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STRUCTURAL HEALTH MONITORING USING TIME-DELAY EMBEDDING AND PHASE-SPACE WARPING

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Abstract. *This paper presents a method to utilize Time-Delay Embedding (TDE) to perform structural health monitoring and obtain a damage metric that tracks deterioration in structural health over time. Proof-of-concept is demonstrated through an experimental investigation. The experimental setup consisted of high-rise building model, made of aluminium, mounted on a digitally-controlled shaker table. Floor accelerations are recorded using accelerometers on each elevation to capture the dynamic response during the base excitation. Natural modes of vibration and their corresponding frequencies were determined experimentally using a frequency sweep of low-amplitude sinusoidal support excitations. The model building was excited at its natural frequency with moderate-amplitude sinusoidal base excitation to establish its "pristine" condition baseline. Subsequently, damage is introduced to columns on the first floor by cutting two notches at mid-height, significantly reducing their cross-sectional areas. The same base motion protocol was re-applied and acceleration data was collected and processed to detect the pre-known damage patterns as well as the total failure using changes in the oscillation orbits. The damage metric is shown to adequately detect structural damage.*

1 INTRODUCTION AND BACKGROUND

With the observed increase in frequency and intensity of seismic activity, there is a need to detect damage in important structures such as bridges and high-rise buildings. This can be done through continuous monitoring for structural integrity purposes. Further, ongoing Structural Health Monitoring (SHM) is not limited to seismic activity but can also be used to maintain long-term safety, viability and value of structures.

SHM is the non-intrusive collection and analysis methods for damage detection and diagnosis. The goal of SHM is to characterize the structure's performance and to help maintain the structural performance over its service life. Vibration-based SHM assumes that the structural dynamic response will depart from a baseline healthy performance pattern(s) when damage occurs in the structure. Thus, damage detection is contingent upon successfully extracting sensitive damage feature(s), patterning such feature(s) and realizing changes in these patterns as damage progresses. To achieve that, a SHM system incorporates four components: data acquisition, damage feature extraction, damage pattern recognition (diagnosis) and damage prognosis. These components relate to the collection of data from sensors or sensor networks, the identification of a critical feature that is sensitive to the damage state in the structure, the use of the damage feature(s) to quantify damage, and the estimating of the remaining service life using system prognosis methods [1]. Over the past three decades, numerous methods with the objective of extracting sensitive damage feature(s) have been tested on several structures [2]–[6]. Tools for damage detection using structural dynamics analysis, such as modal update and Fourier and wavelet transforms, have been examined over a large span of structures ranging from simple cantilevers to large bridges and multi-story buildings [7], [8]. By nature, tall buildings are particularly susceptible to strong ground motions propagating from long distances with ample energy levels at low predominant frequencies [9]. Moreover, tremors can cause low level damage that might not be apparent initially but will lead to damage accumulation and long-term impact on the structural integrity of the building.

Time-Delay Embedding (TDE) was introduced in the 1980s as a method to reconstruct inaccessible states of dynamic systems [10]–[12]. The method uses the time-history of a single accessible state in a coupled dynamic system to reconstruct the full system states. To reconstruct the other states, the time-history of the available state is delayed by a *'proper'* time step. For example, when using TDE one can measure the acceleration of one floor in a building to estimate the states of the key parts in the coupled dynamic system, such as other floors accelerations. The reconstructed phase-space can then be used to track damage evolution. One approach to achieve this is the so-called Phase-Space Warping (PSW) [13]–[15]. PSW uses the healthy system response to construct local linear maps in phase-space that describe the characteristics of the healthy system response. It then compares evolution of the actual system response in phase-space to the healthy system local maps to extract a damage tracking metric.

In this paper, a novel method is developed to detect structural damage in high-rise buildings using TDE and PSW.

2 TIME-DELAY EMBEDDING AND PHASE-SPACE WARPING

A simple representation of an n-story frame is a coupled MDOF oscillator:

$$\begin{aligned} m_1\,\ddot{x}_1 + c_1\,\dot{x}_1 + k_1\,x_1 - k_2\,x_2 &= f_1(x_1,\dot{x}_1,x_2,\dot{x}_2,t) \\ m_2\,\ddot{x}_2 + c_2\,\dot{x}_2 - k_2\,x_1 + k_3\,x_2 &= f_2(x_1,\dot{x}_1,x_2,\dot{x}_2,t) \\ m_n\,\ddot{x}_n + c_n\,\dot{x}_n - k_n\,x_{n-1} + k_{n+1}\,x_n &= f_n(x_{n-1},\dot{x}_{n-1},x_n,\dot{x}_n,t) \end{aligned} \tag{1}$$

where x_1 to x_n are the horizontal displacements of the lumped mass of the n-floors, c_1 to c_n are the damping coefficients, k_1 to k_n are the stiffness of the floors, and the functions f_1 to f_2

contain nonlinear coupling terms between the n-floors motions and external forces. The vibration of this system is excited due to external loads such as ground excitations. Typically, the system periodic motions are on the order of 0.1 to 1 second. On the other hand, the damage processes we are concerned with are slow, leading over long-time, to performance degradation. The typical half-life for these processes is on the order of thousands of hours (10^6 seconds). As far as the dynamic system in n-equations is concerned, this is a slow-variation in the system parameters that modulates the dynamic response. Conversely, damage is also driven by the system fast dynamics (oscillations). Assuming, for simplicity, that a single-damage process appears in the system, we can represent it by the state variable y. As a result, the system evolution is now composed of fast observable dynamics, the native states, and a very slow unobservable dynamic process, the damage state $\dot{y}$; an additional equation can be added as:

$$\dot{y} = \varepsilon g(x_{n-1}, \dot{x}_{n-1}, x_n, \dot{x}_n, y) \tag{2}$$

where ε is a small non-dimensional parameter describing the slow evolution of the damage process compared to the fast oscillations of the system and the function g describes the evolution of the damage mechanism over time. The goal is to use the observable part of the dynamics $(x_{n-1}, \dot{x}_{n-1}, x_n, \dot{x}_n)$ to reconstruct the unobservable part of the dynamics, y. Therefore, damage detection over time can be paused as a system identification problem for a hierarchal dynamic system containing fast-time and slow-time scales. Since the system response is coupled, the observable fast-time dynamics is interrogated to identify the slow-time dynamics (damage) of the oscillator.

To achieve this goal, the method of Phase-Space Warping (PSW) is employed which tracks the orbits of the system in phase space as they evolve over time. The orbits are constructed using the observable states $(x_{n-1}, \dot{x}_{n-1}, x_n, \dot{x}_n)$. Degradation is a process that drives the orbits progressively away, in shape and/or location in the phase space, from the healthy (baseline) orbit of the system. This ‘drift’ (distance between the current orbit and the baseline orbit) is measured as the orbit evolves over time and quantified as the scalar tracking metric ε(t). It has been demonstrated [13], [14] that a one-to-one relationship exists between ε(t) and y(t). The difference between the time scales of the oscillator's native fast dynamics and the damage process's slow dynamics is proven to be useful through periodic sampling the system dynamics. Each sample (snapshot) can then be used to construct an ‘averaged’ orbit, assuming that the damage process is stationary (y(t) = constant) over the medium time-scale (100s of periods). Finally, the average orbits are compared to the baseline orbit over the long time-scale (10,000s of periods) and the difference is quantified as the tracking metric ε(t). This approach has the advantages of reducing data collection and computational costs by periodic sampling over the long time-scale and improving the signal-to-noise ratio by averaging over the medium time-scale, thereby facilitating low-cost early detection of damage or degradation. As the drift and the tracking metric ε(t) grow to exceed the signal noise level, given a pre-set confidence level, damage detection is declared. For brevity, the intricate mathematical details of the procedure are not presented herein. However, the reader is referred to the aforementioned references for further details.

3 EXPERIMENTAL INVESTIGATION

The experimental setup consisted of model 4-story building made of aluminium mounted on a digitally-controlled shaking table as can be seen in Figure 1. Floor accelerations are recorded using accelerometers on each elevation to fully capture the dynamic response during the base shaking. Natural modes of vibration and their corresponding frequencies are determined experimentally using a frequency sweep of low-amplitude sinusoidal support excitations. The model building is excited at its fundamental frequency (4.37Hz) with moderate-amplitude sinusoidal base excitation to establish its "healthy" or "pristine" condition baseline using the collected and processed data.

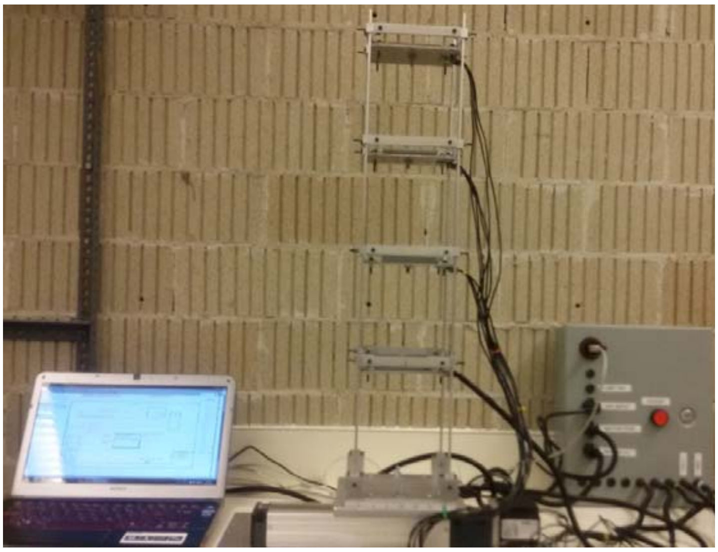

Figure 1: 4-story model building mounted to the shaker table and controller setup.

Subsequently, damage is introduced to columns on the first floor by cutting two notches at the mid-height, significantly reducing their cross-sectional areas (Figure 2). The same base motion protocol is re-applied and acceleration data is collected and processed to detect the pre-known damage patterns as well as the total failure using changes in the oscillation orbits.

Figure 2: Column damage introduced by notched cross-section (cut-out).

4 RESULTS AND DISCUSSION

The healthy or pristine baseline state of the structure is captured through its frequency signature (Fast Fourier Transform, FFT), which is presented in Figure 3. It can be seen that FFT amplitude is highest for the first harmonic and exhibiting descending peaks for the higher harmonics except for the third harmonic. The dominance of a third harmonic among the higher harmonics indicates that the building is symmetric along the direction of oscillations.

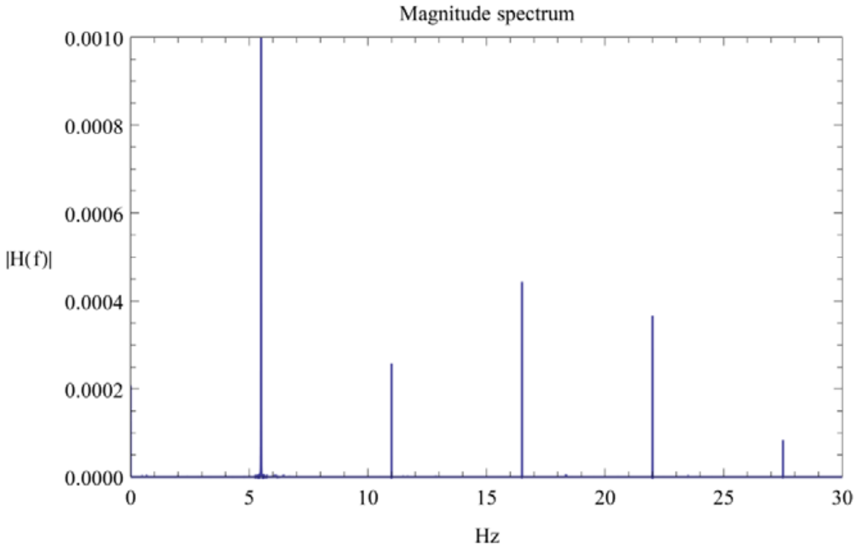

Figure 3: FFT for the model building healthy state.

Figure 4 illustrates the effect of the inflected damage to the columns at the first floor by notching their cross-sections. Inspection of the FFT signature, in comparison to the healthy state, it can be readily appreciated that the harmonics have wider bases. i.e. the damage-induced nonlinearities blur the otherwise narrow-banded peaks at the structure’s harmonics. This is most evident in the first harmonic. This indicates that the introduced nonlinearities are in fact significant and relevant to the building response as the system has suffered noticeable change in characteristics (reduced stiffness). It is also seen that the damage is manifested through the re-ordered amplitudes of higher harmonics (the fourth harmonic is more dominant than the third). This is in agreement with the loss of symmetry that the building model underwent physically.

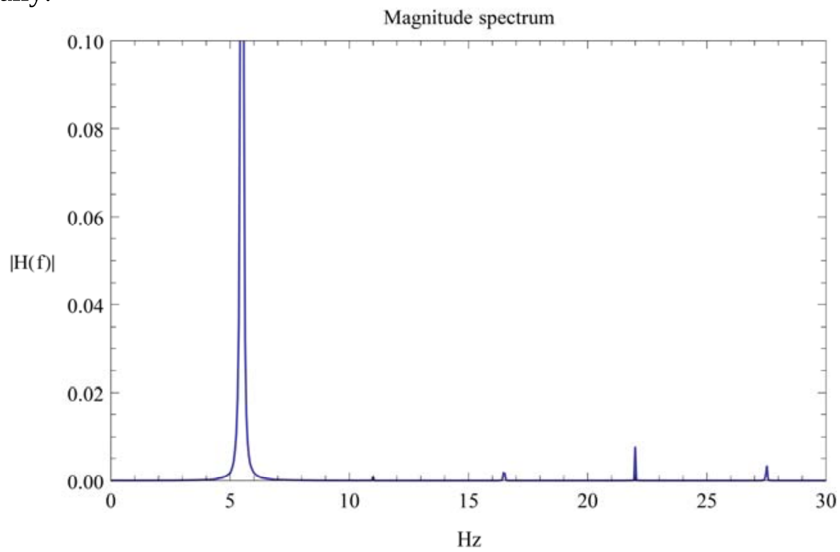

Figure 4: FFT for the model building state with notched columns.

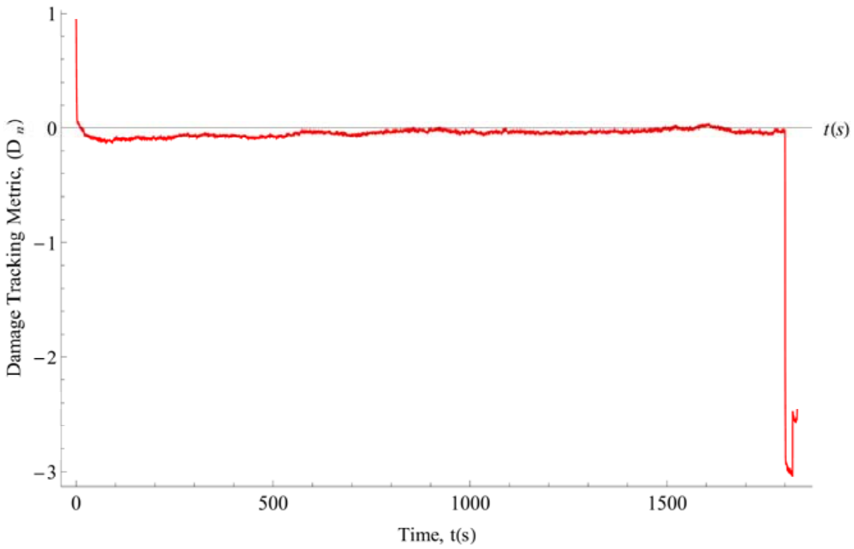

Figure 5: PSW damage metric

The PSW damage metric depicts the evolution and time-history of structural damage as demonstrated in Figure 5. The damage metric has very small values throughout the ground excitation application all the way till the introduction of the notches to the first story columns. At which point, the damage metric drops abruptly to -2.8, clearly indicating a significant abrupt damage. Increasing levels of damage induced by fatigue in the notched column cross-sections are also apparent from the increasing negative slope the fill factor assumes as it approaches -3. After many oscillations with increasing damage, one of the notched columns broke. This was immediately identified by the damage metric changing abruptly again to approximately -2.1. The severe inflected damage through the loss of one of the two notched columns magnified the horizontal irregularity (lack of symmetry) in the model. Subsequent oscillations exerted higher demands on the remaining column and in turn inflected increasingly higher rate of damage. This higher rate is demonstrated through the steeper negative slope in the damage metric which ultimately results in accelerated failure in the remaining notched column. The final collapse of the building is signified by a third abrupt jump in the fatigue metric.

5 CONCLUSIONS

A novel method to track structural damage in MDOF systems is presented. The method assumes that damage is a slowly changing variable in the measured floor acceleration signal. A damage metric is estimated from the slowly varying in the shape and position of phase space orbits as time passes. As a first step, the method of Time-Delay Embedding (TDE) is used to reconstruct the pseudo phase-space from acceleration measurements. Then, the tracking procedure is carried out using the method of Phase-Space Warping (PSW). Results show that the methods used are capable of identifying slowly changing damage using acceleration signals. The damage index revealed information about the health state of the overall coupled dynamic system under test, a 4-story model building in this case.

It is shown that TDE and PSW methods can provide an accurate overall ‘big picture’ estimator of structural damage. Unlike conventional vibration-based damage detection techniques described in the current literature (such as comparisons of FFT signatures only), this method can supply information about damage evolution in an insightful and unique manner. Another major strength to the presented SHM approach is that it provides an ongoing estimate of the *current* health of the structure and with slight modifications, can also provide structural damage prognosis in the form of an estimated time to failure. This opens doors to real-time SHM encompassing diagnosis, monitoring and prognosis of important structures such as bridges and high-rise buildings.

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REFERENCES

[1] C. R. Farrar and D. A. Jauregui, “Comparative study of damage identification algorithms applied to a bridge: I. Experiment,” *Smart Materials and Structures*, vol. 7, no. 5, pp. 704–719, Oct. 1998.

[2] F.-K. Chang, *Structural Health Monitoring 2000*. CRC Press, 1999, p. 1062.

[3] D. L. Balageas, Ed., “Structural Health Monitoring 2002,” in *Proceedings of the 1st International European Workshop on Structural Health Monitoring*, 2002.

[4] C. Boller and W. J. Staszewski, Eds., “Structural Health Monitoring 2004,” in *Proceedings of the 2nd International European Workshop on Structural Health Monitoring*, 2004.

[5] A. Güemes, Ed., “Structural Health Monitoring 2006,” in *Proceedings of the 2nd International European Workshop on Structural Health Monitoring*, 2006.

[6] F. K. Chang, Ed., “Structural Health Monitoring 2007: Quantification, Validation, and Implementation,” in *Proceedings of the 6th International Workshop on Structural Health Monitoring*, 2007.

[7] L. A. N. Laboratory, “A Review of Structural Health Monitoring Literature: 1996-2001 (LA-13976-MS),” 2004.

[8] M. Taha and A. Noureldin, “Wavelet transform for structural health monitoring: a compendium of uses and features,” *Journal of Structural Health Monitoring*, 2006.

[9] M. AlHamaydeh, K. Galal, and S. Yehia, “Impact of lateral force-resisting system and design/construction practices on seismic performance and cost of tall buildings in Dubai, UAE,” *Earthquake Engineering and Engineering Vibration Journal*, 2013.

[10] A. Fraser and H. Swinney, “Independent coordinates for strange attractors from mutual information,” *Physical review A*, 1986.

[11] T. Buzug and G. Pfister, “Optimal delay time and embedding dimension for delay-time coordinates by analysis of the global static and local dynamical behavior of strange attractors,” *Physical review A*, 1992.

[12] M. Kennel, R. Brown, and H. Abarbanel, “Determining embedding dimension for phase-space reconstruction using a geometrical construction,” *Physical review A*, 1992.

[13] D. Chelidze, J. Cusumano, and A. Chatterjee, “A dynamical systems approach to damage evolution tracking, part 1: description and experimental application,” *Journal of Vibration and Acoustics*, 2002.

[14] J. Cusumano, D. Chelidze, and A. Chatterjee, “A dynamical systems approach to damage evolution tracking, part 2: model-based validation and physical interpretation,” *Journal of Vibration and Acoustics*, 2002.

[15] D. Chelidze and M. Liu, “Multidimensional Damage Identification Based on Phase Space Warping: An Experimental Study,” *Nonlinear Dynamics*, vol. 46, no. 1–2, pp. 61–72, Aug. 2006.