COMPDYN 2015
5th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, V. Papadopoulos, V. Plevris (eds.)
Crete Island, Greece, 25–27 May 2015

IMPLEMENTATION OF ROBUST CONTROL THEORY IN SMART STRUCTURE DESIGN USING FREQUENSY DOMAIN DATA

A. J. Moutsopoulou[1*], A. D. Pouliezos[2] and G. E. Stavroulakis[3]

[1]Department of Civil Engineering Technological Educational Institute of Crete

Estavromenos Heraklion Crete, amalia@staff.teicrete.gr

[2,3]Department of Production Engineering and Management,

Technical University of Crete, Kounoupidianna GR-73100 Chania, Greece,

gestavr@dpem.tuc.gr, tasos@dpem.tuc.gr

Key words: *H infinity* control, structural control, frequency domain data, smart beam

Abstract. *This paper presents an implementation of a design method of robust H_∞ optimal control to a structural control system. To obtain the best possible performance in the face of uncertainties, robust H_∞ optimal control for the active control structure is used. Relevant numerical techniques, which have been implemented with the help of MATLAB routines, are applied to solve the formulated structural control problem. By proper selection of the weight factor, the seismic response of the building structure can be reduced considerably. Numerical results show high robust performance of the proposed method.*

1. INTRODUCTION

The control of structural vibrations is an important goal for the structural engineer. Several control techniques have been developed for these purposes. Classical engineering design based on appropriate choice of materials and of dimensions of the structure provides only a partial solution to the problem because of their limited control action. The aim of this research is to design a robust controller to suppress adverse vibrations of smart structures due to earthquake and wind excitations. The design specifications of H_∞ control are given in the frequency domain, and thus it is easy for H_∞ control to deal with the uncertainty at high frequencies and to guarantee the robust stability and robust performance [10, 11, 23].

A short literature review gives a deep insight into the research work done on the smart structures so far. Culshaw [1] discussed the concept of smart structure, its benefits and applications. Rao and Sunar explained the use of piezo materials as sensors and actuators in sensing vibrations in their survey paper [2]. Hubbard and Baily [3] have studied the application of piezoelectric materials as sensor / actuator for flexible structures. Hanagud et.al. [4] developed a Finite Element Model (FEM) for a beam with many distributed piezoceramic sensors/ actuators.

Hwang and Park [5] presented a new finite element (FE) modeling technique for flexible beams. Continuous time and discrete time algorithms were proposed to control a thin piezoelectric structure by Bona, et.al. [6]. Schiehlen and Schonerstedt [7] reported the optimal control designs for the first few vibration modes of a cantilever beam using piezoelectric sensors/ actuators. S.B. Choi et.al. [8] have shown a design of position tracking sliding mode control for a smart structure. Distributed controllers for flexible structures can be seen in Forouza Pourki [9].

2. MODELLING

The dynamical description of the system is given by,

$$\boldsymbol{M}\ddot{\boldsymbol{q}}(t)+\boldsymbol{D}\dot{\boldsymbol{q}}(t)+\boldsymbol{K}\boldsymbol{q}(t)=\boldsymbol{f}_m(t)+\boldsymbol{f}_e(t) \tag{1}$$

Where $\boldsymbol{M}$ is the generalized mass matrix, $\boldsymbol{D}$ the viscous damping matrix, $\boldsymbol{K}$ the generalised stiffness matrix, $\boldsymbol{f}_m$ the external loading vector and $\boldsymbol{f}_e$ the generalised control force vector produced by electromechanical coupling effects. For a model simplified beam model of a composite beam with piezoelectric sensors and actuators the independent variable vector $\boldsymbol{q}(t)$ is composed of transversal deflections w_i and rotations ψ_i, i.e for [10, 11],

$$\boldsymbol{q}(t)=\begin{bmatrix} w_1 \\ \psi_1 \\ \vdots \\ w_n \\ \psi_n \end{bmatrix}$$

where n is the number of finite elements used in the analysis. Vectors $\boldsymbol{w}$ and $\boldsymbol{f}_m$ are positive upwards.

To transform to state-space control representation, let (in the usual manner),

$$\boldsymbol{x}(t)=\begin{bmatrix} \boldsymbol{q}(t) \\ \dot{\boldsymbol{q}}(t) \end{bmatrix}$$

Furthermore to express $\boldsymbol{f}_e(t)$ as $\boldsymbol{Bu}(t)$ we write it as $\boldsymbol{f}_e^*\boldsymbol{u}$, where $\boldsymbol{f}_e^*$ is the piezoelectric force for a unit applied on the corresponding actuator, and $\boldsymbol{u}$ represents the voltages on the actuators. Lastly $\boldsymbol{d}(t)=\boldsymbol{f}_m(t)$ is the disturbance vector. Then,

$$\dot{\boldsymbol{x}}(t)=\begin{bmatrix} 0_{2n\times 2n} & I_{2n\times 2n} \\ -\boldsymbol{M}^{-1}\boldsymbol{K} & -\boldsymbol{M}^{-1}\boldsymbol{D} \end{bmatrix}\boldsymbol{x}(t)+\begin{bmatrix} 0_{2n\times n} \\ \boldsymbol{M}^{-1}\boldsymbol{f}_e^* \end{bmatrix}\boldsymbol{u}(t)+\begin{bmatrix} 0_{2n\times 2n} \\ \boldsymbol{M}^{-1} \end{bmatrix}\boldsymbol{d}(t)$$

$$= \boldsymbol{Ax}(t) + \boldsymbol{Bu}(t) + \boldsymbol{Gd}(t)$$

$$= \boldsymbol{Ax}(t) + [\boldsymbol{B}\ \boldsymbol{G}] \begin{bmatrix} \boldsymbol{u}(t) \\ \boldsymbol{d}(t) \end{bmatrix}$$

$$= \boldsymbol{Ax}(t) + \tilde{\boldsymbol{B}}\tilde{\boldsymbol{u}}(t) \qquad (2)$$

We can augment this with the output equation. For example, if we assume that displacements are only measured then,

$$\boldsymbol{y}(t) = [x_1(t)\ x_3(t)\ \ldots\ x_{n-1}(t)]^T = \boldsymbol{Cx}(t) \qquad (3)$$

with,

$$\boldsymbol{C} = \left[\begin{array}{cccc|ccc} 1 & 0 & \cdots & 0 & 0 & & \\ 0 & 0 & 1 & \cdots & & \vdots & \\ & & \ddots & & & \vdots & \\ 0 & \cdots & 0 & 1 & & & 0 \end{array}\right]$$

In this formulation $\boldsymbol{u}$ is $n\times1$ (at most, but can be smaller), while $\boldsymbol{d}$ is $2n\times1$. The units used are m, rad, sec and N.

The *control problem* is to keep the beam in equilibrium (: zero displacements and rotations) in the face of external disturbances, noise and model inaccuracies, using the available measurements (displacement) and controls [12, 13].

3. CLASICAL CONTROL

In the sequel we will analyse the behaviour of a cantilever beam, with four pairs of piezoelectric patches bonded symmetrically at the top and bottom surfaces of each beam element [14, 23].

The open loop system is as shown in Fig. 1. Using (2), (3) the transfer function from disturbance to position is,

$$\boldsymbol{H_o}(s)=\boldsymbol{C}(s\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{G} \tag{4}$$

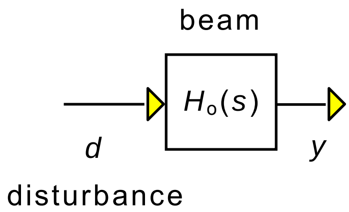

Figure 1 Open loop system

Firstly we note that the $\boldsymbol{A}$ matrix is badly conditioned with condition number = 5.624703967123330e-013 Fig. 2. This means some preconditioning would be beneficiary to sensitive calculations (like pole placement). A way around this problem is to balance the system matrix. MATLAB provides the routine[15],

[T, S] = *balance*(A)

which produces a diagonal transformation matrix $\boldsymbol{T}$ whose elements are integer powers of 2, and matrix B such that,

$$\boldsymbol{A}=\boldsymbol{TST}^{-1}$$

In this way some of the bad conditioning is transferred to $\boldsymbol{T}$. Letting,

$$\boldsymbol{z}=\boldsymbol{T}^{-1}\boldsymbol{x}\Rightarrow\boldsymbol{x}=\boldsymbol{Tz}$$

(2) becomes,

$$\begin{aligned}\boldsymbol{T}z(t) &= \boldsymbol{ATz}(t)+\boldsymbol{Bu}(t)+\boldsymbol{Gd}(t)\\ \Rightarrow \boldsymbol{z}(t) &= \boldsymbol{T}^{-1}\boldsymbol{ATz}(t)+\boldsymbol{T}^{-1}\boldsymbol{Bu}(t)+\boldsymbol{T}^{-1}\boldsymbol{Gd}(t)\\ &= \boldsymbol{Sz}(t)+\hat{\boldsymbol{B}}\boldsymbol{u}(t)+\hat{\boldsymbol{G}}\boldsymbol{d}(t)\end{aligned}$$

Another problem arises from the very small size of the minimum eigenvalue which defines the smallest time constant of the system, which in turns dictates sampling intervals used in simulations. These sampling intervals should be smaller than the smallest time constant. When this happens, arrays involved for example in **lsim** simulations get very big, and special care must be taken if the simulation time is large[15,16].

Also the system is both controllable and observable (in fact the system is both controllable and observable with fewer inputs and measurements).

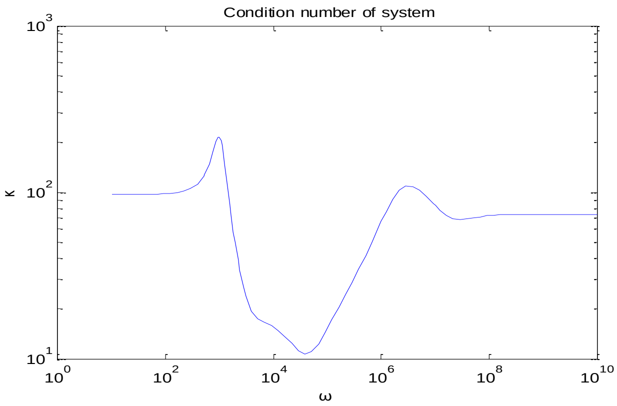

Figure 2 Condition number of the system

Responses to various inputs are in Figs. 2, 4, 5, 6.

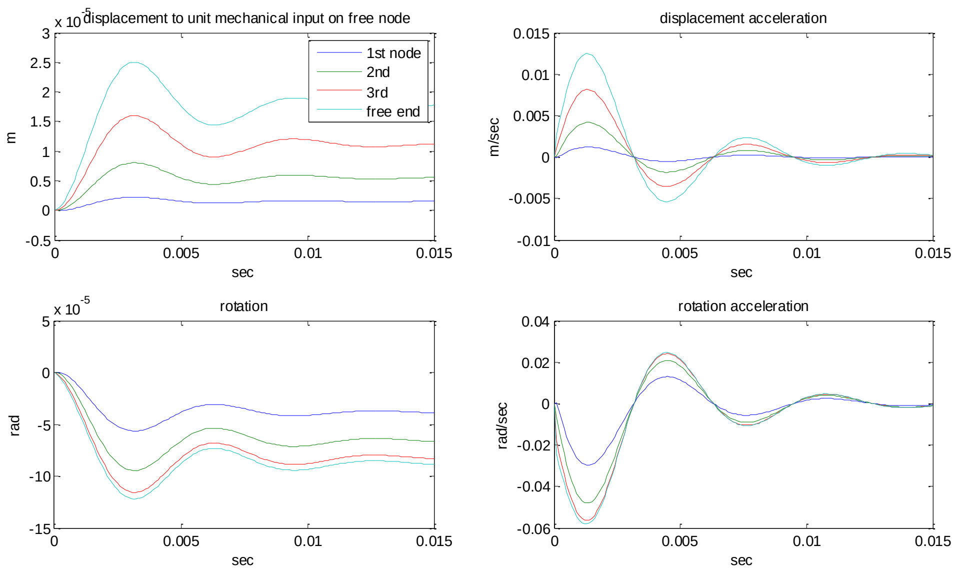

Figure 2 Responses to various mechanical inputs at free end of the beam

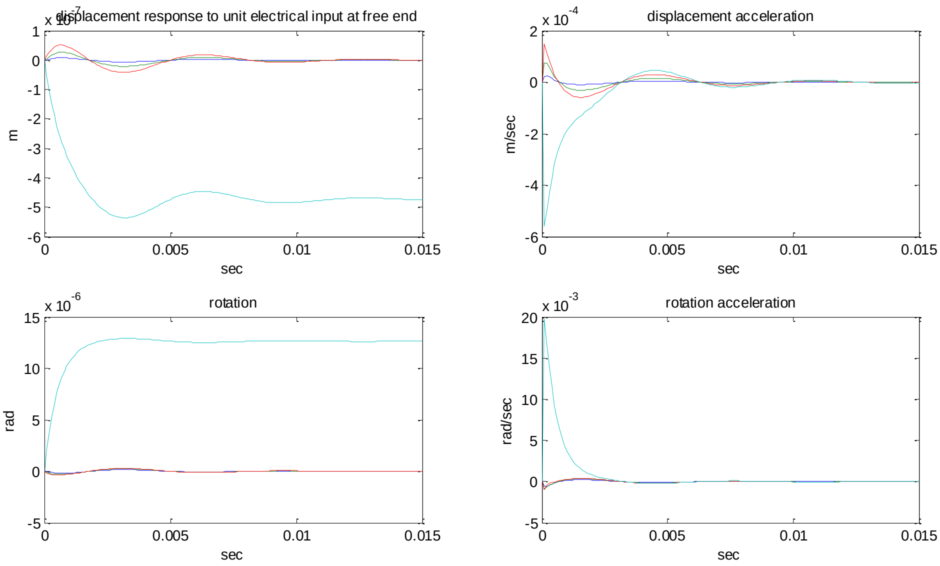

Figure 3 Responses to various electrical inputs at the free end of the system

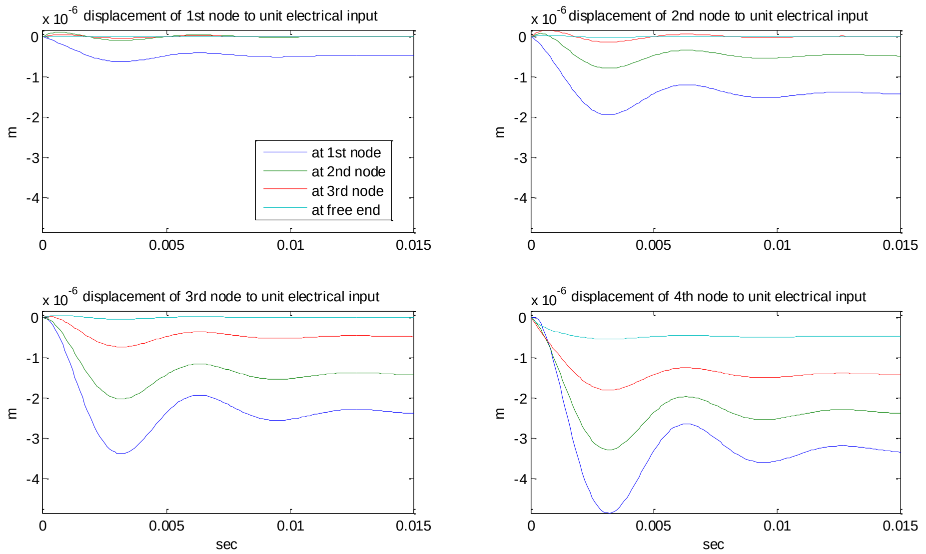

Figure 5 Responses to various electrical inputs at each node of the system

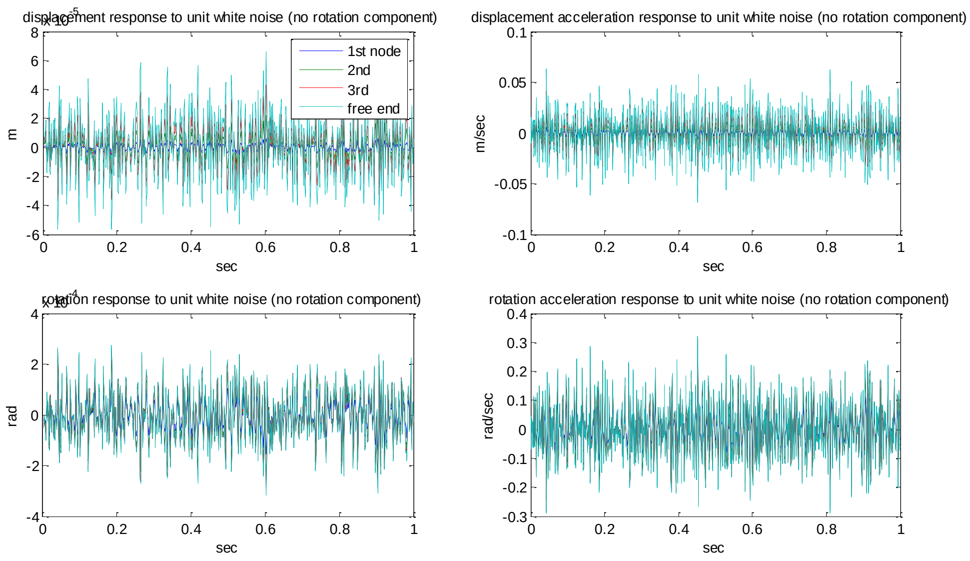

Figure 6 Responses to unit white noise us inputs at each node of the system

3. $H_{infinity}$ CONTROL

To relate the structures used in classical and H_∞ control, let's look at Fig. 4 :

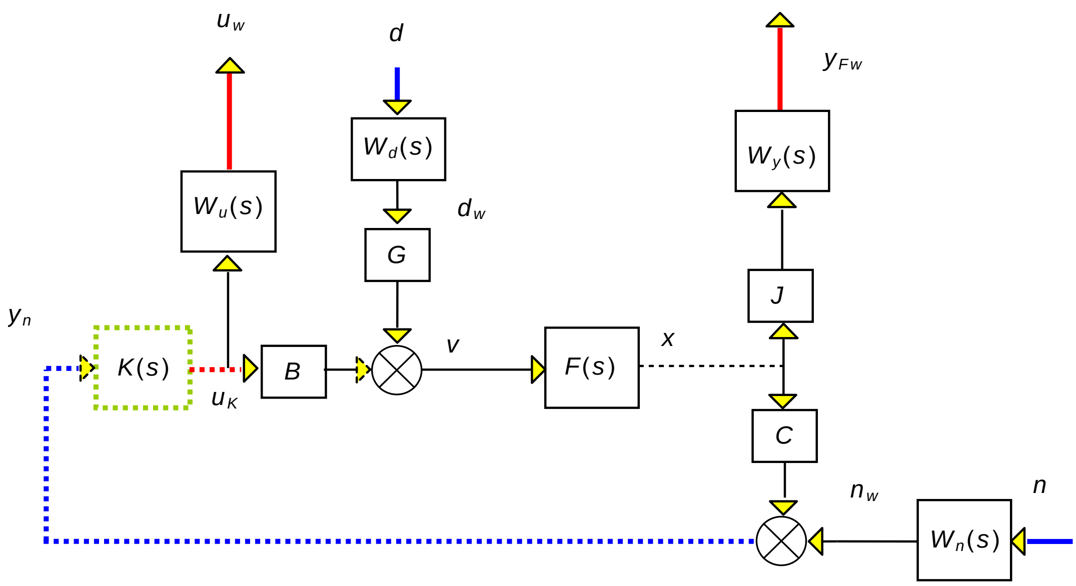

Figure 4 Closed loop diagram in the frequency domain

In this diagram are included all inputs and outputs of interest, along with their respective weighs.

To find the necessary transfer functions [17, 18]:

$$y_{Fw}=W_yJx= W_yJFv=W_yJF(GW_dd+Bu_K)=W_yJFGW_dd+W_yJFBu_K$$

$$u_w=W_uu_K$$

$$y_n=Cx+W_nn=CFv+W_nn=CF(GW_dd+Bu_K)+W_nn=CFGW_dd+CFBu_K+W_nn$$

Combining all these gives,

$$\begin{bmatrix} u_w \\ y_{Fw} \\ \hdashline y_n \end{bmatrix} = \left[\begin{array}{cc:c} 0 & 0 & W_u \\ W_yJFGW_d & 0 & W_yJFB \\ \hdashline CFGW_d & W_n & CFB \end{array}\right] \begin{bmatrix} d \\ n \\ \hdashline u_K \end{bmatrix} \tag{4}$$

Note that the plant transfer function matrix, $F(s)$, is deduced from the suitably reformulated plant equations,

$$\dot{x}(t) = Ax(t) + Iv(t)$$

$$y(t)=Ix(t)$$

where $v(t)=Gd+Bu_k$. Hence,

$$F(s)=(sI-A)^{-1}$$

The equivalent two-port diagram in the closed loop system is fig. 4,

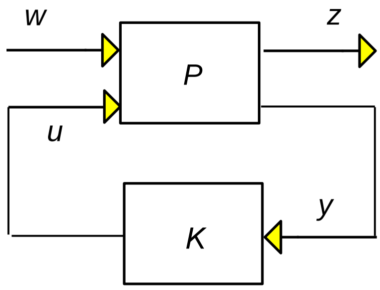

Figure 5 Closed loop system, two port diagram

with,

$$z = \begin{bmatrix} u_w \\ y_{Fw} \end{bmatrix}, \quad w = \begin{bmatrix} d \\ n \end{bmatrix}, \quad y = y_n, \quad u = u_K$$

where z are the output variables to be controlled, and w the exogenous inputs.

Given that P has two inputs and two outputs it is, as usual, naturally partitioned as,

$$\begin{bmatrix} z(s) \\ y(s) \end{bmatrix} = \begin{bmatrix} P_{zw}(s) & P_{zu}(s) \\ P_{yw}(s) & P_{yu}(s) \end{bmatrix} \begin{bmatrix} w(s) \\ u(s) \end{bmatrix} \overset{\text{ορ}}{=} P(s) \begin{bmatrix} w(s) \\ u(s) \end{bmatrix} \tag{5}$$

Also,

$$u(s)=K(s)y(s) \tag{6}$$

Using (4) the transfer function for *P* is,

$$P(s) = \left[\begin{array}{cc:c} 0 & 0 & W_u \\ W_y JFGW_d & 0 & W_y JFB \\ \hdashline CFGW_d & W_n & CFB \end{array}\right] \tag{7}$$

while the closed loop transfer function $M_{zw}(s)$ is,

$$M_{zw}(s)=P_{zw}(s)+P_{zu}(s)K(s)\ (I-P_{yu}(s)K(s))^{-1}P_{yw}(s) \tag{8}$$

Equation (8) is the well known lower LFT for M_{zw}.

To express *P* in state space form, the natural partitioning [19, 20],

$$P(s) = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right] = \begin{bmatrix} P_{zw}(s) & P_{zu}(s) \\ P_{yw}(s) & P_{yu}(s) \end{bmatrix} \tag{9}$$

is used (where the packed form has been used), while the corresponding form for *K* is,

$$K(s) = \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right]$$

Eq. (10) defines the equations,

$$\dot{x}(t) = Ax(t) + \begin{bmatrix} B_1 & B_2 \end{bmatrix} \begin{bmatrix} w(t) \\ u(t) \end{bmatrix}$$

$$\begin{bmatrix} z(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} x(t) + \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} w(t) \\ u(t) \end{bmatrix}$$

and,

$$\dot{x}_K(t)=A_K x_K(t)+B_K y(t)$$

$$u(t)=C_K x_K(t)+D_K y(t)$$

To find the matrices involved, we break the feedback loop and use the relevant equations.

Therefore the equations relating the inputs, outputs, states and input/output to the controller are[21]:

$$\begin{aligned}
&\dot{x}_F = Ax_F + (Gd_w + Bu), \qquad y_F = x_F \\
&\dot{x}_u = A_u x_u + B_u u, \qquad u_w = C_u x_u + D_u u \\
&\dot{x}_{yF} = A_{yF} x_{yF} + B_{yF} J y_F, \quad y_{Fw} = C_{yF} x_{yF} + D_{yF} y_F \\
&\dot{x}_n = A_n x_n + B_n n, \qquad n_w = C_n x_n + D_n n \\
&\dot{x}_d = A_d x_d + Gd, \qquad d_w = C_d x_d + D_d d \\
&y_n = C y_F + n_w
\end{aligned} \tag{10}$$

$$x = \begin{bmatrix} x_F \\ x_u \\ y_{Fw} \\ x_n \\ x_d \end{bmatrix}, \quad y = y_n, \quad w = \begin{bmatrix} d \\ n \end{bmatrix}, \quad z = \begin{bmatrix} u_w \\ y_{Fw} \end{bmatrix}, \quad u = u_K$$

By taken d_w, n_w και y_F from equation (11) we have,

$$\dot{x} = \begin{bmatrix} A_G & 0 & 0 & 0 & GC_d \\ 0 & A_u & 0 & 0 & 0 \\ BC_F & 0 & A_{yF} & 0 & 0 \\ 0 & 0 & 0 & A_n & 0 \\ 0 & 0 & 0 & 0 & A_d \end{bmatrix} x + \begin{bmatrix} GD_d & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & B_n \\ B_d & 0 \end{bmatrix} w + \begin{bmatrix} B \\ B_u \\ 0 \\ 0 \\ 0 \end{bmatrix} u$$

$$z = \begin{bmatrix} 0 & C_u & 0 & 0 & 0 \\ D_{yF}C_F & 0 & C_{yF} & 0 & 0 \end{bmatrix} x + 0w + \begin{bmatrix} D_u \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} C_F & 0 & 0 & C_n & 0 \end{bmatrix} x + \begin{bmatrix} 0 & D_n \end{bmatrix} w + 0u$$

Therefore the matrices are:

$$A_1 = \begin{bmatrix} A_F & 0 & 0 & 0 & GC_d \\ 0 & A_u & 0 & 0 & 0 \\ BC_F & 0 & A_{yF} & 0 & 0 \\ 0 & 0 & 0 & A_n & 0 \\ 0 & 0 & 0 & 0 & A_d \end{bmatrix}, \quad B_1 = \begin{bmatrix} GD_d & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & B_n \\ B_d & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} B \\ B_u \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0 & C_u & 0 & 0 & 0 \\ D_{yF}C_F & 0 & C_{yF} & 0 & 0 \end{bmatrix}, \quad D_{11} = 0, \quad D_{12} = \begin{bmatrix} D_u \\ 0 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} C_F & 0 & 0 & C_n & 0 \end{bmatrix}, \quad D_{21} = \begin{bmatrix} 0 & D_n \end{bmatrix}, \quad D_{22} = 0$$

4. RESULTS

Various tests were performed with disturbance and noise profiles (used in time responses) as shown in Fig. 9.

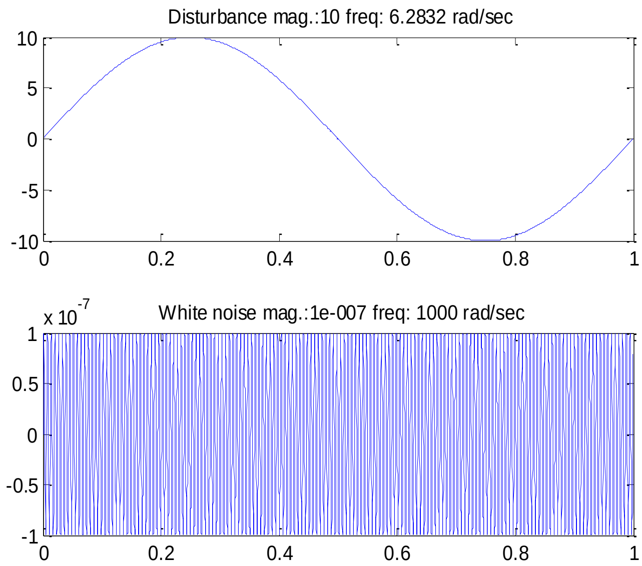

Figure 9 Disturbances of the system

To evaluate robustness we use m_p=10^{-5}, k_p=10^{-10}, d_p=0.005(m_p+k_p).

4.1 Without weights

In the simplest approach no weights are placed on any of the input/output quantities. This means that the H_∞ controller ensures [21, 22],

$$\left\| \begin{bmatrix} e \\ u \end{bmatrix} \right\|_2 \leq 1$$

as long as,

$$\left\| \begin{bmatrix} d \\ n \end{bmatrix} \right\|_2 \leq 1$$

Figs. (10)-(11) show the results of this run.

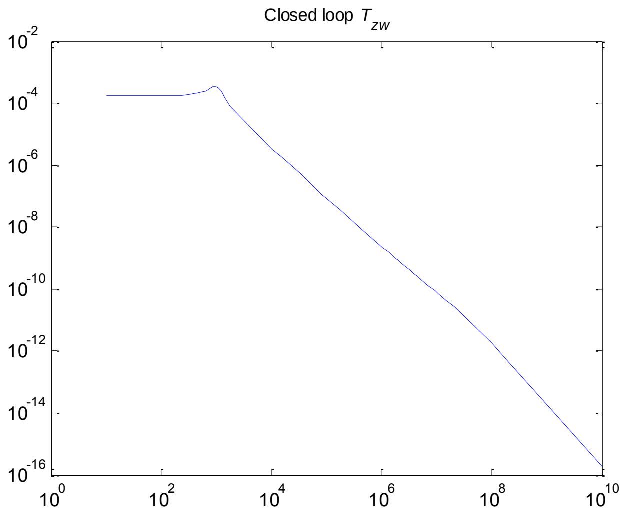

Figure 10 Closed loop T_{zw} for all frequencies

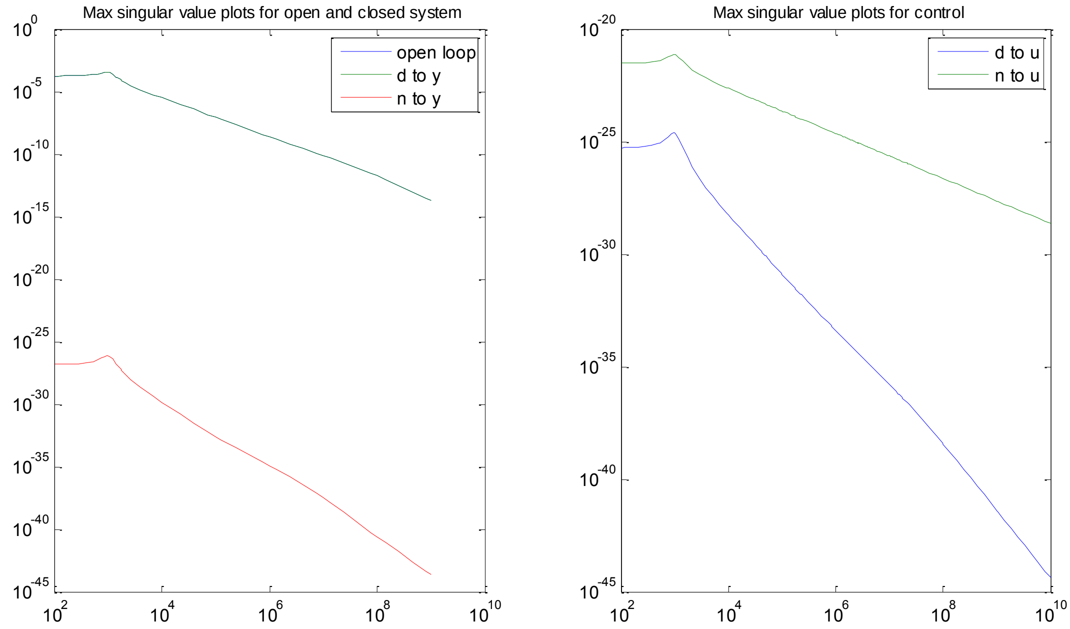

Figure 11 Max singular values for closed and open loop

Figure 10 shows that the price of the singular value of the un weighted system is very small for all frequencies (much lower than one). Fig 11 shows a satisfactory effect of the disturbance on the size of the control scheme (the design could be improved, if it were possible to reduce noise effect for frequencies of 1000 Hz). There are no difference is observed between the frequency plots of open and closed loop for the unweighted system.

4.2. With weights

Next we try constant weights, in particular let,

$$W_n=10^{-8},\ W_u=1/500,\ W_e=10^3$$

Figure 12 and 13promises a marked improvement in performance

Figures 12 shows that the value of T_{zw} is low then one for all frequencies.

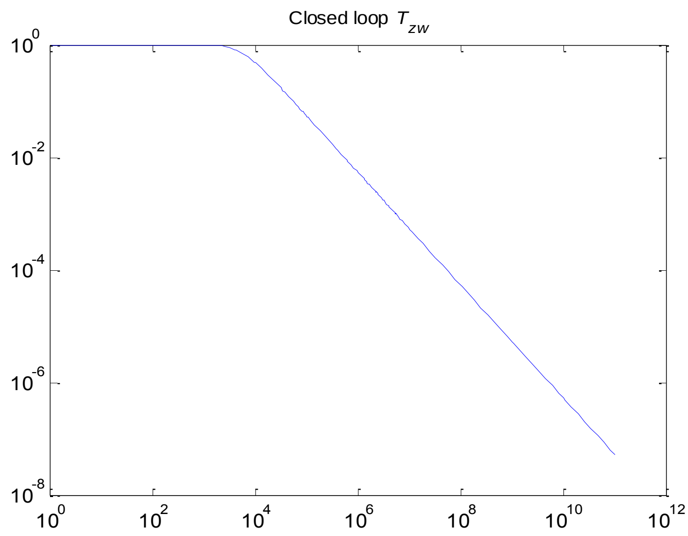

Figure 12 Closed loop T_{zw} for all frequencies

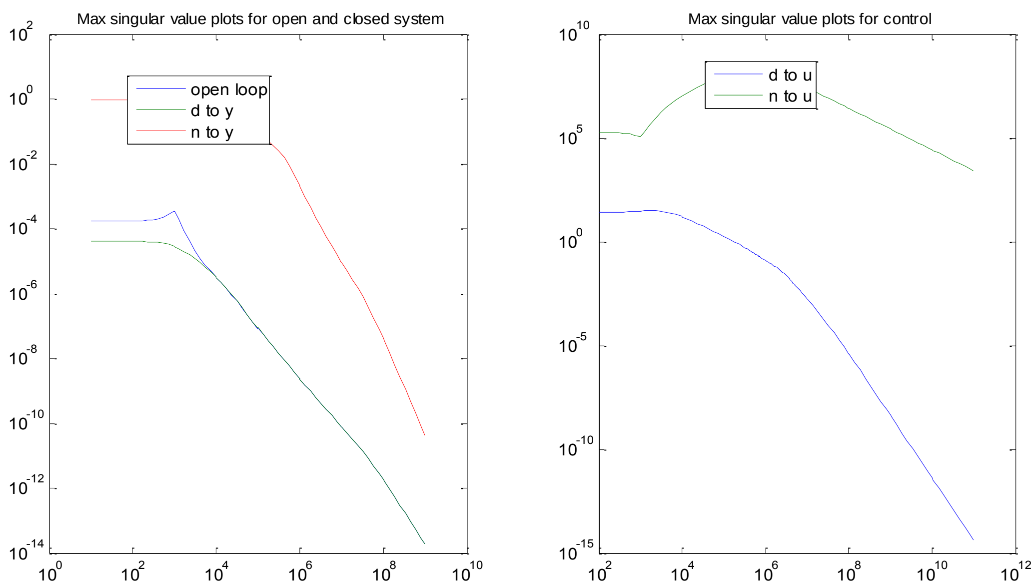

Figure 13 Max singular values for closed and open loop

Fig. 13 shows a satisfactory effect of the disturbance on the size of the control scheme (the design could be improved, if it were possible to reduce noise effect for frequencies of 1000 Hz). There are no difference is observed between the frequency plots of open and closed loop system.

As shown in Fig. 13 a, there is a significant improvement in the effect of disturbance on error up to the frequency of 1000 Hz. In Fig. 13a , there seems to be little effect of noise on error for frequencies beyond 1000 Hz. In Fig. 13b, shows a satisfactory effect of the disturbance on the size of the control scheme (the design could be improved, if it were possible to reduce noise effect for frequencies of 1000 Hz).

CONCLUSIONS

In this paper, a robust control design problem has been formulated within a linear fractional transformation framework using the $H_{infinity}$ technique. The $H_{infinity}$ norm of the closed loop transfer matrix from all disturbances to the errors to be minimized has been chosen as the cost functional. A suboptimal controller has been used for numerical modeling. The open loop and the closed-loop controlled system has been simulated using a periodic impulsive command input, periodic isolated influences. $H_{infinity}$ techniques have the advantage over classical control techniques in that they are readily applicable to problems involving multivariate systems with cross-coupling between channels. Simultaneously optimizing robust performance and robust stabilization is difficult. One method that comes close to achieving this is $H_{infinity}$ loop-shaping, which allows the control designer to apply classical loop-shaping concepts to the multivariable frequency response to get good robust performance, and then optimizes the response near the system bandwidth to achieve good robust stabilization. It must be emphasized that the framework of the structural control employed in this paper is quite general and covers interesting cases of practical importance.

REFERENCES

[1] B. Culshaw, *Smart structures: A concept or a reality*, Journal of Systems and Control Engg, Vol. 26, No. 206, 1992.

[2] S. Rao, M. Sunar, (994), *Piezoelectricity and its uses in disturbance sensing and control of flexible structures: A survey*, Applied mechanics Review, Vol. 17, No..2, pp. 113-119.

[3] T. Baily, J. E. Jr. Hubbard, *Distributed piezoelectric polymer active vibration control of a cantilever beam*, Journal of Guidance, Dynamics and Control, Vol. 8, No. 5, pp. 605-611, 1985.

[4] S. Hanagud, M. W. Obal, and A. J. Callise, *Optimal vibration control by the use of piezoelectric sensors and actuators*, J. Contr. Guidance, Vol. 15, No. 5, pp.1199-1206, 1992.

[5] W. S. Hwang and H. C.l Park, *Finite element modeling of piezoelectric sensors and actuators*, AIAA journal J, Vol. 31, No. 5, pp. 930-937, 1993.

[6] B. Bona, M. Indri, A. Tornamble), *Flexible piezoelectric structures approximate motion equations and control algorithms*, IEEE Auto. Contr., Vol. AC-42, No. 1, pp. 94-101, 1997.

[7] W. Schiehlen, and H. Schonerstedt, *Controller design for the active vibration damping of beam structure*, Proc. Smart Mechanical Systems Adaptronics SAE International. USA, pp. 137-146, 1998 .

[8] S.-Bok Choi, Chae-Cheon Cheong, and C.-Hea Lee, *Possition tracking control of a smart flexible structure featuring a piezofilm actuator*, Journal of Guidance and Control, Vol. 19, No. 6 , pp. 1364-1369, 1996.

[9] Forouza Pourki, , *Distributed controllers for flexible structures using piezoelectric actuators/ sensors*, Proc. 32nd IEEE CDC Conf., Texas, USA, pp. 1367-1369, 1993.

[10] A. J. Moutsopoulou, A. Pouliezos, and G. E. Stavroulakis, *Modelling with Uncertainty and Robust Control of Smart Beams*, Paper 35, Proceedings of the Ninth International Conference on Computational Structures Technology. B.H.V. Topping and M. Papadrakakis, (Editors), Civil Comp Press, 2008.

[11] B. Miara, G. E. Stavroulakis, V. Valente, *Topics on mathematics for smart systems*, Proceedings of the European Conference. Rome, Italy, 26-28 October 2006, World Scientific Publishers, Singapore, International, 2007.

[12] B. Bandyopadhyay, T.C. Manjunath, and M. Unapathy, *Modeling, Control, and Implementation of Smart Structures*, Springer ISBN-10 3-540-48393-4, 2007.

[13] A.J. Moutsopoulou, A. Pouliezos, G. E. Stavroulakis, *Nonsmooth and nonconvex optimization for the design and order reduction of robust controllers used in smart structures*, In: M. Papadrakakis et all., *Computational Methods in Erthquake Engineering*, Vol. 2, Springer ,2012.

[14] Benjeddou, M.A. Trindade, and R. Ohayon, *New shear actuated smart structure beam finite element*, Journal of Guidance. Dynamics and Control.37, 378-383, 1999.

[15] B. Bona, M. Indri, and A. Tornamble, *Flexible piezoelectric structures approximate motion equations and control algorithms*, IEEE Auto.Control 42(1), 94-101, 1997.

[16] O. Bosgra, and H. Kwakernaak, *Design methods for control systems*, Course notes, Dutch Institute for Systems and Control 67, 2001.

[17] J. Burke, A. Lewis, and M.L. Overton, A *robust gradient sampling algorithm for nonsmooth nonconvex optimization*, Siam Journal on Optimization 15, 751-779, 2005.

[18] H. Kimura, *Robust stability for a class of transfer functions*, IEE Transactions on Automatic Control 29, 788–793, 1984.

[19] H. Kwakernaak, *Robust control and H1 optimization*, Tutorial hper JFAC. Automatica, 29(2), 255-273, 1993.

[20] M. Millston, *HIFOO 1.5: Structured control of linear systems with a non trivial feedthrough*, M.S. thesis, Courant Institute of Mathematical Sciences, New York University, 2006.

[21] A. Packard, J. Doyle, and G. Balas, *Linear, multivariable robust control with a μ perspective*, ASME Journal of Dynamic Systems, Measurement and Control, 50th Anniversary Issue, 115(2b), 310–319, 1993.

[22] G. Zames, *Feedback minimax sensitivity and optimal robustness*, IEE Trans. Autom. Control, 28, 585–601, 1983.

[23] G. Foutsitzi, G. Stavroulakis and E. Hadjigeorgiou, *Modelling and Simulation of Controlled Smart Structures*, Proceedings of the 9th HSTAM International Congress on Mechanics pp. 511-518 , Limassol, Cyprus, 2010.