

ON THE PERFORMANCE OF A TECHNIQUE TO ACCELERATE TIME INTEGRATION WHEN APPLIED TO SPACE STRUCTURES ANALYSES

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Abstract. *The true behaviors of structures, specifically when subjected to earthquakes, are dynamic. Time integration is a versatile tool in analysis of the semi-discretized equations of motion. Although the implementation of time integration is simple, the computational costs are considerable and the responses are inexact. The integration step size is the main parameter affecting the accuracies and computational costs. When the excitations are available as digitized records, conventionally, the integration step sizes are to be set not larger than the excitation steps. In 2008, a technique is proposed, providing the capability to analyze with steps larger than the excitation steps, leading to considerable less computational cost. This technique is implemented in analyses of different structural systems, including bridges, tall buildings, residential buildings, etc., and the observed losses of accuracies were negligible. In this paper, after brief theoretical discussions, the technique is applied to the analysis of the three space structures, where the structural members are sized, with attention to the existing national codes. The average acceleration method of Newmark is considered as the integration method; an El Centro as well as a Loma Prieta strong ground motion record are considered as the excitations applied in the vertical direction, and the analyses are carried out considering the linear behaviors. To address the main consequence, though the smallest oscillatory periods with considerable contribution in the response were estimated approximately, the losses of accuracy in cases with the applicability of the technique were small, still depending on the structural system and excitation.*

1 INTRODUCTION

In many cases, the true performance of structural systems is dynamic that, cannot be simplified to static. In order to study dynamic behaviors of structures, the widely accepted approach is to define the structural models, set the mathematical models, and, discretizing them in space then semi-discretized model is as follows:

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{f}_{\text{int}} = \mathbf{f}(t), \quad 0 \leq t \leq t_{\text{end}}$$

$$\text{initial conditions: } \begin{cases} \mathbf{u}(t=t_0) = \mathbf{u}_0 \\ \dot{\mathbf{u}}(t=t_0) = \dot{\mathbf{u}}_0 \\ \mathbf{f}_{\text{int}}(t=t_0) = \mathbf{f}_{\text{int}0} \end{cases}, \quad (1)$$

$$\text{additional constraints: } \mathbf{Q}$$

Where, t and t_{end} imply the time and the duration of the dynamic behavior; \mathbf{M} is the mass matrix; \mathbf{f}_{int} and $\mathbf{f}(t)$ stand for the vectors of internal force and excitation; $\mathbf{u}(t)$, $\dot{\mathbf{u}}(t)$, and $\ddot{\mathbf{u}}(t)$ denote the vectors of displacement, velocity, and acceleration; \mathbf{u}_0 , $\dot{\mathbf{u}}_0$, and $\mathbf{f}_{\text{int}0}$ define the initial status of the model (regarding the essentiality of considering $\mathbf{f}_{\text{int}0}$ in Equations (1), also see [1]); and finally, \mathbf{Q} represents some restricting conditions, e.g. additional constraints in problems involved in impact or elastic-plastic behavior [2, 3], all in view of the degrees of freedom set for the model.

Equation (1) can be analyzed with an appropriate method [4-6]. The most versatile method to analyze equation (1) is direct time integration [7]. However, the time integration methods are accompanied by some errors and are inexact. The integration step size is the main parameter affecting the accuracies and computational costs. In general, the accuracies of obtained results depended on the integration step size and the shorter one, the more accuracy can be obtained. However, time integration analysis using so small time step significantly increases computational costs. One of the broadly accepted comments is as follows:

$$\Delta t = \text{Min}(h_s, \frac{T}{10}, f\Delta t) \quad (2)$$

Where, T is the smallest dominant period in the response, in general, approximated with the smallest natural period of the system at $t = 0$, likely effectual in the response, h_s is the largest value of the integration step, providing numerically stable responses ($h_s = \infty$ for unconditionally stable methods), and $f\Delta t$ is the digitization step size for excitations available as digitized records. Considering equation (2), if $f\Delta t$ is much smaller than $\text{Min}(h_s, \frac{T}{10})$, the time integration step size must be smaller than be required for convergence or accuracy. Hence, computational cost should be expensive. In 2008, a technique is proposed, providing the capability to analyze with steps larger than the excitation steps, leading to considerable reduction of computational cost. The technique changes time step $f\Delta t$ to the longer one $f\Delta t'$. In fact, the technique replaces the excitation with a new one which is digitized at time steps equal to $f\Delta t'$ responses convergence and the rate of convergence are preserved after the replacement of excitation. This technique implemented in analyzing for some different structural systems, i.e. a tall building [8], a fuel storage tank [9], and a silo [10], and also subjected to further theoretical investigation [11].

In this paper, we aim to evaluate this method for some specific kinds of space structures. A brief review of the technique is presented in the following section.

2 THE BRIEF REVIEW OF THE TECHNIQUE

The main idea backing this technique is to replace the digitized records of seismic excitations, with records, digitized, at larger steps, such that the loss of integration accuracy resulting from considering the new excitation (instead of the original excitation) is tolerable. in brief, provided the assumptions bellows:

1. The excitation steps, ${}_f\Delta t_i, i = 1, 2, \dots$, are equally sized,

$$\forall i, j \quad {}_f\Delta t_i = {}_f\Delta t_j = \Delta t = {}_f\Delta t > 0 \quad (3)$$

2. The integration steps, $\Delta t_i, i = 1, 2, \dots$, are equally sized,

$$\forall i, j \quad \Delta t_i = \Delta t_j = \Delta t > 0 \quad (4)$$

3. The excitation steps are embedded by the integration steps (the first time station, i.e. t_0 , is a station for both excitation and integration),

$$\exists n \in \mathbb{Z}^+ \quad \frac{\Delta t}{{}_f\Delta t} = n < \infty \quad (5)$$

4. The $\mathbf{f}(t)$ in equations (1) is a digitized representation of an actual excitation, $\mathbf{g}(t)$, smooth [12] with respect to time, i.e.,

$$\begin{aligned} \mathbf{f}(t) &= \mathbf{g}(t)\delta(t - \alpha_i) \\ \mathbf{g}(t): &\text{smooth with respect to time} \\ \alpha_i &= i {}_f\Delta t, \quad i = 1, 2, \dots \\ \delta(t - \alpha_i) &= \begin{cases} 1 & t = \alpha_i \\ 0 & t \neq \alpha_i \end{cases} \end{aligned} \quad (6)$$

We can replace the excitation in equations (1), \mathbf{f} , with the new excitation, $\tilde{\mathbf{f}}$, digitized at steps equal to $n {}_f\Delta t$, according to:

$$\begin{aligned} t_i: \quad \tilde{\mathbf{f}}_i &= \mathbf{f}(t_i) \\ 0 < t_i < t_{end}: \quad \mathbf{f}_i &= \frac{1}{2}\mathbf{f}(t_i) + \frac{1}{4\dot{n}} \\ t_i = t_{end} \quad \tilde{\mathbf{f}}_i &= \mathbf{f}(t_i) \end{aligned} \quad (7)$$

where,

$$\begin{aligned} t = \Delta t: \quad \dot{n} &= n - 1 \\ \Delta t < t < t_{end} - \Delta t: \quad \dot{n} &= \left\{ \begin{array}{c} \frac{n}{2} \\ \frac{n-1}{2} \end{array} \right\} \\ t = \Delta t: \quad \dot{n} &= n - 1 \end{aligned} \quad (8)$$

and $\Delta t, n (n \in \mathbb{Z}^+)$ are the largest values satisfying

$$\Delta t = n {}_f\Delta t \leq \text{Min}\left(h_s, \frac{T}{10}\right) < t_{end} \quad (9)$$

with generally small loss of accuracy in time integration. Since, the new excitation, $\tilde{\mathbf{f}}$, is digitized at steps equal to $n {}_f\Delta t$, when considered instead of the original excitation, can lead to a reduction in computational cost.

3 NUMERICAL STUDY

In this paper, three different space structures, a barrel vault, a diamatic dome, and a double layer grid, as shown in figure (1), are considered. The span of the diamatic dome and barrel vault is 46.3 and 30 meters, respectively and the height of them is 36 and 18 meters respec-

tively. For double layer grid the span is 10 meters and the layer thickness is 1.5 meters. Young's modules, mass density, and yield stress are 2.1×10^{10} kg/m², 7850 kg/m³, and 2.4×10^6 kg/m², respectively. These structures are designed considering national code [13].

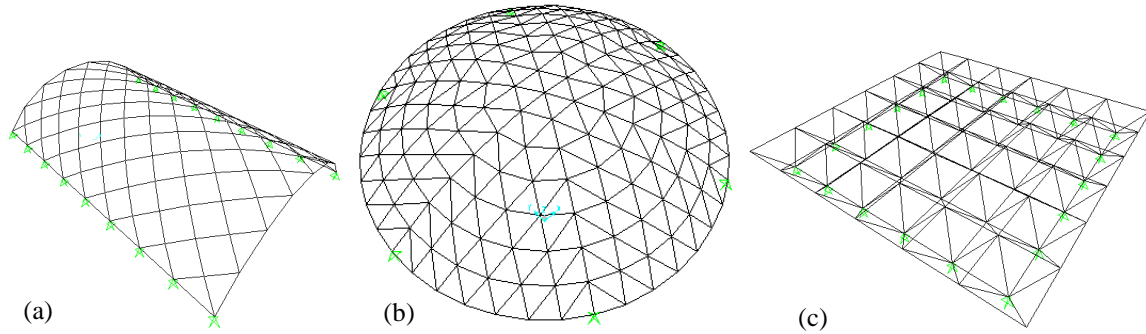


Figure 1: The three space structures under consideration: (a) barrel vault, (b) diamatic dome, (c) double layer grid.

The average acceleration method of Newmark is considered as the integration method, an El Centro and a Loma Prieta strong ground motion record are considered and their time step sizes are 0.02 and 0.005 Sec, respectively, shown in figure 2. Considered to study the performance of the technique.

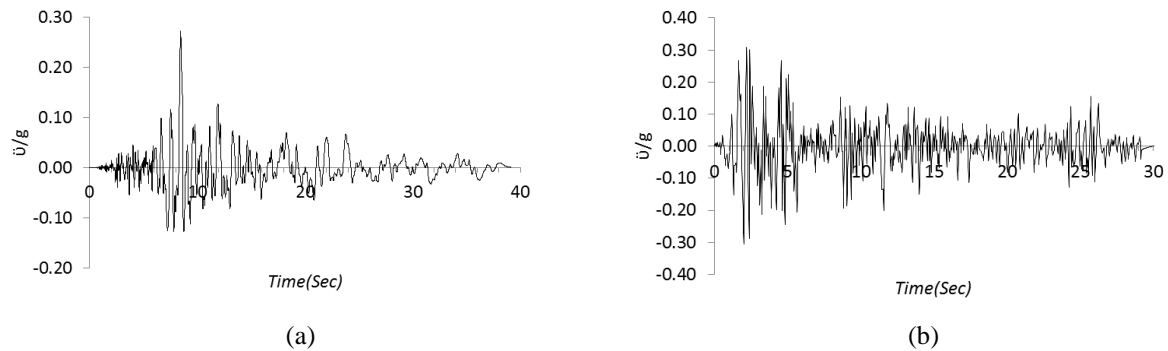


Figure 2: The ground motion applied to the structural model, (a) the Loma Prieta and (b) El Centro strong ground motion record

These models are subjected to the considered excitations applied in vertical direction. In view of the approximate response partially reported in Figure 3, studying the response shows the technique is applicable for the barrel vault for both considered excitation and double layer grid only for the Loma Perieta ground motion record. In other hand, dominate period of structures considered is small and the technique application is limited, hence, the time history analysis implement only for $n=2$. A point displacement is shown in figure (4). The results show the application of this technique can reduce computational costs without considerably compromising the accuracy of responses. Although the technique considering Loma Perieta strong ground motion record is not applicable, comparison the displacement results after implementing the technique are shown in figure (4c).

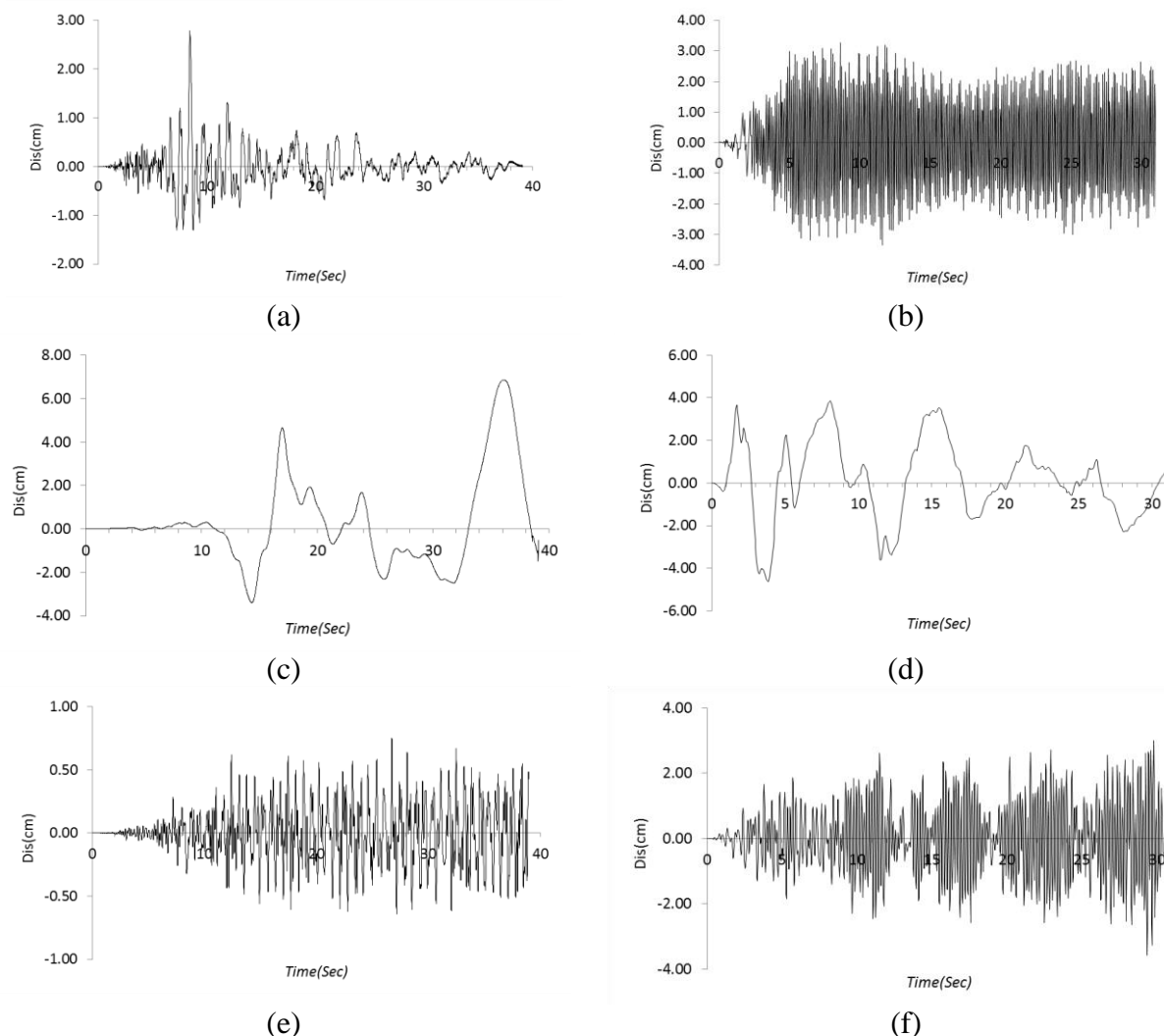


Figure 3: approximate point displacement, (a) and (b) the double layer grid, (c) and (d) the barrel vault, and (e) and (f) the diaphragm dome when applying the Loma Preita and El Centro ground motion record respectively.

4 CONCLUSION

The performance of the technique proposed for accelerating of seismic analysis by time integration is examined considering space structures analysis. The results show, though the smallest oscillatory periods with considerable contribution in the response were estimated approximately, the losses of accuracy in cases with the applicability of the technique were small, still depending on the structural system and excitation. The results show, in some cases, the performance of the technique depends on the excitation that needs further study to investigate this effect. Further study in this regard, considering different ground strong motions, and non-linear time history analysis is suggested.

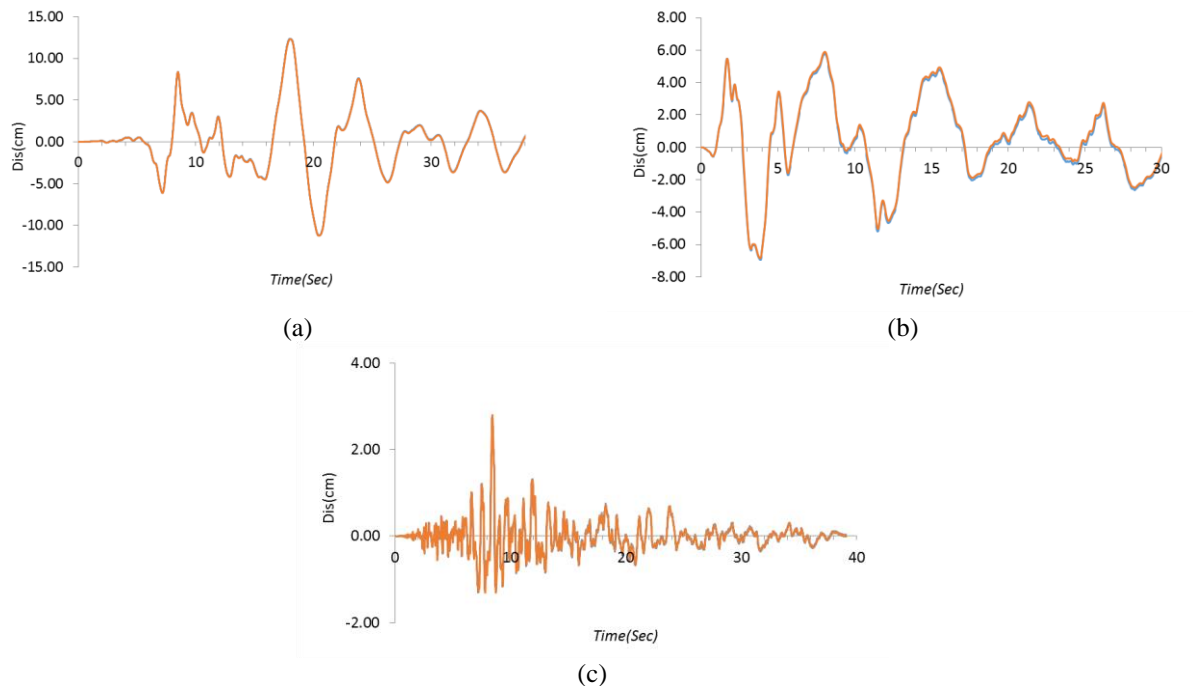


Figure 4: The performance of the technique when applied to, (a) and (b), the barrel vault subjected to the El Centro and Loma Prieta strong ground motion record respectively, and (c) double layer grid subjected to Loma Prieta strong ground motion record.

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