COMPDYN 2015
5th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, V. Papadopoulos, V. Plevris (eds.)
Crete Island, Greece, 25–27 May 2015

THE USE OF MODEL REDUCTION TECHNIQUES IN COMPLEX SIMULATION-BASED PROBLEMS INVOLVING FINITE ELEMENT MODELS

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Keywords: Bayesian updating, Finite element models, Model reduction techniques, Reliability-based design, Stochastic simulation

Abstract. *This work presents a strategy for integrating a class of model reduction techniques into two types of complex problems: finite element model updating using dynamic response data; and reliability-based design of stochastic finite element models. The solution of this type of problems is computationally very demanding due to the large number of finite element model analyses required. Substructure coupling techniques for dynamic analysis are proposed to reduce the corresponding computational cost involved in these processes. The idea is to carry out the updating and design processes efficiently in a reduced space of generalized coordinates.*

1 INTRODUCTION

Finite element model updating using measured system response has a wide range of applications in areas such as structural response prediction, structural control, structural health monitoring, reliability and risk assessment, etc. [1, 2, 3, 4]. For a proper assessment of the updated model all uncertainties involved in the problem should be considered. The ability to quantify the uncertainties accurately and appropriately is essential for a robust prediction of future responses and reliability of structures [5]. In this context, a fully probabilistic Bayesian model updating approach provides a robust and rigorous framework for model updating due to its ability to characterize modeling uncertainties associated with the underlying structural system [6, 7, 8]. A stochastic simulation approach called transitional Markov chain Monte Carlo method is considered here for identification purposes [9]. Such algorithm requires a large number of finite element re-analyses to be performed over the space of model parameters. Thus, the computational demands depend on the number of re-analyses and the time required for performing each finite element re-analysis.

On the other hand, under uncertain conditions the field of reliability-based optimization provides a realistic and rational framework for structural optimization which explicitly accounts for the uncertainties [10, 11]. Of particular interest in this work is the case of structural design problems involving finite element models under stochastic loading. The design problem is formulated as the minimization of an objective function subject to multiple design requirements including standard and reliability constraints. The solution of this type of problems requires a large number of finite element analyses to be perform during the design process. These analyses correspond to finite element re-analyses over the design space (required by the optimizer), and system responses over the uncertain parameter space (required by the simulation technique for reliability estimation). Consequently, the computational demands depend highly on the number of finite element analyses and the time taken for performing an individual finite element analysis. Thus, the computational demands in solving these reliability-based design problems may be large or even excessive [12, 13, 14, 15].

In this framework, it is the main objective of this work to present a strategy for integrating a class of model reduction techniques into the updating and design processes in order to reduce the computational cost involved in the corresponding dynamic re-analyses of the finite element models. Specifically, a class of model reduction techniques known as substructure coupling for dynamic analysis is considered in the present implementation [16]. Such technique can be used to carry out system analyses in a significantly reduced space of generalized coordinates. A particular parametrization scheme in terms of the model parameters and design variables is implemented in the present work. Such scheme reduces drastically the computational demands involved in the updating and design processes without compromising the solution accuracy.

2 FINITE ELEMENT MODEL UPDATING

2.1 Problem Formulation

In this section the first type of problems to be considered is introduced. It corresponds to finite element model updating using dynamic response data. To this end, consider a finite element model class M of a structural system parameterized by a set of model parameters $\boldsymbol{\theta} \in \Theta \subset R^{n_d}$. The plausibility of each model within a class M based on data D is quantified by the updated joint probability density function $p(\boldsymbol{\theta}|M, D)$ (posterior probability density function).

By Bayes' Theorem the posterior probability density function of $\boldsymbol{\theta}$ is given by

$$p(\boldsymbol{\theta}|M, D) = \frac{p(D|M, \boldsymbol{\theta})\ p(\boldsymbol{\theta}|M)}{p(D|M)} \tag{1}$$

where $p(D|M)$ is the normalizing constant which makes the probability volume under the posterior probability density function equal to unity, $p(D|M, \boldsymbol{\theta})$ is the likelihood function based on the predictive probability density function for the response given by model class M, and $p(\boldsymbol{\theta}|M)$ is the prior probability density function selected for the model class M which is used to quantify the initial plausibility of each predictive model defined by the value of the parameters $\boldsymbol{\theta}$, allowing in this manner prior information to be incorporated. In what follows, it is assumed that D contains input dynamic data and output responses from measurements on the system. Specifically, let $x_n(t_j, \boldsymbol{\theta})$ denotes the output at time t_j at the n^{th} observed degree of freedom predicted by the proposed structural model, and $x_n^*(t_j)$ denotes the corresponding measured output. The prediction and measurement errors $e_n(t_j, \boldsymbol{\theta}) = x_n^*(t_j) - x_n(t_j, \boldsymbol{\theta})$ for $n = 1, ..., N_o$, and $j = 1, ..., N_T$, where N_o denotes the number of observed degrees of freedom and N_T denotes the length of the discrete time history data, are modeled as independent and identically distributed Gaussian variables with zero mean [17]. Using the above probability model for the prediction error it can be shown that the likelihood function $p(D|M, \boldsymbol{\theta})$ can be expressed in terms of a measure-of-fit function $J(\boldsymbol{\theta}|M, D)$ between the measured response and the model response at the measured degrees of freedom. Such function is given by [18, 19]

$$J(\boldsymbol{\theta}|M, D) = \frac{1}{N_t N_o} \sum_{n=1}^{N_o} \sum_{j=1}^{N_t} [x_n^*(t_j) - x_n(t_j, \boldsymbol{\theta})]^2 \tag{2}$$

2.2 Simulation-Based Approach

An efficient method called transitional Markov chain Monte Carlo is implemented for Bayesian model updating [9]. Validation calculations have shown the effectiveness of this approach in a series of practical Bayesian model updating problems [9, 20, 21]. The method can be applied to a wide range of cases including high-dimensional posterior probability density functions, multimodal distributions, peaked probability density functions, and probability density functions with flat regions. The method iteratively proceeds from the prior to the posterior distribution. It starts with the generation of samples from the prior distribution in order to populate the space in which also the most probable region of the posterior distribution lies. For this purpose a number of non-normalized intermediate distributions $p_j(\boldsymbol{\theta}|M, D), j = 1, ..., J$, are defined as

$$p_j(\boldsymbol{\theta}|M, D) \propto\ p(D|M, \boldsymbol{\theta})^{\alpha_j} p(\boldsymbol{\theta}|M) \tag{3}$$

where the parameter α_j increases monotonically with j such that $\alpha_0 = 0$ and $\alpha_J = 1$. The parameter α_j can be interpreted as the percentage of the total information provided by the dynamic data which is incorporated in the j^{th} iteration of the updating procedure. The first step $(j = 0)$ corresponds to the prior distribution and in the last stage $(j = J)$ the samples are generated from the posterior distribution. The idea is to choose the values of exponents α_j in such a way that the change of the shape between two adjacent intermediate distributions be small. This small change of the shape makes it possible to efficiently obtain samples from $p_{j+1}(\boldsymbol{\theta}|M, D)$ based on the samples from $p_j(\boldsymbol{\theta}|M, D)$. The value of the parameter α_{j+1} is chosen such that the coefficient of variation for $\{p(D|M, \boldsymbol{\theta}_j^k)^{\alpha_{j+1}-\alpha_j}, k = 1, ..., N_j\}$ is equal to some prescribed target value. The upper index $k = 1, ..., N_j$ in the previous expression denotes the sample number in the j^{th} iteration step ($\boldsymbol{\theta}_j^k, k = 1, ..., N_j$). Once the parameter α_{j+1} has been determined, the samples are obtained by generating Markov chains where the lead samples are selected from the distribution $p_j(\boldsymbol{\theta}|M, D)$. Each sample of the current stage is generating by applying the Metropolis-Hastings algorithm [22]. The lead sample of the Markov chain is a sample from the previous step that is selected according to the probability equal to its normalized weight $\bar{w}(\boldsymbol{\theta}_j^k) = w(\boldsymbol{\theta}_j^k)/\sum_{l=1}^{N_j} w(\boldsymbol{\theta}_j^l)$, where $w(\boldsymbol{\theta}_j^k)$ represents the plausibility weight which is given by $w(\boldsymbol{\theta}_j^k) = p(D|M, \boldsymbol{\theta}_j^k)^{\alpha_{j+1}-\alpha_j}$. The proposal probability density function for the Metropolis-Hastings algorithm is a Gaussian distribution centered at the preceding sample of the chain and with a covariance matrix equal to a scaled version of the estimate covariance matrix of the current intermediate distribution $p_j(\boldsymbol{\theta}|M, D)$. The procedure is repeated until the parameter α_j is equal to 1 ($j = J$). At the last stage the samples ($\boldsymbol{\theta}_J^k, k = 1, ..., N_J$) are asymptotically distributed as $p(\boldsymbol{\theta}|M, D)$. For a detailed implementation of the transitional Markov chain Monte Carlo method the reader is referred to [9].

3 RELIABILITY-BASED DESIGN OPTIMIZATION

3.1 Design Problem

The second type of problems to be considered consists in the reliability-based design of finite element models under stochastic excitation. The problem is formulated in terms of the constrained non-linear optimization problem

$$\begin{aligned} \text{Min}_{\boldsymbol{\theta}} \;\; & c(\boldsymbol{\theta}) \\ \text{s.t.} \;\; & g_i(\boldsymbol{\theta}) \leq 0 \;\; i = 1, ..., n_c \\ & P_{F_i}(\boldsymbol{\theta}) - P_{F_i}^* \leq 0 \; , i = 1, ..., n_r \\ & \boldsymbol{\theta} \in \Theta \end{aligned} \tag{4}$$

where $\boldsymbol{\theta}, \theta_i, i = 1, ..., n_d$ denotes the vector of design variables with side constraints $\theta_i^l \leq \theta_i \leq \theta_i^u$, $c(\boldsymbol{\theta})$ is the objective function, $g_i(\boldsymbol{\theta}) \leq 0, \; i = 1, ..., n_c$ are standard constraints, and $P_{F_i}(\boldsymbol{\theta}) - P_{F_i}^* \leq 0$ are the reliability constraints which are defined in terms of the failure probability functions $P_{F_i}(\boldsymbol{\theta})$ and target failure probabilities $P_{F_i}^*, \; i = 1, ..., n_r$. It is assumed that the objective and constraint functions are smooth functions of the design variables. For structural systems under stochastic excitation the probability that design conditions are satisfied within a particular reference period T provides a useful reliability measure (first excursion probability). In this context, a failure event F_i can be defined as $F_i(\boldsymbol{\theta}, \mathbf{z}) = d_i(\boldsymbol{\theta}, \mathbf{z}) > 1$, where d_i is the so-called normalized demand function defined as $d_i(\boldsymbol{\theta}, \mathbf{z}) = \max_{j=1,...,l} \max_{t \in [0,T]} \mid r_j^i(t, \boldsymbol{\theta}, \mathbf{z}) \mid / r_j^{i^*}$, where $\mathbf{z} \in \Omega_{\mathbf{z}} \subset R^{n_z}$ is the vector of uncertain variables involved in the problem (characterization of the excitation and system parameters), $r_j^i(t, \boldsymbol{\theta}, \mathbf{z}), j = 1, ..., l$ are the response functions associated with the failure event F_i, and $r_j^{i^*}$ is the acceptable response level for the response r_j^i. The response functions r_j^i are obtained from the solution of the equation of motion that characterizes the structural model. The uncertain variables $\mathbf{z}$ are modeled using a prescribed probability density function $p(\mathbf{z})$. The probability of failure evaluated at the design $\boldsymbol{\theta}$ is formally defined as

$$P_{F_i}(\boldsymbol{\theta}) = P[\max_{j=1,...,l} \max_{t \in [0,T]} \frac{\mid r_j^i(t, \boldsymbol{\theta}, \mathbf{z}) \mid}{r_j^{i^*}} > 1] \tag{5}$$

where $P[\cdot]$ is the probability that the expression in parenthesis is true. Equivalently, the failure probability function evaluated at the design $\boldsymbol{\theta}$ can be written in terms of the multidimensional probability integral

$$P_{F_i}(\boldsymbol{\theta}) = \int_{d_i(\boldsymbol{\theta},\mathbf{z})>1} p(\mathbf{z})d\mathbf{z} \tag{6}$$

3.2 Reliability Estimation and Optimization Strategy

The reliability constraints of the nonlinear constrained optimization problem (4) are defined in terms of the first excursion probability functions $P_{F_i}(\boldsymbol{\theta}), i = 1, ...n_r$. In general, these reliability measures are given in terms of high-dimensional integrals in the framework of dynamical systems under stochastic excitation. The difficulty in estimating these quantities favors the application of advanced simulation techniques. In particular, a general applicable method named subset simulation is adopted in the present formulation [23]. In this advanced simulation technique the failure probabilities are expressed as a product of conditional probabilities of some chosen intermediate failure events, the evaluation of which only requires simulation of more frequent events. The intermediate failure events are chosen adaptively using information from simulated samples so that they correspond to some specified values of conditional failure probabilities. Therefore, a rare event simulation problem is converted into a sequence of more frequent event simulation problems. The method uses a Markov chain Monte Carlo method based on the Metropolis algorithm for sampling from the conditional probabilities [23, 24]. Validation calculations have shown that subset simulation can be applied efficiently to a wide range of dynamical systems [25, 26, 27]. Regarding the solution of the reliability-based optimization problem, it can be obtained, in principle, by a number of techniques such as standard deterministic optimization schemes or stochastic search algorithms [28, 29, 30, 31]. Of particular importance are interior point algorithms based on descent feasible direction [32, 33]. By construction these algorithms generate a sequence of steadily improved feasible designs. The above schemes have proved to be quite effective for a wide range of applications including stochastic optimization problems [34, 35].

4 MODEL REDUCTION TECHNIQUE

The proposed Bayesian model updating technique is computationally very demanding due to the large number of dynamic finite element analyses required. In fact, the model updating technique based on the transitional Markov chain Monte Carlo method involves drawing a large number of samples (of the order of thousands) for populating the importance region in the uncertain parameter space. Similarly, the solution of the reliability-based optimization problem is also very expensive from a computational point of view. The reliability estimation at each design requires the evaluation of the system response at a large number of samples in the uncertain parameter space (of the order of hundreds or thousands). In addition, the iterative nature of the optimization strategy may impose additional computational demands. Consequently, the computational cost may become excessive when the computational time for performing a dynamic analysis is significant. To cope with this difficulty, a model reduction technique is considered in the present formulation. In particular, a method based on component-mode synthesis is implemented in order to define a reduced-order model for the structural system [16, 36, 37]. The basic equations and relationships involved in the model reduction technique are reviewed in this section.

4.1 Reduced-Order Model

The so-called fixed-interface normal modes and interface constrain modes are considered here to define a reduced-order model [16]. To this end, the following partitioned form of the mass matrix $\mathbf{M}^c \in R^{n^c \times n^c}$ and stiffness matrix $\mathbf{K}^c \in R^{n^c \times n^c}$ of the component $c, c = 1, ..., N_c$ is considered

$$\mathbf{M}^c = \begin{bmatrix} \mathbf{M}^c_{ii} & \mathbf{M}^c_{ib} \\ \mathbf{M}^c_{bi} & \mathbf{M}^c_{bb} \end{bmatrix} \ , \ \mathbf{K}^c = \begin{bmatrix} \mathbf{K}^c_{ii} & \mathbf{K}^c_{ib} \\ \mathbf{K}^c_{bi} & \mathbf{K}^c_{bb} \end{bmatrix} \tag{7}$$

where the indices i and b are sets containing the internal and boundary degrees of freedom of the component c, respectively. The internal coordinates are kept in the set $\mathbf{x}^c_i(t) \in R^{n^c_i}$ while all boundary coordinates are kept in the set $\mathbf{x}^c_b(t) \in R^{n^c_b}$. The displacement vector of physical coordinates of the component c is given by $\mathbf{x}^c(t)^T =< \mathbf{x}^c_i(t)^T, \mathbf{x}^c_b(t)^T >\in R^{n^c}$, where $n^c = n^c_i + n^c_b$. The fixed-interface normal modes are obtained by restraining all boundary degrees of freedom and solving the eigenproblem $\mathbf{K}^c_{ii}\mathbf{\Phi}^c_{ii} - \mathbf{M}^c_{ii}\mathbf{\Phi}^c_{ii}\mathbf{\Lambda}^c_{ii} = \mathbf{0}$, where the matrix $\mathbf{\Phi}^c_{ii}$ contains the complete set of n^c_i fixed-interface normal modes, and $\mathbf{\Lambda}^c_{ii}$ is the corresponding matrix containing the eigenvalues. The fixed-interface normal modes are normalized with respect to the mass matrix $\mathbf{M}^c_{ii}$. On the other hand, the interface constraint modes are defined by setting a unit displacement on the boundary coordinates $\mathbf{x}^c_b(t)$ and zero forces in the internal degrees of freedom. From this condition the interior partition of the interface constraint-mode matrix can be written as $\mathbf{\Psi}^c_{ib} = -\mathbf{K}^{c\ -1}_{ii}\mathbf{K}^c_{ib}$. Based on the previous normal and constraint modes the following displacement transformation matrix is introduced to define a set of generalized coordinates [16, 36]

$$\mathbf{x}^c(t) = \left\{ \begin{array}{c} \mathbf{x}^c_i(t) \\ \mathbf{x}^c_b(t) \end{array} \right\} = \begin{bmatrix} \mathbf{\Phi}^c_{ik} & \mathbf{\Psi}^c_{ib} \\ \mathbf{0}^c_{bk} & \mathbf{I}^c_{bb} \end{bmatrix} \left\{ \begin{array}{c} \mathbf{v}^c_k(t) \\ \mathbf{v}^c_b(t) \end{array} \right\} = \mathbf{\Psi}^c \ \ \mathbf{v}^c(t) \ \ , c = 1, ..., N_c \tag{8}$$

where $\mathbf{\Phi}^c_{ik} \in R^{n^c_i \times n^c_k}$ is the interior partition of the matrix $\mathbf{\Phi}^c_{ii}$ of the n^c_k kept fixed-interface normal modes, $\mathbf{v}^c(t)$ represents the component generalized coordinates composed by the modal coordinates $\mathbf{v}^c_k(t)$ of the kept fixed-interface normal modes and the boundary coordinates $\mathbf{v}^c_b(t) = \mathbf{x}^c_b(t)$, $\mathbf{\Psi}^c \in R^{n^c \times \hat{n}^c}$ is a transformation matrix with $\hat{n}^c = n^c_k + n^c_b$, and all other terms have been previously defined. Based on the previous transformation two vectors of generalized coordinates are introduced. First, the vector of generalized coordinates for all the N_c components $\mathbf{v}(t)^T =< \mathbf{v}^1(t)^T, ..., \mathbf{v}^{N_c}(t)^T >\in R^{n_v}$, where $n_v = \sum_{c=1}^{N_c} \hat{n}^c$, and the vector $\mathbf{u}(t)$ that contains the independent generalized coordinates consisting of the fixed-interface modal coordinates $\mathbf{v}^c_k(t)$ for each component and the physical coordinates $\mathbf{v}^l_b(t), l = 1, ..., N_b$ at the N_b interfaces $\mathbf{u}(t)^T =< \mathbf{v}^1_k(t)^T, ..., \mathbf{v}^{N_c}_k(t)^T, \mathbf{v}^1_b(t)^T, ..., \mathbf{v}^{N_b}_b(t)^T >\in R^{n_u}$, where $n_u = \sum_{c=1}^{N_c} n^c_k + \sum_{l=1}^{N_b} n^l_b$, and n^l_b is the number of degrees of freedom at the interface l ($l = 1, ..., N_b$). These two vectors are related by the transformation $\mathbf{v}(t) = \mathbf{T}\mathbf{u}(t)$, where the matrix $\mathbf{T} \in R^{n_v \times n_u}$ is a matrix of zeros and ones that couples the independent generalized coordinates $\mathbf{u}(t)$ of the reduced system with the generalized coordinates of each component. The assembled mass matrix $\hat{\mathbf{M}} \in R^{n_u \times n_u}$ and the stiffness matrix $\hat{\mathbf{K}} \in R^{n_u \times n_u}$ for the independent reduced set $\mathbf{u}(t)$ of generalized coordinates take the form

$$\hat{\mathbf{M}} = \mathbf{T}^T \begin{bmatrix} \mathbf{\Psi}^{1^T}\mathbf{M}^1\mathbf{\Psi}^1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{\Psi}^{N_c{}^T}\mathbf{M}^{N_c}\mathbf{\Psi}^{N_c} \end{bmatrix} \mathbf{T} \, , \tag{9}$$

$$\hat{\mathbf{K}} = \mathbf{T}^T \begin{bmatrix} \mathbf{\Psi}^{1^T}\mathbf{K}^1\mathbf{\Psi}^1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{\Psi}^{N_c{}^T}\mathbf{K}^{N_c}\mathbf{\Psi}^{N_c} \end{bmatrix} \mathbf{T} \tag{10}$$

where all terms have been already defined. Note that the dimension of the reduced-order matrices can be much more smaller than the dimension of the unreduced matrices, e.g. $n_u \ll n^c$.

4.2 Model Parametrization

In what follows, a particular parametrization scheme in terms of the model parameters and design variables is considered. Let C_0 be the set of components that do not depend on the vector of uncertain system parameters $\boldsymbol{\theta}$ (model parameters or design variables). In this case, the component fixed-interface normal modes and interface constraint modes are independent of the system parameter value. Thus, only a single analysis is required to estimate the interface modes for a particular component $c \in C_0$. The modes of these components are computed once during the updating and design processes. On the other hand, let C_j be the set of components that depend on the system parameters. Specifically, it is assumed that the stiffness matrix depends on only one of the system parameters and it takes the form $\mathbf{K}^c = \bar{\mathbf{K}}^c h_K^j(\theta_j)$, where the matrix $\bar{\mathbf{K}}^c$ is independent of θ_j, and $h_K^j(\theta_j)$ is a linear or nonlinear function of θ_j. This parametrization scheme is often encountered in several practical applications of model updating and structural optimization. From Eq. (7) it is clear that the partitions of the stiffness matrix $\mathbf{K}^c$ admits the same parametrization. Then, the eigenvalues and eigenvectors associated with the kept fixed-interface normal modes can be expressed as

$$\mathbf{\Lambda}_{kk}^c = \bar{\mathbf{\Lambda}}_{kk}^c \; h_K^j(\theta_j) \quad , \quad \mathbf{\Phi}_{ik}^c = \bar{\mathbf{\Phi}}_{ik}^c \tag{11}$$

where the matrices $\bar{\mathbf{\Lambda}}_{kk}^c$ and $\bar{\mathbf{\Phi}}_{ik}^c$ are the solution of the eigenproblem $\bar{\mathbf{K}}_{ii}^c\bar{\mathbf{\Phi}}_{ik}^c = \bar{\mathbf{M}}_{ii}^c\bar{\mathbf{\Phi}}_{ik}^c\bar{\mathbf{\Lambda}}_{kk}^c$, where the matrices $\bar{\mathbf{\Phi}}_{ik}^c$ and $\bar{\mathbf{\Lambda}}_{kk}^c$ are independent of θ_j. Furthermore, the interface constraint modes are also independent of θ_j since $\mathbf{\Psi}_{ib}^c = -\bar{\mathbf{K}}_{ii}^{c\,-1} \; h_K^{j\,-1}(\theta_j)\bar{\mathbf{K}}_{ib}^c \; h_K^j(\theta_j) = -\bar{\mathbf{K}}_{ii}^{c\,-1}\bar{\mathbf{K}}_{ib}^c = \bar{\mathbf{\Psi}}_{ib}^c$. Therefore, a single component analysis is required to provide the exact estimate of the normal and constraint modes for any value of θ_j. This is a very important result since the computationally intensive re-analyses for estimating the fixed-interface constrained modes and the interface constraint modes of each component for different values of θ_j required during the updating and design processes are completely avoided. Based on the previous parametrization, it can be shown that the partitions for the component stiffness matrix $\hat{\mathbf{K}}^c = \mathbf{\Psi}^{cT}\mathbf{K}^c\mathbf{\Psi}^c \in R^{\hat{n}^c \times \hat{n}^c}, c \in C_j$ (see Eq. (10)) follows the parametrization

$$\hat{\mathbf{K}}_{kk}^c = \bar{\mathbf{\Lambda}}_{kk}^c h_K^j(\theta_j) \quad , \quad \hat{\mathbf{K}}_{kb}^c = \hat{\mathbf{K}}_{bk}^{c^T} = \mathbf{0}_{kb}^c \quad , \quad \hat{\mathbf{K}}_{bb}^c = (\bar{\mathbf{K}}_{bb}^c + \bar{\mathbf{K}}_{bi}^c\bar{\mathbf{\Psi}}_{ib}^c)h_K^j(\theta_j) \tag{12}$$

The previous parametrization allows the stiffness matrix of the reduced-order system to be written as

$$\hat{\mathbf{K}} = \hat{\mathbf{K}}_0 + \sum_{j=1}^{n_d} \hat{\hat{\mathbf{K}}}_{1j} h_K^j(\theta_j) \tag{13}$$

where the matrices $\hat{\mathbf{K}}_0$, and $\hat{\hat{\mathbf{K}}}_{1j}$, $(j = 1, ..., n_d)$ are independent of the values of the vector $\boldsymbol{\theta}$, and n_d is the number of independent model parameters or design variables. These matrices can be obtained directly by replacing the parametrization defined in Eq. (12) into Eq. (10). It is noted that the previous formulation guarantees that the reduced-order model is based on the exact component modes for all values of the system parameters.

4.3 Implementation Aspects

The dynamic response of the original unreduced finite element model, which is needed for the updating and design processes, is approximated by the modal solution of the reduced-order model. Such approach is quite effective since the dimension of the reduced-order model can be substantially smaller than the dimension of the original model. In other words, dividing the structural system into components and reducing the number of physical coordinates to a much smaller number of generalized coordinates have an important impact in the computational effort of the overall processes. The computational efficiency of the proposed approach can be increased additionally by considering high performance computing techniques at the computer hardware level. In fact, the proposed updating and design processes are very-well suited for parallel implementation in a computer cluster [21]. For example, the Markov chains generated at the different stages during the updating and design processes are independent from each other, which means that the generation of different chains can be performed in parallel. Parallelization techniques can also be used at the model level. In this case the definition of all component matrices in generalized coordinates can be carry out concurrently, reducing the computational time of the proposed implementation even further.

5 EXAMPLE PROBLEM

5.1 Formulation

For illustration purposes the effectiveness of the proposed strategy is demonstrated in the second class of problems, that is, reliability-based design of finite element models under stochastic excitation. Applications of the proposed model reduction technique for updating finite element models can be found in [38]. The structural model to be considered consists of a three-span two dimensional frame structure shown in Figure (1). The structure has a total length of 30.0 m and a constant floor height of 5.0 m, leading to a total height of 40.0 m. The finite element model comprises 160 two dimensional beam and column elements of square cross section with 140 nodes and a total of 408 degrees of freedom. The dimension of the square cross section of the column elements is equal to 0.4 m. The axial deformation of the column elements is assumed to be small and they are neglected in the model. The Young's modulus of the different elements is equal to $E = 2.0 \times 10^{10}$ N/m^2. A 5% of critical damping for the modal damping ratios is introduced in the model. The structural system is excited horizontally by a ground acceleration modeled as a non-stationary stochastic process. In particular, a stochastic point-source model characterized by a series of seismicity parameters such as the moment magnitude and rupture distance is considered in the present implementation [39, 40].

The input for the stochastic excitation model involves a white noise sequence and a series of seismological parameters as previously pointed out. Details of the entire procedure can be found in [39, 41]. The duration of the excitation is equal to $T = 30$ s with a sampling interval equal to $\Delta T = 0.01$ s. Based on the characterization of the point source model, the generation of the stochastic ground motions involves more than 3000 random variables for the duration and sampling interval considered. Thus, the vector of uncertain parameters $\mathbf{z}$ involved in the problem (see Section 3) has more than 3000 components.

The cost represented by the total volume of the column elements is chosen as the objective function for the design problem. The design variables comprise the dimension of the column

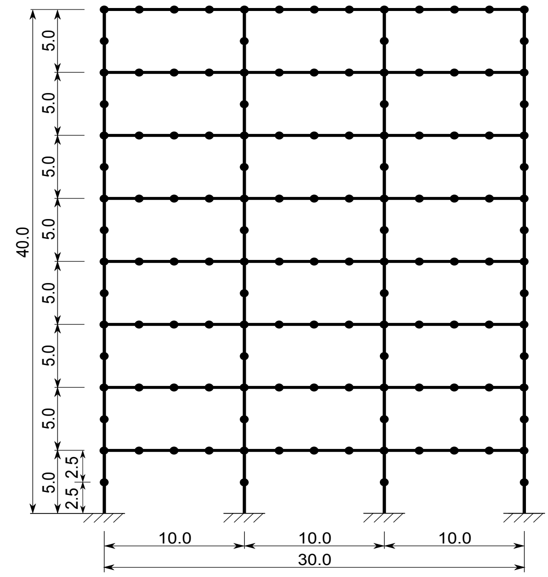

Figure 1: Three-span eight-story structural model

elements square cross section of the different floors. In particular, the dimension of the column elements square cross section are linked into two design variables in this example problem. Design variable number one (θ_1) is related to the dimension of the column elements of floors 1 to 4, while the second design variable (θ_2) controls the dimension of the column elements of floors 5 to 8. To control serviceability, the design criteria are defined in terms of the relative displacements of the first, fifth and eighth floor with respect to the ground. The failure probability functions are defined as

$$P_{F_i}(\boldsymbol{\theta}) = P[\max_{t\in[0,T]} \frac{\mid \delta^i(t,\boldsymbol{\theta},\mathbf{z}) \mid}{\delta^{i*}} > 1] \ \ , \ \ i = 1,2,3 \tag{14}$$

where $\delta^i(t,\boldsymbol{\theta},\mathbf{z}), i = 1,2,3$ are the relative displacements of the first, fifth and eighth floor with respect to the ground, respectively, and $\delta^{i^*}, i = 1,2,3$ are the corresponding critical threshold levels. The threshold levels are defined in terms of a percentage of the total height of the frame structure. The following values are considered: 0.05%; 0.3%; and 0.5% for the threshold levels corresponding to the failure events associated with the first, fifth and eighth floor, respectively. The design problem is formulated as

$$\begin{aligned} \text{Min}_{\boldsymbol{\theta}} \ \ & c(\boldsymbol{\theta}) \\ \text{s.t.} \ \ & P_{F_i}(\boldsymbol{\theta}) \le 10^{-4} \quad i = 1,2,3 \\ & 0.3\text{m} \le \theta_i \le 0.8\text{m} \quad i = 1,2 \end{aligned} \tag{15}$$

Note that the estimation of the probability of failure for a given design $\boldsymbol{\theta}$ represents a high-dimensional reliability problem.

5.2 Reduced-Order Model

The structural model is subdivided into sixteen components as shown in Figure (2). Components 1 to 8 are composed by the column elements of the different floors, while components 9 to 16 correspond to the beam elements of the different floors. Based on this subdivision it is noted that components 1 to 4 depend on the design variable θ_1, components 5 to 8 depend on the design variable θ_2, while components 9 to 16 are independent of the design variables. In connection with the previous section, the nonlinear functions of the design variables θ_j are given by $h_K^j(\theta_j) = \theta_j^4, j = 1, 2$. For each component it is selected to retain all fixed-interface normal modes that have frequency ω such that $\omega \leq \alpha\omega_c$, with α being a multiplication factor and ω_c is a cut-off frequency which is taken equal to 41.0 rad/s in this case (4^{th} modal frequency of the unreduced model). The multiplication factor is selected to be 10.0 for components 1 to 8, and 2.9 for components 9 to 16. With this selection of parameters only one fixed-interface normal mode is kept for each component. Thus, the reduced-order model comprises a total of 16 generalized coordinates out of the 312 internal degrees of freedom of all components (95% reduction in terms of the internal degrees of freedom). On the other hand, the total number of interface degrees of freedom is equal to 96 in this case. Based on this parametrization, the errors between the modal frequencies using the unreduced reference finite element model and the modal frequencies computed using the reduced-order model are quite small. In fact, the error for the lowest 4 modes fall bellow 0.1%. The comparison with the lowest 4 modes seems to be reasonable since validation calculations show that the contribution of the higher order modes (higher than the 4^{th} mode) in the dynamic response of the model is negligible. It is observed that an important reduction in the number of generalized coordinates is obtained with respect to the number of the degrees of freedom of the original unreduced finite element model. In fact, an almost 75% reduction is obtained in this example problem.

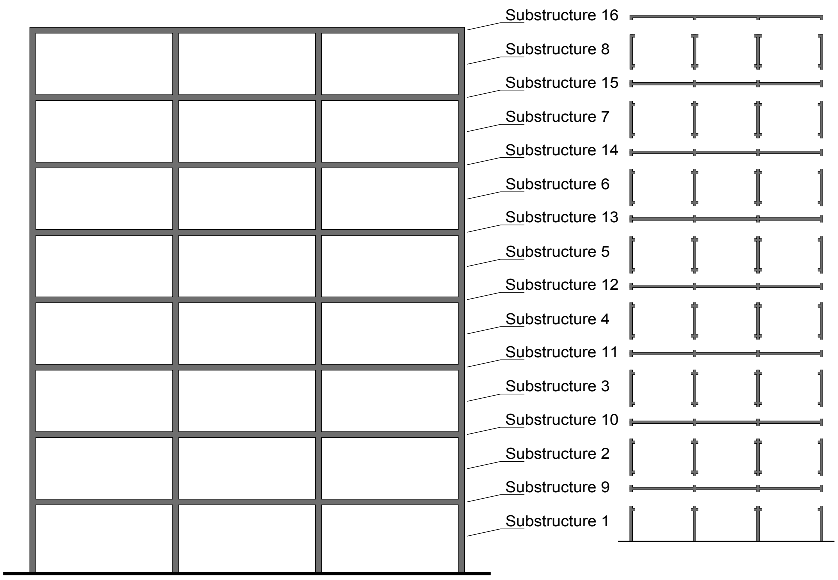

Figure 2: Components (substructures) of the finite element model for design purposes

5.3 Results of the Example Problem

The effectiveness of the model reduction technique in the context of the design problem is shown in Figures (3) and (4). These Figures show some iso-probability curves in the design space constructed by using the original unreduced finite element model and the reduced-order model, respectively. It is observed that the iso-probability curves obtained by the full and reduced-order model are very similar. There is only a slight difference between the iso-probability curves corresponding to low probability events ($\leq 10^{-5}$). The small differences at low probability levels are due mainly to the advanced simulation scheme used to estimate such probabilities. The previous results imply that the final design for the example at hand can be obtained with high accuracy by using the reduced-order model instead of the full model. On the other hand, the shape and structure of the iso-probability curves associated with the different failure events give a valuable insight into the interaction and effect of the design variables on the reliability of the structural model. For example, the iso-probability curves associated with the relative displacement of the first floor (P_{F_1}) show a weak interaction between the dimension of the column elements square cross section of the lower and upper floors. Contrarily, the iso-probability curves associated with the roof displacement (eighth floor) (P_{F_3}) show a substantial non-linear interaction between the dimension of the column elements square cross section of all floors.

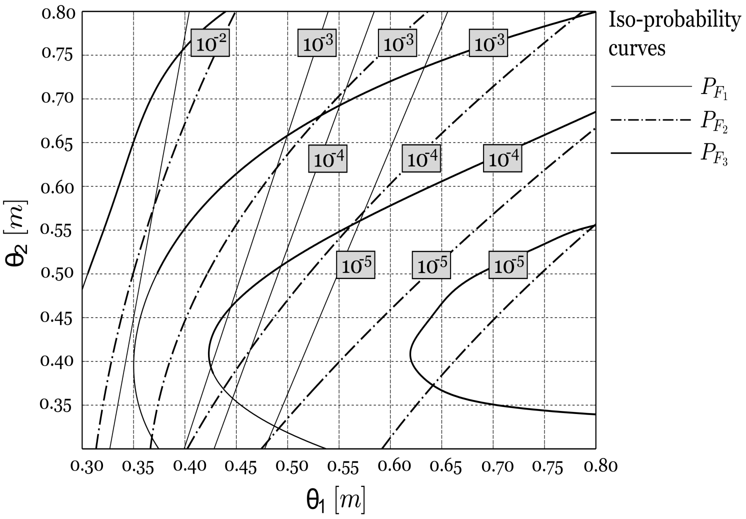

Figure 3: Iso-probability curves associated with the failure events of the first, fifth, and eighth floor (unreduced finite element model)

Figure (4) also shows some normalized objective contours, the feasible domain and the final design. It is observed that the side constraints are inactive at the final design. The reliability constraints related to the relative displacement of the first and eighth floor with respect to the ground are active at the final design. In terms of the computational cost, the number of finite element runs involved during the design process depends mainly on the number of iterations and the number of simulations necessary to estimate the failure probability for the different designs required by the optimizer. Numerical results show that the speedup achieved by the proposed formulation is about 8. In this context, the speedup is defined as the ratio of the execution time of the design process by using the unreduced original model and the execution time of the design process by using the reduced-order model. This reduction in computational effort is achieved without compromising the accuracy of the final design.

6 CONCLUSIONS

A methodology that integrates a model reduction technique into a two types of complex problems, namely, finite element model updating using dynamic response data, and reliability-based design of stochastic finite element models has been presented. In particular, a method based on fixed-interface normal modes and interface constraint modes is considered in this

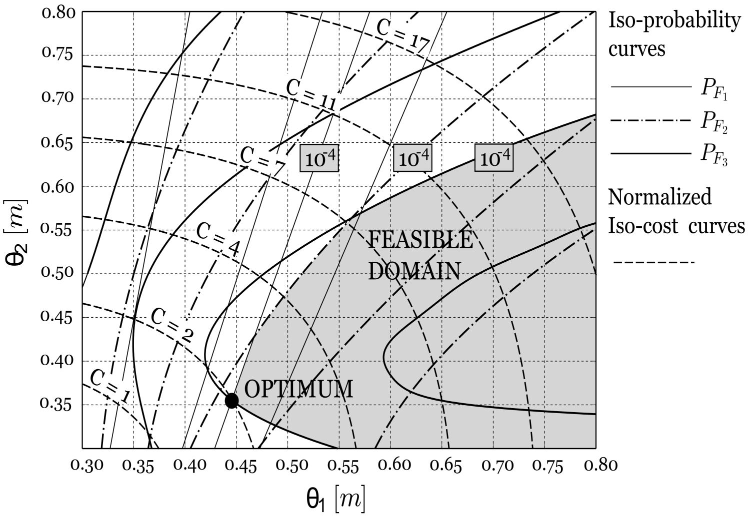

Figure 4: Iso-probability curves associated with the failure events of the first, fifth, and eighth floor (reduced-order model)

work. The method produces highly accurate models with relatively few component modes. It is demonstrated that under certain parametrization schemes, the fixed-interface normal modes and the interface constraint modes of each component are computed once from a reference finite element model. In this manner the re-assembling of the reduced-order system matrices is avoided during the updating and design processes. Results show that the computational effort for updating and designing the reduced-order model is decreased drastically by one or more orders of magnitude with respect to the unreduced model, that is, the full finite element model. Furthermore, the drastic reduction in computational efforts is achieved without compromising the predictive capability of the proposed updating and design processes.

ACKNOWLEDGMENTS

The research reported here was supported in part by CONICYT (National Commission for Scientific and Technological Research) under grant number 1110061, and by the European Union Project PIRSES-GA-2010-269222: Analysis and Design of Earthquake Resistant Structures (ADERS) of the FP7-PEOPLE-2010-IRSES, Marie Curie Actions. This support is gratefully acknowledged by the authors.

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