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SENSITIVITY ANALYSIS OF DISCRETE-FORCE IDENTIFICATION ALGORITHMS TO MEASUREMENT NOISE

A. Rezayat [1], V. Nassiri [2], K. Maes [3], G. Lombaert [3], S. Vanlanduit [1,4], P. Guillaume [1]

[1] Acoustics & Vibrations Research Group (AVRG), Vrije Universiteit Brussel (VUB)
Pleinlaan 2, 1050 Brussels, Belgium
e-mail: arezayat@vub.ac.be

[2] University of Leuven (KUL) & Universiteit Hasselt, I-BioStat, Leuven, Belgium

[3] Department of Civil Engineering, KU Leuven, Belgium

[4] Op3Mech Research Group, University of Antwerp, Belgium

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Abstract. *The increasing complexity of mechanical structures makes the measurement of dynamic loadings on the structure non practical, since the applied forces can not be measured in a direct way. Recently, a vast category of inverse algorithms have been developed in order to solve the load identification problems. In contrast with forward methods that compute the output (effect) of a system subject to inputs (cause), inverse methods offer the possibility to compute the input parameters of a system with known output data and the system model. Inverse problems suffer in general from instability problems due to high condition number. They are almost always under-determined and highly sensitive to measurement errors. Optimization techniques based on regularized cost functions are the most popular remedies to this problem. This paper deals with a sensitivity analysis of the classical regularization techniques, compared to a newly developed method (G-FISTA). The aim is to reconstruct applied loads on structures in noisy environments. Multiple simulation studies are done considering several situations of noise level, point force locations and sensor configurations. This study shows that the location and time history of discrete forces on structures can be better estimated using structured-sparse penalty functions techniques. A detailed noise study is done using the pseudo-inverse, Tikhonov and G-FISTA methods in frequency domain, and their performances are evaluated.*

1 Introduction

The direct measurement of parameters such as the applied forces on a structure is not always practical. In this case, another set of (more accessible) parameters are often measured, such as accelerations or strains. The force identification using structural vibration data has attracted a lot of interest. The inverse load identification using system responses is especially of interest in civil engineering and structural mechanics. This importance is more pronounced at the design stage of structures. For structures such as tall buildings, wind turbines and aircrafts, the knowledge of the active dynamic loads may be useful for structural health monitoring, i.e. fatigue assessment. In almost all cases, these loads cannot be measured directly, or the direct measurement is not practical and cost-effective. Many attempts were made to solve this problem by using indirect measurement techniques. The inverse approach consists of computing the input parameters of a system, with known output data and the structure model.

The great challenge with inverse problems is that they are not mathematically well-posed in almost all cases. A problem is called well-posed in the sense of *Hadamard*, if there exists a unique solution to the problem, which continuously depends on its data. As soon as the number of equations becomes smaller than the amount of unknowns, the problem becomes under-determined. When this condition satisfied, a high condition number makes the problem ill-conditioned. The strong sensitivity of the system to measurement errors causes instability in the calculation of the solution. This lack of stability is related to the fact that the behavior of the structure with respect to a specific excitation force (e.g. impact) can be exactly reproduced using another force configuration, with different force location and amplitude. The mathematical challenge will be to find the most realistic and physical force combination among all the infinite amount of possibilities. This is usually done by performing optimizations. In the particular case of mechanical structures, the inverse load identification aims to estimate force locations and reconstruct their time history on the basis of measured structural response and the dynamic model of the structure.

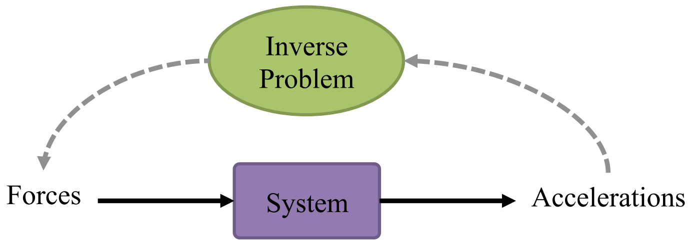


Figure 1: The inverse problem consists of reconstructing input excitations, based on a model and the vibration data.

In the literature, the load identification problems are treated differently depending on the choice of working domain. In some studies such as in [1–5], time domain techniques are used to solve the problem. Recently, there have been great advances in the development of new time domain techniques, especially in the joint input and state identification [6]. The joint input-state estimation algorithm is introduced in structural dynamics as an extension of the joint input-state estimation algorithm proposed in [7], including the correlation between process noise and measurement noise. This correlation is inherently present for civil engineering applications when accelerations are measured in presence of ambient excitation [9]. The conditions for system inversion on structural dynamics were recently derived in [8]. The algorithm is used for the identification of hammer and artificial muscle forces applied to the bridge deck.

In the frequency domain, a classical solution for this problem is using the pseudo-inverse of the system model, [10, 11, 13, 15]. Although the pseudo-inverse approach can be computed easily, the outcome of this method is not satisfying in most cases, due to poor localization capability of the pseudo-inverse operator. In optimization based techniques, the cost function can include a penalization term or not (regularization). In the study of Bond and Adams [14], the force identification is done after an external study on the signal entropy. Among the non-penalized methods, Parloo et al. [15] has suggested a technique that is based on ℓ_p-norm cost functions for localized forces. The value of p tends to zero in an iterative manner until convergence is achieved (non-convex problem). On the other hand, the cost function can contain a penalization term to stabilise the solution. The most common way of regularization is the addition of the ℓ_2-norm to the cost function, such as in the study of Tikhonov and Arsenin [16]. The problem appears when the excitations are applied only on a few locations. There is a clear need for an adequate cost function that can ensure both localization and amplitude reconstruction for point forces. Recently, new techniques have been suggested to solve the ill-conditioned problem in frequency domain [17]. The proposed algorithm (G-FISTA) appears to have good localization capabilities. This method is based on the minimization of a cost function containing a mixed penalty function. The optimization algorithm used in this study is based on the work of Beck et al. [18]. Making use of grouped or structured sparsity, G-FISTA promotes solutions with a sparse pattern over the force point locations, and continuity in the frequency content of the force vector.

In the scope of this paper, we focus our main interest on discrete dynamic forces, acting at multiple point locations. Several methods are used to reconstruct the forces applied on a beam structure. The sensitivity of these algorithms to noise is evaluated using the simulation data of a cantilever beam. Different force/sensor configurations and noise levels are investigated in this work. In the following sections, the load reconstruction problem will be reformulated for penalized least squares optimization. The solution method will be explained and simulation results will be discussed in details.

2 Problem statement

While the forward problem consists of calculating the vibration response of structures to input forces, the inverse force identification problem deals with the reconstruction of unknown dynamic forces acting on a structure, using the output responses. More particularly, the impact force identification consists of estimating the impact force locations and their time evolution as a set of two unknown parameters. A special case of dynamic loading is the impact force produced by a hammer.

In this case study, the structure consists of a cantilever beam of rectangular section in laboratory conditions. By means of system identification techniques, a linear system model describing the beam behaviour is obtained from the roving hammer experiment. This model is then used together with some simulated acceleration data in order to reconstruct impact loads on the beam. The acceleration data is then polluted with a zero-mean Gaussian noise $N_x(f)$ in different signal to noise ratios.

The problem will be solved by pseudo-inverse (Moore-Pnrose), Tikhonov and G-FISTA

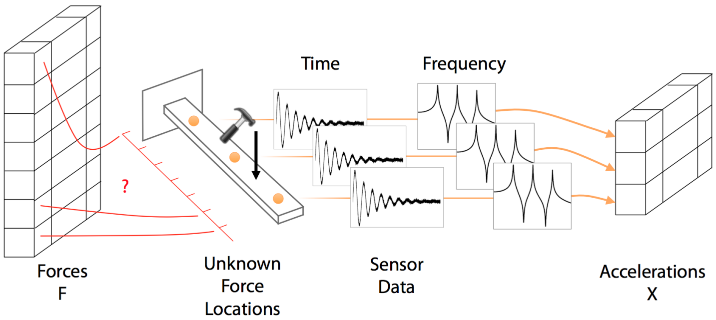


Figure 2: While the number of of unknown candidate force locations is big, only a few vibration sensor data is used for the localization process.

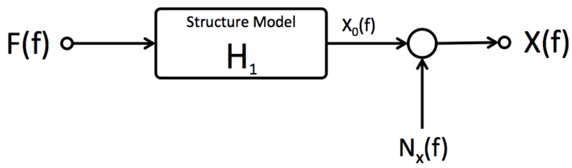


Figure 3: Gaussian white noise with different SNR is added to the simulated accelerations.

methods. Different sensor configurations are examined in this study. A comparative study is done on the effect of the number of point forces. Each sensor configuration is tested using acceleration data containing noise.

3 Solution methods

The inverse force reconstruction problem described in the previous section is solved by an optimization technique in frequency domain, as presented in [17]. Equation 1 represents the linear governing equations of the problem. The matrices $\bar{F}$ and $\bar{X}$ are created by concatenating respectively the force and accelerations of each frequency f into a column vector. The $\bar{H}$ matrix is a block diagonal matrix obtained by placing all the system (FRF) matrices on the diagonal.

$$\bar{X}(f) = \bar{H}(f)\bar{F}(f) \ , \ f \in \{f_1, \ldots, f_N\} \tag{1}$$

3.1 Direct inversion

The most classic solution to this problem is the Pseudo-inverse or the Moore-Penrose inversion. This method is very fast, but it fails to localize the loads in case of discreet forces. Although this problem is almost always ill-posed, there are some situations where the direct inversion of the transfer function can produce results, such as for Uslu et al. [13]. Despite the uniqueness property of the estimated solution, the obtained results are not fully satisfying, because the estimated forces are smeared out over all candidate force locations (poor localization). This is not an unusual result, because the pseudo-inverse does not produce a sparse solution.

$$\bar{F}(f) = \bar{H}^{\dagger}(f)\bar{X}(f) \ , \ f \in \{f_1, \ldots, f_N\} \tag{2}$$

3.2 Regularization

The solution of this problem can be estimated using least squares as in equation 3.

$$\hat{F} = \text{argmin}_{\bar{F}} \left\{ \frac{1}{2} \|\bar{H}\bar{F} - \bar{X}\|_2^2 \right\} \tag{3}$$

Since the number of rows in $\bar{H}$ is smaller than the number of columns, the problem in (3) is ill-posed. One promising method to overcome the ill-posedness of this problem is to penalize the least squares, as follows:

$$\hat{F} = \text{argmin}_{\bar{F}} \left\{ \frac{1}{2} \|\bar{H}\bar{F} - \bar{X}\|_2^2 + \lambda g(\bar{F}) \right\} \ , \ \lambda \geq 0 \tag{4}$$

This expression is known regularized least squares problem, and the function g represents the penalty term. Penalized least-squares is an effective and popular method proposed to solve ill-posed systems of linear equations. Different choices of penalty functions lead to different solutions, therefore the selection of penalty function g should be made in an efficient way [19–21].

3.2.1 Tikhonov

Tikhonov method consists of regularizing the cost function by an ℓ_2-norm as penalty function [2,3,16]. The solution is described as the following:

$$\hat{F} = \text{argmin}_{\bar{F}} \left\{ \frac{1}{2} \|\bar{H}\bar{F} - \bar{X}\|_2^2 + \lambda \|\bar{F}\|_2^2 \right\} \ , \ \lambda \geq 0 \tag{5}$$

3.2.2 G-FISTA

As described in [17], the use of l_2-norm in the first argument of the cost function is motivated by the effectiveness of least-squares in dealing with Gaussian noises on smooth data. While the data fidelity is evaluated by an l_2-norm, an l_1-norm can promote sparsity in the solution, which is more convenient in case of discrete (or localized source) problems.

Considering the physics of the problem, we suggest that an adequate penalty function should possess at least the following three properties:

1. **Sparsity constraint:**
   Since the external forces are usually applied only at a few unknown (discrete) locations on the structure, the force vector is assumed to be zero on most of the candidate force locations. In this sense, the force vector is expected to resemble a high contrast pattern. In other words, along the force locations, a sparse solution is desirable for each frequency. Therefore, using an ℓ_1-penalized model (over the candidate locations) seems to be more suitable (sparsity).

2. **Continuity constraint:**
   As the structure is excited with localized external forces (such as hammer or shaker), the force vector should retain its continuity over the frequency axis. In case of an impulse excitation (ex. hammer), the force will appear in almost every frequency due to the broad-band frequency content of the impact force. To ensure the continuity of the force vector in the frequency axis, we suggest the use of the ℓ_2-norm penalty. This means that at a given location on the structure, the probability for an element in the frequency axis to be non-zero depends on other elements of that particular location in the frequency axis. In other words, for each force location, the estimated force vector should be zero or non-zero along the frequencies (non-sparsity). Therefore, the use of ℓ_2-norm is promoted along the frequencies.

3. **Convex cost function:**
   The convexity of the problem in equation (4) will guarantee a unique solution. For $\lambda \geq 1$, the cost function $g(\lambda)$ is convex.

The continuity constraint (ℓ_2-norm penalty) is applied within each group elements, and the sparsity constraint (ℓ_1-norm penalty) is applied between the groups. Therefore, the new penalty function becomes a hybrid $\ell_{1\&2}$-norm, as described in equation 6, where k is the number of unknown force locations. As this penalty function is convex, it contains all the desirable properties mentioned in previous sections. The new penalized least squares problem is defined in equation 7.

$$\|\bar{F}\|_{1\&2} = \sum_{i=1}^{k} \|\bar{F}_{G_i}\|_2 \tag{6}$$

$$\hat{F} = \text{argmin}_{\bar{F}} \left\{ \frac{1}{2}\|\bar{H}\bar{F} - \bar{X}\|_2^2 + \lambda\|\bar{F}\|_{1\&2} \right\}. \tag{7}$$

The FISTA optimization algorithm proposed by [18] solves an ℓ_1-norm penalized least squares problem for real parameters. In summary, the algorithm can be interpreted as a special case of Majorize-Minimization algorithm (MM) that tries to find the minimum of a surrogate cost function instead of the actual cost function [22]. The detailed algorithm is presented in [19].

4 Validation strategy

The numerical validation process consists of simulating a cantilever beam subject to point forces, i.e. hammer impacts. This steel beam has the following dimensions: $100 \times 1 \times 3$ cm. The structural model has been obtained experimentally in lab conditions, using the roving-hammer test. The system contains 7 degrees of freedom. A force vector is applied numerically to this model and the accelerations are recorded. The number of applied impact points, their amplitudes and the noise contamination levels can be controlled in the simulations.

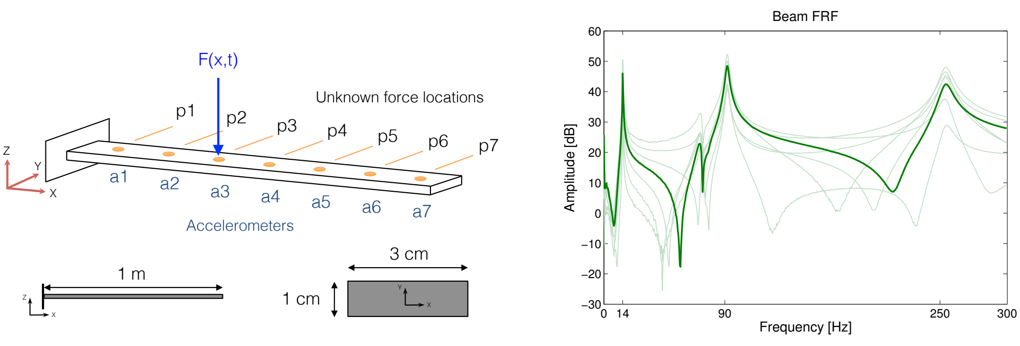


Figure 4: The simulated cantilever beam contains 7 unknown force locations. Some FRF curves corresponding to locations 1 till 7 are presented in the right side figure as examples.

The first step of the validation process is to estimate the location of the applied loads on the beam. In this step only a limited range of frequencies around the resonances are used to decrease the computation time. As a second step, a reduced system of equations is created, taking into account only the force locations estimated in the previous step. Then, the new system is solved using classical methods such as pseudo-inverse over the complete frequency range to reconstruct the complete force time history.

In a general case when there is no prior information on the applied forces, model selection techniques should be used in order to find the best regularization parameter. In this paper though, in order to have a fair comparison between all the techniques, we evaluate the force estimations over a wide range of λ (regularization parameter) and choose the solution corresponding to the best match between imposed and estimated force indexes. In such a way, the λ_{opt} for each method is defined as $argmin_{\lambda}\left\{\|\hat{\beta}-\beta_0\|_2^2\right\}$, where $\hat{\beta}$ and β_0 are the estimated and exact force indexes respectively (see β definition in [15]).

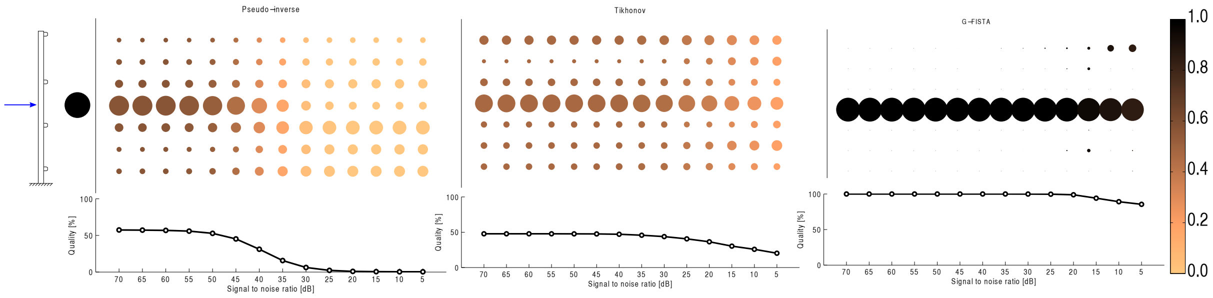
5 Results and discussions

The simulation results are illustrated in this section. The size of the circles in the figures correspond to the $\hat{\beta}$ value, and the color represents the difference between the imposed and estimated force index. These results clearly show that the classical inversion methods such as pseudo-inverse does not produce satisfying force estimations, as predicted from the theory. The resultant force indexes are more distributed than localized. The G-FISTA method is stronger in localizing forces on the beam, even if multiple impact forces are applied. The effectiveness of G-FISTA in localizing forces from noisy data is well pronounced in the simulations. The misfit of the estimated and the exact force vector is represented by a parameter called $Quality$, which is calculated for each SNR values as $Quality = \|\hat{\beta}-\beta_0\|_2 \times 100$. This parameter emphasises on the localization of forces.

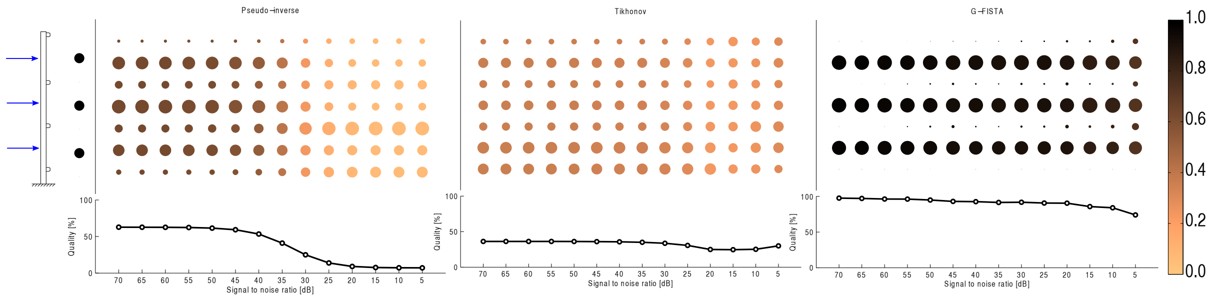
The simulation studies indicate that the consideration of a structured sparse penalty in the cost function (as in G-FISTA) highly improves the force estimations accuracy. The strength of G-FISTA in localizing point forces relies on the fact that this algorithm constitutes a general case with a mixed penalty function that can be simplified in either ℓ_1 or ℓ_2-norms as special cases. In other words, if only one big group (consisting of all components of F together) is defined, then the structured sparsity will act the same as usual ℓ_2 regularization (i.e. Tikhonov). However, if each frequency component of the force vector F is allocated to a separate group, (i.e. one element per group) the structured sparsity will become equivalent to the ℓ_1 penalised case. Therefore, G-FISTA is a general algorithm that can deal with all cases of point force identification problems. This makes G-FISTA a strong toolbox to deal with different force configurations without the need for prior knowledge on the type of the applied force.

Another aspect of this works concerns the investigation of the number of used sensors. Two types of force/sensor configurations are investigated. In the first case, one point force is applied at location 4 on the beam, and the result qualities are compared in different noisy simulations. For all the evaluated methods except the pseudo-inverse, the localization quality increases with the number of considered sensors (see figure 8). The results illustrate that the G-FISTA method delivers more accurate force localization estimations among all tested methods. The latter assumption is true in case of high amount of noise SNR.

In another study, the number of sensors is fixed to four, and different amount of point forces are applied to the beam. Figures 5a till 7 show that the pseudo-inverse and Tikhonov have already a poor force localization for the least noisy data set. In the other hand, G-FISTA appears to be more robust in terms of data noise contamination. For all of the tested algorithms, the estimation quality decreases when more forces are applied on the structure. This can be explained in the case of sparsity based methods: these algorithms generally promote sparse pattern in the force vector. The more non-zero elements are present in the force vector, the more difficult would be for the algorithm to converge. The overall characteristics of G-FISTA still shows that this method can produce better localization estimations, compared to other evaluated techniques. For cases where the number of applied point forces is bigger than the amount of sensors, all methods fail.

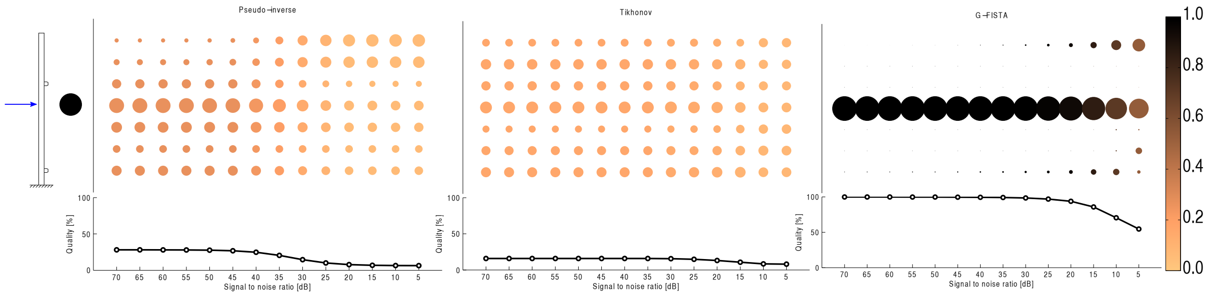


(a) Force at point $\{4\}$ and accelerometers at $\{1, 3, 5, 7\}$.

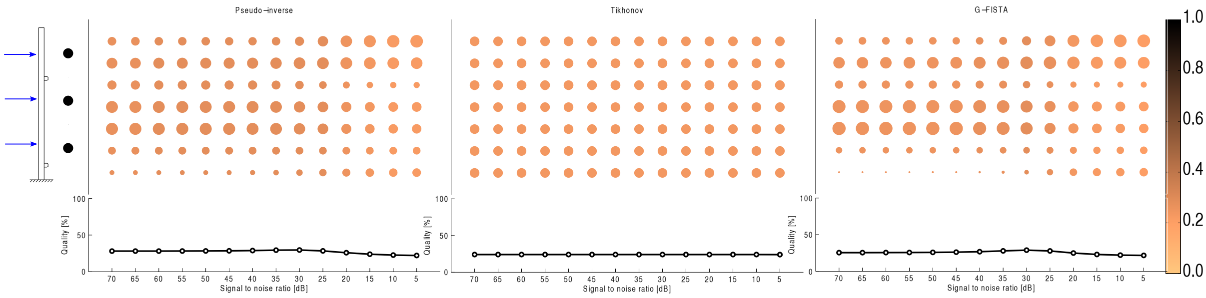


(b) Forces at points $\{2, 4, 6\}$ and accelerometers at $\{1, 3, 5, 7\}$.

Figure 5: Simulation results show that the unknown point forces are better estimated using G-FISTA method. The size and color of the bubbles represent respectively the estimated force index $\hat{\beta}$ and fitting quality. Four sensors are considered.



(a) Force at point $\{4\}$ and accelerometers at $\{1, 5\}$.



(b) Forces at points $\{2, 4, 6\}$ and accelerometers at $\{1, 5\}$.

Figure 6: Simulation results show that the estimation quality decreases with noise level. In this configuration all methods fail to localize forces. The size and color of the bubbles represent respectively the estimated force index $\hat{\beta}$ and fitting quality. Only two sensors are considered.

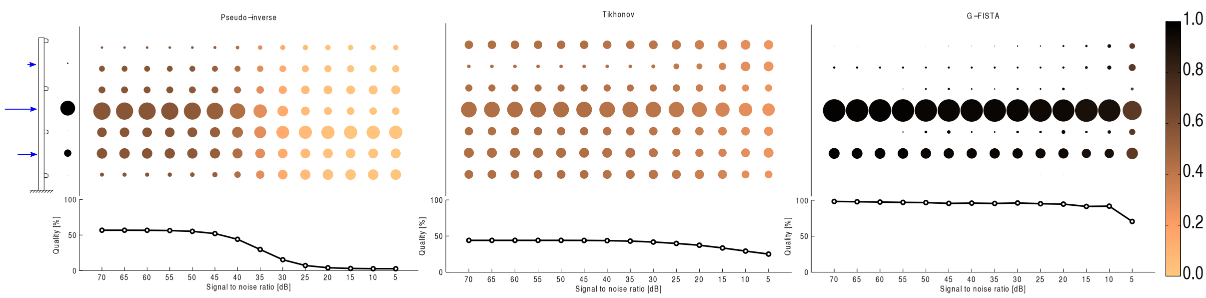
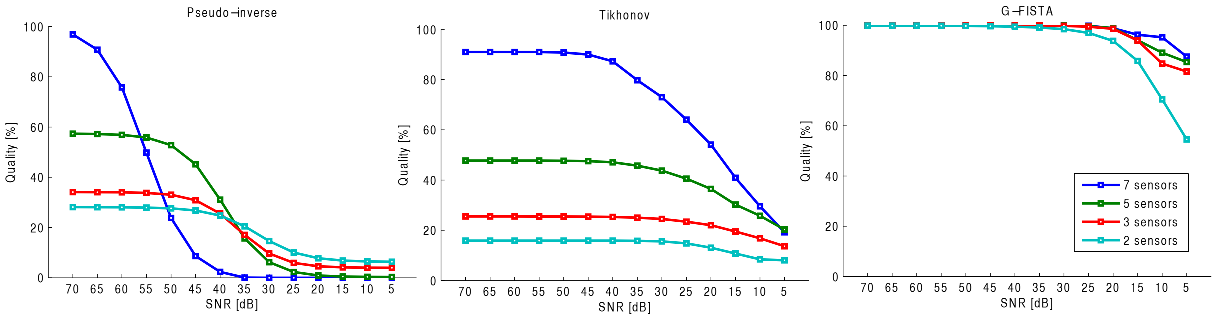
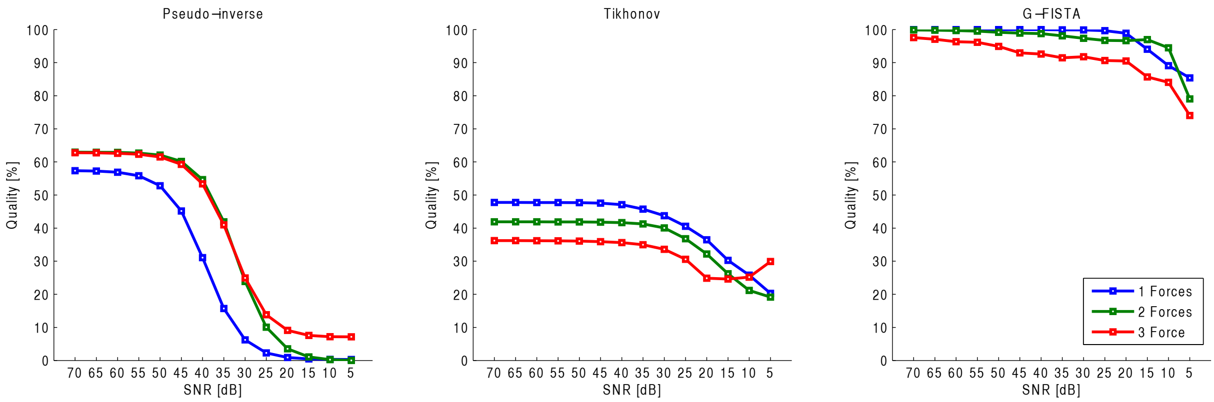


Figure 7: The simulation of multiple forces with different amplitudes show that G-FISTA method produces better estimation quality. Forces are applied at points $\{2, 4, 6\}$ with amplitudes $(50N, 100N, 10N)$, and accelerometers placed at $\{1, 3, 5, 7\}$.



(a) Force at point $\{4\}$ and different sensor settings: $\{1-7\}$, $\{1,3,5,7\}$, $\{1,4,7\}$ and $\{2,5\}$.



(b) Sensors placed at $\{1,3,5,7\}$ and different active force locations: $\{4\}$, $\{3,5\}$ and $\{2,4,6\}$.

Figure 8: The influence of number of sensors and forces is investigated using simulations. The results show that G-FISTA is less sensitive to the number of used sensors, even at high noise levels. The quality decreases when number of active forces increase.

6 Conclusion

In this paper, multiple numerical studies are conducted, aiming at localization and reconstruction of dynamic loads on a cantilever beam structure. The sensitivity of several algorithms to measurement noise is investigated. The influence of number of forces/sensors is also studied in this paper. The ill-conditioned load identification problem is reformulated as a new mathematical problem. The problem is then solved by means of pseudo-inverse, Tikhonov regularization and G-FISTA. The latter method is based on a particular mixed cost function taking advantage of structured sparsity.

The results show generally that the classical methods such as the pseudo-inverse and Tikhonov fail to localise the loads correctly. It is shown that the amount of considered sensors will generally increase the force localization quality. In all simulations, G-FISTA seems to be more accurate and it estimates the correct force locations, even in situations where the number of installed sensors is low. It can be concluded that the amount of sensors needed for an accurate estimation is at least equal to the number of force points applied to the structure. In terms of sensitivity to noise, the simulations illustrate the robustness of G-FISTA approach in localizing impact forces on a beam structure.

In the present study, the G-FISTA delivers the best overall force estimations in comparison to pseudo-inverse and Tikhonov regularization. Although the computation cost of this method is relatively higher than other force identification techniques, it still remains the best choice for point force reconstruction for off-line data processing.

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