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DETERMINATION OF THE PERIODS OF COUPLED SHEAR WALLS BY RECURSIVE DIFFERENTIATION METHOD

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Abstract. *Coupled shear walls are commonly used in the constructions to meet the architectural requirements and to improve the lateral resistance of the structure. Evaluation of the behavior of the structures with coupled shear walls under earthquake loads is carried out by dynamic analysis. For the dynamic analysis, identification of the dynamic properties of the coupled shear walls is important. The period of the structure is an important design parameter used in the dynamic analysis. There are various methods used for the determination of the periods. In this study, the periods of the coupled shear walls are determined by using Recursive Differentiation Method. At the end of study, good results are received by the example that is solved to investigate the presented method.*

1 INTRODUCTION

There are several studies with regard to the analysis of coupled shear walls [1] – [11]. In this study, Recursive Differentiation Method is proposed for the determination of the periods of coupled shear walls.

The assumptions made in the study are: the material is linearly elastic and the displacements are small, P-Δ effects are neglected, the influence of shear deformation of the shear walls is neglected, material and geometrical properties of the shear walls are same throughout the height of the structure.

2 METHOD

In this part of the study, Recursive Differentiation Method that is detailed in the literature [12] is applied to the determination of the periods of coupled shear walls and the method is explained. Equations of the coupled shear walls can be written as in Equation (1) and (2).

$$EI\frac{d^4y}{dx^4} - K\left(\frac{d^2y}{dx^2} - \frac{d^2y_b}{dx^2}\right) + m\omega^2 y = 0 \tag{1}$$

$$D\frac{d^3y}{dx^3} = -K\left(\frac{dy}{dx} - \frac{dy_b}{dx}\right) \tag{2}$$

Here, y is the total displacement, y_b is the displacement due to bending and x is the vertical axis. Boundary conditions are

$$y(0) = 0 \tag{3}$$

$$\frac{dy}{dx}_{x=0} = 0 \tag{4}$$

$$\frac{dy_b}{dx}_{x=0} = 0 \tag{5}$$

$$EI\frac{d^2y}{dx^2}_{x=H} = 0 \tag{6}$$

$$D\frac{d^2y_b}{dx^2}_{x=H} = 0 \tag{7}$$

$$EI\frac{d^3y}{dx^3}_{x=H} - K\frac{dy}{dx}_{x=H} + K\frac{dy_b}{dx}_{x=H} = 0 \tag{8}$$

EI is the total flexural rigidity of the coupled shear wall represented in Figure (1) and calculated by Equation (9).

$$EI = \sum_{i=1}^{n}(EI)_i \tag{9}$$

K is the shear rigidity and D is the global flexural rigidity.

$$K = K_b\frac{K_c}{K_c+K_b} \tag{10}$$

$$K_c = \sum_{i=1}^{n-1}\frac{12EI_{c,i}}{h^2} \tag{11}$$

$$K_b = \sum_{i=1}^{n}\frac{6EI_{b,i}\left[(l+s_1)^2+(l+s_2)^2\right]}{l^3h\left(1+\frac{12kEI_b}{l^2GA_b}\right)} \tag{12}$$

$$D = E\sum_{j=1}^{n}A_{c,j}\,t_j^2 \tag{13}$$

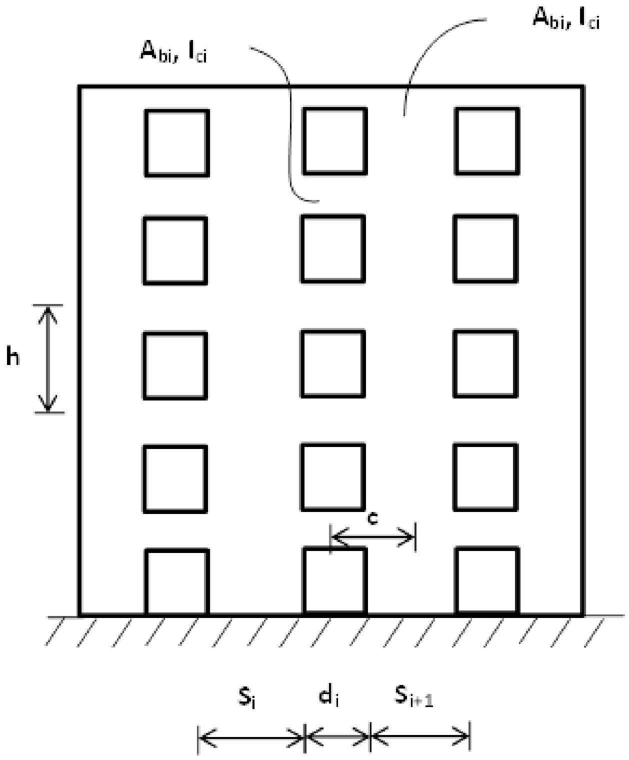

Figure 1: Coupled shear wall.

To get the non-dimensional forms of Equation (1) and (2), Ɛ=x/H transform is applied and Equation (14) and (15) are obtained.

$$\frac{d^4y}{d\mathcal{E}^4} - a\left(\frac{d^2y}{d\mathcal{E}^2} - \frac{d^2y_b}{d\mathcal{E}^2}\right) + py = 0 \tag{14}$$

$$\frac{d^3y}{dx^3} = -b\left(\frac{dy}{dx} - \frac{dy_b}{dx}\right) \tag{15}$$

a, b and p parameters are found according to Equations (16), (17) and (18)

$$a = H^2\frac{K}{(EI)} \tag{16}$$

$$b = H^2\frac{K}{D} \tag{17}$$

$$p = \frac{H^4\omega^2 m}{EI} \tag{18}$$

With the purpose of the reduction of the order of the differential equation, $\frac{dy_b}{dx}$ rotation is named as θ_b

$$\frac{dy_b}{dx} = \theta_b \tag{19}$$

Thus, the differential equations return into the equations below:

$$\frac{d^4y}{d\mathcal{E}^4} - a\left(\frac{d^2y}{d\mathcal{E}^2} - \frac{d\theta_b}{dx}\right) + py = 0 \tag{20}$$

$$\frac{d^2\theta_b}{dx^2} = -b\left(\frac{dy}{dx} - \theta_b\right) \tag{21}$$

To put the equation into final form, boundary conditions are as below:

$$y(0) = 0 \tag{22}$$

$$\frac{dy}{d\varepsilon}_{x=0} = 0 \tag{23}$$

$$\theta_b(0) = 0 \tag{24}$$

$$\frac{d^2y}{d\varepsilon^2}_{\varepsilon=1} = 0 \tag{25}$$

$$\frac{d\theta_b}{d\varepsilon}_{\varepsilon=1} = 0 \tag{26}$$

$$\frac{d^3y}{d\varepsilon^3}_{\varepsilon=1} - a\frac{dy}{d\varepsilon}_{\varepsilon=1} + \theta_{b\,\varepsilon=1} = 0 \tag{27}$$

In the Recursive Differentiation Method, expressions of the derivatives taken can be written recursively by the Equations (20) and (21)

$$\frac{d^{(i+4)}y}{d\varepsilon^{(i+4)}}_1 = a\left(\frac{d^{(i+2)}y}{d\varepsilon^{(i+2)}} - \frac{d^{(i+1)}\theta_b}{d\varepsilon^{(i+1)}}\right) - p\frac{d^iy}{d\varepsilon^i} \tag{28}$$

$$\frac{d^{(i+2)}\theta_b}{d\varepsilon^{(i+2)}} = -b\left(\frac{d^{(i+1)}y}{d\varepsilon^{(i+1)}} - \frac{d^i\theta_b}{d\varepsilon^i}\right) \tag{29}$$

Displacement and flexural rotation of the top of the coupled shear wall are written as in the equations below:

$$y(1) = y(0) + \frac{y^{\iota}(0)}{1} + \frac{y^{\iota\iota}(0)}{2!} + \cdots + \frac{y^k(0)}{k!} + \cdots.. \tag{30}$$

$$\theta_b(1) = \theta_b(0) + \frac{{\theta_b}^{\iota}(0)}{1} + \frac{{\theta_b}^{\iota\iota}(0)}{2!} + \cdots + \frac{{\theta_b}^k(0)}{k!} + \cdots.. \tag{31}$$

As the boundary conditions are written by means of these two equations, Equations (32) - (34) are obtained.

$$y''(1) = y''(0) + \frac{y^{\iota\iota\iota}(0)}{1} + \frac{y^{IV}(0)}{2!} + \cdots + \frac{y^k(0)}{(k-2)!} + \cdots.. = 0 \tag{32}$$

$$\theta_b{}'(1) = \theta_b{}'(0) + \frac{{\theta_b}^{\iota\iota}(0)}{1} + \frac{{\theta_b}^{III}(0)}{2!} + \cdots + \frac{{\theta_b}^k(0)}{(k-1)!} + \cdots.. = 0 \tag{33}$$

$$y'''(1) - ay'(1) + a\theta_b(1) = \left(y'''(0) + \frac{y^{IV}(0)}{1} + \frac{y^V(0)}{2!} + \cdots + \frac{y^k(0)}{(k-3)!} + \cdots..\right) - a\left(y'(0) + \frac{y''(0)}{1} + \frac{y^{III}(0)}{2!} + \cdots + \frac{y^k(0)}{(k-1)!}\right) + a\left(\theta_b(0) + \frac{{\theta_b}^{\iota}(0)}{1} + \frac{{\theta_b}^{\iota\iota}(0)}{2!} + \cdots + \frac{{\theta_b}^k(0)}{k!} + \cdots\right) = 0 \tag{34}$$

In this equation system derivatives are written in terms of y''(0), y'''(0), θ_b'(0) and with the help of Equations (28) and (29), matrix equation below is obtained.

$$[A] * \begin{Bmatrix} y''(0) \\ y'''(0) \\ \theta_b{}'(0) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{35}$$

In this matrix equation, the determinant of A should be zero for non-trivial solution.

Angular frequencies are obtained by Equation (36).

$$\omega_i = \frac{1}{H^2}\sqrt{\frac{p_iEI}{m}} \tag{36}$$

3 EXAMPLE

The coupled shear wall for the example is given in Figure 2. The properties of the coupled shear wall system are given in Table1.

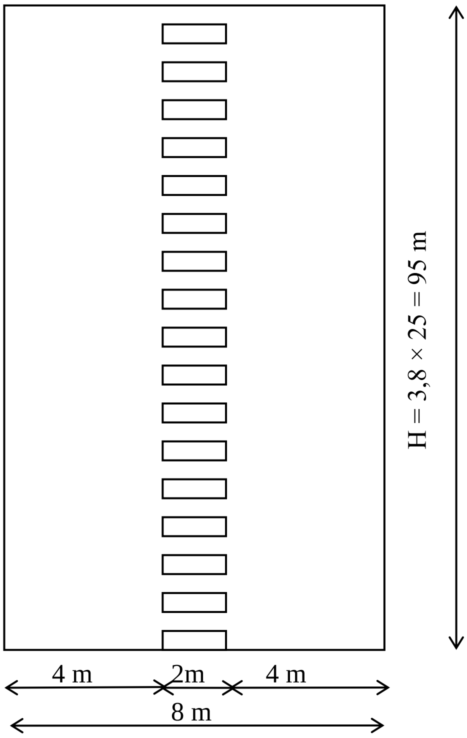

Figure 2: Coupled shear wall.

Title 1	Title 2
EI	$2{,}9808*10^{8}$ kNm^2
K	$4550446*10^{5}$ kN
D	$1{,}58976*10^{9}$ kNm^2
m	831,6 t
a	13,7774
b	5,5833

Table 1: Properites of the coupled shear wall system in the example.

Mod No	Proposed Method **1**	Kuang **2**	Ratio **1 / 2**
1	0,660	0,667	0,990
2	2,897	2,925	0,990
3	7,104	7,159	0,992
4	13,20	13,28	0,994
5	21,33	21,44	0,995
6	31,48	31,61	0,996
7	43,67	43,82	0,997
8	57,88	58,04	0,997
9	74,12	74,30	0,998
10	92,41	92,57	0,998

Table 2: Natural frequencies of the example

With the solution of the example, good results are received and the differences between the results of the method and the literature [10] are given in Table 2.

4 CONCLUSIONS

In this study, Recursive Differentiation Method is proposed for the determination of the periods of coupled shear walls. The method for the solution of the periods according to the Recursive Differentiation Method is explained in detail. In consequence of the comparison of the presented method and the literature by the example at the end of the study, good results are received.

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