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DRY FRICTION INFLUENCE ON STRUCTURE DYNAMICS

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Abstract. *In this paper we have analysed the influence of dry friction on the response of a two story building model. We have assumed that this structure is mounted on two FPSs (Friction Pendulum Systems) and have modelled these devices by a spring and a dry friction force.*
We have shown how dry friction influences the response of the system and determined numerically the dynamical response under periodic ground excitation.

1 INTRODUCTION

FPSs are devices by which we can mitigate the effects of seismic excitations on buildings or structures. These devices allow us to make more safe structures in a very easy way provided that we have machines for installing them. These devices are very simple mechanical systems. They are made of two bodies which can slide one on other along a path with assigned form and able to dissipate energy by a friction mechanism [1, 2]. By the dynamical viewpoint these devices can be modelled by a linear spring and dry friction devices. The spring acts as filter of vibrations while friction device acts as a damper.
This devices can be divided into active and passive devices and according to the material they are made of. For example, Rubber Bearings, used extensively in bridge structures and prestressed and precast concrete buildings are made of EPDM, SBR and Natural Rubber, Neoprene Rubber, etc. Such devices allow vibration dampening, prevent sound transfer, reduce the destructive action of the vibrations in the structures by allowing displacements caused by normal expansion costranction and materials [3, 4]. In this work we have investigated the use of a particular passive system for the mitigation of vibrations in structures, the Friction Pendulum System (FPS). This device uses friction to ensure flexibility to the structural system and to add damping to the isolated structure. This device represents the simplest sliding system that works without the aid of active restoring forces. The radius of curvature of the dishes of FPS influences in a decisive manner relative displacements between the upper plate connected to the structure and the lower one connected to ground so that fundamental period of the base-isolated structure can be shifted further away from the predominant period of near-fault ground motions [5]. We therefore decided to use such a device, to investigate possible forms of instability that may arise due to the presence of dry friction between the moving parts of FPS [6, 7].
We organized the paper as follows. On The Response Of A Mass-Spring-Dry Friction section we studied the influence of discontinuous and continuous friction model on describing the dynamical behaviour of the system. In Mathematical Model section we discussed the dimensionless equation used to describe the motion of the structure considered in presence of dry friction. In Numerical Simulations we have reported some details about the values used to describe different configurations of the device in different scenarios with the simplifying assumptions made. In Final section we presented the conclusions.

2 ON THE RESPONSE OF A MASS-SPRING-DRY FRICTION

Let us consider the system indicated in figure 1. The equation of motion can be written in the following dimensionless form:

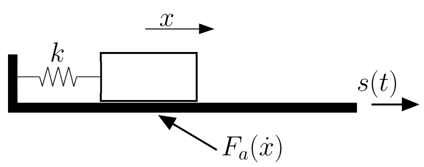


Figure 1: One Degree of Freedom System

$$\ddot{z} + f(\dot{z} + 1) + z = 0 \tag{1}$$

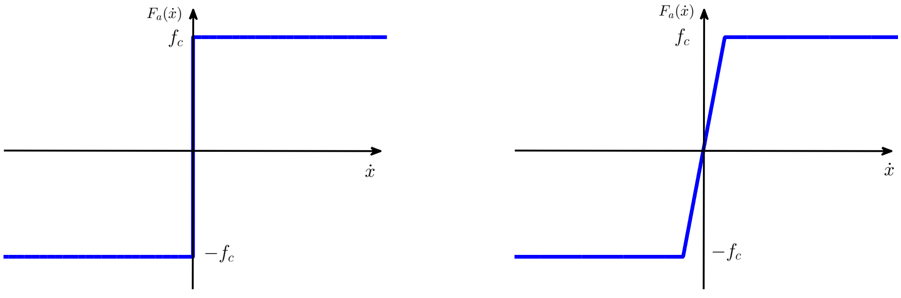
having put

$$\overline{z} = x - s; \qquad \tau = \omega_n t; \qquad z = \frac{\overline{z}}{\frac{\dot{s}}{\omega_n}};$$

$$\omega_n^2 = \frac{k}{m}; \qquad \frac{F(\dot{z}+1)}{m\omega_n \dot{s}} = f(\dot{z}+1).$$

By assuming the friction force between the slide and ground described by a Coulomb friction model (see figure 2(a)) we have calculated the response and indicated it in figure 3 without ground excitation.

We can observe that every point which belongs to the set $-f_c$, f_c is an equilibrium point of the



(a) (b)

Figure 2: Dry Friction Coulomb Model

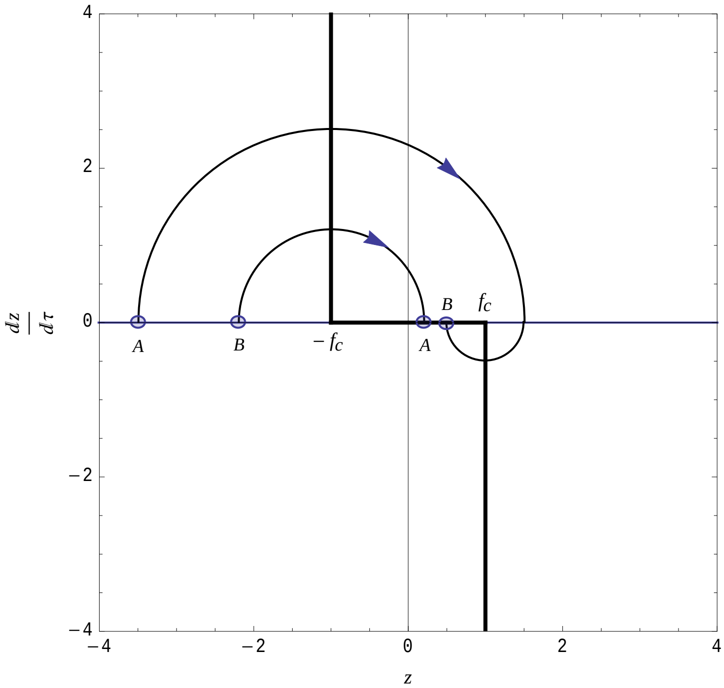


Figure 3: Dynamical Response with Discontinuous Friction Characteristic

system.

On the other hand, if we make continuous the friction characteristic, so as indicated in figure 2(b), the response of the system will be that of the figure 4.

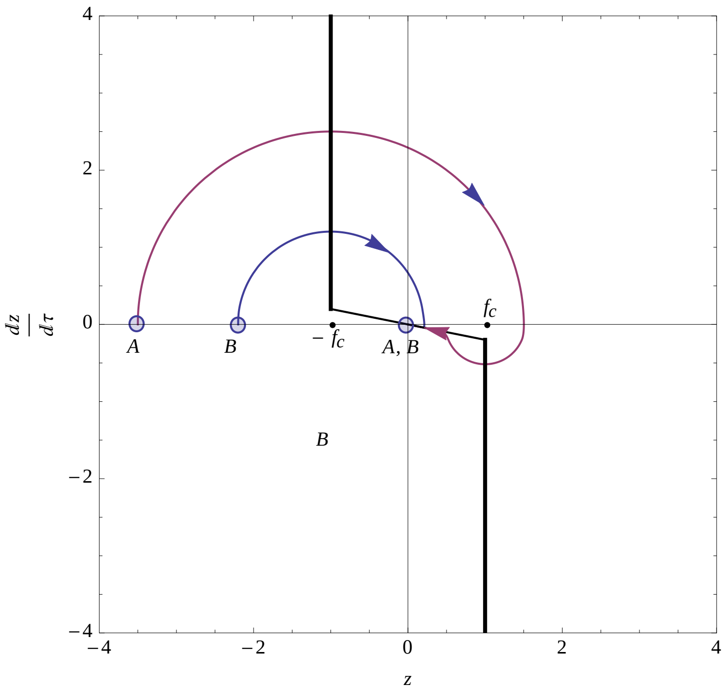


Figure 4: Dynamical Response with Continuous Friction Characteristic

By comparing figures 3 and 4 we can conclude that by making continuous the Coulomb friction in that way is a bad approximation about describing the dynamical behaviour of the system 1. If we consider the system excited by a ground motion, the system has not equilibrium point and, except singular solutions, the body is never undergone to stopping conditions (stick phase). On the basis of these last consideration we can assume that when the system is excited by external force we can make continuous the friction characteristic without drawbacks on the numerical simulations.

3 RESPONSE OF A TWO STORY BUILDING ON FPSs

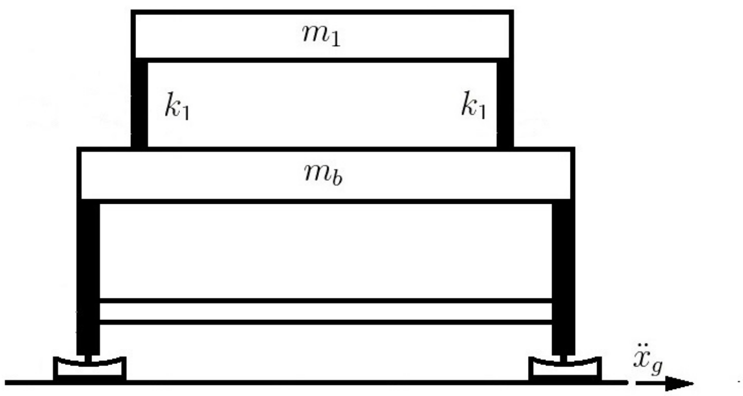


Figure 5: Mechanical System

In figure 5 is reported the mechanical model of the two degrees of freedom system used to study the dynamic behaviour in presence of dry friction. The equation of motion is reported by equations 2:

$$\begin{aligned} & m_b\ddot{\tilde{x}}_b(t) + k_b(\tilde{x}(t) - \tilde{x}(t)) + f_{a,b}(\dot{\tilde{x}}_b(t) - \dot{\tilde{x}}_g(t)) + k_1(\tilde{x}(t) - \tilde{x}_1(t)) = 0; \\ & m_1\ddot{\tilde{x}}_1(t) + k_1(\tilde{x}_1(t) - \tilde{x}(t)) = 0. \end{aligned} \tag{2}$$

If we choose to measure displacement, velocity and acceleration of each mass compared to the ground, $\tilde{x}, \dot{\tilde{x}}_g, \ddot{\tilde{x}}_g$ respectively, we introduce relative measurements as showed below. We can now write a new set of equation in matrix notation as reported in 3

$$\begin{cases} \tilde{x} = x_b + \tilde{x}_g \\ \tilde{x}_1 = x_1 + \tilde{x}_g \end{cases}$$

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) + \mathbf{F_a}(\dot{\mathbf{x}}_b) = -\mathbf{M}\ddot{\tilde{\mathbf{x}}}_g(t); \tag{3}$$

where $\mathbf{x}(t)$,$\dot{\mathbf{x}}(t)$ and $\ddot{\mathbf{x}}(t)$ are vectors of relative displacement, velocity and acceleration and $\ddot{\tilde{x}}_g$ represents the ground acceleration. With $\mathbf{M}$ and $\mathbf{K}$ are reported respectively the mass and stiffness matrix and with $\mathbf{F_a}$ the dry friction terms.

$$\mathbf{M} = \begin{bmatrix} m_b & 0 \\ 0 & m_1 \end{bmatrix} \tag{4}$$

$$\mathbf{K} = \begin{bmatrix} k_b + k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \tag{5}$$

$$\mathbf{F_a} = \begin{Bmatrix} f_{a,b}(\dot{x}_b) \\ 0 \end{Bmatrix} \tag{6}$$

For the discontinuous function used to describe dry friction we have chosen the function reported in figure 2(a) having the following characteristic:

$$f_a(\dot{x}_b) = \begin{cases} \mu_b(m_b + m_1)g & \dot{x}_b > 0 \\ -\mu_b(m_b + m_1)g < f_{a,b} < \mu_b(m_b + m_1)g & \dot{x}_b = 0 \\ -\mu_b(m_b + m_1)g & \dot{x}_b < 0 \end{cases}$$

The equation that describes the dynamic motion of the base mass becomes the following:

$$m_b\ddot{x}_b + (k_b + k_1)x_b - k_1x_1 + f_{a,b}(\dot{x}_b) = -m_b\ddot{\tilde{x}}_g \tag{7}$$

while the equation that describes the motion of the other mass is the following:

$$m_1\ddot{x}_1 + k_1x_1 - k_1x_b = -m_1\ddot{\tilde{x}}_g \tag{8}$$

making explicit the friction term, equation 7 becomes:

$$m_b\ddot{x}_b + (k_b + k_1)x_b - k_1x_1 + \mu(m_b + m)g\frac{\dot{x}_b}{|\dot{x}_b|} = -m_b\ddot{\tilde{x}}_g \tag{9}$$

Dividing equation 9 by m_b and equation 8 by m_1 we obtain:

$$\ddot{x}_b + \frac{k_b + k_1}{m_b}x_b - \frac{k_1}{m_b}x_1 + \mu\frac{m_b + m_1}{m_b}g\frac{\dot{x}_b}{|\dot{x}_b|} = -\ddot{\tilde{x}}_g \tag{10}$$

$$\ddot{x}_1 + \frac{k_1}{m_1}x_1 - \frac{k_1}{m_1}x_b = -\ddot{\tilde{x}}_g \tag{11}$$

By performing the following positions:

$$\omega^2{}_b = \frac{k_b + k_1}{m_b}; \qquad \omega^2{}_1 = \frac{k_1}{m_1}; \qquad \tau = \omega_1 t;$$

$$(') = \frac{d}{d\tau}; \qquad \Omega = \frac{\omega}{\omega_1}; \qquad y_i = \frac{x_i}{|\tilde{x}''_g|};$$

dividing equations 10 and 11 by $\omega^2{}_1$, assuming for the external forcing a cosinosuidale law and introducing the following parameters:

$$\beta_1 = \frac{k_b}{k_1}; \qquad \beta_2 = \frac{m_1}{m_b}; \qquad \beta_3 = \frac{m_1 g}{k_1};$$

$$\alpha_1 = (1+\beta_1)\beta_2; \qquad \alpha_2 = \mu_b\beta_3; \qquad \alpha_3 = (1+\beta_2)\alpha_2.$$

the equations can be rewritten in dimensionless form as follows:

$$\begin{aligned} &y''{}_b + \alpha_1 y_b - \beta_2 y_1 + \alpha_3 \frac{y'{}_b}{|y'{}_b|} = cos(\Omega\tau); \\ &y''{}_1 + y_1 - y_b = cos(\Omega\tau). \end{aligned} \tag{12}$$

4 NUMERICAL ANALISYS

We have developed a numerical procedure to investigate the influence of dry friction on the dynamic behaviour of a structure. The parameters considered to carry out this investigation are stiffness ratio β_1 between k_b and k_1, the mass ratio β_2 between m_1 and m_b and the friction coefficient μ_b between upper body and the base. For β_1 we have considered values $[0.1, 0.01]$ assuming that the equivalent stiffness of the friction pendulum system should be at least one order of magnitude lower compared to the stiffness of the pillars and for β_2 we have considered the following values in brackets $[1, 0.1]$ and are reported in Tab. 1. Further for the friction coefficient, we considered four scenarios with μ_b equal to values reported in brackets $[0.1, 0.3, 0.5, 0.7]$, using values frequently encountered in most cases. The assumptions made in this paper are the following:

$\beta_2 \setminus \beta_1$	0.1	0.01
1	$k_b = 0.1k_1$ $m_b = m_1$	$k_b = 0.01k_1$ $m_b = m_1$
0.1	$k_b = 0.1k_1$ $m_b = 0.1m_1$	$k_b = 0.01k_1$ $m_b = 0.1m_1$

Table 1: Several Configurations of the Two Degrees of Freedom System Considered

- Linear law for elastic constants;
- Friction force remains constant with respect to sliding velocity;
- Sinusoidal law for ground acceleration;
- Order of magnitude of the upper structure mass m_1 is one order of magnitude lower than the elastic constant k_1.

The simulations for the cases considered, have been carried out by the use of calculation software Matlab, in Simulink environment. Of the four simulations carried out, the three most significant graphs of the coefficient of amplification have been reported for the first and second mass respectively in fig 6 and 7.

5 CONCLUSIONS

In this paper the presence of eventual forms of instability due to the presence of dry friction in the dynamics of a mechanical system were investigated . Through appropriate positions and appropriate hypothesis the system under consideration has been brought back to a two degrees of freedom system that can represent a single-storey building isolated by fps, friction pendulum system. The reported results, clearly demonstrate how the dynamic behaviour of a mechanical systems subject to dry friction for low values of the friction coefficient is not affected if not in a partial way of the presence of friction while, for high values of the friction coefficient, the response of the system is significantly different.

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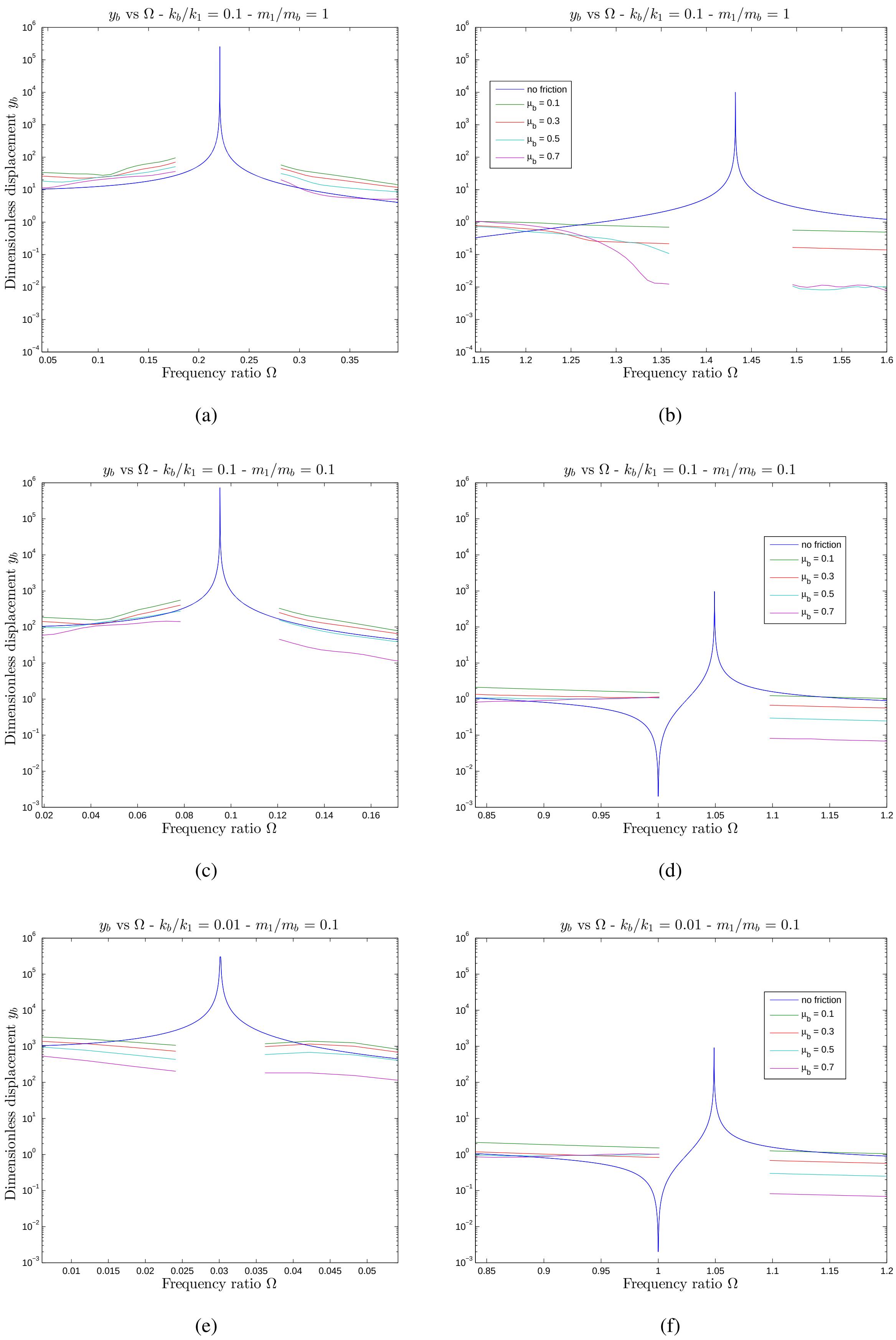

Figure 6: Response of the Base Floor of the Reduced System

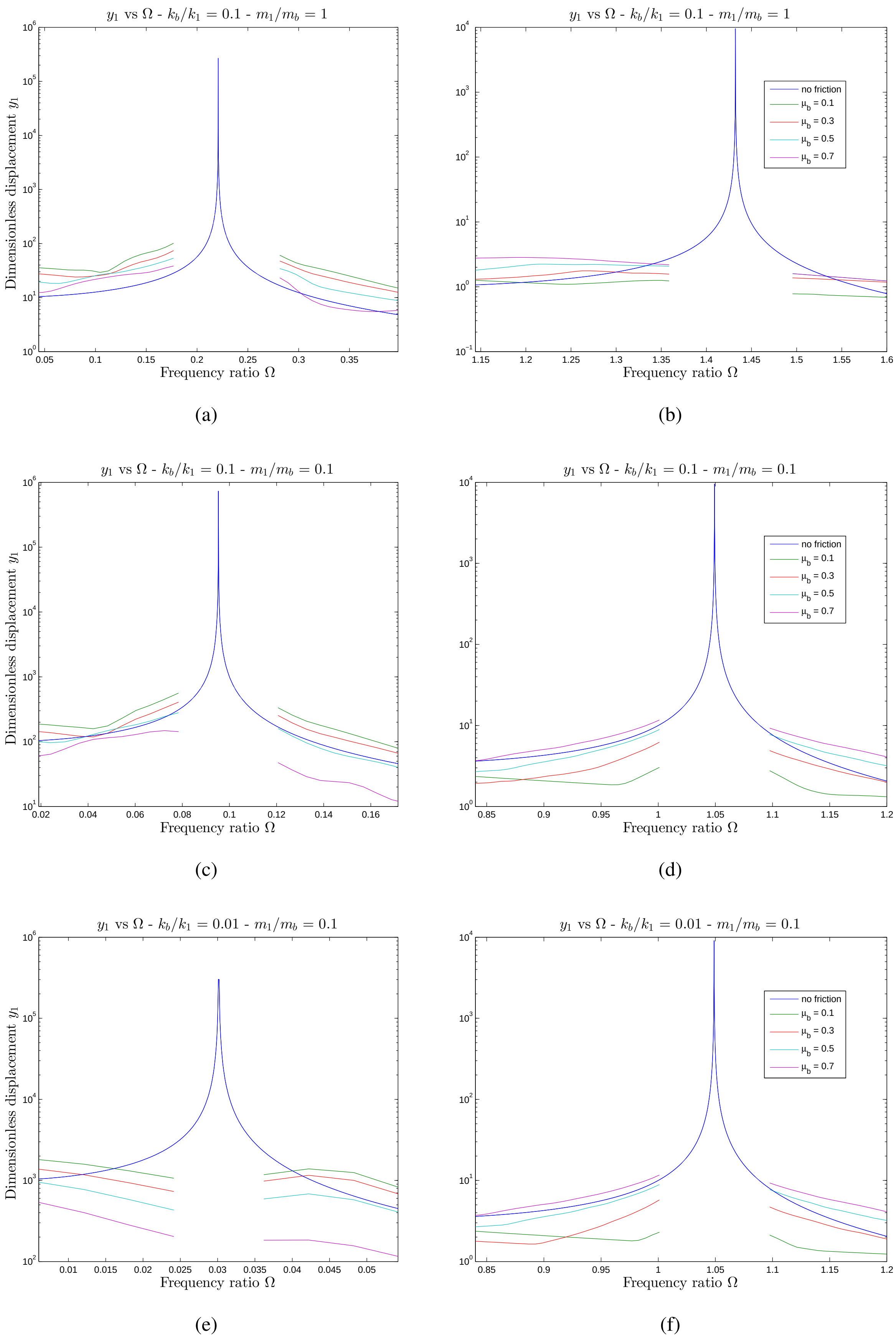

Figure 7: Response of the First Floor of the Reduced System