COMPDYN 2015
5th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, V. Papadopoulos, V. Plevris (eds.)
Crete Island, Greece, 25–27 May 2015

SAFETY ASSESSMENT OF A SMALL SPAN HIGH-SPEED RAILWAY BRIDGE USING AN EFFICIENT PROBABILISTIC METHODOLOGY

João M. Rocha[1], António A. Henriques[2], and Rui Calçada[2]

[1] University of Porto - Faculty of Engineering
Rua Dr. Roberto Frias, s/n, 4200-465 Porto, Portugal
e-mail: jmsrocha@fe.up.pt

[2] University of Porto - Faculty of Engineering
Rua Dr. Roberto Frias, s/n, 4200-465 Porto, Portugal
{aarh,ruiabc}@fe.up.pt

Keywords: Efficient probabilistic methodology; Simulation methods; Extreme value theory; Optimized extrapolation; Railway bridges; High-speed traffic.

Abstract. *The behaviour of small span railway bridges is known to be particularly difficult to predict due to the complexity of the coupled train-track-bridge system, as well as for being particularly sensitive to resonant phenomena. The objective of this paper is to present an efficient methodology to evaluate the safety of small span bridges in high-speed railway lines, bearing in mind the real variability of the parameters that influence the dynamic response of the train-track-bridge coupled system. This requires the development of adequate numerical models for the bridge, the track and the train subsystems, as well as the definition of the distributions and variability of all the variables related to the structure, the train, the track and also the wheel-rail contact. Track irregularities are also accounted for in the dynamic analysis. Canelas railway bridge, located in the North of Portugal, was selected as case study. The bridge has six simply supported spans of 12 m each, leading to a total length of 72 m. The deck is a composite structure with two half concrete slab decks with nine embedded rolled steel profiles HEB 500, each supporting one rail track. In order to assess the safety of the bridge two simulation methods were used: the Monte Carlo method and the Latin Hypercube method. Furthermore, both simulation methods are combined with two different approaches to enhance efficiency. One based on the extreme value theory that uses the Generalized Pareto Distribution to model the tail of the distribution. The other uses an approximation procedure based on the evaluation of the failure probabilities at moderate levels to estimate the target probability of failure by extrapolation. The track stability safety due to the deck vibrations level is used as the safety criterion to validate the proposed methodology. The results are extremely promising and indicate the feasibility of this methodology due to the very reasonable computational costs that are required.*

1 INTRODUCTION

Safety has always been one of the main Engineering concerns, particularly for important structures such as bridges. Due to the generalized use of computers and their continuous evolution, the safety assessment techniques have become increasingly sophisticated and complex, allowing more realistic and accurate analysis. In recent years, the use of probabilistic methods to assess the safety of new and existing structures has become more frequent.

Due to the degree of complexity of the safety assessment of railway bridges the use of such methods is not very common. Simulation methods are generally used when probabilistic methodologies are employed to study these problems. The Monte Carlo method [1] is usually selected, but more refined methods like the Latin Hypercube [2] can also be applied. Regardless of the selected method, the objective is always to obtain an accurate assessment using an efficient methodology. The efficiency of a methodology represents its capacity to reduce computational costs without compromising accuracy. Besides accuracy, the quantification of the uncertainties in the predictions is also important.

Despite the potential of the simulation methods, it is observed that for several problems the use of such technique may be computationally prohibitive. For this reason several authors proposed complementary procedures in order to enhance the efficiency of such methods. Naess et al. [3] proposed an approach that estimates the probability of failure by extrapolating the tail probabilities based on the estimates of the failure probabilities at moderate levels. Ramu et al. [4], on the other hand, proposed the estimation of the probability of failure by modelling the tails of the obtained distributions.

In previous research works the safety due to track instability caused by excessive deck vibrations [5] and the running safety of trains [6] on this type of bridges have already been studied using probabilistic approaches. In the present study the safety assessment of a short span composite high-speed railway bridge is performed. The current paper is focused on the analysis of the efficiency of different probabilistic methodologies in the safety assessment of short span railway bridges. Besides the standard Monte Carlo simulation, that has been used in previous works, in this paper a more refined simulation technique, namely the Latin Hypercube sampling method, is also applied. Furthermore, both simulation methods are combined with two different approaches to enhance efficiency. One is based on the extreme value theory and uses the Generalized Pareto Distribution to model the tail of the distribution. The other uses an approximation procedure based on the evaluation of the failure probabilities at moderate levels to estimate the target probability of failure by extrapolation.

The safety of the train-track-bridge system is assessed through the analysis of the track instability due to excessive deck vibrations. The results obtained by different methods are compared in order to identify the most efficient approach.

2 STRUCTURAL RELIABILITY ANALYSIS

The use of analytical techniques for the reliability analysis of complex structural systems, which are often characterized by non-linear limit-state functions, can be extremely difficult. In this section, several alternatives to asses this problem are presented and discussed. The different investigated techniques are used in the safety assessment of a short span high-speed railway bridge presented in Section 3. The obtained results are compared in order to identify the most efficient approach.

2.1 Tail modelling - Generalized Pareto Distribution (GPD)

Since structural reliability problems are determined by the tail of the obtained statistical distributions, the computational cost can be significantly reduced if an extrapolation of the Cumulative Distribution Function (CDF) is made using tail modelling techniques [4]. The classical tail modelling is based on the extreme value theory and consists on approximating the tail portion of the CDF above a certain threshold, *u*, by the Generalized Pareto Distribution [7]. The approximation function, $F_{\xi,\psi}(z)$, can be written as [4]:

$$F_{\xi,\psi}(z) = \begin{cases} 1 - \left(1 + \frac{\xi}{\psi} \cdot z\right)^{-\frac{1}{\xi}} & if \ \xi \neq 0 \\ 1 - \exp\left(-\frac{z}{\xi}\right) & if \ \xi = 0 \end{cases} \tag{1}$$

where z is the exceedance, ξ and ψ are the shape and scale parameters, respectively. This method has been applied in previous research works [5] and proved its efficiency in the analysis of complex multi-modal response problems. However, the tail needs to be modelled accurately, as small variations in the tail of the distribution can result in a variation by an order of magnitude of the safety level. Furthermore, the method relies significantly in the most extreme values, which are the ones that present the largest uncertainty. For this reason the estimated probability of failure may require, in some cases, a larger number of simulations until it stabilizes.

2.2 Enhanced simulation method (ES)

Naess et al. [3] proposed an enhanced simulation method which is able to overcome some limitations of high computational cost due to large samples needed for a robust estimation as in the previously presented method. It exploits the regularity of the tail probabilities to set up an approximation procedure based on the estimates of the failure probabilities at more moderate levels for the prediction of the far tail failure probabilities. The safety margin, *M*, represents the difference between the capacity and the demand to define the probability of failure as $p_f = \mathrm{Prob}(M \leq 0)$ and is extended to a parameterized class of safety margins in the following way:

$$M(\lambda) = M - \mu_M \cdot (1 - \lambda) \tag{2}$$

where μ_M is the mean value of the safety margin *M* and λ is the scaling parameter that assumes values in the interval $0 \leq \lambda \leq 1$, putting the emphasis on the more reliable data points. Thus, the original system is obtained for $\lambda = 1$ while $\lambda = 0$ represents a system highly disposed to failure.

For a sample of size *N* an empirically estimated probability of failure is given by:

$$\hat{p}_f(\lambda) = \frac{N_f(\lambda)}{N} \tag{3}$$

where $N_f(\lambda)$ represents the number of realizations where there was failure for the given λ level. The 95% confidence interval for the value $p_f(\lambda)$ can be reasonably estimated by:

$$C^{\pm}(\lambda) = \hat{p}_f(\lambda) \cdot \left[1 \pm 1.96 \cdot C_v\left(\hat{p}_f(\lambda)\right)\right] \quad (4)$$

Introducing an approximation function fitted to the estimates allows estimating the target probability of failure by extrapolation. As proposed by [3], it is assumed that the probability of failure in the tail is dominated by a function that can be written as a function of λ:

$$\underset{\lambda \to 1}{p_f}(\lambda) \approx q(\lambda) \cdot \exp\left\{-a \cdot (\lambda - b)^c\right\} \quad (5)$$

where $q(\lambda)$ is a function that varies slowly compared with the exponential function $exp\left\{-a(b-\lambda)^c\right\}$. Thus, for practical applications it can be assumed that $q(\lambda) = q$ and can then be applied in the following form for a suitable value of λ_0 [3]:

$$p_f(\lambda) \approx q \cdot exp\left\{-a \cdot (\lambda - b)^c\right\} \quad (6)$$

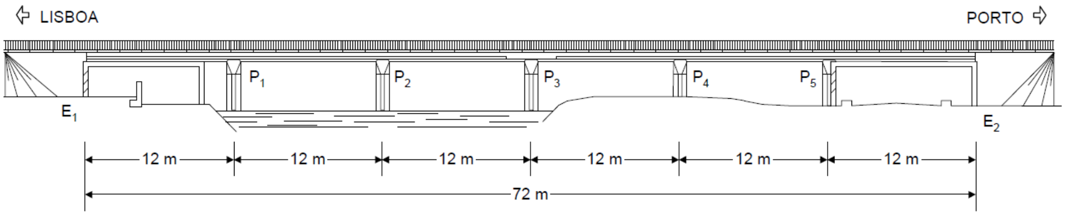
Therefore, an important part of the method is the identification of this suitable λ_0 so that Eq. (5) represents a good approximation of $p_f(\lambda)$ for $\lambda \in [\lambda_0, 1]$, and at least such that $\lambda_0 > b$. The optimum values for the four parameters q, a, b and c can be obtained through a least square optimization method using the Levenberg-Marquardt algorithm to the failure probabilities obtained by the Monte Carlo simulation.

For the estimation of the confidence intervals the empirical confidence band is re-adjusted to the optimal curve through Eq. (4). Then, the optimized confidence intervals are determined using a similar optimization procedure based on the re-adjusted confidence band. The fact that the confidence intervals can also be obtained by extrapolation, reducing significantly the influence of the sample size, is an important feature of this method. This enables it to be used to determine the accuracy of the estimated probability of failure. The fitted curves, extrapolated to the level of interest, will determine an optimized confidence interval of the estimated target failure probability.

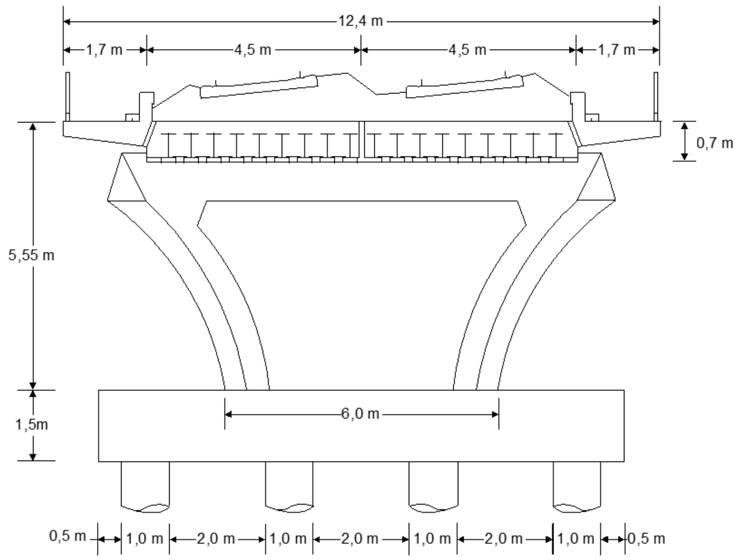
3 CASE STUDY

3.1 Canelas Bridge

Canelas Bridge is selected as case study. It is located in the Northern line of the Portuguese railway and is composed of six simply supported spans of 12 m each, with a total length of 72 m. The bridge deck is a filler beam consisting of two half concrete slab decks, each supporting a single ballasted track, with nine embedded rolled steel profiles HEB 500. Laminated neoprene elastomeric bearings are placed underneath each steel profile, in a total of 9 bearings per half deck. A longitudinal view of the bridge used as case study and the typical deck cross section is shown in Figure 1.



a) Longitudinal view



b) Typical cross section

Figure 1: Canelas railway Bridge.

The main reason for the selection of this bridge is that this is a very common structural system used for the European railway lines. A detailed description of the case study bridge can be found in [8]. The numerical model developed for the bridge is similar to the one used in [5] and is presented in the following section.

3.2 Numerical models

The Finite Element Method is used for the numerical modelling of both the bridge and the train. Regarding the bridge a 2D model is developed and discretized using beam elements. A single span is modelled and only one track is analysed, as the two half slab decks are independent. Furthermore, the model assumes that the bridge is inserted in a straight section of a high-speed railway line.

The deck is modelled as a beam positioned at the corresponding centre of gravity. The bearings are also included in the numerical model as springs positioned at the corresponding centre of rotation. The vertical and horizontal stiffness of the bearings are calculated according to [9]. With respect to the ballasted track, the model includes the rails, modelled as beams, the sleepers which are simulated by lumped masses and also the rail pads and the ballast layer, modelled as sets of springs and dampers. The shear behaviour of the ballast layer due to composite effects is also accounted for and is computed in accordance with the elastic range of the bi-linear behaviour proposed by [10]. A more detailed description of the numerical model can be consulted in [5]. A schematic representation of the track-bridge numerical model is shown in Figure 2. The zoom in Figure 2 corresponds to the same section that is composed by an overlap of the two represented elements.

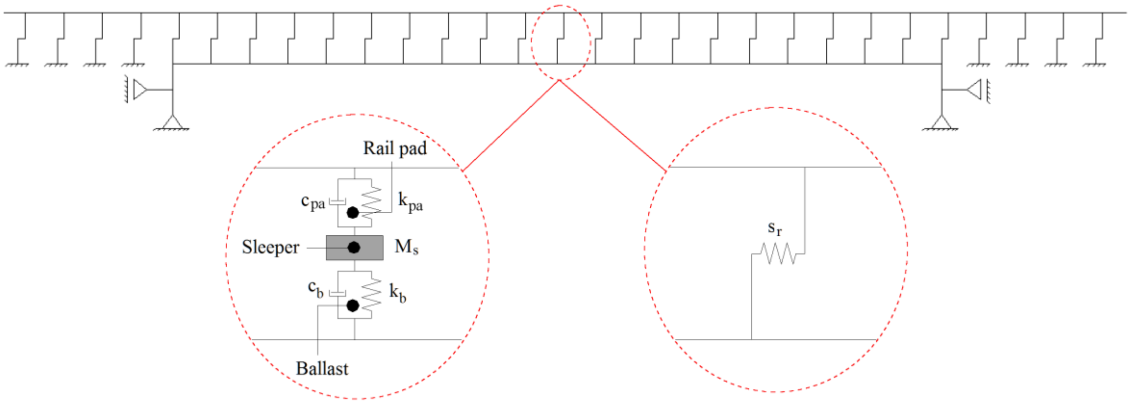


Figure 2: Track-bridge numerical model.

The existence of track irregularities is also taken into account in this paper and they are randomly generated using power spectral density functions according to SNCF proposal, which is limited to wavelengths between 2 m and 40 m [11]. In the current paper the wavelength range D1 defined by [12] is considered, limiting the analysis to wavelengths between 3 m and 25 m. The numerical process used to generate track irregularities can be consulted in [5]. In order to comply with the quality demands of EN13484-5 [12] the maximum standard deviation of the generated track irregularities profiles is limited to 1.5 mm over a track extension of 200 m.

Bearing in mind the bridge case study, variables that might have relevant nondeterministic properties which can lead to a significant change of the dynamic response were identified. This results in the selection of the random variables presented in Table 1. Similarly, the set of selected random variables for the track is specified in Table 2.

Variable	Distribution	Mean (gaussian) or Min. (uniform)	Std. Deviation (gaussian) or Max. (uniform)
Concrete density weight (γ_c)	Gaussian	2.5 t/m^3	0.1 (CV = 4%)
Concrete elasticity modulus (E_c)	Gaussian	36.1 GPa	2.888 (CV = 8%)
Concrete height (h_c)	Gaussian	Nominal value	10 mm
Concrete width (b_c)	Gaussian	Nominal value	5 mm
HEB 500 area (A_s)	Gaussian	Nominal value	0.04 x nominal area
Neoprene shear modulus (G)	Uniform	0.75 MPa	1.2 MPa
Structural damping (ζ)	Gaussian	2 %	0.3 %

Table 1: Bridge random variables.

Variable	Minimum	Maximum
Ballast density weight (γ_b)	1.5 t/m^3	2.1 t/m^3
Ballast elasticity modulus (E_b)	80 MPa	160 MPa
Ballast height (h_b)	0.30 m	0.60 m
Ballast load distribution angle (α)	15 º	35 º
Sleeper weight (M_s)	220 kg	325 kg
Rail pad stiffness (k_p)	100 kN/mm	600 kN/mm
Track shear resistance (R_t)	20 kN/m	60 kN/m
Irregularity amplitude (A)	160	275

Table 2: Track random variables.

Regarding the train, the dynamic behaviour of the bridge is assessed for the crossing of a TGV double high-speed train. This is an articulated train that is composed by a total of 20 coaches: four power cars, four power passenger cars and twelve passenger cars. The train model includes the car bodies, the bogies, the primary and secondary suspensions, the wheel-sets and also the wheel-rail contact stiffness. A schematic representation of the train and the corresponding numerical model is illustrated in Figure 3. A more detailed description of the train numerical model can be consulted in [5].

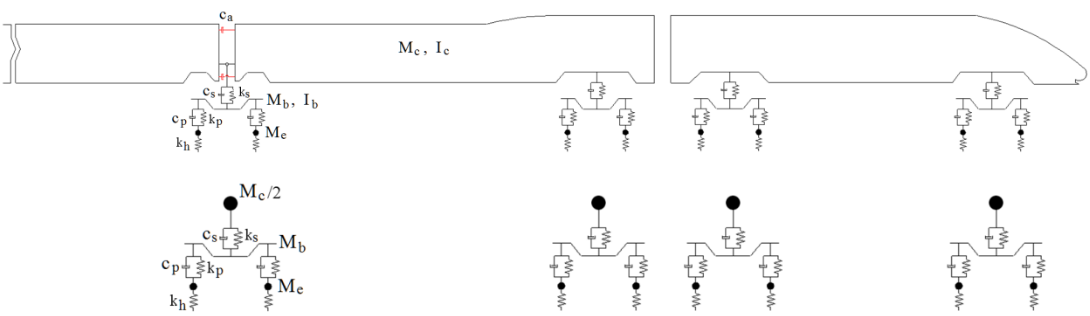


Figure 3: Schematic representation of the TGV double train numerical model.

Adopting the same principle that is considered for the bridge and the track, the random variables selected for the train model are indicated in Table 3. It should be mentioned that the dynamic properties of the train are defined according to the values presented by [10].

Variable	Distribution	Mean (gaussian) or Min. (uniform)	Std. Deviation (gaussian) or Max. (uniform)
Occupancy rate	Uniform	0 %	100 %
Bogie mass	Uniform	2.32 t	3.48 t
Wheel set mass	Uniform	1.6 t	2 t
Primary suspension stiffness	Uniform	1300 kN/m	3900 kN/m
Primary suspension damping	Uniform	6 kN.s/m	18 kN.s/m
Secondary suspension stiffness	Uniform	290 kN/m	870 kN/m
Secondary suspension damping	Uniform	10 kN.s/m	30 kN.s/m
Wheel-rail contact stiffness	Gaussian	1.61x10^6 kN/m	4.1x10^4 kN/m

Table 3: Train random variables.

4 SAFETY ASSESSMENT

In this section the safety assessment of the bridge is discussed. The different methodologies presented in Section 2 are used and the results compared in terms of efficiency. Both Monte Carlo (MC) and Latin Hypercube (LH) simulation are used. Furthermore, each of these simulation methods are combined with the Generalized Pareto Distribution (GPD) fit and also with the Enhanced Simulation method (ES) proposed by [3]. The safety of the train-track-bridge system is assessed through the analysis of the track instability due to excessive deck vibrations. The train-bridge system is considered to be safe if the estimated probability of failure is lower than 10^{-4}, which corresponds to a reference value in the Joint Committee on Structural Safety (JCSS) Probabilistic Model Code [13].

4.1 Track instability

The safety due to track instability is assessed by the bridge deck acceleration. For each simulation the maximum acceleration along the bridge deck is identified and selected as representative to assess the safety. The track instability is characterized by the loss of interlock between ballast grains, leading to the loss of the lateral resistance of the sleepers. Laboratory tests confirmed that the ballast instability only tends to occur for deck accelerations higher than 7 m/s^2 [14]. Below this limit the sleeper hardly moves and no loss of lateral resistance is observed. However, EN1991-2 [15] limits deck acceleration to a maximum of 3.5 m/s^2 for ballasted tracks, which indicates that a safety factor of 2 is applied. Since the safety of the train-bridge system is only at risk when deck accelerations are over 7 m/s^2, the probability of failure of the train-bridge system is established as the probability of the bridge deck reach this acceleration level.

It is observed that the obtained distributions are very similar for both simulation methods, as expected. However, the obtained distribution is not unimodal, due to the existence of two distinct types of response: resonant and non-resonant behaviour. This provides an interesting opportunity to check on how the complexity of the response affects the efficiency of the different methodologies. The estimated probabilities of failure for the different methods are summarized in Table 6, whereas Table 7 shows the required number of simulations to accurately assess the probability of failure due to track instability.

Speed (km/h)	MC	LH	GPD-MC	GPD-LH	ES-MC (C^- ; C^+)	ES-LH (C^- ; C^+)
275	0.20	< 0.1	0.05	0.14	0.08 (0.06 ; 0.31)	0.08 (0.03 ; 0.22)
280	0.40	0.50	0.58	0.70	0.44 (0.26 ; 1.47)	0.48 (0.24 ; 1.24)
285	1.40	1.33	1.32	1.03	1.45 (0.81 ; 3.55)	1.19 (0.82 ; 3.49)
290	5.80	2.67	5.17	4.93	4.40 (2.94 ; 8.17)	3.76 (2.11 ; 8.24)
295	10.30	8.33	13.00	7.81	10.80 (7.45 ; 21.1)	10.30 (8.21 ; 14.4)

Table 6: Estimated probability of failure due to track instability ($\times 10^{-4}$).

Speed (km/h)	GPD-MC	GPD-LH	ES-MC	ES-LH
275	8000	8000	3000	9000
280	14000	7000	3000	4000
285	15000	25000	3000	3000
290	6000	7000	3000	3000
295	5000	5000	2000	9000

Table 7: Number of simulations required to accurately assess the probability of failure due to track instability.

Compared to the GPD approach the ES proves to be more efficient, resulting in reducing the necessary number of simulations to slightly less than 30% (less 34,000 simulations) for the MC simulation and a little over 50% (less 24,000 simulations) for the LH simulation. Regarding the influence of the complexity of the limit state function it became evident that if the response is not monotonic the LH may cause problems thus affecting the efficiency of the ES procedure. Furthermore, it was shown that if the computational costs can be reduced through a refined simulation method then the use of the ES approach will result in even further benefits. Another important feature of the ES procedure that should be pointed out is the fact that it enables quantifying the uncertainties of the estimated probability of failure, thus providing an extra tool to assess the quality of the failure estimation. The GPD approach seems to be much more sensitive to the level of irregularity of the tail of the distribution rather than the complexity of the limit state.

Globally, the safety assessment due to excessive deck acceleration can be accurately analysed with 14,000 simulations by enhanced MC simulation for the critical speed range (275 – 295 km/h). In order to guarantee the track stability the train speed over the bridge should be limited to 280 km/h.

5 CONCLUSIONS

The current work is focused on assessing the efficiency of different probabilistic methodologies for the safety assessment of short span railway bridges. A composite bridge with six simply supported spans of 12 m and ballasted track was selected as case study for the crossing of the TGV-double high-speed train. The variability of parameters related to the bridge, the track and the train were taken into account along with the existence of track irregularities. The selected case study offers the possibility of analysing a failure modes that is characterized by a non-unimodal distribution limit state functions.

Two different simulation techniques (Monte Carlo and Latin Hypercube) are combined with two different procedures: a tail modelling approach based on the extreme value theory that uses the Generalized Pareto Distribution to model the upper tail of the obtained distribution, and an Enhanced Simulation technique which uses an approximation procedure based on the estimates of the failure probabilities at moderate levels for the prediction of the far tail failure probabilities.

The ES approach proves to be more efficient than the GPD, resulting in reducing the necessary number of simulations to slightly less than 30% (less 34,000 simulations) for the MC simulation and a little over 50% (less 24,000 simulations) for the LH simulation. It also became evident that if the response is not monotonic the LH may cause problems thus affecting the efficiency of the ES procedure. Furthermore, it was shown that if the computational costs can be reduced through a refined simulation method then the use of the ES approach will result in even further benefits. The ES procedure benefits from another important feature which is the fact that it enables quantifying the uncertainties of the estimated probability of failure. This provides an extra tool to assess the quality of the failure estimation. Regarding the GPD approach it seems to be much more sensitive to the level of irregularity of the tail of the distribution rather than the complexity of the limit state.

Globally, the safety assessment due to excessive deck acceleration can be accurately analysed with 14,000 simulations by enhanced MC simulation for the critical speed range (275 – 295 km/h). In order to guarantee the track stability the train speed over the bridge should be limited to 280 km/h.

REFERENCES

[1] Rubinstein, R.Y., *Simulation and the Monte Carlo method*. New York: John Wiley and Sons, 1981.

[2] Mckay M. D., Beckman R. J., Conover W. J., A comparison of three methods for selecting values of input variables in the analysis of output from a computer code. *Technometrics*, **21**, 2: 239–245, 1979.

[3] Naess, A., Leira, B.J., Batsevych, O., System reliability analysis by enhanced Monte Carlo simulation. *Structural Safety*, **31**, 349-355, 2009.

[4] Ramu, P., Kim, N. H., Haftka, R. T., Multiple tail median approach for high reliability estimation. *Structural Safety*, **32**, 124-137, 2010.

[5] Rocha, J. M., Henriques, A. A., Calçada, R., Probabilistic safety assessment of a short span high-speed railway bridge. *Engineering Structures*, **71**, 99-111, 2014.

[6] Rocha, J. M., Henriques, A. A., Calçada, R. Probabilistic assessment of the train running safety on a short span high-speed railway bridge. *Structure and Infrastructure Engineering*, http://dx.doi.org/10.1080/15732479.2014.995106, 2014.

[7] Castillo E., *Extreme value theory in engineering*. San Diego, USA: Academic Press, 1988.

[8] Rocha, J. M., Henriques, A. A., Calçada, R., Safety assessment of a short span railway bridge for high-speed traffic using simulation techniques. *Engineering Structures*, **40**, 141-154, 2012.

[9] Manterola J., *Puentes: apuntes para su diseño, cálculo y construcción*. Servicio de Publicaciones del Colegio de Caminos, Canales y Puertos, 2006 [in Spanish].

[10] ERRI D-214/RP 9. *Rail bridges for speeds > 200 km/h. Final report*. Utrecht: European Rail Research Institute (ERRI), 1999.

[11] Frỳba, L. Dynamics of railway bridges. London: Thomas Telford, 1996.

[12] EN13484-5. *Railway applications - Track - Track geometry quality - Part 5: Geometric quality levels*. European Committee for Standardization (CEN), Brussels, 2008.

[13] Joint Committee on Structural Safety (JCSS). *Probabilistic Model Code – 12th draft*, 2001. (http://www.jcss.ethz.ch/).

[14] Zacher, M., Baeβler, M., Dynamic behaviour of ballast on railway bridges. In: *Proceedings of Dynamics of High-Speed Railway Bridges*. London: Taylor & Francis, 2009.

[15] EN1991-2. *Eurocode 1 - part 2: Actions on structures - traffic load on Bridges*. European Committee for Standardization (CEN), Brussels, 2003.