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DEFINITION OF MECHANICAL PROPERTIES OF EXISTING MASONRY ACCOUNTING FOR EXPERIMENTAL KNOWLEDGE BY BAYESIAN UPDATING

Stefano Bracchi[1], Maria Rota[2], Andrea Penna[2,3] and Guido Magenes[2,3]

[1] ROSE Programme, UME School, Institute for Advanced Studies
via Ferrata, 1 I-27100 Pavia, Italy
stefano.bracchi@umeschool.it

[2] European Centre for Training and Research in Earthquake Engineering
via Ferrata, 1 I-27100 Pavia, Italy
maria.rota@eucentre.it

[3] University of Pavia, Department of Civil Engineering and Architecture
via Ferrata, 3 I-27100 Pavia, Italy
{andrea.penna, guido.magenes}@unipv.it

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Abstract. *The current approach of the Italian and European building codes to account for knowledge-based uncertainties in the seismic assessment of masonry buildings consists in selecting a knowledge level and reducing material strengths by means of the associated value of confidence factor. Previous works showed that, in the case of masonry structures, the codified approach leads in many cases to inconsistent and unrealistic results and does not properly consider the experimental tests performed. This paper proposes a methodology for taking into account the knowledge on material properties acquired by experimental tests, using Bayesian updating techniques. The use of the Bayesian approach allows to update the values of the material properties assumed a priori as knowledge on the building increases, by taking into account all the information (experimental or judgement-based) gained during the assessment process. The material parameters resulting from this method could be used as input in numerical models, with the aim of calibrating confidence factors on material properties. Finally, analytical expressions are provided, which approximate the results obtained by Bayesian updating, allowing the analyst to obtain the values of material properties to be used in the analysis, as a function of the experimental information gained, without the need of performing a case-by-case Bayesian updating.*

1 INTRODUCTION

The approach currently proposed by the European [1] and Italian [2][3] codes for the seismic assessment of an existing building consists in the definition of three discrete knowledge levels (KLs) and the association to each of them of a value of the so-called confidence factor (CF). The latter has to be applied to material strengths and it is supposed to account for all sources of uncertainty involved in the assessment. This approach was proven to provide in many cases inconsistent and unexpected results, at least for the case of nonlinear static analysis, for both reinforced concrete and stone masonry buildings [4][5].

Some attempts have been proposed in the literature to overcome the main limitations of the current code approach. For example, [6] proposed a probabilistic approach to the seismic assessment of masonry buildings considering all the uncertainties involved, whereas Cattari et al. [7] developed a procedure based on systematic use of a simplified sensitivity analysis, leading to the definition of confidence factors to be applied to the parameters most influencing the response. Neither of these approaches, however, specify how the available experimental results should be used to improve the definition of the material parameters to be adopted in the numerical analyses.

With respect to the aforementioned works, which do not account for the experimental information eventually gained, the Commentary to the Italian code [3] includes, at least for KL3, a rough procedure for updating the initial information on mechanical properties (based on the masonry typology selected at KL1 and KL2), as a function of the experimental value of strength measured at KL3. For example, in case one or two experimental values of strength are available, the Commentary suggests to use the maximum/mean value of the codified interval for the selected typology or the mean of the experimental results, depending on the location of the mean of the experimental results with respect to the proposed interval. However, this seems to represent only a first attempt of updating the available knowledge, since it is not applied to the case of KL2 and it only regards the material property that is directly measured (i.e. the shear strength at KL3).

Starting from these considerations, Rota et al. [8] proposed a probabilistic framework for the seismic assessment of existing masonry buildings, in which the effects of the different sources of uncertainty are explicitly taken into account by means of variability factors and of the confidence factor accounting for uncertainty in material properties. Calibration of the variability factors accounting for modelling uncertainties and for the uncertainty in the definition of limit state thresholds is presented in previous works [8][9]. The confidence factors accounting for uncertainty on material properties, CF_{mat}, take into account the imperfect knowledge on material parameters and, hence, they should explicitly consider any additional information, acquired for example by means of experimental tests. However, appropriate values of these CF_{mat} were not yet calibrated, nor a methodology allowing to take advantage of the experimental information was proposed. This work moves therefore from the need to develop a rigorous and efficient procedure for taking into consideration the additional information obtained from experimental tests at the higher levels of knowledge (KL2 and KL3). This methodology, based on Bayesian updating, allows integrating the additional information on material properties with the prior information. The adoption of the mechanical parameters deriving from Bayesian updating into appropriately defined numerical models will allow to calibrate the confidence factors on material properties and hence to complete the methodology proposed in [8].

Bayesian updating techniques have been recently applied to the problem of the assessment of existing buildings (e.g. [10], [11]), with the aim of updating the distributions of material parameters and the corresponding evaluation of structural reliability, as a function of the experimental tests and inspections performed. In some cases (e.g. [11], [12]), these techniques were applied to calibrate new values of confidence factors. However, the largest part of previous literature work was concerned with reinforced concrete buildings, whereas, to the author's knowledge, the only attempt to apply Bayesian updating with reference to a masonry structure was performed in [13], but with reference to a church. Furthermore, in previous works, a direct experimental information is always required to update the value of any mechanical property. Although this could be the case in reinforced concrete structures, where an experimental information is usually available for the mechanical parameters of interest (i.e. the strengths of both concrete and steel), this is rarely the case for masonry structures, for which very often only a single mechanical property is measured during the assessment process. This work is hence intended to propose a methodology based on Bayesian inference to update the knowledge on material properties taking advantage of the available experimental results. Based on the results obtained, analytical expressions will be calibrated, which provide the values of the material properties representing the input for the numerical model of the assessed building, as a function of the additional information gained through experimental tests. These equations would allow to take advantage of the results of a rigorous Bayesian updating approach, without the need to perform it on a case by case basis, with a view on the possible consideration of this approach in a future revision of the code.

2 PROPOSED PROCEDURE TO ACCOUNT FOR UNCERTAINTY IN MATERIAL PROPERTIES BY BAYESIAN UPDATING

As already mentioned, the aim of this work was to take into account the additional information obtained by in-situ tests for the definition of the material properties to be used in the seismic assessment at the higher levels of knowledge (KL2 and KL3). To this aim, a Bayesian updating framework was developed, allowing to modify the a-priori knowledge of the different mechanical parameters (represented by the so-called prior distributions), taking into account any additional information acquired either experimentally or by exploiting empirical correlations of material parameters with other parameters that have been directly measured. The result of the updating are the so-called posterior distributions.

The proposed procedure is discussed in the following, for the three knowledge levels considered in the Italian code [2][3]. It results in values of the different mechanical parameters, to be used as input in the numerical models of the buildings to be assessed.

2.1 KL1

At knowledge level 1, the knowledge of the structure is only based on a visual survey of the building, since no experimental test is required. The first step of the assessment consists in choosing a masonry typology among those proposed by [3]. In this work, only stone masonry typologies were considered. The possibility of selecting a typology that does not correspond to the actual typology of the building (as highlighted in [14]) was taken into account by defining, for each actual typology, a list of typologies that could be erroneously selected, with the corresponding probability of selection (Table 1).

At KL1, the values of the mechanical parameters to be used as input for numerical models can be taken as the central values of the intervals suggested by [3] for the selected masonry typology. It is recalled that [3] indicates to use the minimum value of the interval for the shear strength τ_0 and the masonry compressive strength f_m, but we would suggest to use the central value, for consistency with the other mechanical parameters and knowledge levels. Finally, for the unit weight of masonry, the single value given for the chosen typology could be used.

Actual typology	Description	Selected typology	Probability
1	Rubble stone masonry (pebbles, fieldstones, irregular stones)	1	90%
		2	10%
2	Undressed stone masonry with facing walls of limited thickness and infill core (sacco)	1	10%
		2	60%
		3	20%
		4	10%
3	Partially dressed stone masonry with good bonding (usually double-leaf)	2	20%
		3	60%
		4	20%
4	Soft stone masonry (tuff, limestone, etc.)	2	10%
		3	20%
		4	60%
		5	10%
5	Dressed rectangular stone masonry	4	10%
		5	90%

Table 1: Typologies that can be selected and probability of selection for the five stone masonry typologies of [3].

2.2 KL2

At knowledge level 2, in addition to the visual survey of the building, [3] recommends to perform at least one double flat-jack test, obtaining a direct experimental measure of the elastic modulus E. The experimental test was simulated by randomly selecting a structural member of one of the perfectly known structures[1] on which the test is performed (taking into account the different probability of choosing an internal vs. external wall, as well as a wall at a different storey) and extracting the experimental result from a uniform distribution defined on the interval identified by the exact value plus or minus the error intrinsic in this type of test, which was estimated as being plus or minus 15% of the actual result (based on experimental results, e.g. [15]). For further details about this simulation approach, the reader is referred to [5], in which a similar procedure was followed.

Four random variables were considered, consisting of elastic modulus E, compressive strength f_m, shear modulus G and shear strength τ_0 (shear strength associated with diagonal cracking to be used within the Turnsek-Sheppard strength criterion [16]). The four parameters considered in the application of the Bayesian approach consist of the mean of the distributions of these four random variables, i.e. μ_E, μ_{fm}, μ_G and $\mu_{\tau 0}$.

At KL2, it is assumed that a single observation of the random variable E is available, consisting of the experimental measure obtained by a single double flat-jack test (in case of multiple observations the procedure would not change significantly). This experimental observation will be simply indicated as E, whereas the parameters μ_{fm}, μ_G and $\mu_{\tau 0}$ will be indicated for simplicity as f_m, G and τ_0, respectively. Since observations of the other mechanical properties are not available, empirical correlations between the different parameters were exploited to derive the posterior distributions of the four parameters, starting from prior distributions and the single observation of E. The empirical correlation between f_m and μ_E was expressed by the variable α,

[1] The perfectly known structures represent the real building to be assessed and are defined with material properties varying within different sub-intervals of the intervals defined in [3] for the masonry typology of interest, taking into account element-to-element variability.

assumed to vary in the interval between 500 and 1000 and defined as the ratio between μ_E and f_m.

Since, for stone masonry, significant statistical analyses are not available in the literature, the interval of variation of α was assumed to be consistent with available data and dispersion for other masonry typologies (e.g. [17], [18]).

Similarly, the empirical correlation between the parameters G and μ_E was represented through the variable β, defined as the ratio between G and μ_E and assumed to vary between 0.15 and 0.40. The upper limit of the interval of β (0.4) corresponds to a Poisson's ratio equal to 0.25, a value commonly accepted for this type of material [19]. The lower limit (0.15) corresponds instead to a ratio E/G equal to 6, which is a value historically used and codified for masonry buildings [20]. It should also be noticed that the ratios implicit in the values reported in [3] are around 0.33 and thus they are included in the defined range.

Since a reliable empirical correlation between the shear strength τ_0 and one of the other parameters involved in this problem is not available in the literature, a linear correlation between τ_0 and μ_E was assumed, based on the intervals of variation reported in [3], under the assumption that, for the same masonry typology, if a material has for example a value of E higher than the central value of the interval, it is likely to have as well a value of τ_0 higher than the central value. For simplicity, this correlation can be represented by the variable η, defined as the ratio between μ_E and τ_0.

Four prior distributions were defined for μ_E, μ_α, μ_β and μ_η, which will be indicated as $\pi(\mu_E)$, $\pi(\alpha)$, $\pi(\beta)$, $\pi(\eta)$. Notice that μ_α, μ_β and μ_η were replaced by α, β and η and this notation will be used in the following for simplicity. The prior distribution of μ_E was defined as a normal distribution, with mean μ' equal to the central value of the interval given by [3] for the typology selected at KL1 and standard deviation σ' equal to the 25% of the width of the same interval. This value of standard deviation corresponds, for a normal distribution, to the assumption that about 95% of the values fall inside the considered interval of variation. As regards the random variable E, Bayesian inference was applied only to μ_E, assuming the standard deviation of E, σ, to be equal to 7.5% of the experimental observation of E obtained from the test, i.e. equal to one fourth of the width of the interval of variation of the error of the experimental test.

The prior distributions of α and β were assumed to be normal distributions, with mean equal to the central value of the interval of variation of each parameter and standard deviation equal to 1/8 of the width of this range of variation. The low value of standard deviation is necessary to limit the number of values falling outside the defined interval of variation during Monte Carlo sampling and to avoid the presence of values too far from the defined range. The prior distribution of η was calculated numerically, with values of μ_E extracted from its prior distribution and values of τ_0 extracted from a normal distribution with mean equal to the central value of the interval indicated by [3] and standard deviation equal to 1/4 of the width of the same interval.

The posterior distributions of the four parameters can be calculated numerically, starting from the joint posterior distribution conditioned to the experimental observation E of the random variable E, $\pi(f_m, G, \tau_0, \mu_E|E)$, given by:

$$\pi(f_m, G.\tau_0, \mu_E \mid E) = \frac{f(E \mid \mu_E) \cdot \pi(\mu_E, f_m, G, \tau_0)}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(E \mid \mu_E) \cdot \pi(\mu_E, f_m, G, \tau_0) d\mu_E dG df_m d\tau_0} \tag{1}$$

In the following it is assumed that the parameters μ_E and α (and similarly μ_E and β and μ_E and η) are independent and consequently uncorrelated, whereas the four mechanical parameters

are dependent and the correlation among them was taken into account by means of the empirical correlations previously discussed.
The posterior distribution of μ_E can be easily calculated from the prior distribution of μ_E, the observation E and the standard deviation σ. In particular, in case n observations are available, the posterior distribution is a normal distribution with mean and variance given as in Eq. (2):

$$\mu_E \mid E = N\left(\frac{(\sigma')^2}{\sigma^2/n + (\sigma')^2}\,\overline{E} + \frac{\sigma^2/n}{\sigma^2/n + (\sigma')^2}\,\mu', \frac{\sigma^2/n}{\sigma^2/n + (\sigma')^2}(\sigma')^2 \right) \tag{2}$$

where μ' and σ' are the mean and standard deviation of the prior distribution, $\overline{E}$ is the mean value of elastic modulus measured from the tests (equal to the measured E in case of a single test), σ is the standard deviation of the random variable E and n is the number of observations (in this case $n = 1$).

The posterior distributions of the other three parameters can be calculated numerically by extracting, at each step of a Monte Carlo procedure (with Latin hypercube sampling) a value of μ_E from its posterior distribution (Eq. (2)) and a value of α, β and η from their prior distributions. The corresponding values of f_m, G and τ_0 are hence obtained using the empirical correlations previously defined. The posterior distributions of the four considered parameters obtained at KL2 are reported in Figure 1.

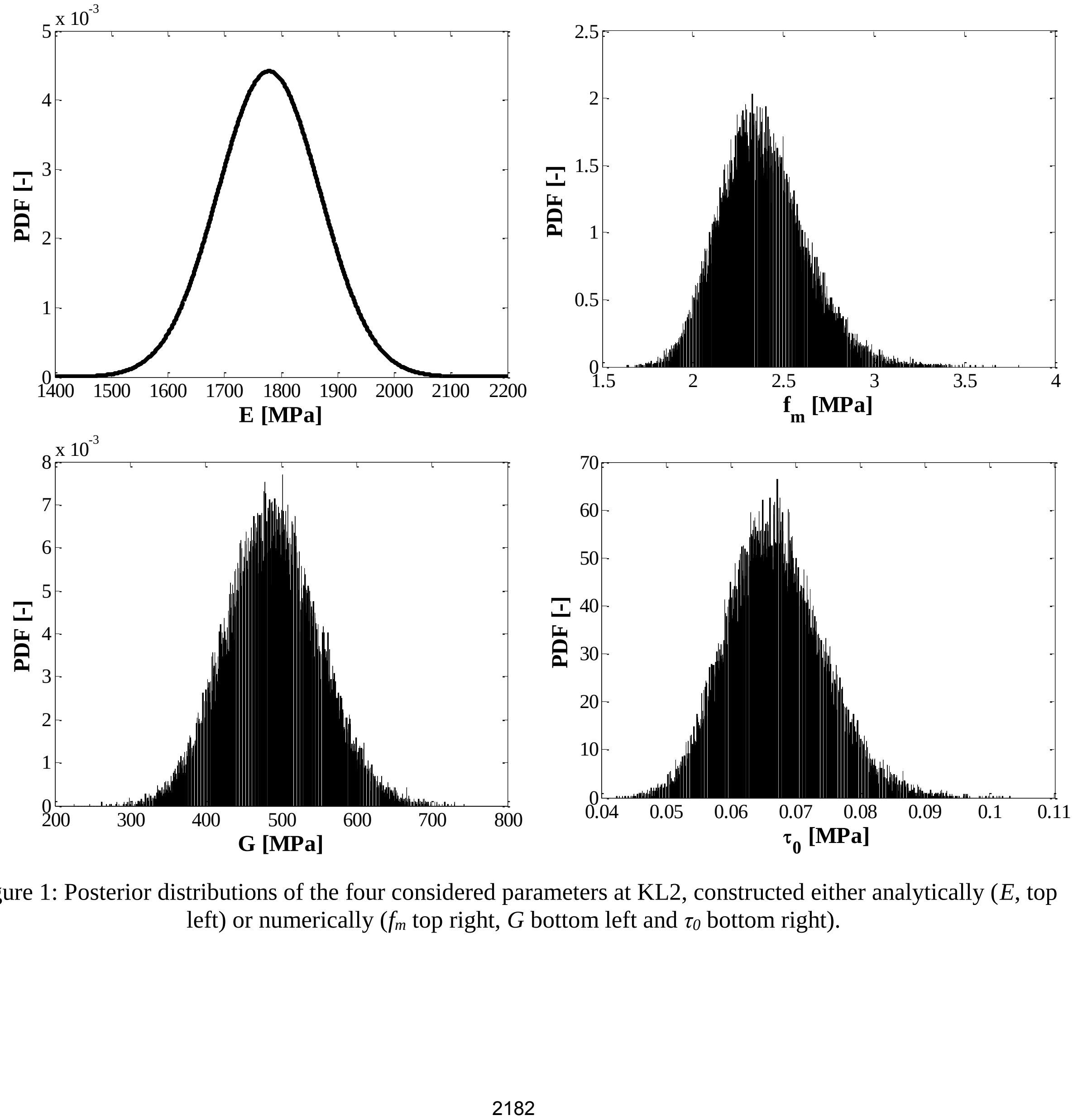

Figure 1: Posterior distributions of the four considered parameters at KL2, constructed either analytically (E, top left) or numerically (f_m top right, G bottom left and τ_0 bottom right).

2.3 KL3

At knowledge level 3, in addition to the visual survey and the test already performed at KL2, [3] requires the performance of additional experimental tests, such as diagonal compression tests. It was assumed that either one, two or three tests, with different probabilities, could be performed, providing a direct measure of the shear strength τ_0. Similarly to KL2, each experimental test was simulated by randomly selecting a structural member of one of the reference structures assumed to be perfectly known on which the test is performed (taking into account the different probability of choosing an internal vs. external wall, as well as a wall at a different storey) and considering an intrinsic error of plus or minus 20% of the actual result (based on [15]). The diagonal compression test theoretically allows to measure also the shear modulus G, but it is not easy to obtain a reliable measure of G and thus it is not very common to measure it.

At KL3, Bayesian updating was used to find the posterior distributions of the four mechanical properties, starting from the prior knowledge (represented by the posterior knowledge obtained at KL2) and from the available observations of τ_0. Similarly to KL2, prior distributions were introduced for the mean value of τ_0 and for the three parameters defining the empirical correlations between the different properties, i.e. α, β and η, defined as for KL2. The prior distribution of $\mu_{\tau 0}$ was modelled as a normal distribution, with mean equal to the mean of the numerically built posterior distribution of τ_0 at KL2 and standard deviation equal to the standard deviation of the same distribution divided by the square root of the ratio k between the number of double flat-jack tests and diagonal compression tests performed. This ratio, proposed in [12], allows to take into account the different reliability of the two types of test performed.

As regards the random variable τ_0, Bayesian inference was applied only to $\mu_{\tau 0}$, assuming the standard deviation of τ_0 equal to

$$\sigma = \frac{1}{n}\sqrt{\sum_{i=1}^{n}\sigma_i^2} \tag{3}$$

where n is the number of diagonal compression tests performed and σ_i is the standard deviation (defined based on the error intrinsic in this type of test) associated with the results of each test, which are assumed to be a random variable. The prior distributions of α and β were assumed equal to the ones used at KL2, whereas the prior distribution of η was defined as done at KL2, with the only difference that the values of E and τ_0 were extracted from the posterior distributions obtained at KL2.

As for KL2, the parameters $\mu_{\tau 0}$ and α (and similarly β and η) were assumed to be independent and consequently uncorrelated, whereas the correlation between the four mechanical parameters was taken into account. The joint posterior distribution of the four parameters $\mu_{\tau 0}$, f_m, G and E, conditioned to the experimental observations of τ_0 was hence derived. Using the formulas of the Bayesian inference already employed at KL2 (Eq. (2)), with the prior distribution of $\mu_{\tau 0}$ previously defined, the posterior distribution of $\mu_{\tau 0}$ turned out to be a normal distribution with mean equal to

$$\mu'' = \frac{\tau_0 \cdot \zeta + k \cdot \mu'}{\zeta + k} \tag{4}$$

where

$$\zeta = \left(\frac{\sigma'}{\sigma}\right)^2 \tag{5}$$

In the previous equations μ' and σ' indicate the mean and standard deviation of the prior distribution, before dividing the standard deviation by the square root of k, τ_0 is the mean of the observations and σ is the standard deviation of τ_0. This formula corresponds to the one used in [12] for combining data from different types of tests. The standard deviation of the posterior distribution was instead given by:

$$\sigma'' = \sqrt{\frac{\sigma^2}{k\sigma^2 + (\sigma')^2}(\sigma')^2} \tag{6}$$

Similarly to KL2, the posterior distributions of the other three parameters could be calculated numerically by extracting a value of $\mu_{\tau 0}$ from its posterior distribution (Eq. (4) and (6)) and a value of α, β and η from their prior distributions. The corresponding values of f_m, G and E were hence obtained using the empirical correlations previously defined for KL2. For both KL2 and KL3, the values of mechanical properties to be used as input for the numerical model of the building to be assessed should be taken as the expected (mean) values of the different posterior distributions obtained after Bayesian updating. It should be remarked that [3] also provides (single) values of the weight of the masonry, for each masonry typology. This parameter was not considered in the Bayesian updating, mainly because it is not common to measure it experimentally and it is impossible to define a correlation between the weight and any other mechanical property.

2.4 Considerations on the dispersion of the mechanical parameters

As already discussed, some of the posterior distributions were calculated numerically using a Monte Carlo approach. The number of Monte Carlo simulations was selected as to guarantee stability of the mean of the four mechanical properties of interest. Furthermore, if the obtained parameters were to be used in numerical analyses of a building, the proposed Bayesian approach would need to be applied a number of times sufficient to reach stability of the mean and standard deviation of the parameter representing the result of the assessment procedure (e.g. the acceleration leading the building to a limit state of interest).

The effectiveness of the proposed Bayesian approach was evaluated by looking at the distribution of values of the four mechanical properties representing the input for the numerical model. As an example, Figure 2 shows the distributions of E and τ_0 obtained for one of the considered case study buildings, for KL2 and KL3. The grey bars in the figure represent the values obtained by applying the proposed procedure, the shaded areas indicate the range of variation of the mechanical properties of the perfectly known structure and the continuous black lines denote the mean values obtained for the considered property. The continuous grey lines and the dashed grey lines represent the mean plus or minus one and two standard deviations of the obtained results, respectively. It is recalled that the perfectly known structure represents the real structure to be assessed, in which element-to-element variability of the mechanical properties within some specified ranges is taken into account. Figure 2 shows that, moving from KL2 to KL3, the dispersion of E and τ_0 decreases, the distribution and its mean value shift towards the range of values of the reference structure and a larger fraction of the values tends to be included in the range of the reference structure. Therefore, the applied Bayesian updating framework appears to be effective.

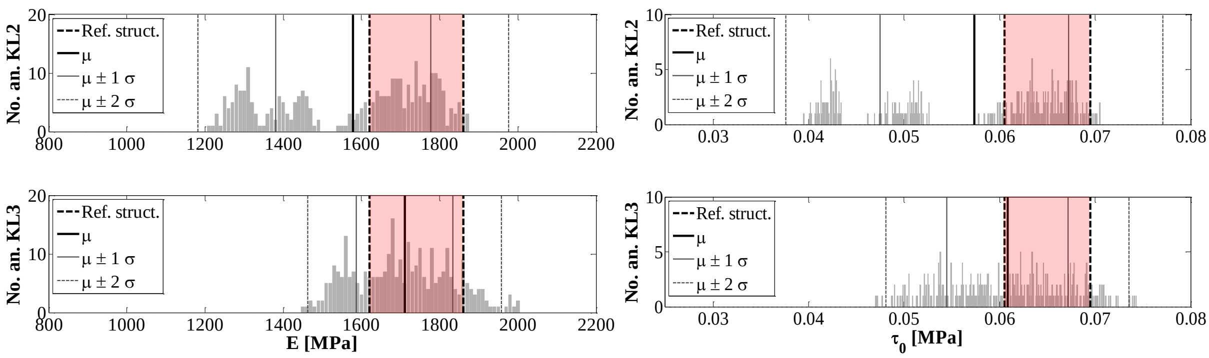

Figure 2: Distribution of E (left) and τ_0 (right) obtained for one of the case study buildings, at KL2 (top) and KL3 (bottom). The shaded areas indicate the range of variation of the mechanical properties of the perfectly known structure, continuous black lines indicate the mean values, whereas continuous and dashed grey lines indicate mean plus or minus one and two standard deviations, respectively.

3 ANALYTICAL EXPRESSIONS PROVIDING VALUES OF THE MATERIAL PROPERTIES AS A FUNCTION OF EXPERIMENTAL RESULTS

For each of the five stone masonry typologies considered in [3], the results of the proposed Bayesian approach were used to calibrate equations providing the values of mechanical properties representing the input for structural analyses, as a function of the experimental information gained at KL2 and KL3.

At KL2, for each typology, an interval of variation of the experimental value of E (E_{exp}) was defined as to cover all the intervals of variation of E corresponding to the typologies which can be confused with the considered typology, accounting also for the intrinsic error of the experimental test. This interval was then discretized into sufficiently small sub-intervals and the central value of each sub-interval was considered as a possible result of the double flat-jack test carried out at this knowledge level. Bayesian approach was then applied, using the prior distributions previously defined and the central value of each sub-interval as experimental observation of E. For each value of E_{exp}, Bayesian updating provided four values of the mechanical parameters representing the input for the analyses, as shown in Figure 3 (referring to typology 2 of Table 1, selected as an example). The shaded areas in the figure indicate the intervals used for the definition of the prior distribution of μ_E. As expected, when the experimental value of E is lower than the mean of the prior distribution of μ_E , the input value of E is larger than E_{exp}, whereas when E_{exp} is larger than the mean of the prior, the input value of E is lower than E_{exp} .

At KL3, for each typology, the experimental value of E was defined with the same sub-intervals used at KL2. The interval of variation of the experimental value of τ_0 ($\tau_{0,exp}$) was discretized into sub-intervals and it was defined as to cover all the intervals of variation of τ_0 corresponding to the typologies which can be confused with the considered typology. Similarly to KL2, the central value of each sub-interval was considered as a possible result of the experimental test. The Bayesian approach was then applied for each couple of experimental values of E and τ_0, using a prior distribution of τ_0 equal to the posterior distribution obtained at KL2. Figure 4 shows the input values of the mechanical properties, as a function of the experimental values of both E and τ_0.

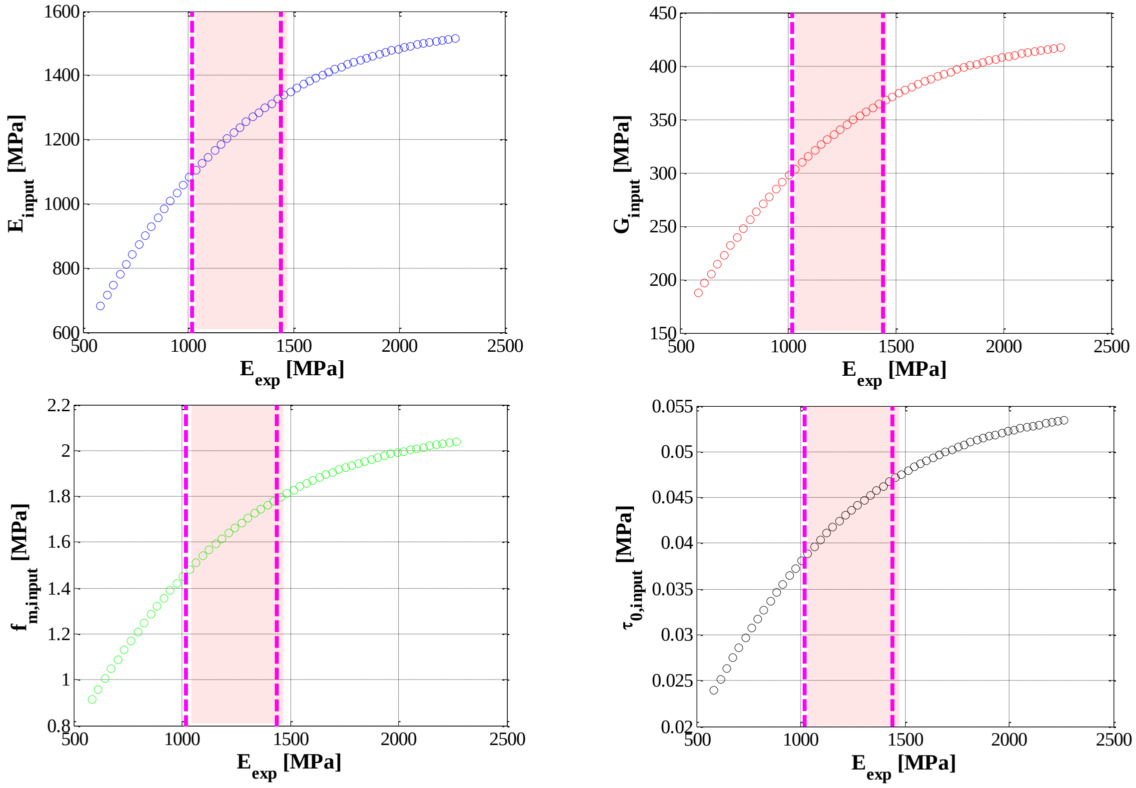

Figure 3: Input values of the four mechanical properties obtained from Bayesian updating at KL2 for typology 2.

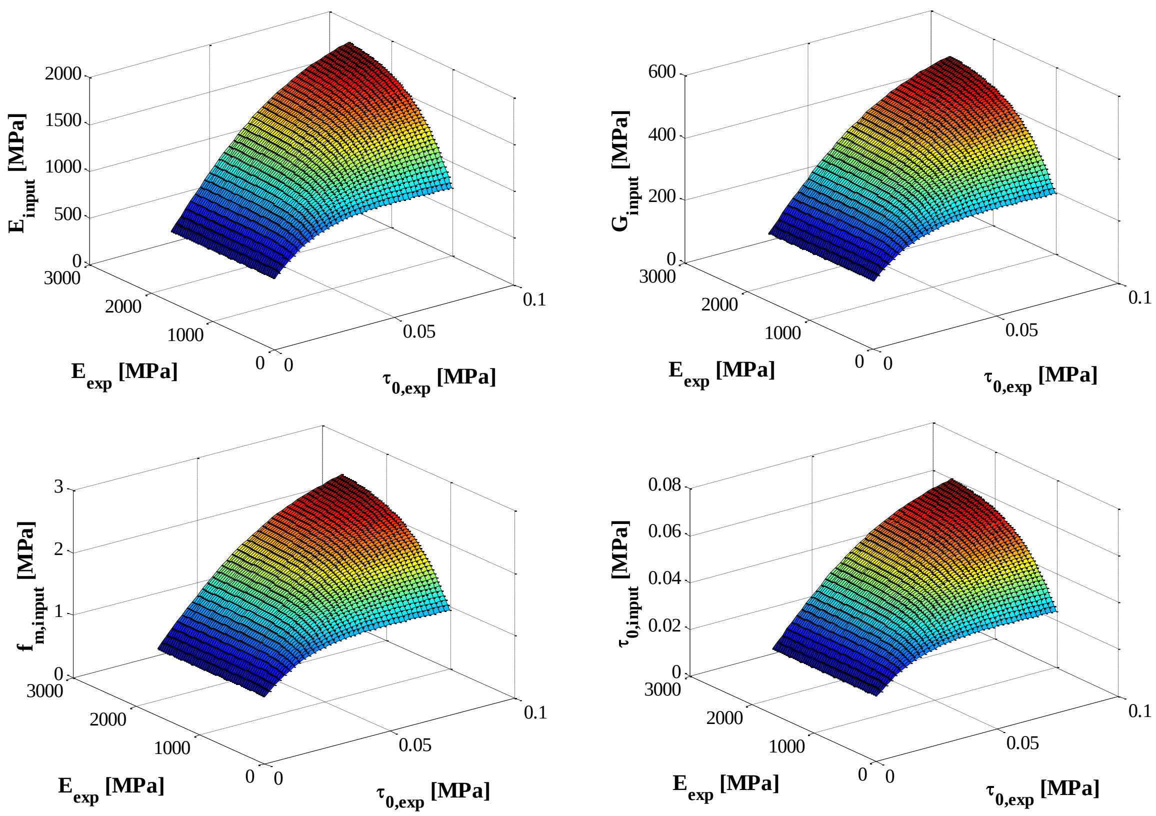

Figure 4: Input values of the four mechanical properties obtained from Bayesian updating at KL3 for typology 2.

The curves and the surfaces obtained from Bayesian updating at KL2 and KL3 were then fitted using a simple mathematical model, in order to find analytical equations providing the values of the mechanical properties representing the input for the structural model, as a function of the obtained experimental values of E and τ_0.

As regards KL2, a 2nd order polynomial provides a good fitting for all the typologies. It should be remarked that, although the use of a polynomial of a higher degree always provides a better fitting of the available data, a compromise has to be found between the goodness-of-fit and the number of coefficients defining the polynomial equation. In this work, it was decided to use a polynomial of low degree in order to limit the number of coefficients. The model adopted for the fitting is thus represented by the following equation:

$$y = p_1 \cdot x^2 + p_2 \cdot x + p_3, \tag{7}$$

where y is the a posteriori value, x is the value of the observation and p_1, p_2 and p_3 are the polynomial coefficients specific for the considered typology.

The parameters calibrated for typology 2, selected as an example, are reported in Table 2, together with the coefficient of determination R^2, which represents a classically adopted measure of the goodness of the fitting (a value closer to 1 indicates a better fitting).

Mechanical property	p_1	p_2	p_3	R^2
E	-3.24E-04	1.39E+00	-4.02E+00	0.998
G	-8.90E-05	3.83E-01	-1.10E+00	0.998
f_m	-4.35E-07	1.87E-03	-5.41E-03	0.998
τ_0	-1.14E-08	4.91E-05	-1.44E-04	0.998

Table 2: Parameters of the models fitting the data and values of R^2 at KL2 for typology 2.

As regards KL3, a 2nd order polynomial surface was selected for all the typologies, in order to limit the number of coefficients, as explained for KL2. The adopted model is now represented by the following equation:

$$y = p_{00} + p_{10} \cdot x_1 + p_{01} \cdot x_2 + p_{20} \cdot x_1^2 + p_{11} \cdot x_1 \cdot x_2 + p_{02} \cdot x_2^2, \tag{8}$$

where x_1 and x_2 are the values of the observations at KL2 and KL3, respectively.

The polynomial coefficients p_{00}, p_{10}, p_{01}, p_{20}, p_{11}, and p_{02} calibrated for typology 2 are reported in Table 3, together with the corresponding values of the coefficient of determination R^2.

Mech. property	p_{00}	p_{10}	p_{01}	p_{20}	p_{11}	p_{02}	R^2
E	-2.77E+02	2.67E+04	5.94E-01	-2.24E+05	8.57E+00	-2.68E-04	0.990
G	-7.61E+01	7.34E+03	1.63E-01	-6.17E+04	2.36E+00	-7.36E-05	0.990
f_m	-3.72E-01	3.59E+01	7.98E-04	-3.01E+02	1.15E-02	-3.59E-07	0.990
τ_0	-9.57E-03	9.21E-01	2.05E-05	-7.74E+00	2.96E-04	-9.25E-09	0.990

Table 3: Parameters of the models fitting the data and values of R^2 at KL3 for typology 2.

In order to check the efficiency of the fitting, the percentage differences between the values of the mechanical properties obtained from Bayesian updating and the ones obtained from the calibrated equations were calculated. For the considered typology, the mean values of these percentage differences are approximately equal to 0.7%, with a maximum of about 3%, for KL2 and 3%, with a maximum of 29%, for KL3. Although the maximum value is significantly larger at KL3, the fitting is still acceptable, since this maximum difference is concentrated only at the corners of the surfaces, as confirmed by the low mean values.

The calibrated equations could be useful to provide the input values of the mechanical properties, updated as a function of the experimental information, without requiring the analyst to perform the Bayesian updating previously described.

4 CONCLUSIONS

This paper proposed a procedure for updating the initial knowledge of mechanical parameters, taking into account additional experimental information by means of a Bayesian approach. In particular, the procedure moves from the selection, performed at KL1, of one of the five stone masonry typologies proposed by [3] and the definition of the prior information based on the intervals provided for each typology. At the higher knowledge levels, this prior information is updated based on the results of the performed experimental tests. The mean values of the obtained posterior distributions could be used as input for the numerical model of the building to be assessed.

For each masonry typology and for each KL, the developed methodology was then used to calibrate analytical expressions providing the input parameters for the numerical model, as a function of the results of the experimental tests. These relationships could be used by the engineer during the assessment of an existing building in order to obtain the material properties to be used in the analysis of the building, taking into proper consideration the results of the Bayesian updating approach, without the need of applying it on a case-by-case basis.

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