COMPDYN 2015
5th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, V. Papadopoulos, V. Plevris (eds.)
Crete Island, Greece, 25–27 May 2015

THE INFLUENCE OF DEGRADATION OF SHEAR MODULUS OF SOIL ON PREDICTED RAILWAY VIBRATIONS IN TUNNELS EXCAVATED WITH TBM: A NUMERICAL ANALYSIS

J. Fernández Ruiz [1], P. Alves Costa [2], P. Lopes [3], L. Medina Rodríguez[4]

[1] University of La Coruña
Campus de Elviña, 15071 La Coruña (Spain)
e-mail: jesus.fernandez.ruiz@udc.es

[2]University of Porto
Rúa Dr. Roberto Frías, s/n 4200-465 Porto (Portugal)
e-mail: pacosta@fe.up.pt

[3]University of Porto
Rúa Dr. Roberto Frías, s/n 4200-465 Porto (Portugal)
e-mail: cpl@isep.ipp.pt

[4] University of La Coruña
Campus de Elviña, 15071 La Coruña (Spain)
e-mail: lmedina@udc.es

Keywords: Railway vibrations in tunnels, G_s soil degradation due to TBM

Abstract. *In this paper the influence of the stress state within the soil on predicted railway vibrations in tunnels is studied, regarding Gs modulus degradation that occurs when the tunnel is excavated with Tunnel Boring Machine (TBM). The adopted procedure is uncoupled, using a 2D FEM model to simulate the process of tunnelling which incorporates the constitutive model known as Hardening Soil with small strain stiffness (HS-small). The Gs modulus values are then used in a dynamic numerical model based on a 2.5D FEM-PML approach in order to simulate the vibrations induced by railway traffic. Simulations are then performed assuming two different scenarios for the elastic properties of the ground: i) assuming Gs modulus for low strain deformation for whole domain, i.e., neglecting the perturbation of the soil due to the tunnel construction; ii) assuming values of Gs modulus computed taking into account the strains induced by the tunnel construction. The numerical results show a clear influence of tunnelling process, being more relevant at closest distances to the tunnel.*

1. INTRODUCTION

Vibrations induced by railway traffic in tunnels is a special interest and development topic in recent years, mainly in urban areas where nuisance generated can be common if no preventive measures are taken or if the initial design is not right. It is reasonable to affirm that transport infrastructure in the 21st century has much more requirements than previously, since environmental aspects have gained a significant importance. Nowadays, the impact of vibrations on working and living environments is considered as an important problem of modern societies [1]. Moreover, the relevant growth of population around metropolitan areas implies the need for more sustainable underground transport systems. One of the key advances to minimize the impact of railway systems is the development of the numerical models that are capable of predicting correctly the level of vibrations on free-field or on buildings and to help in finding technical solutions for mitigating possible nuisances. For that, in last recent years several models have been developed in order to understand the complex problem of railway vibrations in tunnels. It is worth mentioning semi-analytical models such as those proposed by Hussein and Hunt [2], Kuo et. al [3] and Muller [4], whose main advantage is a high computational efficiency although they could not be rigorously applied in some situations. Complex numerical models have also been developed, such as those presented by Clouteau et al. [5] and Gupta et al. [6, 7], among others.

An interesting approach of the problem has been presented by Hung and Yang [8] and Lopes et al. [9, 10] in which 2.5D models are developed. The former is a 2.5D FEM-IEM model while the latter is a 2.5D FEM-PML model. The 2.5D models can be applied for longitudinally invariant structures, obtaining the 3D solution by numerical discretization of the cross-section combined with the Fourier transformation of the domain along the orthogonal direction. In addition to these three last researches, similar models have also been applied for the study of several railway lines both in surface and in tunnels [11-14].

In the above-mentioned studies the soil behaviour has been considered as linear-elastic as the vibrations induced by railway traffic in tunnels generate very small strains in soils. This approach seems to be clear and reasonably correct. Nevertheless, none of them has considered the possible effect of the tunnel construction on the degradation of the shear modulus of the soil. In fact and to authors' knowledge, there are no studies about railway vibrations in tunnels in which this phenomenon had been taken into account. During tunnelling process the shear modulus of soil does not remain constant and its degradation magnitude depends on multiple factors such as diameter and depth of tunnel, excavation method and soil stiffness, among others. It is not easy to quantify standard values of shear strains during tunnel excavation although they could range between 10^{-4}-10^{-2} [15]. Even though tunnel excavation with TBM is a three dimensional problem, Panet and Guénot [16] demonstrated that 3D ground response can be approximately analysed with a plane strain approach, introducing a stress release coefficient that could be evaluated through 2D and 3D numerical comparisons. However, such comparisons are very limited in the technical literature. Because of underground railway vibrations is the main focus of this paper, a 2D simplified method has been applied for studying G_s degradation during tunnelling process where the tunnel lining grouting has not been considered due to G_s degradation induced by this process is very located in the soil attached to the tunnel lining.

Regarding the complete behaviour of the soil, from very small strains to large strains range, several constitutive models have been developed such as the *Hyperbolic model* [17], the *Equivalent linear model* [18], the *Hardening soil model* [19] and, recently, the *Hardening soil model with small-strain stiffness* [20], among others. The latter is applied in this paper in order to take into account G_s degradation during tunnelling process.

2. NUMERICAL APPROACH

2.1 Generalities

The applied computational scheme is uncoupled, using a 2D FEM static model to simulate the tunnelling process and calculate G_s degradation within soil by using the PLAXIS commercial code. The *Hardening soil model with small-strain stiffness* (HS-small) has been used to take into account the G_s modulus degradation. Once calculated the resulting stress state in soil after tunnel construction, a 2.5D FEM-PML model formulated in the wavenumber/frequency domain is used to obtain the free-field response, assigning on every finite element the resulting soil stiffness from the 2D FEM static model. This stiffness (different for each finite element) remains constant during the computing process with the 2.5D FEM-PML model, since the strains induced by the traffic are very small. The computed results are compared with those obtained without regarding tunnel excavation, i.e. neglecting the perturbation of the soil due to tunnel construction. In the followings sections these numerical models are briefly described.

2.2 2D Numerical simulation of tunnelling with TBM

To simulate tunnel construction with TBM, a 2D FEM model has been used through software PLAXIS. A 2D simplified method is applied in order to calculate the stress state within soil. Some of the 2D available methods are: Core Support Method (α-method), Lining Reduction Method (δ-method) and Stress Reduction Method (β-method), among others [21]. In this research, β-method has been used to simulate the construction process of tunnel, whose main feature is to take into account arching effect that occurs within the soil due to near presence of tunnel face. The basis of this method is schematized in *Fig. 1* and the idea is that the initial stresses p_k that act in the contour of the tunnel are divided into two parts: the first is $(1-\beta)\cdot p_k$ that is applied to the unsupported tunnel and the second is $\beta\cdot p_k$ that is applied to the supported tunnel. In PLAXIS code this approach is performed through three construction phases, controlling in each one the staged construction process (through parameter ∑Mstage), as follows:

1- Phase 1: Generate the initial stress field with K_0-procedure in case of horizontal soil layers
2- Phase 2: Eliminate tunnel clusters applying an ultimate level of ∑Mstage equal a 1-β
3- Phase 3: Activate the lining, allowing a complete staged construction (∑Mstage = 1)

The main uncertainty of this method applied to tunnel excavation is the β value. This parameter can oscillate between 0.2-0.8 depending on excavation method, tunnel diameter, among others factors. In the TBM cases, β value is usually between 0.4-0.8.

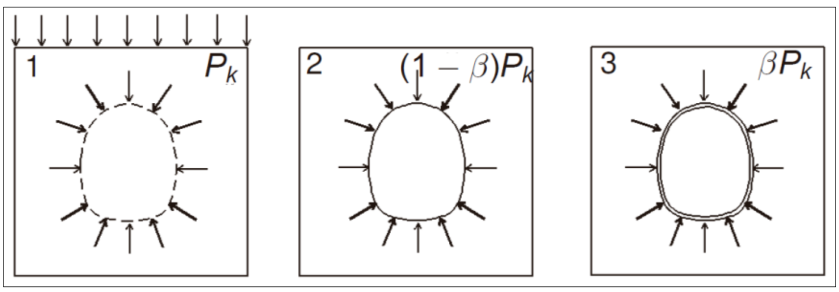

Fig. 1. Schematic representation of the β-method for the analysis of tunnels

2.3 2.5D FEM-PML approach for simulation of traffic vibrations

Contrarily to what happens on the tunnelling operations simulation, where a reasonable solution can be found using a 2D approach, the problem of vibrations induced by railway traffic is fully 3D and, consequently is not compatible at all with a 2D plane strain state approach. However, assuming that the system is invariant and infinitive along the longitudinal direction, the 3D solution can be achieved adopting a 2.5D procedure, i.e., only the cross-section of the problem needs to be discretised into finite elements (in the present case a finite elements approach is considered; alternative approaches using BEM or MFS can be found on [13, 22] since a Fourier transformation is applied regarding the longitudinal coordinate.

For the present case study, the 2.5D FEM approach seems to be advantageous since non-homogeneous cross-sections are considered. However, there is a pitfall when using finite elements approaches on dealing with dynamics of unbounded domains: the demands of limitation of the discretized domain gives rise to the need for treatment of the artificial boundaries in order to prevent the spurious reflexion of waves. In the present approach this requirement is fulfilled using a 2.5D PML's approach along the artificial boundaries.

Due to the length limitation of the paper, the mathematical formalism inherent to the 2.5D FEM-PML approach is not here presented. However, it can be found in other papers of the authors, namely in: [9, 10, 23].

The prediction of vibrations due to railway traffic in tunnels constitutes a challenge by several reasons. One of these particular aspects is the train-track interaction, which is provided by the track irregularities, the out-of-roundness of the wheels or by the non-homogeneous support of the rail. These heterogeneities of the system give rise to inertial forces on the train, i.e., to a train-track interaction force which is variable along time. By that reason, the structural behaviour of the train needs to be also simulated on the prediction model. In the present model, a sub-structured approach is followed, where the train simulated by a multi-body approach and the remaining system by the 2.5D FEM-PML model described above. *Fig. 2* shows a schematic representation of the distinct models considered on the sub-structured approach. The dynamic interaction of the different models is also depicted on the same figure. Once again, the limitation of length of the paper prevents an exhaustive description of the train-track interaction model, but the reader can found a detailed description on previous papers of the authors, namely in: [9, 10, 24]

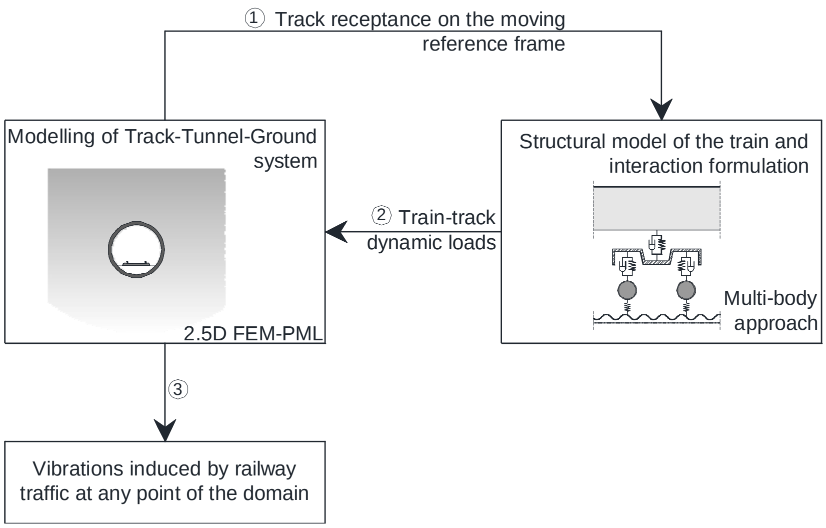

Fig. 2. Schematic representation of the sub-structured model used for prediction of vibration due to railway traffic

Despite the ability of the model for dealing with different sources of excitation of the train, as for instance the out-of-roundness of the wheels, in the present application it is assumed that the only source of dynamic excitation corresponds to the track unevenness.

3. APPLICATION EXAMPLE

3.1. Objectives

As emphasized above, the main objective of this research is to study the influence of G_s degradation that occurs within the soil during the excavation process of tunnel with TBM on predicted railway vibrations in tunnels. For that, an only case has been analysed with a homogeneous soil for the whole domain, where β-value has been considered as 0.5.

3.2. Model description

*Fig. 2*shows the geometry of the finite element mesh used to simulate the tunnel construction with PLAXIS code. Although it usually works with unstructured mesh, in this case it has been structured in order to use a similar mesh to study both tunnel construction and railway vibrations. The size of mesh to analyse the tunnel excavation has been estimated according to Meissner [24]. The depth of tunnel has been chosen in order to consider a realistic geometry in which is possible to build it with TBM method.

Fig. 4. shows the 2.5D FE-PML model to study railway vibrations. The size of finite element in the soil has been estimate regarding 6 elements per wavelength, limiting frequencies up to 80 Hz. The interest domain is limited by PML's in order to avoid the spurious reflexion of the waves that impinges the artificial boundaries.

3.3. Soil and tunnel properties

The soil constitutive model applied is the *HS-small* (only in 2D FEM model for simulating tunnel construction) which is based on the *Hardening soil model* but considering the complete stress-strain curve including the very small strain range. The *Hardening soil model* and the *Hardening soil model with small-strain stiffness* are explained in detail by Schanz et al. [19] and Benz [20] respectively. The soil properties are shown in table 1 according to the *HS-small model.*

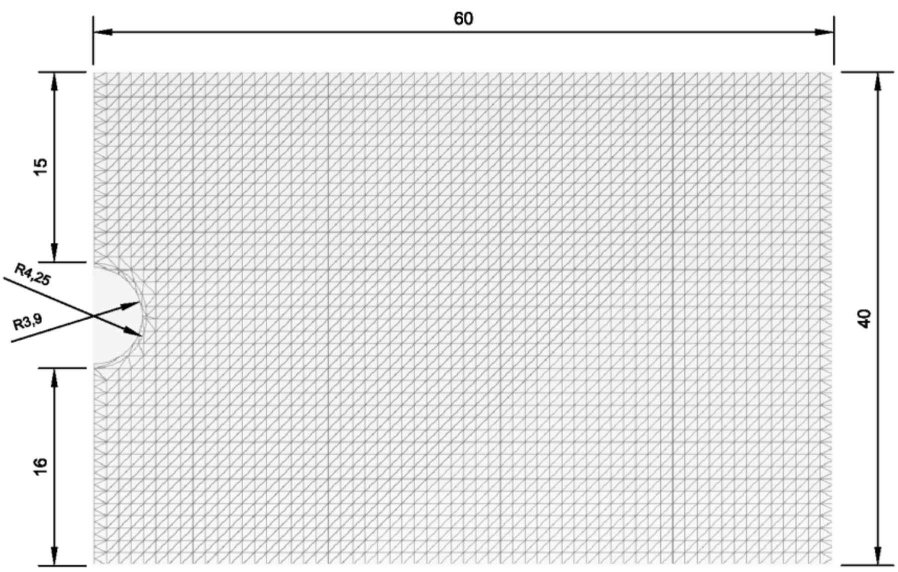

Fig. 3. Finite element mesh (in metres) for tunnelling

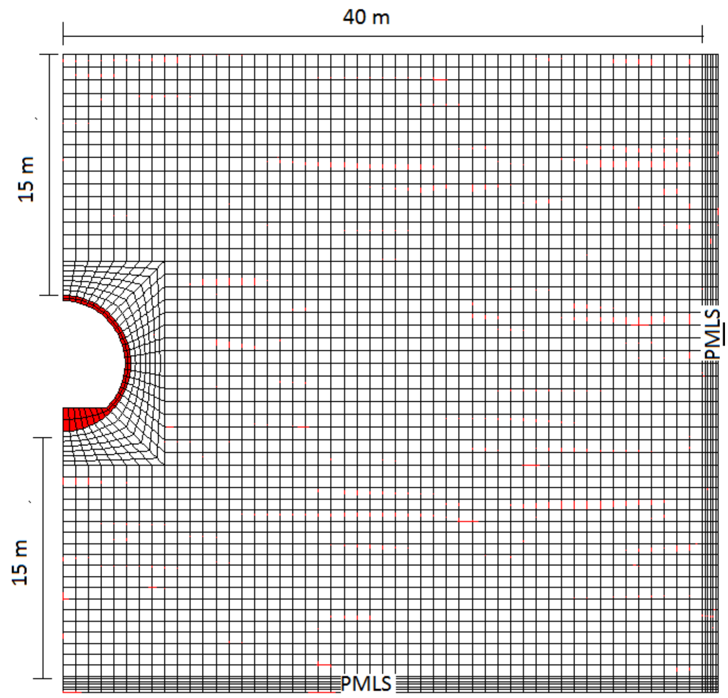

Fig. 4. 2.5D FEM-PML mesh adopted for the analysis of vibrations due to railway traffic

γ_{ap} (kN/m^3)	20
E_{50}^{ref} (kN/m^2)	$200\cdot10^3$
E_{oed}^{ref} (kN/m^2)	$200\cdot10^3$
E_{ur}^{ref} (kN/m^2)	$400\cdot10^3$
m	0.5
c' (kN/m^2)	25
φ'	35º
ψ	5º
ν_{ur}'	0.2
K_o^{nc}	0.4264
$\gamma_{0.7}$	$1.8\cdot10^{-4}$
G_0^{ref} (kN/m^2)	$310\cdot10^3$
P_{ref} (kN/m^2)	100
R_f	0.9
C_p (m/s)	643
C_s (m/s)	394

Table 1: Soil properties in HS-small model and P and S-waves velocity

The relationship between E_{50}^{ref} and E_{oed}^{ref} has been estimated according to Schanz [25] and the relationship between E_{50}^{ref} and E_{ur}^{ref} is supposed to be equal to 2. G_0^{ref} has been calculated as follows:

$$G_0^{ref} = \frac{E_0^{ref}}{2(1+\upsilon_{ur})} \tag{1}$$

Where E_0^{ref} has been calculated according to Alpan [26]. The relationships given by Alpan between E_0 and E_{ur} are shown in *Fig. 5*.

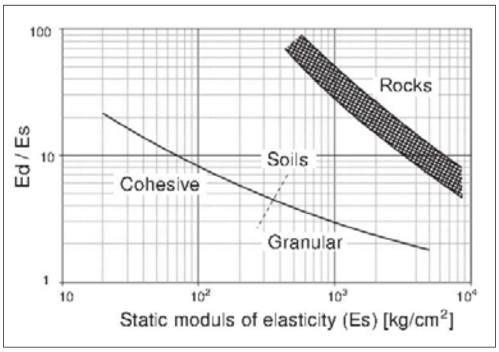

Fig. 5. Relation between dynamic stiffness (E_d=E_0) and static soil stiffness (E_s=E_{ur}) [27]

The "m" exponent has been estimated according to Benz [19]. The soil has been considered as normally consolidated and in this way K_0 has been calculated according to Jaky [27]. Although $\gamma_{0.7}$ is dependent on confining stress, in this case it has been considered to be constant for the whole domain and is obtained as follows [20]:

$$\gamma_{0.7} = \frac{0.385}{4G_0}\left[2c'(1+\cos(2\varphi')) + \sigma'_1(1+K_o)\sin\ (2\varphi')\right] \tag{2}$$

Where σ'_1 is the effective vertical stress in the middle of considered soil layer in the numerical model. *Fig. 6* shows the G_s/G_0-γ_s relationship for the soil in the *HS-small* constitutive model and the tunnel properties are summarized in table 2.

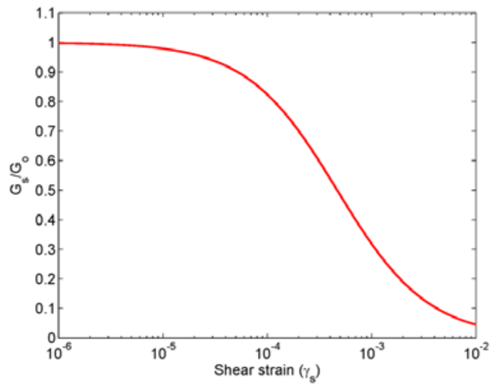

Fig. 6. G_s/G_0-γ_s relationship in HS-small

Thickness (m)	0.35
Inner radius (m)	3.9
Density (kg/m^3)	2500
Elastic modulus (kN/m^2)	$30 \cdot 10^6$
Poisson ratio	0.2

Table 2: Tunnel properties

3.4. Stress state within soil after tunnelling

Fig. 7 shows computed G_s/G_0 and shear strains (γ) contours after numerical simulating of tunnel construction, in which can be seen as G_s degradation is more evident closed to lining, even reaching the ground surface although the effect of tunnelling process at ground surface is limited until a distance of 25-30 meters from symmetric axis of the model.

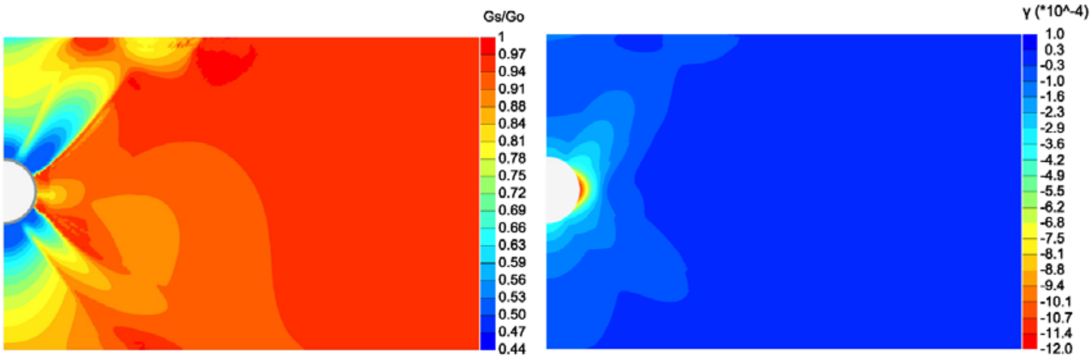

Fig. 7. Computed contours G_s/G_o (left) and shear strains (γ) (right)

3.5. Track and train properties

In what concerns the modelling of vibrations induced by railway traffic, additional description of the model is required. Two distinct computations are performed: i) non disturbed ground; ii) disturbed ground due to tunnel construction operations. In the former case, the shear modulus of the ground is assumed constant with value of G_0 indicated in Table 1. On the other hand, the second computation set corresponds to the disturbed ground, where the distribution of G depicted in *Fig. 7* is adopted. In both situations, the volumetric mass was assumed as 2000 kg/m3; the Poisson ratio equal to 0.2 and a hysteretic damping ratio of 0.04.

The railway track is composed by a continuous concrete slab track 0.3 m thick and 2.5 m wide, with a longitudinal bending stiffness of $1.62x10^8$ N/m2 and a mass of 3000 kg per unit of length. The rails, materialized by UIC60 profiles are continuously supported by railpads with a stiffness of $2.5x10^8$ N/m2 and a damping coefficient of $6x10^4$ Ns/m2. It is assumed a floating slab solution, introducing a resilient mat between the slab and the tunnel invert. The stiffness of the mat is $0.153x10^9$ N/m2 per meter in the longitudinal direction and a damping of $5.5x10^4$ Ns/m2 is considered.

In what concerns to the rolling stock, it was assumed the passage of the Alfa-Pendular train at a running speed of 40 m/s. The main geometrical and mechanical properties of the train Alfa-Pendular are summarized in *Fig. 8* and in Table 3.

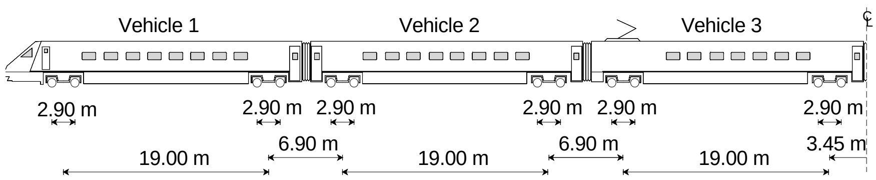

Fig. 8. Alfa-Pendular geometry

Axles	**Mw (kg)**	1538-1884
Primary suspension	**Kp (kN/m)**	3420
	Cp (kNs/m)	36
Bogies	**Mb (kg)**	4712-4932
	Jb (kg/m^2)	5000-5150
Car body	**Mc (kg)**	32900-35710

Table 3: Mechanical properties of the train

As mentioned above, the first step of the calculation procedure concerns the solving of the train-track interaction problem. The quasi-static and dynamic excitation mechanisms are uncoupled, being the solution obtained through a superposition procedure.

In the present study, the source of vibration is due to the track unevenness. An artificial unevenness profile was generated taking into account the power spectral density of amplitude of the track unevenness for a range of wavelengths between 28 m and 0.55 m. The following equation was used for address the PSD of the track unevenness:

$$S(k_1) = S(k_{1,0})\left(\frac{k_1}{k_{1,0}}\right)^{-w} \tag{3}$$

where, $k_{1,0}$=1 rad/s, w=3.5 and $S(k_{1,0})$ was assumed equal to $1x10^{-8}$ m^3.

Once the train speed was assumed to be 40 m/s, the unnevess profile considered excitates the train in the frequency range between 1.4 Hz to 72 Hz..

4. RESULTS AND DISCUSSION

4.1. Transfer functions for stand still loads

Firstly, to the study of the impact of construction operations of the tunnel on the vibration levels perceived at the ground surface, a simple analysis is presented where a unitary stand still load is applied at the centre of tunnel invert and the dynamic response is recorded at different positions at the ground surface. This simple analysis allows observing how the shear modulus degradation affects the transfer functions between the tunnel invert and the ground surface. In the present case it is assumed that the load is applied at the cross section defined by x=0 m (x is the longitudinal axle), and the receiver points are located at the same cross-section.

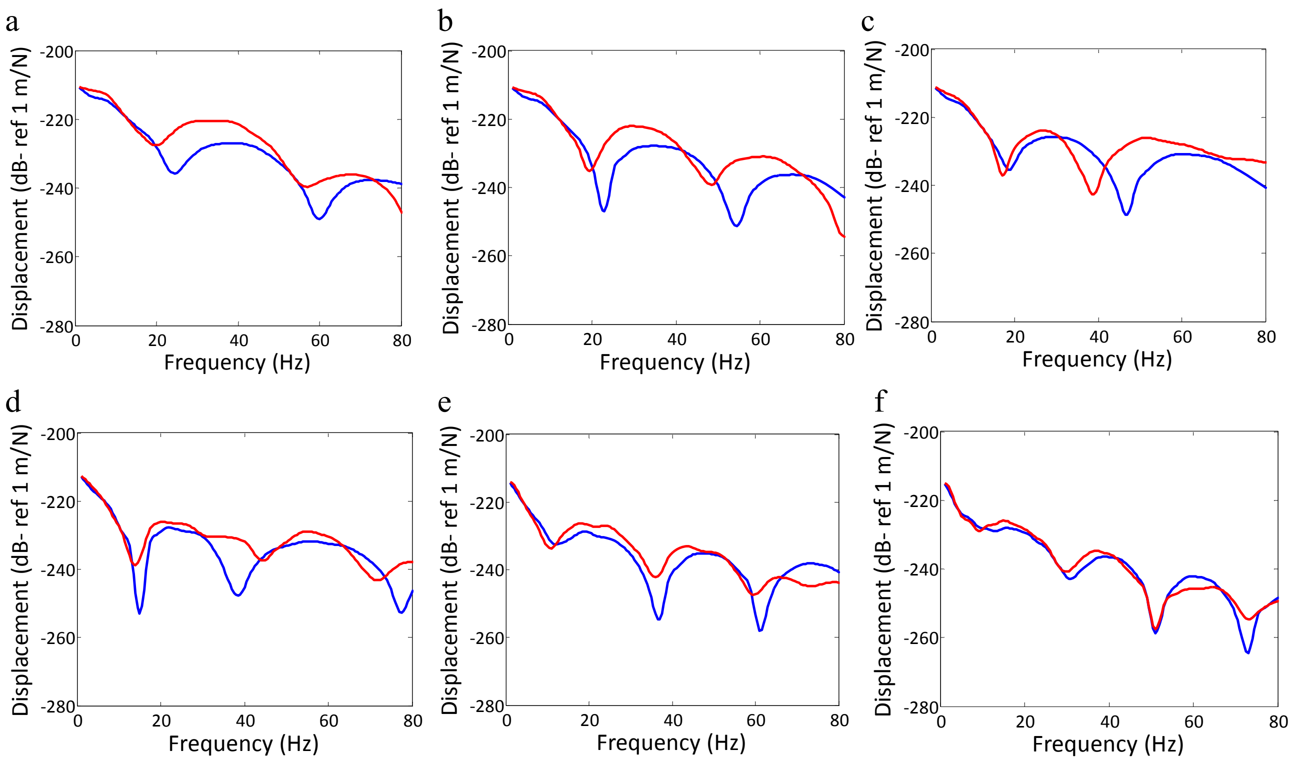

Fig. 9. Vertical displacement transfer functions due to a load applied at the tunnel invert and evaluated at distinct positions of ground surface: a) P1(0,0,0); b) P2 (0,5,0); c) P3 (0,10,0); d) P4 (0,15,0);e) P5 (0,20,0) f) P6 (0,40,0) (blue line – non disturbed soil; red line – disturbed soil)

The analysis of *Fig. 9* allows to see an agreement of the response obtained for both scenarios in the lower frequency range, i.e., up to 15 Hz. Actually, the wavelengths generated in this frequency range are quite large, being not so much affected by point variations of the ground stiffness. On the other hand, for the receiver points located close to the tunnel, and for frequencies above the mentioned limit, a relevant influence of the construction operations on the transfer function is detectable. On *Fig. 9*.a to d, it is possible to see a change of the frequency of the troughs of the transfer function when the stiffness degradation due to the tunnel construction is taken into account. On the other hand, it seems that the difference between the two scenarios evanesces with the increase of the distance between the source and the receiver. This aspect is quite evident in *Fig. 9*f, where, for frequencies up to 50 Hz, there is a good agreement between the results obtained from both scenarios. However, for frequencies above this limit, differences remain, being justified by the fact that the wavelengths generated are short enough to be influenced by local changes on soil stiffness.

4.2. Vibrations due to railway traffic

Figs. 10 and *11* show time record and one-third octave spectrum of vertical velocity due to passage of the Alfa Pendular train respectively. It is worth mentioning that only 4 points at ground surface are shown, located at several distances from symmetric axis of the model: 0, 5, 20 and 40 metres.

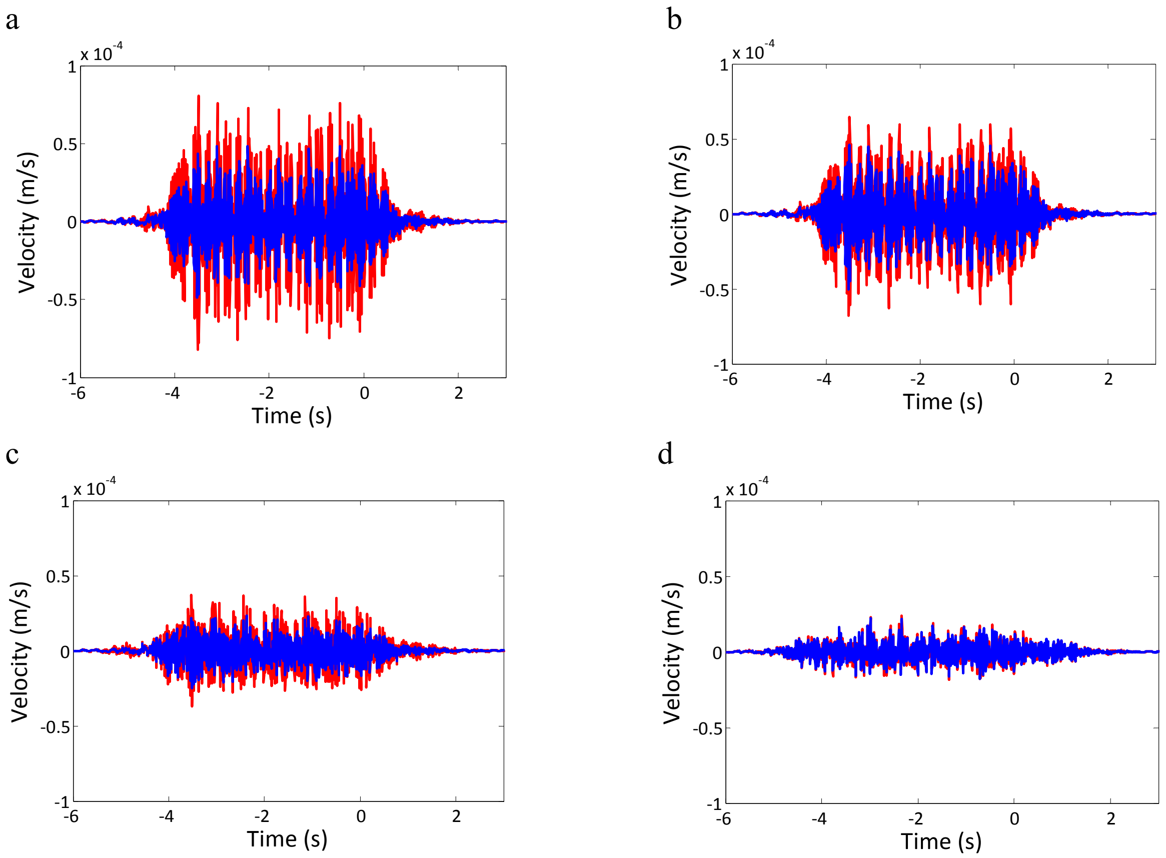

Fig. 10. Time record of vertical velocity at distinct points located at the ground surface: a) P1(0,0,0); b) P2 (0,5,0); c) P5 (0,20,0) d) P6 (0,40,0) (blue line – non disturbed soil; red line – disturbed soil)

Regarding the compared response in the time domain is noteworthy that the effect of taking into account the tunnelling process is more notorious at points located closer to the tunnel, resulting minor differences between the two scenarios with the increase of the distance to the tunnel. In fact, the difference found at a distance of 40 metres is almost negligible. However, at the other points (0, 5 and 20 metres) the differences are noteworthy, being more important for smaller distances to the tunnel.

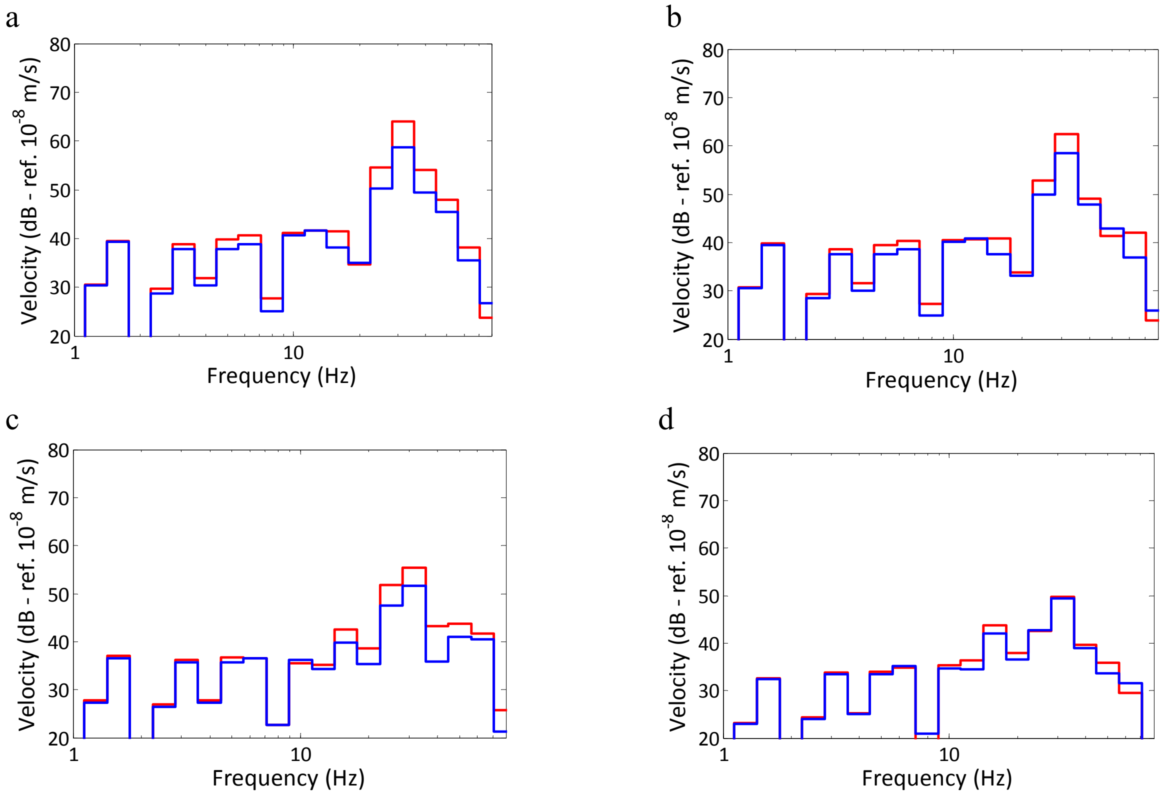

Fig. 11. One-third octave spectrum of vertical velocity at distinct points located at the ground surface: a) P1(0,0,0); b) P2 (0,5,0); c) P5 (0,20,0) d) P6 (0,40,0) (blue line – non disturbed soil; red line – disturbed soil)

A more clear comparison between the two considered scenarios is provided by *Fig. 11*, in which it can seen that the influence of tunnelling process is more evident with the decrease of the distance between the source and the receiver. However, regarding one-third octave spectrum it is also possible to comment that there are more differences in frequencies above 15 Hz than below this value, due to the wavelengths generated below 15 Hz are quite large, being not so much affected by point variations of the ground stiffness. Similar conclusion was also found for the analysis of transfer functions depicted on *Fig. 9*. Comparing the two scenarios in terms of decibels, it is possible to conclude that in frequencies below 15-20 Hz the maximum difference is 3 dB while above 20 Hz is it possible to achieve differences around 7 dB for the points located close to the tunnel. However, the influence of tunnelling process evanesces with the increase of the distance source-receiver. Actually, the differences found on *Fig. 11*d are almost negligible.

5. CONCLUSIONS

In this paper, the influence of the stress state within the soil on predicted railway vibrations in tunnels has been studied, regarding the degradation of shear modulus that occurs when the tunnel is excavated with Tunnel Boring Machine (TBM). In this way, the main conclusions are:

- The influence of the degradation of shear modulus of soil during tunnelling process with TBM on predicted railway vibrations can be relevant, providing a higher level of vibrations at the ground surface when G_s degradation is considered.
- This fact is more notorious at the closest points of the tunnel, being practically negligible at distances greater than 20 metres from the line of centre of the tunnel.

- The influence is more important for frequencies above 15 Hz than below this value. So, the maximum difference above this mentioned limit has been 7 dB while for frequencies up to 15 Hz has been 3 dB.

The numerical results show that the effect of degradation of shear modulus of the soil during tunnelling process on predicted railway vibrations should be taken into account in design process in order to find more accurate results.

The research is on-going and numerical studies will be performed for other geotechnical conditions in order to accomplish more conclusive remarks.

REFERENCES

[1] M.G. Smith, L. Cray, M. Ögren, K. Persson Waye, On the Influence of Freight Trains on Humans: A Laboratory Investigation of the Impact of Nocturnal Low Frequency Vibration and Noise on Sleep and Heart Rate. *PLoS ONE, 2013.* **8**(2): p. e55829.

[2] M. Hussein, H. Hunt, A numerical model for calculating vibration from a railway tunnel embedded in a full-space. *Journal of Sound and Vibration*, 2007. **305**(3): p. 401-431.

[3] K.A. Kuo, H.E.M. Hunt, M.F.M. Hussein, The effect of a twin tunnel on the propagation of ground-borne vibration from an underground railway. *Journal of Sound and Vibration*, 2011. **330**(25): p. 6203-6222.

[4] K. Muller, *Dreidimensionale dynamische Tunnel-Halbraum-Interaktion*, in *Lehrstuhl fur Baumechanik*. 2007, Technische Universitat Munchen: Munchen.

[5] D. Clouteau, M. Arnst, T.M. Al-Hussaini, G. Degrande, Free field vibrations due to dynamic loading on a tunnel embedded in a stratified medium. *Journal of Sound and Vibration*, 2005. **283**(1-2): p. 173-199.

[6] S. Gupta, M.F.M. Hussein, G. Degrande, H.E.M. Hunt, D. Clouteau, A comparison of two numerical models for the prediction of vibrations from underground railway traffic. *Soil Dynamics and Earthquake Engineering*, 2007. **27**(7): p. 608-624.

[7] S. Gupta, Y. Stanus, G. Lombaert, G. Degrande, Influence of tunnel and soil parameters on vibrations from underground railways. *Journal of Sound and Vibration*, 2009. **327**: p. 70-91.

[8] H. Hung, Y. Yang, Analysis of ground vibrations due to underground trains by 2.5D finite/infinite element approach. *Earthquake Engineering and Engineering Vibration*, 2010. **9**(3): p. 327-335.

[9] P. Lopes, P. Alves Costa, M. Ferraz, R. Calçada, A. Silva Cardoso, Numerical modelling of vibrations induced by railway traffic in tunnels: from the source to the nearby buildings. *Soil Dynamics and Earthquake Engineering*, 2014. **61-62**: p. 269-285.

[10] P. Lopes, P. Alves Costa, R. Calçada, A. Silva Cardoso, Influence of soil stiffness on vibrations inside buildings due to railway traffic: numerical study. *Computers & Geotechnics*, 2014. **61**: p. 277-291.

[11] P. Galvín, S. François, M. Schevenels, E. Bongini, G. Degrande, G. Lombaert, A 2.5D coupled FE-BE model for the prediction of railway induced vibrations. *Soil Dynamics and Earthquake Engineering*, 2010. **30**(12): p. 1500-1512.

[12] Y. Yang, H. Hung, Soil Vibrations Caused by Underground Moving Trains. *Journal of Geotechnical and Geoenvironmental Engineering,* 2008. **134**(11): p. 1633-1644.

[13] P. Alves Costa, R. Calçada, A. Silva Cardoso, Track–ground vibrations induced by railway traffic: In-situ measurements and validation of a 2.5D FEM-BEM model. *Soil Dynamics and Earthquake Engineering*, 2012. **32**(1): p. 111-128.

[14] S. François, P. Galvín, M. Schevenels, G. Lombaert, G. Degrande, A 2.5D coupled FE–BE methodology for the dynamic interaction between longitudinally invariant structures and a layered halfspace. *Computer Methods in Applied Mechanics and Engineering*, 2010. **199**(23-24): p. 1536-1548.

[15] R. J. Mair, Developments in geotechnical engineering research: application to tunnels and deep excavations. *Proceedings of Institution of Civil Engineers: Civil Engineering*, 1993. **93**: p. 27-41.

[16] M. Panet, A. Guenot, Analysis of convergence behind face of a tunnel, *in Proc. Tunnelling'82*. 1982. p. 197-203.

[17] J. M. Duncan, C.Y. Chang, Nonlinear analysis of stress and strain in soil. *ASCE Journal of the Soil Mechanics and foundations Division*, 1970. **16**: p. 1629-1653.

[18] J. Lysmer, T. Udaka, C. Tsai, H. Bolton Seed, *FLUSH: a computer program for approximate 3-D analysis of soil-structure interaction problems*, R.E. 75-30, Editor. 1974, University of California: Berkeley.

[19] T. Schanz, P.A. Vermeer, P.G. Bonnier, The hardening soil model: Formulation and verification, in *Beyond 2000 in Computational Geotechnics-10 YEARS of Plaxis*. Rotterdam 1999.

[20] T. Benz, *Small-strain stiffness of soils and its numerical consequences.* 2006, Institut für Geotechnik der Universität Stuttgart.

[21] K. Schikora, T. Fink, Berechnungsmethoden moderner bergmännischer bauweisen beim u-bahn-bau. *Bauingenieur*, 1982. **57**: p. 193-198.

[22] P. Amado Mendes, L. Godinho, P. Alves Costa, 2.5D modeling of soil-structure interaction using a coupled MFS-FEM formulation, in *11th World Congress on Computational Mechanics*, E. Onate, J. Oliver, and A. Huerta, Editors. 2014: Barcelona.

[23] P. Lopes, P. Alves Costa, R. Calçada, A. Silva Cardoso, Numerical Modeling of Vibrations Induced in Tunnels: A 2.5D FEM-PML Approach, in *Traffic Induced Environmental Vibrations and Controls: Theory and Application*, H. Xia and R. Calçada, Editors. 2013, Nova. p. 133-166.

[24] P. Alves Costa, R. Calçada, A. Silva Cardoso, *I*nfluence of train dynamic modelling strategy on the prediction of track-ground vibrations induced by railway traffic. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* 2012. **226**(4): p. 434-450.

[25] T. Schanz, *Modellierung des Mechanischen Verhaltens von Reibungsmaterialen.*, in Stuttgart Universität. 1998.

[26] I. Alpan, *The geotechnical properties of soils. Earth-Science Reviews* 1970. **6**: p. 5-49.

[27] J. Jaky, Pressure in silos., in *Proc. 2nd International Conference on Soil Mechanics and Foundation Engineering*. 1948: Rotterdam. p. 103-107.