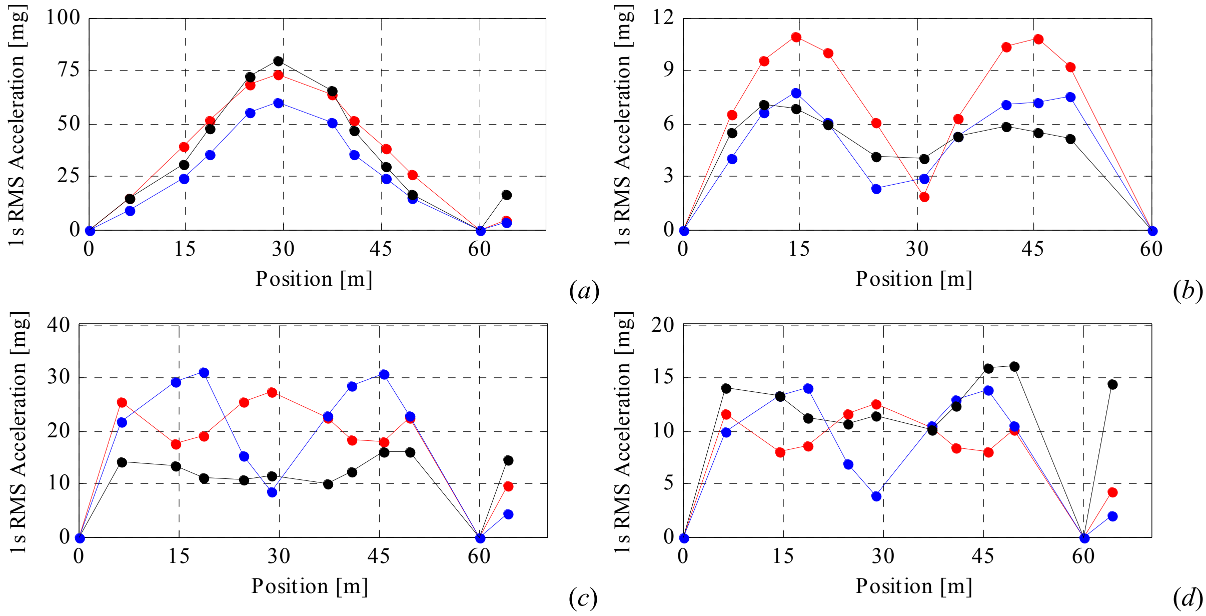



Figure 15: Acceleration levels recorded at the different measuring positions and for different test conditions: 12 pedestrians walking at a step frequency of 1.45 Hz (*a*); a single pedestrian running at a pacing frequency of 2.2 Hz (*b*); 44 pedestrians walking at a pacing frequency of 1.2 Hz, with a synchronization factor equal to *N* (*c*) or 0.5*N* (*d*). Black line: experimental RMS accelerations; red line: RMS accelerations from Model I-B; blue line: RMS accelerations from Model II

compared in Figure 14. The amplification factors are in good agreements and both models well represent the dynamic behaviour of the footbridge. The experimental results are accurately reproduced also with reference to acceleration levels recorded at the different measuring positions. Similar to Figure 13, Figure 15 shows the RMS acceleration values caused by 12 pedestrians walking at a step frequency of 1.45 Hz (Figure 15*a*) and by a single runner crossing the footbridge with a pacing frequency of 2.2 Hz (Figure 15*b*). Simulated results match the experimental ones in both cases; only the response amplitude obtained from the Model I-B is slightly overestimated.

Finally, the recorded accelerations caused by 44 pedestrians walking at a pacing frequency of 1.2 Hz are compared with the simulated ones (Figure 15*c,d*). The frequency of 1.2 Hz indicates a very slow velocity of pedestrian, typical of a crowded situation. If a perfect synchronization between the pedestrians is considered (i.e. by multiplying the accelerations caused by one pedestrian by 44), the simulated accelerations become higher than the measured ones (Figure 15*c*). In fact, even if pedestrians were guided by the metronome signal, the synchronization was not perfectly obtained because of the slow velocity imposed. On the other end, the assumption of total absence of synchronization leads to an underestimation of the induced accelerations, causing maximum values of about 5 mg. It is reasonable to assume that the effective synchronization was intermediate between the perfect and the unsynchronized case; a synchronization factor equal to $0.50N$ gives the results reported in Figure 15*d*.

## 8 CONCLUSION

In this paper the dynamic behaviour of the Pasternak footbridge subjected to pedestrian loads is investigated. First of all, an experimental campaign is carried out to characterize the dynamic behaviour of the footbridge subjected to ambient vibrations. Starting from the acquired accelerations, 10 modes (6 vertical and 4 torsional) with frequencies in the range 1.462÷12.061 Hz are clearly identified. Then, several dynamic tests with different-sized groups of pedestrians crossing the footbridge at different step frequencies were performed to investigate the dynamic response of the structure subjected to pedestrian loads. High dynamic amplifications are obtained for a pacing frequency close to the first vertical frequency of the structure; RMS accelerations reach maximum values of about 80 mg considering a group of pedestrians crossing the footbridge with a step frequency of 1.45 Hz. After, a FE model is developed and calibrated, so that numerical frequencies and mode shapes match as well as possible the experimental ones. Then, two mathematical models to simulate the dynamic walking excitations due to a single pedestrian or a crowd of people crossing the footbridge are tested. The first method, based on a periodic load model, is adopted to perform dynamic analyses on both the updated and non-calibrated FE models. In the second approach, the equation of motion is solved via modal decomposition, considering experimental mode shapes and frequencies and the multi-harmonic force model.

Simulation results illustrate the strong dependency of the footbridge acceleration levels on the step frequency of the pedestrian. Accelerations obtained through the mathematical approaches and the experimental tests are generally in good agreements and both simulation models well represent the dynamic behaviour of the footbridge.

As regards the Model II, good agreement between the simulated and the measured response can be noted, mainly due to the direct use of experimental natural frequencies and mode shapes. Moreover, the possibility to take into account the uncertainties in the step frequencies and the walking force (i.e. the inter- and intra-subject variability during walking) is appreciable. As for Model I, the need to develop a proper FE model to obtain numerical accelerations in good agreement with the experimental results is confirmed.

## REFERENCES

[1] P. Dallard, T. Fitzpatrick, A. Flint, S. Le Boutva, A. Low, R. Ridsdill Smith, M. Willford, The London Millennium Footbridge. *The Structural Engineering*, **79**, 17–33, 2001.

[2] F. Danbon, F. Grillaud, Dynamic Behaviour of a Steel Footbridge. Characterization and Modeling of the Dynamic Loading Induced by a Moving Crowd on the Solferino Footbridge in Paris. *Footbridge 2005*, Venice, Italy, December 6-8, 2005.

[3] E. Caetano, Á. Cunha, F. Magalhães, C. Moutinho, Studies for controlling human-induced vibration of the Pedro e Inês footbridge, Portugal. Part 1: Assessment of dynamic behaviour. *Engineering Structures*, **32**(4), 1069–1081, 2010.

[4] E. T. Ingólfsson, C. T. Georgakis, J. Jönsson, Pedestrian-induced lateral vibrations of footbridges: A literature review. *Engineering Structures*, **45**, 21–52, 2012.

[5] K. Van Nimmen, G. Lombaert, G. De Roeck, P. Van den Broeck, Vibration serviceability of footbridges: Evaluation of the current codes of practice. *Engineering Structures*, **59**, 448–461, 2014.

[6] O. S. Bursi, A. Kumar, G. Abbiati, R. Ceravolo, Identification, Model Updating, and Validation of a Steel Twin Deck Curved Cable-Stayed Footbridge. *Computer-Aided Civil and Infrastructure Engineering*, **29**(9), 703–722, 2014.

[7] R. Guidorzi, R. Diversi, L. Vincenzi, C. Mazzotti, V. Simioli, Structural monitoring of a tower by means of MEMS-based sensing and enhanced autoregressive models. *European Journal of Control*, **20**(1), 4–13, 2014.

[8] E. Bassoli, P. Gambarelli, L. Simonini, L. Vincenzi, M. Savoia, Dynamic monitoring of the Pasternak footbridge using MEMS-based sensing system. *7th International Conference on Structural Health Monitoring of Intelligent Infrastructure*, Turin, Italy, July 1-3, 2015. [in press]

[9] P. Gambarelli, L. Vincenzi, A surrogate-assisted evolutionary algorithm for dynamic structural identification. *4th International Conference on Engineering Optimization (ENGOPT 2014)*, Lisbon, Portugal, September 8-11, 2014.

[10] L. Vincenzi, M. Savoia, Coupling Response Surface and Differential Evolution for Parameter Identification Problems. *Computer-Aided Civil and Infrastructure Engineering*, 2015. DOI:10.1111/MICE12124

[11] J. Santana, R. van den Hoven, C. van Liempd, M. Colin, N. Saillen, D. Zonta, D. Trapani, T. Torfs, C. Van Hoof, A 3-axis accelerometer and strain sensor system for building integrity monitoring. *Sensors and Actuators A: Physical*, **188**, 141–147, 2012.

[12] J. S. Bendat, A. G. Piersol, *Engineering applications of correlation and spectral analysis, 2nd Edition.* John Wiley & Sons, Inc., 1993.

[13] R. Brincker, L. Zhang, P. Andersen, Modal identification of output-only systems using frequency domain decomposition. *Smart Material Structures*, **10**(3), 441, 2001.

[14] Y. Jin, M. Olhofer, B. Sendhoff, A framework for evolutionary optimization with approximate fitness functions. *IEEE Transactions on Evolutionary Computation*, **6**(5), 481–494, 2002.

[15] EN 1990:2002/A1:2005, *Eurocode - Basis of structural design. Application for bridges*.

[16] ISO 10137:2007*, Bases for design of structures - Serviceability of buildings and walkways against vibrations*.

[17] Sétra, *Footbridges - Assessment of vibrational behaviour of footbridges under pedestrian loading. Technical guide*.

[18] HiVoSS 2008, *Design of Footbridges*.

[19] Y. Matsumoto, T. Nishioka, H. Shiojiri, K. Matsuzay, Dynamic design of footbridges. *Proceedings of IABSE Conference*, Bergamo, Italy, May, 1978.

[20] P. Young, Improved floor vibration prediction methodologies, *Proceedings of ARUP Vibration Seminar,* London, United Kingdom, October 4, 2001.

[21] H. Bachmann, "Lively" footbridges - A real challenge. *Proceedings of the International Conference on the Design and Dynamic Behavior of Footbridges*, Paris, France, November 20-22, 2002.

[22] S. Živanović, A. Pavic, P. Reynolds, Vibration serviceability of footbridges under human-induced excitation: a literature review. *Journal of Sound and Vibration*, **279**(1–2), 1–74, 2005.

[23] S. Nakamura, Model for Lateral Excitation of Footbridges by Synchronous Walking. *ASCE Journal of Structural Engineering*, **130**(1), 32–37, 2004.

[24] Q. Li, J. Fan, J. Nie, Q. Li, Y. Chen, Crowd-induced random vibration of footbridge and vibration control using multiple tuned mass dampers. *Journal of Sound and Vibration*, **329**(19), 4068–4092, 2010.

[25] S. Živanović, A. Pavić, P. Reynolds, Probability-based prediction of multi-mode vibration response to walking excitation. *Engineering Structures*, **29**(6), 942–954, 2007.

[26] A. Ebrahimpour, A. Hamam, R. L. Sack, W. N. Patten, Measuring and modeling dynamic loads imposed by moving crowds. *Journal of Structural Engineering*, **122**(12), 1468–1474, 1996.

[27] J. M. Brownjohn, A. Pavic, P. Omenzetter, A spectral density approach for modelling continuous vertical forces on pedestrian structures due to walking. *Canadian Journal of Civil Engineering*, **31**(1), 65–77, 2004.