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SEMI-ACTIVE PENDULUM TO CONTROL OFFSHORE WIND TURBINE VIBRATIONS

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Abstract. Wind energy presents itself, nowadays, as one of energy sources in fast development and implementation all over the world. The project, building and maintenance of the so called wind farm still present lots of challenges for engineers and researches. In this context, offshore wind turbine are found: wind turbines installed on ocean next to coast which, between other advantages, gets benefit of more intense and consistent wind with less turbulence in these regions. One of conceptions from this kind of wind turbine is the floating. This kind of structural system can be analyzed like a discrete model of inverted pendulum. This type structure can be vulnerable to excessive vibration caused by wind loads. One of the most used vibration control systems is the Tuned Mass Damper (TMD). It basically consist of a mass-spring-damper system attached to the main structure, the frequency of the damper is tuned to a particular frequency, with the goal of making the TMD vibrate out of phase with the main system, thus transferring the energy system to the damper. However, passive devices only work properly for the design frequency range, and wind forces are random type of excitations. In this sense, a better approach would be design a semi-active device. Semi-active control joins confidence and simplicity typical of passive systems with active control adaptability. It is characterized by not adding mechanical energy due to the structure and to have properties that can change dynamically. These devices can be viewed as controllable passive devices because despite of its changing properties of damping and/or stiffness it action on the structure is passive. This work aims to achieve better results in vibration control of offshore wind turbines proposing a semi-active controller to it, considering an inverted pendulum discrete model.

1 INTRODUCTION

Wind power is a clear, sustainable source of energy. A great number of wind farms have been installed all over the world in recent years. It appears as an attractive option to reduce carbon dioxide emissions. The wind turbine transforms kinetic energy, generated from wind velocities, in electric power. It is constituted of a supporting structure (tower), a nacelle, a gearbox, a cube, blades and a generator.

Wind farms located at the seaboard, the so called offshore wind turbines, have some advantages compared to the onshore ones: do not have land space limitations; do not have audible or visual impact; at the sea occurs less turbulent winds [1]. Meanwhile, offshore wind turbine stability has to be taken into account, since it has shallower foundations and can even be a floating structure with a mobile base. Floating wind turbines also experience an additional source of dynamical loads: the movement of the sea waves at its base.

Wind turbine towers generally are slender and flexible due to its high altitude and can present excessive vibrations caused by its own operation, as well as, wind action. A detailed analysis of the structural behavior of the tower is of great relevance as it can represent up to 30% of the total system cost [2].

A technology that proposes to reduce excessive vibration of the structures is called structural control. It consists in the addition of external devices such as dampers or application of external forces that change properties of stiffness and/or damping. It can be classified as: passive control, active control, hybrid control and semi-active control. [3]

One widely used device is the tuned mass damper (TMD). It reduces the main structure response amplitudes by transferring energy between the structure and an auxiliary mass. TMD consists of mass-spring-dashpot system that is designed tuning it to a given modal frequency, for high towers, usually the first one [4].

The TMD can also be designed as pendulum system [5], it has the vibration period depending on pendulum length and can only be considered as a linear oscillator when amplitude vibrations are small, and otherwise the system turns to be nonlinear.

However, passive control strategies have disadvantages: they only are efficient within the frequency range for which they were designed. It is a little robust control. In the case of structures subjected to wind loads this fact becomes critical, since this type of loading has a random characteristic.

One alternative to passive control is the so called semi-active control. Semi-active devices do not add energy to the structure and its properties can be varied dynamically [6]. The semi-active systems are more reliable and more robust than active systems. Damping and stiffness properties can be controlled in an active way by a control signal. These are controllable passive devices since they don't apply any additional force to the structure.

Chey et al [7] proposed passive and semi-active tuned mass damper (PTMD and SATMD) building systems to mitigate structural response due to seismic loads.

Zemp et al [8] describes the implementation of a proof-of-concept pendulum tuned inertial mass magnetorheological (TM-MR) damper assembly in a tall building in Santiago, Chile, designed primarily to control building motions, and hence indirectly, interstory deformations.

Rafieipour et al [9] proposed a semi-active TMD with an innovative variable stiffness device called folding variable stiffness springs (FVSS), with the capability to change stiffness between lower and upper bounds through a small change between its supports. It is verified that this damper has an effective performance compared to a passive device.

Despite previous studies and reasonable number of practical applications of structural control in bridges, high towers and buildings, the application of structural control techniques in wind turbines is still a new topic [10-12]. However, despite previous studies, obstacles

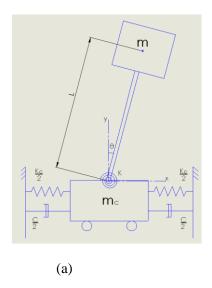
remain to be overcome. These include: reducing cost/maintenance and increase reliability, efficiency and robustness.

On previous studies [13-15] an inverted pendulum model was proposed to study the dynamic behavior and stability of a floating offshore wind turbine. Two passive control systems, a simple pendulum TMD and an inverted pendulum TMD were proposed aiming to reduce the angular amplitude of the main structure. The results obtained showed that inverted pendulum model had a better performance.

In this work a semi-active TMD pendulum is proposed to control excessive vibration of an offshore floating wind turbine. The structure is modeled as an inverted pendulum discrete model. This model is presented as a preliminary model for structural control alternatives studies, the results serve as a basis for real structures design with a more careful modeling. A bang bang control strategy was considered, TMD stiffness and damping values were calculated trough optimal control theory. Satisfactory results were find out compared to those of passive TMD pendulum.

2 FLOATING WIND TURBINE MODEL: INVERTED PENDULUM

For modeling the dynamic behavior of an offshore wind turbine, a discrete model of an inverted pendulum is considered [13] as presented in Fig. 1(a). It is a system with a spring and a damper at the base that simulates fluid resistance of the sea. The excitation of the system is a dynamic force applied to the mass m, simulating the wind force on the turbine blades. Figure 1(b) presents the installation of an inverted pendulum TMD to reduce vibration's amplitude of the structure. Some simplifying hypotheses were assumed [14]: (a) angular amplitude is kept within boundaries for a linear behavior; (b) a two dimension vibration system is considered; (c) wave loading is disregarded (d) a top mass represents nacelle+blades [10].



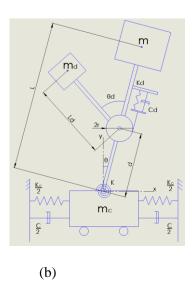


Figure 1: Model of an inverted pendulum over a mobile base (a) with inverted pendulum TMD installed (b).

Where m is the tip mass at the end of the bar, θ is bar angular amplitude, l is bar's length, K is torsional stiffness, m_c is base's mass, k_c is base's stiffness, c is base's damping coefficient, m_d is TMD pendulum's mass, l_d is TMD pendulum's length, d is the distance from pendulum to bar's base, r is TMD disc's radius, c_d is TMD's damping coefficient, k_d is

TMD's stiffness and θ_d is TMD pendulum's angular amplitude related to the bar. The motion equation of the system with control (Figure 1(b)) is given in matrix form:

$$\begin{bmatrix} M_{1,1} & M_{1,2} & M_{1,3} \\ M_{2,1} & M_{2,2} & M_{2,3} \\ M_{3,1} & M_{3,2} & M_{3,3} \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\theta}_d \\ \ddot{u} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & c_d \cdot r^2 & 0 \\ 0 & 0 & c \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{\theta}_d \\ \dot{u} \end{bmatrix} + \begin{bmatrix} K - g \left(m \cdot l + \frac{\rho \cdot l^2}{2} + m_d (d + l_d) - \rho_d \cdot l_d \left(d + \frac{l_d}{2} \right) \right) \\ + m_d \cdot g \cdot l_d + \rho_d \cdot \frac{l_d^2}{2} \cdot g \\ 0 & k_d \cdot r^2 - m_d \cdot g \cdot l_d - \rho_d \cdot \frac{l_d^2}{2} \cdot g & 0 \\ 0 & k_c \end{bmatrix} \begin{bmatrix} \theta \\ \theta_d \\ u \end{bmatrix} = \begin{bmatrix} F(s) \cdot l \\ 0 \\ 0 \end{bmatrix}$$

$$(1)$$

Where:

$$\begin{split} M_{1,1} &= \frac{\rho.l^3}{3} + m. \, l^2 + m_d (d + l_d)^2 + \rho_d. \, d. \, l_d (d + l_d) + \rho_d. \frac{l_d^3}{3} \\ M_{1,2} &= -m_d. \, d. \, l_d - m_d. \, l_d^2 - \rho_d. \, d. \frac{l_d^2}{2} - \rho_d. \frac{l_d^3}{3} \\ M_{1,3} &= m. \, l + \frac{\rho.l^2}{2} + m_d (d + l_d) + \rho_d. \, d. \, l_d + \rho_d. \frac{l_d^2}{2} \\ M_{2,1} &= -m_d. \, d. \, l_d - m_d. \, l_d^2 - \rho_d. \, d. \frac{l_d^2}{2} - \rho_d. \frac{l_d^3}{3} \\ M_{2,2} &= m_d. \, l_d^2 + \rho_d. \frac{l_d^3}{3} \\ M_{2,3} &= -m_d. \, l_d - \rho_d. \frac{l_d^2}{2} \\ M_{3,1} &= m. \, l + \frac{\rho.l^2}{2} + m_d (d + l_d) + \rho_d. \, d. \, l_d + \rho_d. \frac{l_d^2}{2} \\ M_{3,2} &= -m_d. \, l_d - \rho_d. \frac{l_d^2}{2} \\ M_{3,3} &= m_c + m + m_d + \rho. \, l + \rho_d. \, l_d \end{split}$$

Usually, the formulation and solution of control problems present equations of motion Equation (1) as state equations:

$$\dot{z}(t) = Az(t) + Bu(t) + Ef(t)$$
(2)

where z(t) is the state vector 4n + 2m; A corresponds to the matrix system state $4n + 2m \times 4n + 2m$; B and E are the control and disturbance input matrices; Finally, the vector $\dot{z}(t)$ represents the state of the structural system which contains the relative velocity and responses to acceleration of the structure with respect to the ground. The details of each vector and matrix are listed as follows:

$$z(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \quad A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ M^{-1}D \end{bmatrix} \quad E = \begin{bmatrix} 0 \\ M^{-1}H \end{bmatrix}$$
(3)

Where x(t) and x(t) are system displacement and velocities vectors, respectively. Matrices D and H define the location of control forces and of external loading, respectively.

3 SEMI-ACTIVE CONTROL STRATEGY

The Bang Bang control, also called control ON / OFF control, is a feedback controller that suddenly changes between two limit values. This device compares the input with a target value, so that if the output exceeds the input, the actuator is switched off, otherwise, the actuator is now on. Figure (2) shows an example of a bang bang controller block diagram. This is a low cost controller, further its simplicity and convenience.

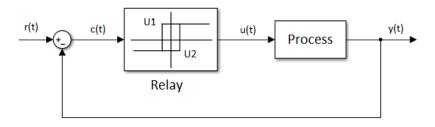


Figure 2: Bang bang control [16].

The control strategy is to control structural response using bang bang control, varying pendulum TMD stiffness k_d and damping c_d , switching from one extreme set of values to the other. The optimal parameter values (k_d and c_d) are obtained based on linear optimal control algorithm (linear quadratic regulator – LQR) [17].

First a LQR controller is designed assuming an active pendulum TMD system and neglecting the actuator dynamics. The optimal actuator force u(t) is defined by the gain matrix G. The actuator force is not really applied at the TMD, this force is applied through a semi-active damper.

The linear optimal control problem consist in finding the control vector $\mathbf{u}(t)$ that minimizes the performance index J subject to the constraint (3). In structural control, the performance index is usually chosen as a quadratic function in $\mathbf{z}(t)$ and $\mathbf{u}(t)$, as follows

$$J = \int_{t_0}^{t_f} \left[\mathbf{z}^{\mathrm{T}}(t) \mathbf{Q} \mathbf{z}(t) + \mathbf{u}^{\mathrm{T}}(t) \mathbf{R} \mathbf{u}(t) \right] dt$$
(4)

Where Q is a $2n \times 2n$ positive semi-definite matrix and R is a $m \times m$ positive definite matrix. The matrices Q and R are referred to as weighting matrices, whose magnitudes are assigned according to the relative importance attached to the state variables and to the control forces in the minimization procedure. The minimization problem leads to a Riccati differential equation system:

$$\dot{\mathbf{P}}(t) + \mathbf{P}(t)\mathbf{A} - \frac{1}{2}\mathbf{P}(t)\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}(t) + \mathbf{A}^{\mathrm{T}}\mathbf{P}(t) + 2\mathbf{Q} = \mathbf{0},$$

$$\mathbf{P}(t_f) = \mathbf{0}$$
(5)

where P(t) is the Ricatti matrix. The control vector u(t) is linear in z(t). In this case, the linear optimal control law is

$$\mathbf{u}(t) = \mathbf{G}(t)\mathbf{z}(t) = -\frac{1}{2}\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}(t)\mathbf{z}(t)$$
(6)

where $\mathbf{G}(t) = -\frac{1}{2}\mathbf{R}^{-1}\mathbf{B}^{\mathsf{T}}\mathbf{P}(t)$ is the control gain. In most structural applications, numerical analysis show that Riccati matrix $\mathbf{P}(t)$ stays constant during the control range, converging fast to zero on the neighborhood of t_f [17]. Thus $\mathbf{P}(t)$, in most cases, can be approximated by a constant matrix \mathbf{P} and Riccati equation (5) is reduced to

$$\mathbf{P}\mathbf{A} - \frac{1}{2}\mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P} + \mathbf{A}^{\mathrm{T}}\mathbf{P} + 2\mathbf{Q} = \mathbf{0},$$
(7)

and the constant control gain is given by

$$\mathbf{G} = -\frac{1}{2}\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P} \tag{8}$$

As it was stated above the control vector $\boldsymbol{u}(t)$ is proportional to state vector $\boldsymbol{z}(t)$ which contains system displacements and velocities. The elements of control gain matrix \boldsymbol{G} that multiplies θ_d and $\dot{\theta}_d$ are considered to set optimum values for pendulum TMD parameters k_d and c_d .

4 NUMERICAL RESULTS

The present work numerical study considered properties of the turbine NREL 5 MW [18], they are exhibited in Table 1.

Rating	5 MW
Rotor, hub diameter	126.3 m
Hub Height	90 m
Rotor mass	110,000 kg
Tower Mass	347,460 kg

Table 1: Wind turbine's properties [18]

Also the following values were considered for system parameters [14]: l = 90 m, $m_c = 240,000$ Kg; g = 9.81 m/s², K = 8.94 x 10^8 N/rad, m = 110,000 kg, $m_d = 49,267$ kg, d = 79.6 m. The wind dynamic loading was modeled as an applied load at the inverted pendulum mass. Two loading cases were considered: harmonic force and white noise force. The system without control fundamental frequency is 0.97 rad/s.

In previous studies [15], parameter values for pendulum TMD were found out through a parametric study: $kd = 5.9 \times 10^6 \text{ N/m}$, $cd = 4.4 \times 10^5 \text{ Ns/m}$ and $l_d = 7.4 \text{ m}$. It results on an efficient vibration control of the wind turbine tower when subjected to a harmonic force approximation of wind load and also for a white noise wind load. In order to improve the performance of this structural control device it is proposed in the present work a semi-active pendulum TMD that varies k_d and c_d properties through a bang bang (ON/OFF) control.

Aiming to design an efficient semi-active controller, the values of parameters k_d and c_d were optimized via classical optimal control algorithm (LQR), assuming an active pendulum TMD system. The optimal actuator force u(t) is defined by the gain matrix G, obtained through solution of Riccati equation (Equation 7). It is not really applied at the TMD, this force is applied through the semi-active damper.

Optimal control effectiveness is directly related to a proper choice of the weighting matrices Q and R. The flexibleness on this matrices choice allows the generation of a family of multiple different controllers; this represents the great advantage, as well as, the great disadvantage of this method. It is extremely important to carry out a detailed parametric study of weighting matrices in order to ensure control robustness [19].

The optimal control algorithm does not lead to a control force truly optimal in certain cases, since the excitation is ignored in obtaining the Riccati matrix [3]. Because of this, in this work two loading cases were considered to obtain the optimum control force u(t): harmonic (HSA) and white noise (WNSA). Parametric studies were performed in both loading cases to set Q and R matrices.

The parametric study for HSA loading case provided the following weighting matrices and the corresponding gain matrix:

As for WNSA loading case, the following weighting matrices and the corresponding gain matrix are:

$$G = [1.444 * 10^6 \ 1.563 * 10^4 \ 1.332 * 10^4 \ -7.220 * 10^7 \ 1.070 * 10^6 \ -1.563 * 10^6]$$

Gain Matrix G coefficients are applied to displacements and velocities θ , θ_d , u, $\dot{\theta}$, $\dot{\theta}_d$ and \dot{u} , however only the coefficients associated to θ_d and $\dot{\theta}_d$, are considered on the semi-active case. The optimum values of k_d and c_d are obtained adding G[2,1] and G[5,1] to passive k_d and c_d values, respectively, obtaining: $k_d = 5.9292 \times 10^6 \text{ N/m}$ and $c_d = 2.1654 \times 10^6 \text{ Ns/m}$ for HSA case and $k_d = 5.9112 \times 10^6 \text{ N/m}$ and $c_d = 1.1886 \times 10^6 \text{ Ns/m}$ for WNSA case.

Comparing those optimum parameter values to passive parameter values ($k_d = 5.9 \times 10^6$ N/m and $c_d = 4.4 \times 10^5$ Ns/m to the optimum ones it can be observed that a significant increase occurs only for c_d , showing that a semi-active device varying only damping parameters would be satisfactory.

A time domain analysis was performed, four situations were considered for analysis: (1) structure without control, (2) system with passive TMD (sTMD) [14] considered as the semi-

active turned OFF, (3) system with semi-active ON, with optimum parameter for harmonic loading (HSA) and (4) system with semi-active ON, with optimum parameter for white noise loading (WNSA). Time domain simulations for harmonic loading were performed varying forcing frequency in a range from 0 to 2 rad/s.

Figure 3 shows efficiency relation of the semi-active device in ON (HSA) and OFF position (sTMD). This efficiency is measured by *rms* values as follows:

$$EFF = \frac{\theta_{OFF} - \theta_{ON}}{\theta_{OFF}} \times 100 \% \tag{9}$$

Figure 3 shows efficiency value as a function of forcing frequency. On the frequency range where efficiency has positive values HSA (ON) is effective and turns to be ineffective for negative values of efficiency.

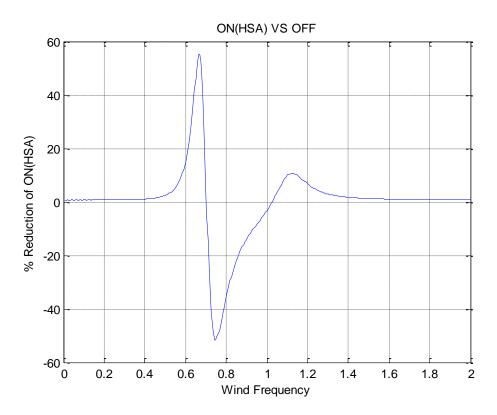


Figure 3: Efficiency of HSA (ON) semi-active control

Figures 4(a) and 4(b) shows angular displacement θ time history for OFF and ON(HSA) cases, considering forcing frequencies of 0.66 rad/s and 0.74 rad/s, respectively. These frequencies values match to maximum and minimum efficiency curve peaks of Figure (3).

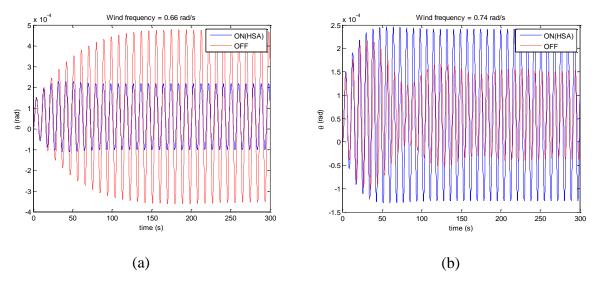


Figure 4: Angular displacement time history (a) ω_f =0.66 rad/s; (b) ω_f =0.74 rad/s

It can be noticed, as expected from Figure 3, that for $\omega_f = 0.74$ rad/s HSA(ON) is less effective than OFF position. The same time histories are presented in Figures 5(a) and 5(b) plotting also the system uncontrolled response, in a way to verify semi-active ON/OFF control effectiveness. It can be verified that the semi-active controller presents a good performance on reducing excessive vibration amplitudes.

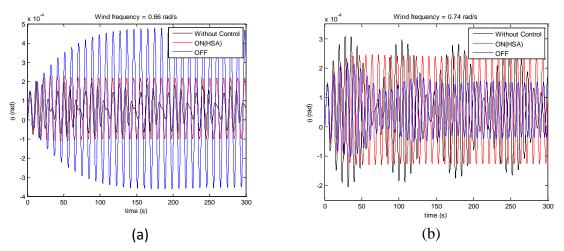


Figure 5: Angular displacement time history with and without control

Figure 6 shows angular displacement time history when the structure is subjected to a white noise wind loading. It is verified that HSA(ON) case presents a better efficiency in most of the time response.

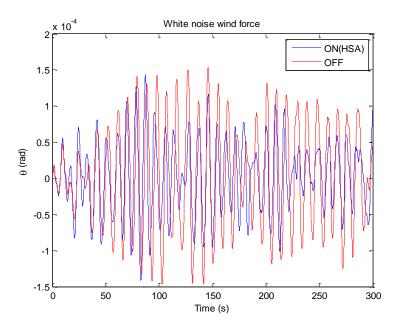


Figure 6: Angular displacement time history: white noise loading.

Similar to Figure 3, Figure 7 shows the efficiency curve for the case ON (WNSA). On the frequency range where efficiency has positive values ON (WNSA) is effective and turns to be ineffective for negative values of efficiency.

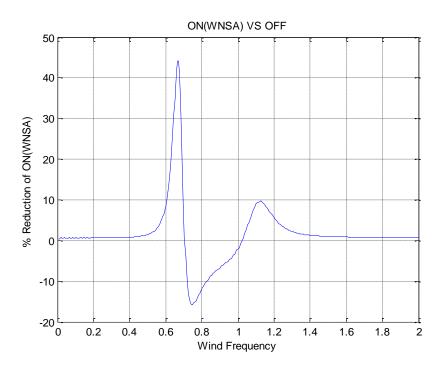


Figure 7: Efficiency of WNSA (ON) semi-active control

Figures 8(a) and 8(b) shows angular displacement θ time history for OFF and ON(WNSA) cases, considering harmonic load with forcing frequencies of 0.66 rad/s and 0.74 rad/s,

respectively. These frequencies values match to maximum and minimum efficiency curve values of Figure 7.

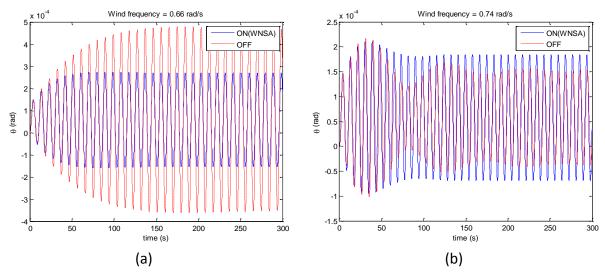


Figure 8: Angular displacement time history (a) ω_f =0.66 rad/s; (b) ω_f =0.74 rad/s

It can be noticed, as expected from Figure 7, that for $\omega_f = 0.74$ rad/s WNSA(ON) is less effective than OFF position. Figure 9 shows angular displacement time history when the structure is subjected to a white noise wind loading. It is verified that WNSA(ON) case presents a better efficiency in most of the time response.

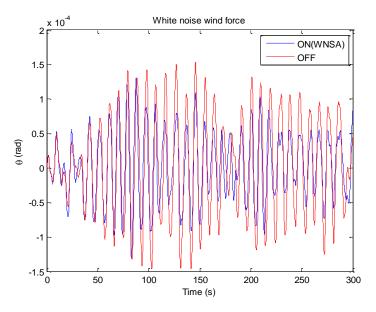


Figure 9: Angular displacement time history: white noise loading.

Comparing efficiency curves on Figures 3 and 7 it can be noticed that HSA present high peak amplitudes than WNSA, this is also visible comparing time histories on Figure 4(a) with Figure 8(a) and Figure 4(b) with Figure 8(b). It can be concluded that HSA(ON) present the

best efficiency and the greater inefficiency compared to WNSA(ON), however semi-active switch ON-OFF supplies this inefficiency.

5 CONCLUSIONS

In this work a semi-active TMD pendulum is proposed to control excessive vibration of an offshore floating wind turbine. The structure is modeled as an inverted pendulum discrete model. This model is presented as a preliminary model for structural control analysis, the results serve as a basis for real structure design with a more careful modeling. A bang bang (ON/OFF) control strategy was considered, semi-active TMD stiffness and damping values were calculated trough optimal control theory. It was held a detailed parametric study of weighting matrices in order to ensure control robustness. Since the excitation is ignored in obtaining the Riccati matrix, two loading cases were considered: harmonic and white noise, leading to a set of two k_d and c_d parameters (HSA and WNSA). It was concluded that HSA parameters are the best choice for setting the semi-active TMD ON position. Satisfactory results were found out compared to those of passive TMD pendulum. Semi-active controller presents a good performance on reducing excessive vibration amplitudes. Further studies would be necessary in order to test other control strategies to semi-active controller design.

6 ACKNOWLEDGEMENTS

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