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ROBUST VIBRATION SERVICEABILITY DESIGN OF A TUNED MASS DAMPER FOR THE PHÉNIX FOOTBRIDGE

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Abstract. *Slender structures such as footbridges may be prone to human induced vibrations such that vibration mitigation devices as TMD's are adopted to increase the structural damping. Both the prediction of the structural response and the tuning of the TMD parameters rely on the modal parameters of the footbridge. These parameters are subjected to uncertainty in design stage, when only an estimation can be made for example using finite element models. After construction, the natural frequency and damping ratio can be measured but variations due to environmental effects such as temperature can result in parameter variations.*

Therefore it is important to take into account these uncertainties for the vibration serviceability assessment and the design of the TMD.

The present paper proposes a robust TMD design which adopts a worst case approach to take into account uncertainties in design stage. The proposed approach is illustrated for the Phénix footbridge. Considering an uncertain natural frequency and damping value for the case study, a worst case approach is adopted to determine the optimal values of the TMD mass, stiffness and damping.

A significant difference is found between the optimal TMD parameters of a nominal and robust tuned TMD. The mass and damping ratio of the robust TMD are found to be much higher than for the TMD tuned at nominal values of the natural frequency and damping ratio. This ensures that the comfort constraints are satisfied in all possible cases.

1 INTRODUCTION

The upcoming of advanced design methods and high strength materials enables the design of slender footbridges characterised by low natural frequencies, possibly in the range of loading frequencies induced by human walking. Recently, several vibration serviceability problems were reported [1, 2].

Currently, the Sétra [3] and HiVoSS [4] guidelines are often used to evaluate the dynamic behaviour of footbridges in design stage. The acceleration levels of the footbridge are evaluated based on calculations with assumptions for both the dynamic behaviour of the construction and the dynamic load.

If the calculated vibration levels exceed the required comfort threshold, a Tuned Mass Damper (TMD) can be included as a passive vibration control device. The TMD serves as an energy absorber and is characterised by its mass, stiffness and damping parameter. The tuning of the TMD parameters is based on the modal parameters of the main structure.

Uncertainties in the modal parameters of the bridge strongly influence the predicted response. Moreover, a TMD tuned for the nominal values of the modal parameters may result in an ineffective response reduction. Therefore, the authors suggest to take into account uncertainties in the modal parameters for the TMD tuning.

In the last decades, two types of optimisation methods were developed to deal with uncertainties in design. A first one is the reliability-based optimisation method (RBO) which optimises the design variables under uncertain conditions to reach a predefined target reliability. A second approach is a robust design optimisation (RDO) resulting in a design less sensitive to the effect of uncertainties [5, 6, 7]. A reliability-based robust design optimisation problem (RBRDO) combines the two aforementioned methods [8].

To obtain a robust design of the TMD that guarantees a prescribed response level despite of the considered uncertainties, a worst case approach is here adopted. This approach is illustrated for the Phénix footbridge in Charleroi. The present contribution compares a TMD tuned at nominal values of the modal parameters with the robust TMD. The paper focuses on the TMD optimisation in design stage. After construction, the uncertainty interval on the modal parameters can be reduced with measurements. A TMD designed in post-construction stage thus may differ from the presented. Also here variations due to environmental effects such as temperature may be expected.

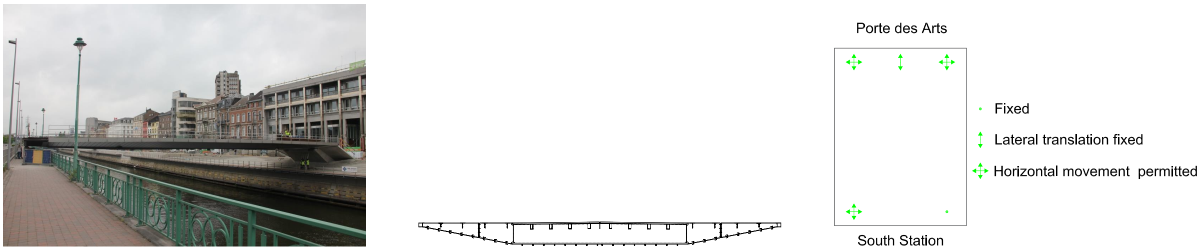
The structure of the paper is as follows. The Phénix footbridge is presented and its vibration serviceability is assessed following the Sétra and HiVoSS guidelines. Subsequently, a deterministic optimisation problem is formulated to determine the parameters of the TMD. The effect of uncertainties on the modal parameters of the footbridge is studied by considering a variation of the natural frequency and damping ratio within a range reasonably expected in design stage. The next section proposes a robust design optimisation of the TMD to deal with these uncertainties. Finally the optimal TMD parameters and the response reduction obtained for the different designs are compared.

2 THE PHÉNIX FOOTBRIDGE - CHARLEROI

2.1 Description of the footbridge

The Phénix footbridge is a slender construction with a single span of 38.25 m and a width of 13.35 m and is situated nearby the railway station of Charleroi (Belgium). The cross section of the bridge consists of three parts. The midspan is a main box of 6.5 m by 1 m with on both sides, cantilevered tapering I-profiles that are mounted on fixed distances. Figures 1(a) and 1(b) show a global view of the footbridge and the cross section of the structure respectively.

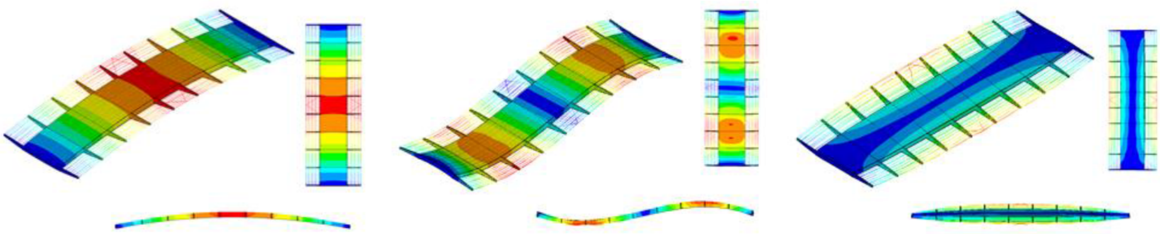
The bridge is supported by four neoprene bearings. Vertical translations are fixed at all corners but horizontal translations are fixed at one. An additional support is added in the middle of one side of the bridge to avoid lateral movements. An overview of the support conditions is given schematically in figure 1(c).



(a) (b) (c)

Figure 1: (a) The Phénix footbridge in Charleroi, (b) Cross section, (c) Support conditions.

The dynamic behaviour of the construction is defined by its modal parameters. The mode shapes as calculated by an initial finite element model (FE-model) are given in figure 2. The first mode has a natural frequency of 1.65 Hz which is in the range of the loading frequencies of the walking load because its natural frequency is lower than 5 Hz.



(a) (b) (c)

Figure 2: Modal parameters of the Phénix footbridge based on the initial FE-model: (a) 1st bending mode ($f_1 = 1.65$ Hz), (b) 2nd bending mode ($f_2 = 5.22$ Hz), (c) 1st torsional mode ($f_3 = 6.15$ Hz)

2.2 Vibration serviceability assessment

For lively footbridges, a vibration serviceability assessment is often performed following the guidelines [3, 4]. The maximal acceleration response is predicted for all sensitive modes assuming resonance conditions disregarding contributions from other modes. A human induced loading for different pedestrian densities is therefore considered. Both the dynamic load and the dynamic behaviour of the structure are discussed next.

2.2.1 Dynamic load

The walking force is considered as a periodic load and thus written as a Fourier series:

$$F_e(t) = G + \sum_{h=1}^{n_h} G\alpha_{eh} \sin\left(2\pi h f_s t - \theta_h\right) \tag{1}$$

with $F_e(t)$ the walking force in direction e (vertical, lateral or longitudinal) in the time domain, G the weight of the pedestrian, h the number of the harmonic load component, α_{eh} the dynamic load factor in direction e for the h^{th} harmonic, f_s the step frequency and θ_h the phase shift for harmonic h. Only the first two harmonic components ($n_h = 2$) are taken into account in the evaluation by the guidelines.

Sétra and HiVoSS simplify the forces due to a group of N random pedestrians into an equivalent load which is uniformly distributed on the bridge deck. Therefore, different traffic classes were introduced representing different pedestrian densities. In table 1, an overview is given of the classes for Sétra and HiVoSS for different pedestrian densities.

	Pedestrian density d [#pers./m^2]					
	15 pers.	0.2	0.5	0.8	1.0	1.5
Sétra			Class III	Class II	Class I	
HiVoSS	TC1	TC2	TC3		TC4	TC5

Table 1: Sétra and HiVoSS traffic classes and corresponding pedestrian densities.

The proposed load model consists of an equivalent number of perfectly synchronised pedestrians N_{eq}. A distinction is made between sparse and dense pedestrian densities. For low pedestrian densities, free movements of the pedestrians are possible and the equivalent number depends on the structural damping ξ_j. For higher densities, the walking behaviour of the pedestrians is obstructed and the levels of synchronisation increase. The higher level of synchronisation results in an increased number of equivalent pedestrians. N_{eq} can be found as follows:

$$N_{eq} = 10.8\sqrt{\xi_j N} \qquad \text{for } d < 1\,\text{pers./m}^2 \tag{2}$$

$$N_{eq} = 1.85\sqrt{N} \qquad \text{for } d \geq 1\,\text{pers./m}^2 \tag{3}$$

The amplitude of the equivalent uniformly distributed load $q_{eq,e}$ $[\text{N/m}^2]$ in direction e is calculated as:

$$q_{eq,e} = \frac{N_{eq}}{S}\alpha_{eh} G \psi_{eh}(f_j) \tag{4}$$

with S the surface of the bridge deck and ψ_{eh} a reduction factor that brings into account the possibility of resonance as a function of the natural frequency f_j of the bridge. The reduction factor depends on the direction of the force (longitudinal, lateral or vertical). Figure 3 shows ψ_{eh} for vertical vibrations as a function of the natural frequency of the system.

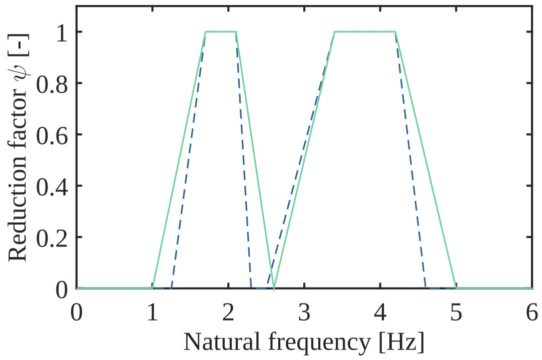


Figure 3: Reduction factor $\psi(f_j)$ for vertical vibrations according to the guidelines Sétra (–) and HiVoSS (- -).

2.2.2 Dynamic behaviour of the construction

The dynamic behaviour of the construction is determined by its modal parameters (natural frequency, mode shape and damping ratio). In design stage, the natural frequencies and mode shapes can be calculated by a FE-model but the damping ratio must be estimated based on prior experience with similar structures. Suggested values for the damping ratio are given by the guidelines. After construction, the modal parameters can be estimated from measurements.

For different pedestrian densities, the natural frequencies of the structure change due to the additional mass of the pedestrians. The guidelines prescribe that, for each traffic class, the natural frequencies and mode shapes must be extracted from a modified FE-model which takes into account the additional mass. For each of these calculated frequencies, the possibility of resonance with the pedestrian's step frequency is evaluated.

If resonance with the first or second harmonic of the load is likely to occur, a response calculation is required. Higher harmonic components of the walking force are not considered in the guidelines.

2.2.3 Response calculation and evaluation

The maximum acceleration $\ddot{u}_{j,e,max}$ for mode j in direction e is calculated as follows:

$$\ddot{u}_{j,e,max} = \frac{q_{eq,e} \sum_i S_i \left|\phi_{j,p(i)e}\right|}{2\xi_j} \max_{p(i)} \left|\phi_{j,p(i)e}\right| \tag{5}$$

with S_i the discretisation of the bridge deck, $p(i)$ the position on the bridge, $\phi_{j,pe}$ the modal displacement on position p in direction e of the j-th mode shape. The latter formula is equivalent to the response calculation assuming resonance for an SDOF-system as follows:

$$\ddot{u}_{max} = \frac{F_{ext}}{2\xi_j m_j} \tag{6}$$

with the following parameters describing mode shape j as an equivalent SDOF-system,

$$m_j = \frac{1}{(\max_{p(i)} \left|\phi_{j,p(i)e}\right|)^2}, k_j = (2\pi f_j)^2 m_j, c_j = 2\xi_j m_j (2\pi f_j) \tag{7}$$

and F_{ext} the amplitude of the external force assumed by the guidelines. Combining equation (5) and (6) allows to calculate F_{ext} as next:

$$F_{ext} = \frac{q_{eq,e} \sum_i S_i |\phi_{j,p(i),e}|}{\max_{p(i)} |\phi_{j,p(i)e}|} \tag{8}$$

Afterwards, the predicted vibration levels are evaluated by means of a comfort scale (maximal, mean, minimal comfort or unacceptable). Note that the vibration serviceability assessment must be performed for the different pedestrian densities considering all excited modes.

2.3 Vibration serviceability assessment for the Phénix footbridge

Because of the low natural frequency of the first mode (f_1 <5 Hz), resonance with the first or second harmonic of the walking force must be considered. For higher natural frequencies, the guidelines assume no risk for resonance with the walking force. Since the Phénix footbridge is a steel structure, the assumed damping ratio is 0.4% as proposed for steel structures [3, 4].

The results of the vibration serviceability assessment for the first mode are given in figure 4. For low pedestrian densities, minimal comfort is guaranteed. In table 2, the acceleration levels are summarised for the different traffic classes of both guidelines. The highest accelerations are predicted for a pedestrian density of $1\,\mathrm{pers./m^2}$ corresponding to Sétra class I.

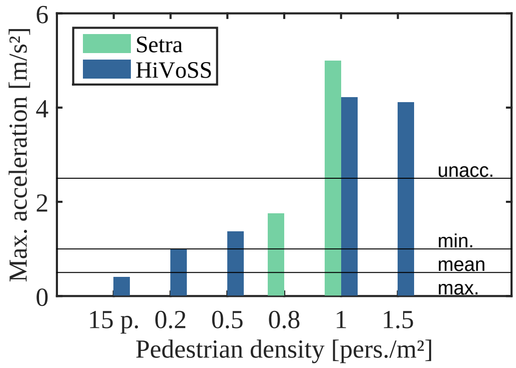


Figure 4: Vibration serviceability assessment of the first bending mode of the Phénix footbridge according to Sétra and HiVoSS (based on initial FE-model).

$d\,[\#\mathrm{pers./m^2}]$	15 pers.	0.2	0.5	0.8	1	1.5
Sétra	/	/	0	1.75	5.00	/
HiVoSS	0.41	0.99	1.37	/	4.22	4.12

Table 2: Predicted acceleration levels $\ddot{u}_{\max}\,[\mathrm{m/s^2}]$ for the different pedestrian densities.

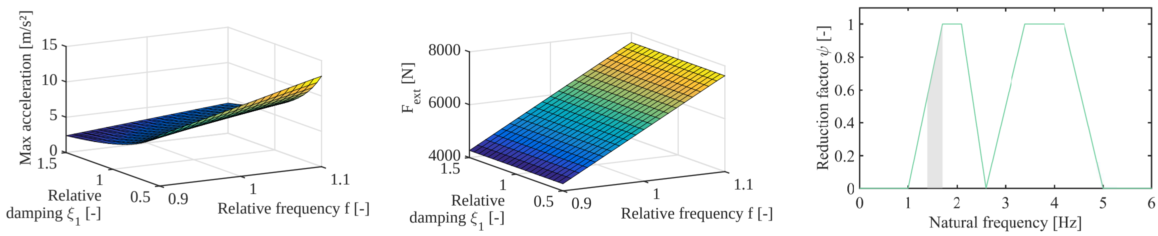
2.4 Effect of uncertainties on the predicted response

The accuracy of the vibration serviceability assessment strongly depends on the knowledge of the structure's modal parameters. The predictions are however highly sensitive to changes in the natural frequencies and damping ratios.

The influence of deviations in the values of the natural frequency and structural damping is illustrated for the Phénix footbridge. For the natural frequencies, a range of 0.9 to 1.1 times the nominal value is considered in line with deviations recently observed in a large number of case studies [9]. For the damping ratio, a larger interval of 0.5 to 1.5 times the proposed value of 0.4% is adopted. The same uncertainty range will be assumed in all analyses next:

$$\begin{aligned} 0.9\, f_{j,\text{nom}} \leq f_j \leq 1.1\, f_{j,\text{nom}} \\ 0.5\, \xi_{j,\text{nom}} \leq \xi_j \leq 1.5\, \xi_{j,\text{nom}} \end{aligned} \tag{9}$$

The influence of the assumed uncertainty is investigated for the Phénix footbridge considering Sétra Class I. Figure 5(a) shows that the maximal acceleration predicted by the guidelines is very sensitive to the deviations in the natural frequency. The influence of the uncertain frequency on the external load F_{ext} is given in figure 5(b). The load F_{ext} does not depend on the structural damping because of the calculation of N_{eq} for $d \geq 1\,\text{pers./m}^2$ (see equation (3)). The load F_{ext} depends on the natural frequency mainly through the reduction factor ψ_{eh} for the considered frequency range (figure 5(c)) since all other terms are frequency independent in this case.



(a) (b) (c)

Figure 5: Sétra - Class I (1p./m^2): (a) Maximal response for structure without TMD as a function of uncertain natural frequency and damping ratio. (b) Maximal equivalent force F_{ext} as a function of uncertain natural frequency and damping ratio. (c) Frequency dependent load reduction factor for Sétra class I with considered uncertainty range in filled area.

3 DETERMINISTIC DESIGN OPTIMISATION OF TMD

In order to reduce the vibration levels and to satisfy the threshold for pedestrian comfort as described in section 2.2, a TMD is added for the Phénix footbridge. The present section discusses a design of the TMD tuned at nominal values disregarding uncertainties on the modal parameters of the footbridge.

3.1 Deterministic problem formulation

In designs, a trade off must be made between performance and cost-efficiency. The design of the TMD can be described as an optimisation problem where a cost function is to be minimised satisfying a set of constraints. A linear relation between the total cost and the TMD mass m_{TMD} is assumed. The objective function is therefore given by the TMD mass. A limit value of $1\,\text{m/s}^2$ is chosen as the maximal allowed vertical acceleration $\ddot{u}_{\text{comfort}}$ and is implemented as a constraint. The response is calculated according to equation (5). Second, a constraint is added to limit the relative displacement of the TMD to a tolerable value Δ_{tol} for which a value of $0.05\,\text{m}$ is assumed. An upper and lower limit for the stiffness k_{TMD} and damping c_{TMD} of the TMD is included to ensure buildability. This leads to the following formulation of the optimisation problem:

$$\begin{aligned} \underset{m_{\text{TMD}},k_{\text{TMD}},c_{\text{TMD}}}{\text{minimise}} \quad & m_{\text{TMD}} \\ \text{subject to} \quad & \ddot{u}_{\text{max}} < \ddot{u}_{\text{comfort}} \text{ (nominal)} \\ & \Delta_{\text{rel}} < \Delta_{\text{tol}} \text{ (nominal)} \\ & k_{\text{min}} \leq k_{\text{TMD}} \leq k_{\text{max}} \\ & c_{\text{min}} \leq c_{\text{TMD}} \leq c_{\text{max}} \end{aligned} \tag{10}$$

with the limit values of the constraints summarised in following table 3. The stiffness and damping constraints are determined in a realistic range so they do not affect the optimal mass.

$\ddot{u}_{\text{comfort}}[\text{m/s}^2]$	1.00
$\Delta_{\text{tol}}[\text{m}]$	0.05
$k_{\text{TMD}}[\text{N/m}]$	[0 - 10^6]
$c_{\text{TMD}}[\text{Ns/m}]$	[0 -10^5]

Table 3: Constraints TMD optimisation problem

3.2 Results nominal TMD design for the Phénix footbridge

Solving the deterministic optimisation problem (10) with the *fmincon* function in Matlab, the TMD parameters are summarised in table 4. The TMD is located at the midpoint of the footbridge, assuming a maximal modal displacement for the damped mode shape on that location. Both the absolute values and the dimensionless TMD parameters are given: mass ratio μ_{TMD} ($= m_{\text{TMD}}/m_1$), frequency ratio ρ_{TMD} ($= f_{\text{TMD}}/f_1$) and damping ratio ξ_{TMD} ($= c_{\text{TMD}}/(2\sqrt{m_{\text{TMD}}k_{\text{TMD}}})$).

μ_{TMD} [%]	ρ_{TMD} [-]	ξ_{TMD} [%]
0.56	0.98	8.56
m_{TMD} [kg]	k_{TMD} [N/m]	c_{TMD} [Ns/m]
835	76239	1364

Table 4: Optimal TMD parameters - nominal design.

A vibration serviceability assessment for the footbridge equipped with the nominal tuned TMD is given in figure 6(a). The acceleration comfort constraint is satisfied for all traffic classes disregarding an uncertain natural frequency and damping ratio of the footbridge. The effect of uncertainties in modal parameters on the response is given in figure 6(b) for Sétra Class I. The maximal acceleration value is very sensitive to deviations in the natural frequency and damping ratio of the footbridge and the worst case scenario is found for a combination of the lowest damping ratio and highest frequency. This can be explained on the basis of figure 5(c).

In order to verify the design constraints, the acceleration response is plotted in figure 6(c). For the nominal set of modal parameters, the response reaches the limit value of $1\,\text{m/s}^2$, which implies that the acceleration constraint is active. An evaluation of the displacements is given in figure 6(d). The relative displacement of the TMD also reaches its upper value of $0.05\,\text{m}$ so this constraint is active as well.

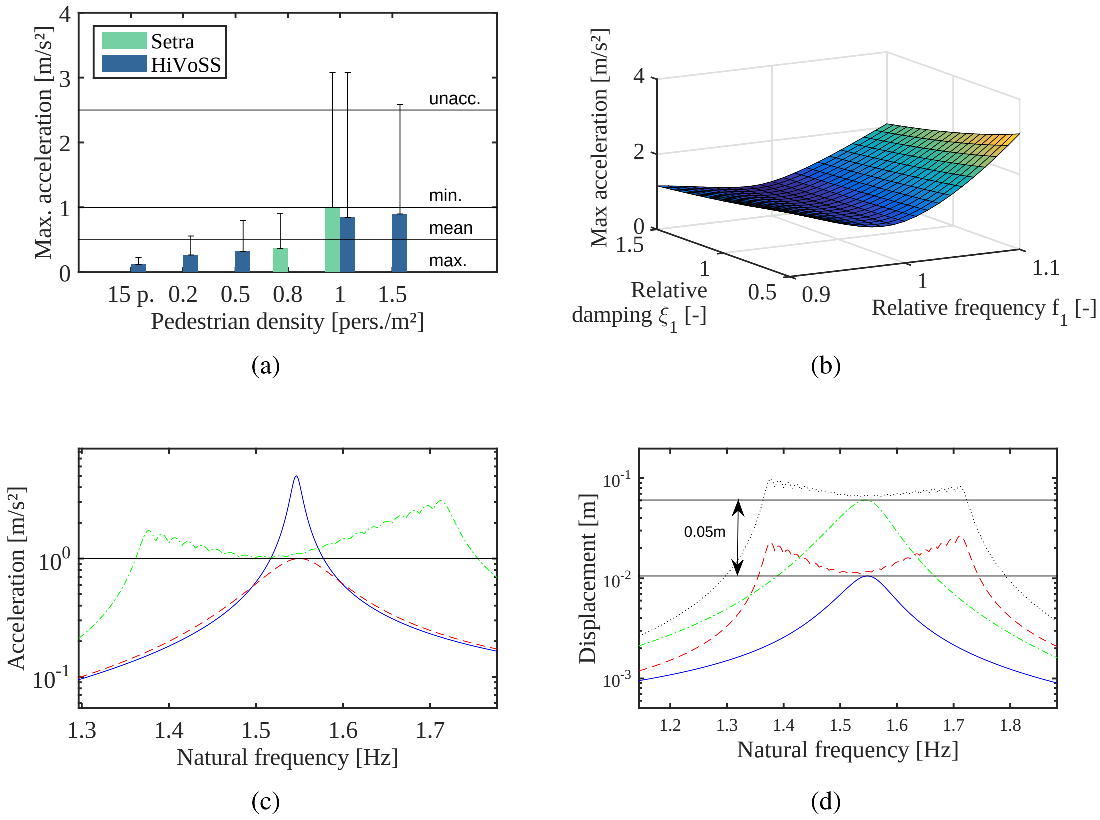


Figure 6: (a) Vibration serviceability assessment for structure with TMD tuned for nominal values of natural frequency and damping ratio. (b) Maximal acceleration response for Sétra Class I under uncertain natural frequency and damping with nominal tuned TMD. (c) Maximal acceleration as a function of frequency for structure without TMD (–), with nominal tuned TMD (- -) and with nominal tuned TMD under uncertain natural frequency and structural damping (enveloping curve of all possible combinations) (-.). In black the $\ddot{u}_{\text{comfort}}$ value. (d) Maximal displacement as a function of frequency: bridge deck (– and - -) and TMD (-. and $\cdots$) displacements respectively with and without uncertainties for nominal tuned TMD.

4 ROBUST DESIGN OPTIMISATION OF A TMD

Both subsections 2.4 and 3.2 show that uncertainties in the natural frequency and damping ratio have a considerable influence on the response evaluation and TMD behaviour. Robust approaches consider the effects of uncertainties in their designs.

In robust design optimisation, Jung and Lee [10] define robustness in a dual way. Sensitivity robustness on the one hand guarantees the insensitivity of the objective function to variations in the design parameters. Feasibility robustness on the other hand, ensures that the final design is situated in a feasible region, in spite of the uncertainties. A good literature review about robust optimisation is given by Beyer and Sendhoff [11]. The authors discuss several techniques including a robust multi-objective optimisation to minimise both the expected value and the standard deviation of the objective function. This formulation is also used by Doltsinis and Kang [7]. Constraints here ensure a specific level of reliability. Very often, a distinction is made between robustness in objective function and robustness in constraints. A frequently used type in the latter class is the worst case approach where the objective function still satisfies the constraints in the worst case [10, 12].

In the present section, a robust optimisation of a TMD is proposed to deal with uncertainties in the natural frequencies and damping ratios of the bridge.

4.1 Robust TMD design

Uncertainties in loading or in the main system modal parameters are usually disregarded in conventional approaches. Several studies were performed to optimise the parameters of the TMD under uncertain conditions. In [13] a conventional TMD design and two robust optimisations (single and multi-objective) are compared. The analysis showed that the TMD parameters of the conventional and the robust solution considerably differ from each other. Zang et al. [14] propose to determine the TMD mass, stiffness and damping by minimising a linear combination of the mean value and the standard deviation of the peak response solving a multi-objective optimisation problem. It is frequently assumed that the loads are the only uncertain parameter [5]. However, the uncertainty of the main system parameters often has a larger influence on the response than uncertainty on the load [15].

This paper suggests to take into account uncertainties in modal parameters to obtain a robust design.

4.2 Robust problem formulation

The response for the nominal designed TMD satisfies the constraints only for the design point. In order to obtain a robust design, it is necessary to account for the influence of an uncertain natural frequency and damping ratio of the bridge in design stage. A *worst case* approach is adopted here to ensure that the TMD performs well for the entire uncertainty intervals. By minimising the TMD mass, the acceleration comfort constraint ($1\,\mathrm{m/s^2}$) must be satisfied for the worst case scenario considering the uncertain natural frequency and damping ratio. The maximal relative displacement in the worst case scenario is limited to $0.05\,\mathrm{m}$. The robust optimisation problem thus is formulated as in (11). The constraints are summarised in table 3.

$$
\begin{aligned}
&\underset{m_{\text{TMD}},k_{\text{TMD}},c_{\text{TMD}}}{\text{minimise}} \; m_{\text{TMD}} \\
&\text{subject to} \qquad \ddot{u}_{\text{max}} < \ddot{u}_{\text{comfort}} \text{ (worst case)} \\
&\qquad\qquad\qquad \Delta_{\text{rel}} < \Delta_{\text{tol}} \text{ (worst case)} \\
&\qquad\qquad\qquad k_{\text{min}} \leq k_{\text{TMD}} \leq k_{\text{max}} \\
&\qquad\qquad\qquad c_{\text{min}} \leq c_{\text{TMD}} \leq c_{\text{max}}
\end{aligned}
\tag{11}
$$

4.3 Results Robust TMD design for the Phénix footbridge

The optimal parameters of the robust optimisation problem are summarised in table 5. The robust TMD design is characterised by a much higher mass ratio than for the nominal design in table 4. To satisfy the constraints in the worst case approach, a strongly increased damping is found. Note that for the TMD frequency ratio ρ_{TMD} an interval is given due to the uncertain natural frequency.

μ_{TMD} [%]	ρ_{TMD} [-]	ξ_{TMD} [%]
1.65	0.92-1.13	12.94
m_{TMD} [kg]	k_{TMD} [N/m]	c_{TMD} [Ns/m]
2458	239792	6285

Table 5: Optimal TMD parameters - robust design.

The pedestrian comfort is guaranteed for all traffic classes considering an uncertain natural frequency and damping ratio of the footbridge. Figure 7(a) shows the vibration serviceability assessment for the Phénix footbridge equipped with the robust tuned TMD. Variations in maximal response are rather small as illustrated in figure 7(b) for Sétra Class I. The limit value is reached only in the worst case scenario (combination of lowest damping and highest natural frequency).

Figure 7(c) compares the acceleration response for the bridge with and without TMD. The nominal response with TMD satisfies clearly the constraints and for the worst case response, the maximal allowable acceleration is reached. The constraint for relative TMD displacement is not active for the robust solution (figure 7(d)).

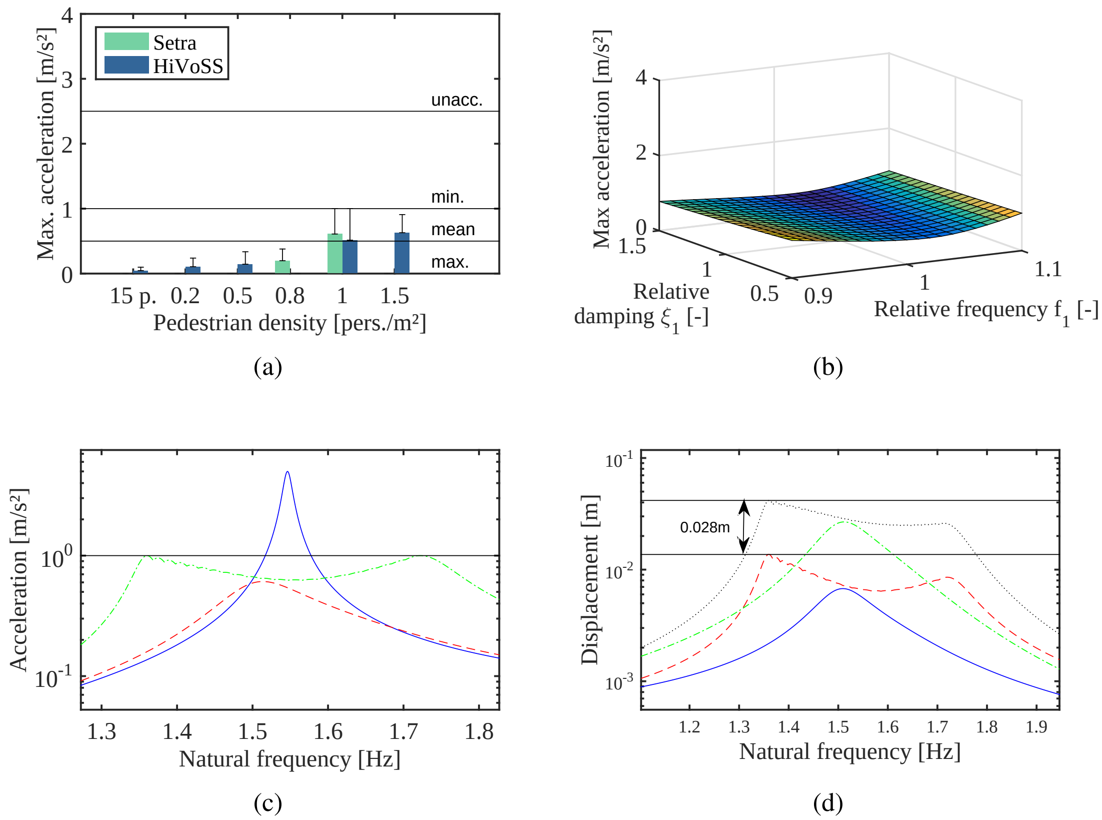


Figure 7: (a) Vibration serviceability assessment for structure with TMD tuned accounting for uncertain natural frequency and damping ratio. (b) Maximal acceleration response for Sétra Class I under uncertain natural frequency and damping with robust tuned TMD. (c) Maximal acceleration as a function of frequency for structure without TMD (–), with robust tuned TMD (- -) and with robust tuned TMD under uncertain natural frequency and structural damping (enveloping curve of all possible combinations) (-.). In black the $\ddot{u}_{\text{comfort}}$ value. (d) Maximal displacement as a function of frequency: bridge deck (– and - -) and TMD (-. and $\cdots$) displacements respectively with and without uncertainties for robust tuned TMD.

5 CONCLUSIONS

Changes in the natural frequency and damping ratio of the footbridge have a significant influence on both the response with and without TMD.

For a TMD tuned at nominal values, the response constraints are satisfied for nominal values of the modal parameters of the footbridge. Considering the proposed uncertainties, the response reduction is not sufficient. In order to make the performance of the TMD robust with respect to uncertainties, a robust TMD optimisation is characterised by a strongly increased TMD mass and damping ratio. The required constraints are guaranteed in the worst case approach.

The proposed method in this paper allows for a robust design of a TMD based on the simplified load model used by the Sétra and HiVoSS design guidelines. Future research may consider more advanced load models for the pedestrian excitation.

6 ACKNOWLEDGEMENTS

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