EVALUATION OF SEISMIC RESPONSE OF BUILDINGS USING DYNAMIC ANALYSIS ON STICK MODELS

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Abstract. A new methodology for the study and evaluation of the seismic response of buildings is presented in this paper. The method involves the use of nonlinear time-history analysis on equivalent multi-degrees of freedom stick models, defined according to the classification given in FEMA440 Report. The steps for implementing the method and obtaining the equivalent model are described in the article. For the validation of the method, the results obtained from the stick models were compared to that obtained by time-history analysis on detailed finite element models of the structures (using the definition of detailed model described in FEMA440). The engineering parameters considered in the comparison are top displacements, interstorey drifts, forces at the floor and base shear. The results of the conducted analyses on the equivalent stick models match satisfactorily, in an excellent way in certain cases, those of the analyses carried out on detailed models. The time-history analyses on the stick models, allowing a considerable saving in terms of both computational effort and time for numerical solution and post-processing of results, could make this kind of analysis more attractive and feasible for a daily design, and comparable in terms of costs with pushover procedures.
1 INTRODUCTION

The evaluation of dynamic response of a structural system and the interpretation of results obtained are the key aspects in the study of a building subjected to seismic action, both in the case of new building design and for the seismic assessment of existing buildings. To this regard, in the early nineties, innovative design criteria based on respect of structural performance criteria were introduced, not only for ultimate limit state analysis but also related to the containment of damage resulting from seismic events [1].

In the light of this, the design criteria based on performance requirements related to the displacements, have led to the definition of “displacement-based design” procedures [2]. The need to properly estimate the displacements resulting from the seismic action, using simplified models of the structure, however, is an issue introduced earlier.

Linked to the problem of limiting the computational efforts in the study of inelastic structures, the attention of researchers was soon addressed to look for simplified equivalent methods able to correctly represent the global response of structures. To this regard it seems useful to quote the Substitute Damping Method proposed in [3], the Method of the Substitute Structure introduced in [4], the Q-Model [5] and the Multi-Rigid-Body Method [6] which, starting from the theory of rigid bodies, defines an equivalent discrete multi-degrees of freedom (MDOF) model for the study of dynamic response of shear-type frames.

Between the various simplified model proposed from the researchers, one of the recurring assumption is to search for an equivalent single-degree of freedom (SDOF) model, that usually is adopted instead of the complete model in the seismic analysis, with the objective to estimate, albeit approximately, the maximum displacements at the top of the structure and the maximum (positive or negative) value of shear force at the base of the structure. This simple model, if on one hand allows an important save of computational efforts, on the other can sometimes introduce considerable errors due to the MDOF effects usually not considered in the SDOF model.

The equivalent model proposed here is a MDOF model with three degrees of freedom associated to each floor: the allowed movements are two translations in the plane of the slab and one rotation around the vertical axis, orthogonal to the mid-plane of the floor. In summary, being N the number of floors of the structure at hand, the numerical model has a total of 3N degrees of freedom.

2 DESCRIPTION OF THE EQUIVALENT STICK MODEL METHOD

The present paper describes a new structural model for the evaluation of nonlinear seismic behavior of framed buildings. The method involves the definition of an inelastic equivalent model characterized by few degrees of freedom, classified as stick model in the FEMA440 Report [7], and obtained starting from a detailed model of the same structure.

The Equivalent Stick Model (ESM), introduced in [8], is based on the determination of the capacity curves of each floor by incremental static analysis, and assigning each curve to the corresponding degree of freedom. The proposed model is then incorporated in a broader context of a procedure called Equivalent Stick Model Method (ESMM), implementing the ESM model in the resolution of nonlinear time-history analysis. The new methodology proposed here consists of the following main steps:

1) Implementation of the structural detailed model;
2) Determination of the capacity curves for each floor;
3) Definition of the cyclic (hysteretic) behaviour of each degree of freedom;
4) Determination of the mass to be assigned to each degree of freedom;
5) Assembling the equivalent stick model (ESM);
6) Time-history analysis on stick model;
7) Determination of dynamic structural response;
8) Estimation of local parameters from the detailed model.

The first five steps lead to the determination of the equivalent model; the other three provide the procedure for the application of the model in the proposed method. In the following, the procedure to achieve the ESM model will be first described and, subsequently, that for its application in the framework of the proposed methodology used for the assessment of structural seismic response. Furthermore, for a first validation of the method, two case studies are described and analyzed. The results reported provide useful indication on the reliability of the proposed method.

2.1 Implementation of the structural detailed model

The implementation of the detailed model, in accordance with the classification given in [7], allows to assess the dynamic parameters of the real structure (frequencies, periods, modal shape, participation factors). The detailed model must fit in terms of distribution of stiffnesses, masses and strengths, with the real structure since, from it, the capacity curves of each floor will be then achieved.

2.2 Determination of the capacity curves for each floor

As one of the fundamental aspects of the procedure, the seismic induces horizontal displacements of each point of a floor are determined by the generalized displacements (displacements and rotation) of a node identified as representative of the floor. The movements of a floor will be two translations in the horizontal plane and a rotation around the vertical axis. The idea of statically condensing the movements of an entire floor in the movements of a single point, considered as representative, is correct only if the floor has a sufficiently high (theoretically infinite) in-plane stiffness.

The selected representative point is the geometric center of mass CM of the floor. It is also assumed that the centers of mass of all the floors are aligned along the axis perpendicular to the slab. The behavior of each floor is identified by three inelastic curves, obtained by nonlinear static incremental analysis, with a prescribed force distribution. In [8] different types of forces distribution were tested: proportional to the translational mass; proportional to a uniform accelerations distribution; proportional to the shear distribution obtained from one or more vibration modes.

The capacity curves related to each floor are in the form "Force-Displacement" for the two translational degrees of freedom while in the form "Torque-Rotation" for the rotational degree of freedom. Force and torque of the curve are those statically applied in the incremental analysis, while displacement and rotation are those registered at the representative point.

2.3 Definition of the behaviour of each degree of freedom

After determining the three backbone capacity curves for each storey, must be defined a damage criteria to take into account the degradation behavior of the structure due to cyclic action. The choice of the most suitable hysteretic model to be adopted depends on the damaging phenomena to be included. In the present work, in order to simplify the capacity curves obtained via incremental analysis, a tetra linear polygonal curve has been adopted. In addition, to represent the cyclic degradation effects, a peak-oriented hysteretic behaviour has been chosen. With the described assumptions, the response of the structural system is calculated by moving along the backbone curve during virgin loading with unloading and reloading process.
that moving through the opposite side of the diagram, points to the value of maximum (or minimum) displacement reached in the past steps of the analysis (see Figure 1).

2.4 Determination of the mass assigned to each degree of freedom

The mass values attributed to the three nodal degrees of freedom of each floor are the translational mass and the rotational moment of inertia of the storey. The calculation of the translational mass of each floor is carried out according to the following rules:

- the mass relative to dead load and live load directly applied on the floor are attributed to the floor itself;
- the mass of structural elements that connect two consecutive storeys is divided equally on two storeys;
- the mass of non-structural elements between two consecutive storeys is attributed equally if they are firmly fixed at both ends, otherwise they are entirely allocated to the underlying floor.

The calculation of rotational moment of inertia to be attributed to the considered floor is made in respect of the following rules:

- the mass considered is the same determining the translational mass;
- the rotational moment of inertia is obtained considering the various distances computated from the center of mass to the actual geometrical position of the elements in the floor.

2.5 Assembling the equivalent stick model (ESM)

The finite element adopted in the definition of the equivalent stick model is a classical lumped plasticity element, having a linear elastic component in series with two inelastic hinges to the extremities. The capacity curves previously obtained are used to define the properties of the elastic portion and the behaviour of inelastic hinges. The boundary conditions adopted are total restraint prescribed at the base section of the model and rotational restraint (along the X-dir and Y-dir) prescribed at each floor level. The generic stick model resulting is schematically shown in Figure 2.

![Figure 1: Peak-oriented tetra linear curve modeling the cyclic behaviour of a floor.](image-url)
2.6 Time-history analysis on stick model

After defined the equivalent structural model, the next step is to conduct the nonlinear time-history analysis for the determination of the seismic response of the structure. The seismic input is a accelerogram applied to the base section of the model, or a pair of accelerograms if the effects of bi-directionality of the earthquake are of interest.

2.7 Determination of dynamic structural response

By means of time-history analysis, the displacements history of each degrees of freedom of the model can be derived, and consequently, the history of global parameters of interest (i.e. referring to the structure) be obtained, such as absolute displacements or interstorey displacements; the same applies to the torsional rotations that provide an estimate of the inelastic seismic demand of rotation on the various floors. Similar considerations can be made with reference to the forces since, from the time-history analysis, it is possible to obtain the storey shear (or storey force) rather than the base shear.

2.8 Estimation of local parameters from the detailed model

The dynamic seismic demand for the individual elements (beams or columns) both in terms of displacements and forces, can be obtained from the detailed model discussed before. In fact, starting from the maximum (or minimum) value of displacement or storey shear required by the seismic input to the stick model, it is possible to determine the corresponding step of the incremental analysis, conducted on the detailed model and discussed at Section 2.2. The value of parameters of interest at that specific step can be considered as an estimate of the value that would be obtained by solving the time-history analysis directly on the detailed model. To set up the structural verification, if of interest, the determination of displacement capacity or strength capacity for the elements, may be obtained following one of the methods prescribed by codes, as well as from the elaboration of data obtained by experimental tests available in the literature [9].

Figure 2: Assembling of the stick model.
3 VALIDATION OF THE MODEL BY NUMERICAL ANALYSIS ON CASE STUDIES

3.1 Case studies description

For the validation of the reliability of the procedure ESMM and of the adequacy of the ESM to predict the seismic demand, two buildings were selected and time-history analyses were conducted on both equivalent stick models and detailed models. The latter are considered as the reference solutions, able to provide the "correct" seismic response of the structures. In the present work, the two case studies are presented: a one-storey structure, symmetrical in both directions; a three-storey structure, symmetrical in both directions. The two buildings have the same plan, with dimensions of 6.0m x 6.0m and interstorey height of 3.0m. A total fixed constraint was prescribed at the base of the columns of the structures.

3.2 Modelling assumption

The detailed model of the structure was built by means of distributed plasticity finite elements, with transversal fibers section. Four integration points for each element have been used. For confined and not confined concrete the model of Kent and Park [10] was adopted as modified in [11]. The model, adequate for monotonically and increasing load, is completed with the unloading and reloading model of Karsan and Jirsa [12], suitable for analyses with cyclic loading. For the steel of reinforcement bars the model of Menegotto and Pinto [13] was used. Table 1 and Table 2 show the properties of the materials adopted and the coefficients used to define the inelastic behavior of the sections according to the mentioned models.

<table>
<thead>
<tr>
<th>CONCRETE</th>
<th>f’c [MPa]</th>
<th>K</th>
<th>εco</th>
<th>εcu</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confined concrete</td>
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<td>1.0</td>
<td>0.002</td>
<td>0.006</td>
<td>266.67</td>
</tr>
<tr>
<td>Not confined concrete</td>
<td>25.0</td>
<td>1.0</td>
<td>0.002</td>
<td>0.0035</td>
<td>1600.0</td>
</tr>
</tbody>
</table>

Table 1: Adopted parameters for the definition of cyclic behaviour of concrete according to [11,12].

<table>
<thead>
<tr>
<th>STEEL</th>
<th>fy [MPa]</th>
<th>E [MPa]</th>
<th>b</th>
<th>Ro</th>
<th>a1</th>
<th>a2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
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<td>200000</td>
<td>0.005</td>
<td>20</td>
<td>18.5</td>
<td>18.5</td>
</tr>
</tbody>
</table>

Table 2: Adopted parameters for the definition of cyclic behaviour of steel according to [13].

3.3 Selection of the seismic input

In the present paper, four registered accelerograms have been used (see Table 3). Not scaled, rather than scaled in terms of PGA (Peak Ground Acceleration), were applied to both models (detailed and ESM), to test the reliability of the ESMM procedure for different intensities of the seismic event. The modeling and the dynamic analyses were carried out by means of software MIDASGen [14].
Table 3: Seismic input adopted in the time-history analysis.

4 CASE STUDY N.1: ONE-STOREY SYMMETRICAL STRUCTURE

The structure has columns with 60cm × 25cm transversal dimensions of and beams with cross section of 50cm × 25cm (WxH). In the detailed model, the sections have the fibers division shown in Figure 3, while the force distribution on the floor is shown in Figure 4. The floor is modeled by reinforced concrete shells element with 20cm thickness. In the analysis of the detailed model, the following vertical loads were considered: for the elements of RC frames a specific weight of 25kN/m³ was assumed; a dead load uniformly distributed on the floor of 6 kN/m²; a live load of 0.6 kN/m².

Since the structure has a single floor, the capacity curves to be determined for the stick model are three, one for each degree of freedom of the stick model. In the simplified case of unidirectional earthquake acting in X direction, the only kinematic parameter to be considered is the displacement $\Delta_x$ in X-dir.

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**Figure 3:** Case study n.1: cross section of columns and beams and their fiber discretization adopted in the detailed model.

**Figure 4:** Case study n.1: force distribution applied to obtain the capacity curve of floor.
The capacity curve in the X-dir is obtained by applying an increasing horizontal forces distribution applied at the floor level (see figure 4), proportional to the storey shear resulting from the first mode of vibration in the X-dir. The capacity curve $F_X - \Delta x$, obtained by increasing the force distribution and conducting a nonlinear static analysis, is shown in Figure 5. The curve, given on the abscissa axis the displacement in the X-dir of the representative point of the floor (center of mass CM of the floor) and on the ordinate axis the resultant of the distribution of forces acting on X-dir.

In Figure 5, the polygonal tetra linear curve used to approximate the numerical one is superimposed. This curve is then used to obtain the relation to be assigned to the plastic hinges of the stick model. The mass allocated to the single degree of freedom of the stick model is equal to $m_1 \Delta x = 35811.8$ kg. The elastic properties of the finite elements of the stick model are defined in such a way to obtain an equivalent model with the same frequencies of the detailed model. Figure 6 summarizes the key features of the equivalent stick model obtained.

To validate the model, dynamic analyses were performed on both models (stick and detailed) and the results obtained were compared. The parameters of interest and used in the comparison, are the displacements of the representative node of the floor monitored on the detailed model (displacement of CM) and the displacements of the top node of the stick model. Moreover, the base shear forces are compared. For the specific cases, the sum of the base shear force of each column for the detailed model and the base shear force registered at the single vertical element for the stick model were compared. In Figure 7 and Figure 8, the results in terms of “storey shear-storey displacement” are shown with reference to the Earth-
quake n.1 and Earthquake n.2 in Table 3. For the Earthquake n.2, the result obtained using a Scale Factor SF=2.65 is shown in Figure 9. Figure 10 and Figure 11 show, respectively, the trend of displacement and storey shear, during the time, for Earthquake n.1. The results show that, even for dynamic actions that impose to the structure a strongly inelastic deformation scenarios, the stick model provides accurate results when compared with those of the detailed model. The comparison in terms of “storey shear-storey displacement” is also reported in Figure 12, obtained by superimposing the structural response of dynamic analysis (time-history) and that of nonlinear static analysis both conducted on the detailed model.

Figure 7: Case study n.1: storey shear-Storey displacement for the Earthquake n.1

Figure 8: Case study n.1: storey shear-Storey displacement for the Earthquake n.2 - SF=1

Figure 9: Case study n.1: storey shear-Storey displacement for the Earthquake n.2 - SF=2.65

Figure 10: Case study n.1: storey displacement - time for the Earthquake n.1.

Figure 11: Case study n.1: storey shear - time for the Earthquake n.1.
This comparison suggests that the curve obtained via static analysis (capacity curve) is a satisfactory approximation of the dynamic request (seismic demand) of the seismic action on the structure. The values of the quantities determined by stick model allow to obtain, for the individual seismic input, the dynamic request in terms of shear and displacement to the storey. The seismic request can be used to set the necessary verification from both local (element or section) and global (structure) point of view. In fact, it is possible to set a procedure for the determination of stress and deformation of a single element rather than a section. For example, with reference to the maximum (positive and negative) horizontal displacement imposed to the representative node of the slab and in the absence of torsional rotations (as in the present case), the displacement imposed at the top of each column is known. Reporting this displacement value on the floor capacity curve, it is possible to pick out the load step of the incremental analysis where this displacement is reached. From the analysis of force and deformations obtained on the detailed model for that load increment, it is possible to determine the structural response at the instant of maximum displacement demand. The values of force and displacement by static analysis may be regarded as an estimate of that obtained for the element (or section) from a time-history on the detailed model.

5 CASE STUDY N.2: THREE-STOREY SYMMETRICAL STRUCTURE

The structure has columns with 60cm × 25cm cross-section and beams with 50cm × 25cm (WxH) cross-section. The sections were divided into fibers in both directions, as shown in Figure 13. The slab modeling is the same of the previous case study.

Since three floors are present, the capacity curves to be determined in order to define the stick model are nine. For a unidirectional earthquake acting in X-direction, the determination of the three capacity curves in the X-dir are enough for the description of the structure. The three curves at hand were obtained again by a static incremental analysis with the application of a distribution of forces proportional to the distribution of the storey shear forces calculated by linear dynamic modal analysis. In the present case, the force distribution from principal vibration mode in the X-dir was considered (see Figure 14).
The capacity curve “storey force-interstory displacement” $S_{X}^{(1)}-\Delta_{X}^{(1)}$, obtained for the floor n.1 (the lower of the three), is shown in Figure 15 as an example. In the same figure, the tetra linear approximating curve used for the set up of the stick model is superimposed.

The mass located at each one of the three degrees of freedom is considered equal to $m_{X}^{(1)}=m_{X}^{(2)}=35940\text{kg}$ and $m_{X}^{(3)}=33690\text{kg}$. Even in this case, the elastic properties of the finite elements of the stick model are established in such a way to obtain, the same frequencies of the detailed model. Figure 16 summarizes the key features of the equivalent stick model.

In the following, some results of time-history conducted on both the structural models (detailed and stick) are reported.

Figures 17a, 17b, 17c and 17d show the results for the floor n.1 in terms of “storey shear-interstorey displacement” with reference to the four earthquakes considered (see Table 3).
Figure 16: Case study n.2: equivalent stick model.

Figure 17a: Case study n.2: storey shear - interstorey displacement of floor n.1 for the Earthquake n.1 (SF=1.5)

Figure 17b: Case study n.2: storey shear - interstorey displacement of floor n.1 for the Earthquake n.2 (SF=3)

Figure 17c: Case study n.2: storey shear - interstorey displacement of floor n.1 for the Earthquake n.3 (SF=3)

Figure 17d: Case study n.2: storey shear - interstorey displacement of floor n.1 for the Earthquake n.4 (SF=2.5)
Figure 18 and Figure 19 show the same quantities, but referred respectively to floor n.2 and floor n.3 for the Earthquake n.1.

In Figure 20 and Figure 21 the time-variation of displacement and storey shear, for the floor n.1 under the action of Earthquake n.1 are illustrated. Such figures show that ESM is able to reproduce, with good approximation, the dynamic global behavior of the structure, matching satisfactorily with the maximum values required at various levels from the “reference” solution, both in terms of displacement and force estimation. The results obtained show that, even for seismic excitations imposing to the structure a deformation scenarios abundantly beyond the elastic range, the model maintains good precision. It is also to be noted that a very simple hysteretic law, such as that adopted, appears to represent with enough accuracy the dynamic behavior of the structure at hand.

Finally, Figure 22 summarizes the comparison in terms of “storey shear-interstorey displacement”, for floor n.1, superimposing the responses obtained by time-history analyses and static incremental analysis both conducted on detailed model. As for the previous case, the capacity curve obtained by static analysis represents an acceptable approximation of the seismic dynamic demand both in terms of force and displacements. More faithful is the static curve matching the dynamic response, more accurate will be the equivalent stick model obtained by the procedure.
Figure 22: Case study n.2: static capacity curve and dynamic demand (Earthquake n.1, n.2, n.3, n.4) for the analysis on detailed model.

6 COMPARISON IN TERMS OF COMPUTATIONAL EFFORTS

The particular benefits and peculiarities of the proposed method lies in the fact that, leading to the definition of a numerical model with reduced number of degrees of freedom allows to set up dynamic analyses extremely advantageous from a computational point of view. Table 4 and Table 5 show the comparison between the computational time needed to solve the nonlinear time-history, presented earlier. The computational time and post-processing necessary to obtain time-history results on the structural stick model turn out to be far fewer than those required to perform the same analysis on the detailed model. The computational time required by the ESMM procedure, and consisting in the definition of the equivalent stick model, is negligible if compared to the computational savings, even if a limited number of time-history is to be performed.

<table>
<thead>
<tr>
<th>Seismic input</th>
<th>Detailed model Solving [s]</th>
<th>Detailed model Post-processing [s]</th>
<th>ESM Total time [s]</th>
<th>ESM Total time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earthq. n.1 (SF=1)</td>
<td>1026</td>
<td>1022</td>
<td>2048</td>
<td>0.37</td>
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<tr>
<td>Earthq. n.2 (SF=1)</td>
<td>700</td>
<td>838</td>
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<td>0.28</td>
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<tr>
<td>Earthq. n.2 (SF=2.65)</td>
<td>717</td>
<td>764</td>
<td>1481</td>
<td>0.29</td>
</tr>
</tbody>
</table>

Table 4. Case study n.1: computational time required to solve the time-history analysis.

<table>
<thead>
<tr>
<th>Seismic input</th>
<th>Detailed model Solving [s]</th>
<th>Detailed model Post-processing [s]</th>
<th>ESM Total time [s]</th>
<th>ESM Total time [s]</th>
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<tbody>
<tr>
<td>Earthq. n.1</td>
<td>2927</td>
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<td>Earthq. n.2</td>
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<tr>
<td>Earthq. n.4</td>
<td>3547</td>
<td>5580</td>
<td>9127</td>
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</table>

Table 5. Case study n.2: computational time required to solve the time-history analysis.
7 CONCLUSIONS

For the considered case studies, used for a preliminary validation of the method, the results obtained by the ESM procedure are generally in good agreement, excellent in some cases, with those obtained by the same analysis but conducted on a structural detailed model. The engineering parameters assessed by the described procedure, both global (with reference to the entire structure or its floor) and local (with reference to an element or a section), can be used effectively to set verification in terms of strength or displacement. As regard the choice of the hysteretic behaviour of a floor, the numerical results presented show that a simple model, computationally cheap, such as the adopted peak-oriented type, leads to good results. This aspect will be further investigated in future research.

The lower computational efforts, if compared to time-history analyses on detailed models, and the lower level of uncertainty with respect to other seismic analysis methods (pushover), place the ESMM procedure in an interesting intermediate position between the existing procedures usually adopted to assess the seismic response of buildings.

The ESM model and the proposed ESMM methodology seem therefore suitable to be used, as new tools of analysis and design, for the estimation of the dynamic response of framed structure buildings.

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